



## SIGNAL CONSTELLATION DISTORTION AND ITS IMPACT ON CUMULANT-BASED AMC PERFORMANCE

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**Abstract:** Automatic modulation classification (AMC) is of crucial importance for a variety of both military and commercial communications, where signal constellation is not a priori known at the receiver. Due to their simplicity, among many AMC algorithms developed so far, the algorithms based on higher-order cumulant structures remain competitive in terms of practical applicability and thus being in focus of authors worldwide. Still, most of research addressing these algorithms particularly is focused on only additive white Gaussian noise, multipath and interference as sources of signal degradation, and corresponding AMC performance. In this paper a set of practical sources of signal constellation distortion is analyzed in context of AMC for the first time, namely: amplitude imbalance, phase imbalance and the presence of phase jitter. Impact of these effects is presented on AMC performance in the cases of standard fourth-order cumulants, standard sixth-order cumulants and unbiased sixth-order cumulants, in noisy environment, with considerations verified via a number of Monte-Carlo simulations.

**Keywords:** automatic modulation classification, cumulants, phase jitter, amplitude imbalance, phase imbalance.

### 1. INTRODUCTION

Automatic modulation classification (AMC) has been in focus of many researchers worldwide for around three decades. Being of crucial importance for a variety of both military (electronic warfare, spectrum surveillance) and commercial applications (intelligent radio, cognitive systems, IoT), it represents an important tool for processing a priori unknown communication signal, logically placed between the points of signal detection within receiver and its further demodulation, i.e. before the extraction of the very information carried by the signal itself [1-3].

There are many AMC algorithms developed so far. Although not being very much new as a concept, due to their extreme simplicity – algorithms based on specific higher-order statistics (HOS) features of the signal, such as moments and cumulants, still remain quite competitive in terms of practical applicability, and “still be considered as state-of-the-art in AMC” [4]. Fourth-order cumulants [5,6] and sixth-order cumulants [7-9] structures are the most frequently considered for this purpose, although other HOS-based solutions can be found in literature as well, nowadays often additionally equipped with complex and powerful classifiers, like neural networks and deep learning engines [10,11].

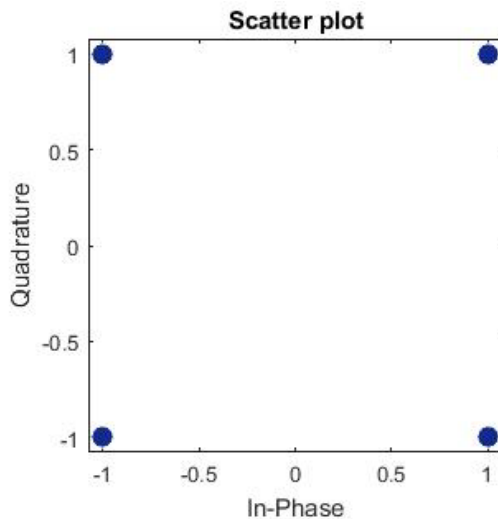
When it comes to the issue of realistic propagation conditions in published AMC research, it can be easily concluded that mostly additive white Gaussian noise (AWGN) was considered as the source of signal degradation. In addition to AWGN, effects of multipath

propagation and interference were considered by some authors, while the structure of normalized higher-order cumulants makes them „naturally“ resistant on effects of flat fading (i.e. fixed amplitude attenuation) and fixed-value phase mismatch – which thus have no relevance for performance of corresponding AMC algorithms. Still, there is a number of sources of signal degradation, quite present in practice, which to the best of our knowledge have not been addressed in AMC research so far.

In this paper a set of practical effects of signal constellation distortion is analyzed in context of AMC, for the first time. Amplitude imbalance, phase imbalance and the presence of phase jitter [12,13] have been set up in context of modulation classification performance, in an AMC problem observed through higher-order cumulants.

### 2. SIGNAL CONSTELLATION DISTORTION

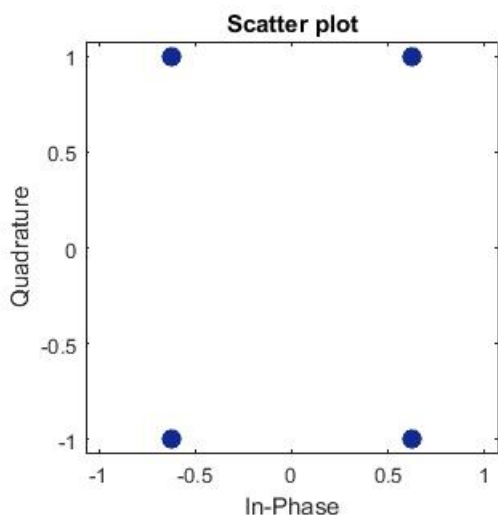
One typical constellation diagram of a signal modulated according to Quadrature Phase Shift Keying - QPSK scheme is presented in Fig. 1: it is characterized by identical voltage level values which represent the signal amplitudes in both I and Q branch, with fixed phase spacing between branches of 90 degrees. Therefore, the constellation diagram has the shape of a regular square, and the positions of individual symbols within this square are marked by precisely grouped constellation points. Strong shape regularity of the formed figure points to conclusion that the presented diagram represents undistorted signal. At the same time, zero dispersion of the values of individual symbols means practically no presence of noise.



**Figure 1.** Ideal QPSK signal constellation diagram

Different impacts can result in disruption of the correct structure shown in Fig. 1, and based on the type of change to which the constellation diagram is exposed, the following phenomena can be distinguished:

