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OPTIMIZING THE CROSS-SECTION OF ELASTIC BEAM OF THE BEARINGLESS ROTOR OF AN ULTRALIGHT HELICOPTER

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Abstract: *The main element of bearingless rotor of a helicopter is elastic beam, which replaces all three hinges (flapping hinge, lead-lag hinge, and feathering hinge), and it has to be well designed in order to provide necessary movements of the rotor blade, without exceeding the permitted stresses in the material. The goal of this study is to find optimal size of cross-section of the head of the bearingless rotor of an unmanned helicopter in respect to feathering function. The study considers the influence of the size of the cross-section to feathering properties, by analyzing the results obtained by finite elements metod. The optimal size of cross-section is found by approximating feathering function with corresponding polynominal with divided differences and using the least squared error and the mean squared error to find the size of cross-section that leads to minimal error. The optimal values for the cross-section obtained by using the the least squared error the mean squared error were compared and it was concluded that both approaches can be applied for optimizing the size of the cross-section.*

Keywords: *Bearingless rotor, Elastic beam, Finite element, Finite differences, Mean squared error*

1. INTRODUCTION

Constant strive of helicopter manufacturers to simplify main rotor construction, has been obtained by implementing compliant elements - elastic beams in the construction [1]. Elastic beam has, in its shape, size, and composition material, replaced three hinges (pitch, flap, and lead-lag hinge), which were used earlier as blades‘ connection to the main rotor hub. In attempts to achieve the best possible rotor characteristics, similar to, or even exceeding those of hinged rotor systems, there are different types of beams proposed in the practice, differing in cross sections and lengths [2]. In order to get improved pitch, flap and lead-lag properties, adequate design of the elastic beam is crucial, as it is practically the only part of the bearingless system that affects control characteristics. Due to highly connected dynamic and structural demands, put in front of the elastic beam, regarding geometrical properties as length, cross-sectional area, different spacing between multiple beams in root and end segment, etc. it is necessary to use mathematical optimisation methods to obtain suitable geometrical values that provide required dynamical properties. Although all of the required parameters are in direct connection, it is difficult to simultaneously observe and analize all of them, having in mind that each of these parameters is very complex by itself. For all above mentioned reasons, this analysis will aim effects of cross-sectional area on the blade pitch change characteristics (elastic beam torsion).

2. TECHNICAL ISSUE

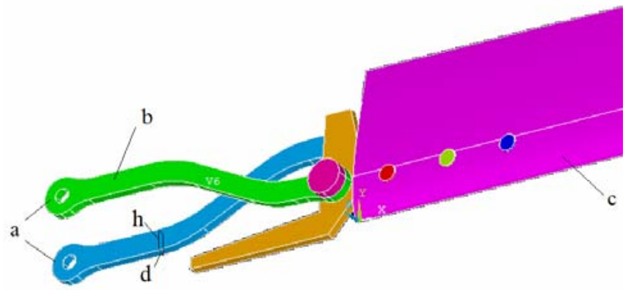

Figure 1. Elastic element and rotor blade: a) elastic element, b) elastic beams, c) rotor blade, d) beam cross-sectional height, h) beam cross-sectional width

The main technical issue in bearingless rotor systems is achieving required fredom of movement by deforming of the elastic element (Figure 1), connecting the rotor blade with the hub. Elastic element consists of two elastic beams, placed one above the other at a certain distance. Beams have composite structure, made of laminated steel bonded by rubber. The steel ensures necessary strength and toughness, while the rubber provides non-linear deformation characteristics. In difference to the present examples of the main rotor elastic elements, consisted mostly of single elastic beam per element, with different cross-section forms like „+“ or „I“ beams or other open or closed shapes, analised beam will have rectangular cross section shape. Sellection of the rectangular cross-section provides great load-bearing capabilities combined with simplified design and manufacture process, intending to lower the overal cost. To obtain required movement properties, pitch and flap [3] characteristics in particular, it is necessary to define the cross-sectional area of the elastic beam [4].

3. OPTIMISATION DEMANDS AND INITIAL INPUT DATA

The primary goal in determining optimal dimension of the elastic beam of the helicopter rotor is to achieve the necessary movement dynamics without exceeding the permitted stresses in the material. The realization of this goal was carried out by optimization [5] of the cross-sectional area of the elastic element, and therefore, of the elastic beam. When determining the optimal size of the cross-section of the elastic beam, the height of the element (d) was varied, while the width (h) was fixed.

Bearing in mind that relatively thin beams may allow for significant or eve exessive deflections, namely torsion and bending, and thus lead to increased vibrations level on rotor blades and hub (which would further demand installation of a dampening mechanism) minimal beam thickness in this analise will be d=7mm.

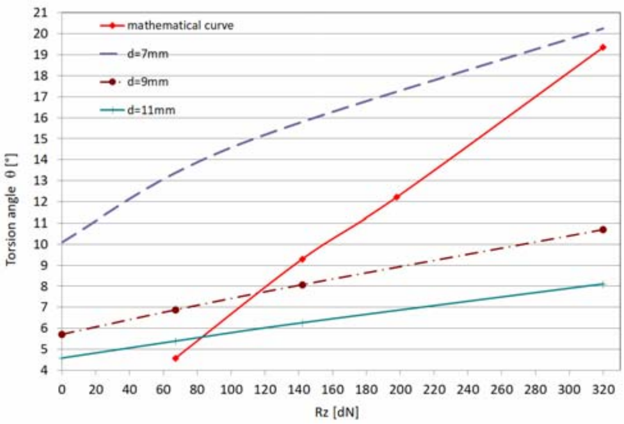

Figure 2. Rotor blade pitch angle value change (torsion angle value) in correlation with lifting force

The initial input data were obtained by finite element analysis [6,7]. That method simulated behavior of elastic beams, l=240mm in length, and of thickness d=7mm/9mm/11mm. Analysis included following conditions:

Root end of elastic beams were fixed (simulating connection to rotor hub), so as to achieve zero movement at their contact surfaces; Centrifugal force, with magnitude of Fc=3000daN, loading the elastic element, acts at blade‘s center of mass, while aerodynamical force acts at the blade‘s center of thrust, located at 0,75R (measured from the axis of rotation). Analysis included blade pitch angle changes at different aerodynamic lift values: Rz=67daN, Rz=142daN and Rz=320daN, representng thrust at blade‘s angular position of 0o in autorotation and forward level flight at 120km/h and 180km/h, respectively. Aquired blade‘s pitch angle values are drawn on Figure 2.

4. INTERPOLATION OF FUNCTIONS WITH MULTIPLE VARIABLES

The blade pitch angle (torsion angle) change function, denoted as θ, is considered as a function of two variables: beam cross-section thickness d and magnitude of aerodynamic force Rz, i.e., θ(d,Rz). The apporximation of the function θ(d,Rz) has been made by using interpolating polynominal of two vatiables with divided differences [8] based on given values $\theta(d_i,Rz_j)$, i=0,...,n, j=0,...,n. The following section will briefly present the interpolation of a function of two variables by using a polinominal with divided differences.

Interpolation is a type of approximation of a function, in which it is required that the values of approximant coincide with the values of function in the given set of points. If the approximant is polynomial, it is named as interpolation polynomial. The construction of interpolation polynomial of two variables is performed as the extension of interpolation of function of one variable. In this study, we will construct interpolation polynomial of two variables with divided differences of the function θ(d, Rz).

Let us assume that the function θ(d, Rz) is given by its values in $(n+1)^2$ points $\theta_{ij}=\theta(d_i,Rz_j)$, i = 0,...,n, j = 0,...,n, which are presented in the table 1:

Table 1. The torsion angle change in relation to aerodynamical force and the elastic beam thickness

θ_{ij}	Rz_0	Rz_1	...	Rz_n
d_0	$\theta(d_0, Rz_0)$	$\theta(d_0, Rz_1)$	...	$\theta(d_0, Rz_n)$
d_1	$\theta(d_1, Rz_0)$	$\theta(d_1, Rz_1)$	...	$\theta(d_1, Rz_n)$
...	...	...	...	...
d_n	$\theta(d_n, Rz_0)$	$\theta(d_n, Rz_1)$	...	$\theta(d_n, Rz_n)$

The divided differences for function θ(d,Rz) are defined in itterative manner [9]. More precisely, divided differences of order i+j (order i regarding the first variable d, and order j regarding the second variable Rz) are defined via two divided differences of order (i-1)+j or via two divided differences of order i+(j-1).

The divided difference of order 0+0 are defined as the value of the function in the given point:

$$\theta[d_0;Rz_0]=\theta(d_0,Rz_0) \tag{1}$$

The divided difference of order 1+0 is defined via two divided differences of order 0+0:

$$\theta[d_0,d_1;Rz_0]=\frac{\theta[d_1;Rz_0]-\theta[d_0;Rz_0]}{d_1-d_0} \tag{2}$$

Similarly, the divided difference of order 0+1 is defined as:

$$\theta[d_0;Rz_0,Rz_1]=\frac{\theta[d_0;Rz_1]-\theta[d_0;Rz_0]}{Rz_1-Rz_0} \tag{3}$$

The divided difference of order 1+1 is calculated as:

$$\theta[d_0,d_1;Rz_0,Rz_1]= =\frac{\theta[d_1;Rz_0,Rz_1]-\theta[d_0;Rz_0,Rz_1]}{d_1-d_0}= =\frac{\theta[d_0,d_1;Rz_1]-\theta[d_0,d_1;Rz_0]}{Rz_1-Rz_0} \quad (4)$$

Note that divided difference is symmetric operator. As it can be seen from (4), divided difference of order 1+1 can be calculated by using either two divided differences of order 1+0 or two divided differences of order 0+1, as it is shown in (2) and (3), respectively.

The divided difference of order 2+0 is defined as the difference of two divided differences of order 1+0:

$$\theta[d_0,d_1,d_2;Rz_0]=\frac{\theta[d_1,d_2;Rz_0]-\theta[d_0,d_1;Rz_0]}{d_2-d_0} \quad (5)$$

Each of the two divided differences of order 1+0 is calculated by using two divided differences of order 0+0, as it is shown in (2).

Similarly, the divided difference of order 0+2 is calculated as:

$$\theta[d_0;Rz_0,Rz_1,Rz_2]=\frac{\theta[d_0;Rz_1,Rz_2]-\theta[d_0;Rz_0,Rz_1]}{Rz_2-Rz_0} \quad (6)$$

Each of the two divided differences of order 0+1 is calculated by using two divided differences of order 0+0, as it was shown in (3).

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In general, the divided difference of order i+j is defined as:

$$\theta[d_0,d_1,...,d_i;Rz_0,Rz_1,...Rz_j]= =\frac{\theta[d_{1,...,}d_i;Rz_{0,...},Rz_j]-\theta[d_0,...,d_{i-1};Rz_0,...Rz_j]}{d_i-d_0}= =\frac{\theta[d_{0,...,}d_i;Rz_{1,...},Rz_j]-\theta[d_0,...,d_i;Rz_0,...Rz_{j-1}]}{Rz_j-Rz_0} \quad (7)$$

The symmetric property of divided differences can be also be observed from (7) . More precisely, divided differences of order i+j are defined via two divided differences of order (i-1)+j or via two divided differences of order i+(j-1). The divided difference of order (i-1)+j is calculated via two divided differences of order (i-2)+j or two divided differences of order (i-1)+(j-1), etc.

If we have the values $\theta(d_i,Rz_j)$, $i,j=0,1,..,n$, $0\le i+j\le n$, then we can calculate all divided differences of order $i+j$, where $i,j=0,1,..,n$ and $0\le i+j\le n$. From the definition of the divided differences, it is obvious that each divided difference can be represented via the values of function in the given set of points.

Let us introduce the notation $a_{ij}=\theta[d_0,...,d_i;Rz_0,...,Rz_j]$ for divided difference of order i+j. Then the polinominal with divided differences [6] that approximates function θ(d,Rz) is as follows:

$$P_\theta(d,Rz)= =\sum_{k=0}^{n}\sum_{i+j=k}a_{ij}(d-d_0)...(d-d_{i-1})\cdot(Rz-Rz_0)...(Rz-Rz_{j-1}) \quad (8)$$

$$P_\theta(d,Rz)=\sum_{k=0}^{n}\sum_{i+j=k}\begin{matrix}\theta[d_0,...,d_i;Rz_0,...,Rz_j]\cdot\\ \cdot(d-d_0)...(d-d_{i-1})\cdot\\ \cdot(Rz-Rz_0)...(Rz-Rz_{j-1})\end{matrix} \quad (9)$$

In this particular case, we have n=2 and the data given in table 2:

Table 2. Pitch angle value in relation to aerodynamic force and elastic beam thickness

θ_{ij}		Rz_0	Rz_1	Rz_2
		67dN	142dN	320dN
d_0	7mm	13.37	15.8	20.25
d_1	9mm	6.88	8.07	10.7
d_2	11mm	5.39	6.26	8.11

Using the values given in table 2, we can construct polinominal with divided differences P_θ(d,Rz) that approximates the function θ(d,Rz). This polynomal of two variables is constucted using (8) and (9), having the following form:

$$\begin{aligned}P_\theta(d,Rz)&=3.37\text{-}3.245\cdot(d\text{-}7)+0.0324\cdot(Rz\text{-}67)+\\&+0.0083\cdot(d\text{-}7)\cdot(Rz\text{-}67)\text{-}\\&\text{-}2.92\cdot 0.0000\cdot(Rz\text{-}67)\cdot(Rz\text{-}142)+\\&+0.626\cdot(d\text{-}7)\cdot(d\text{-}9)\end{aligned} \quad (10)$$

As the mathematical form of the curve θ(Rz) is known (it is derived theoreticaly), the error function can be defined as the sum of the squared deviations [10,11] of the polynomial P_θ(d,Rz) to θ(d,Rz) for Rz_0,....,Rz_n (in our case n=2). The error function defined in this way allows us to determine the optimal value of d in function of pitch angle change. The error function will be a function of the variable d. More precisely, it is obtained as:

$$G(d)=\sum_{i=0}^{n}(\theta(d_i,Rz_i)-\theta(Rz_i))^2 \quad (11)$$

$$G(d)=(P_\theta(d,Rz_0)-\theta(Rz_0))^2+ +(P_\theta(d,Rz_1)-\theta(Rz_1))^2+(P_\theta(d,Rz_2)-\theta(Rz_2))^2 \quad (12)$$

As the mathematical form of the curve θ(Rz) is known, we can easily obtain the values in the required points, More precisely, we have

$\theta(Rz_0)=\theta(67)=4.57$, $\theta(Rz_1)=\theta(142)=9.29$,
$\theta(Rz_2)=\theta(320)=19.35$,

as the values for the curve θ(Rz). It also applies:

$$_{\theta}(d, Rz_0) = \\ = (0.626 \cdot d - 4.382) \cdot (d - 9) - 3.245 \cdot d + 36.085 \quad (13)$$

$$P_{\theta}(d, Rz_1) = \\ = (0.626 \cdot d - 4.382) \cdot (d - 9) - 2.623 \cdot d + 34.158 \quad (14)$$

$$P_{\theta}(d, Rz_2) = \\ = (0.626 \cdot d - 4.382) \cdot (d - 9) - 1.145 \cdot d + 29.58 \quad (15)$$

After performing mathematical calculus, we obtain:

$$G(d) = ((0.626 - 4.382) \cdot (d - 9) - 3.245d + 31.515)^2 \\ + ((0.626 - 4.382) \cdot (d - 9) - 2.62d + 24.87)^2 \\ + ((0.626 - 4.382) \cdot (d - 9) - 1.145d + 10.233)^2 \quad (16)$$

Our goal is to find the value for d for which the minimum of the function error G(d) is achieved, i.e., the optimal pitch angle for which the deviation from the pitch angle function is the smallest. Since it is a quadratic non-negative function, G(d) has a unique global minimum, which is obtained as the zero of the first derivative, as a solution to the nonlinear equation G'(d)=0.

$$G'(d) = (2.5 \cdot d - 22.32)((0.626 \cdot d - 4.38) \cdot (d - 9) - \\ -1.15 \cdot d + 10.23) + 2 \cdot (1.252 \cdot d - 13.26) \cdot \\ \cdot ((0.626 \cdot d - 4.38) \cdot (d - 9) - 3.25 \cdot d + 31.52) + \\ +2 \cdot (1.252 \cdot d - 12.64)((0.626 \cdot d - 4.38) \cdot (d - 9) - \\ -2.62 \cdot d + 24.87) \quad (17)$$

The next step is to solve the nonlinear equation G'(d)=0. By applying some of the numerical methods for finding zeros of a nonlinear function (for example Newton's method [10,11]), we obtain that the optimal value is approximately $d_{opt} = 9.97\text{mm}$.

Let us now use different approach to find the optimal value for d. More precisely, we define the error in the mean square sense over the observed interval for Rz:

$$G_1(d) = \int_{Rz_0}^{Rz_n} [P_{\theta}(d, Rz) - \theta(Rz)]^2 dRz \quad (18)$$

Here, $P_\theta(d,Rz)$ is a polynomial with divided differences (10) that approximates the step change function obtained from Table 1. Note the use of mathematical curve θ(Rz) leads to integral that is difficult to calculate. For this reason, in (18) we will use an approximant of θ(Rz), i.e., $P_2(Rz)$ that is interpolation polynomial of the second degree that approximates the mathematical curve θ(Rz). The polynomial $P_2(Rz)$ is constructed from the known values of the mathematical curve θ(Rz) in the points $Rz_0 = 67$, $Rz_1 = 142$, and $Rz_2 = 320$:

$$\theta(67) = 4.57, \ \theta(142) = 9.29, \ \theta(320) = 19.35$$

In particular, the constructed interpolation polynomial $P_2(Rz)$ is:

$$P_2(Rz) = -0{,}000026 \cdot Rz^2 + 0{,}068234 Rz + 0{,}112177 \quad (19)$$

The error function to be minimized is defined as the deviation error $P_\theta(d,Rz)$ from $P_2(Rz)$ in the mean square sense on the interval $[Rz_0, Rz_2]$:

$$G_1(d) = \int_{Rz_0}^{Rz_n} [P_{\theta}(d, Rz) - P_2(Rz)]^2 dRz \quad (20)$$

$$G_1(d) = \\ = \int_{67}^{320} \left[\begin{pmatrix} 3{,}37 - 3{,}245 \cdot (d-7) \\ +0{,}0324 \cdot (Rz(67)) \\ +0{,}0083 \cdot (d-7) \cdot (Rz(67)) \\ -2.92 \cdot 0.00001 \cdot (Rz(67)) \cdot (Rz(142)) \\ +0.626 \cdot (d-7) \cdot (d-9) \\ -(-0.000026 \cdot Rz^2 + 0.068234 \cdot Rz \\ +0.112177) \end{pmatrix}^2 \right] dRz \quad (21)$$

As $G_1(d)$ is a non-negative quadratic function, it has a unique global minimum which is obtained as the zero of its first derivative $G_1'(d)$.

$$G_1'(d) = 396.58 \cdot d^3 + 11603.77 \cdot d^2 - \\ -106435 \cdot d + 302614.7 \quad (22)$$

By solving the equation $G_1'(d) = 0$ by using Newton's method, we obtain the value $d_{opt} = 9.69$.

5. CONCLUSION

This study shows possibility of elastic beam cross-section optimisation in relation to pitch angle change function, by applying combination of interpolated functions of two variables with divided differences, approximating function with corresponding polynominal with divided differences and using the least squared error and the mean squared error. Input data was obtained by analysis of the behavior of the elastic beam with thickness d=7mm, 9mm and 11mm, using the finite elements method. While the thickness varied for this analysis, the width of the beam was kept unchanged. The goal of this study is to determine the optimal size of the cross-sectional area of the bearingless main rotor hub to satisfy required dynamical characteristics of the rotor. The pitch angle change function was aproximated by polynominal with divided differences. When the magnitude of the deviation from designated curve was calculated by least squared error method, resulting optimal value was 9.97mm, but when mean squared error method was applied, the resulting optimal value was 9.69mm. As resulting optimal values in both cases are close, that leads to conclusion that both of these methods can be used in determining of the optimal thickness of the elastic beam cross-section.

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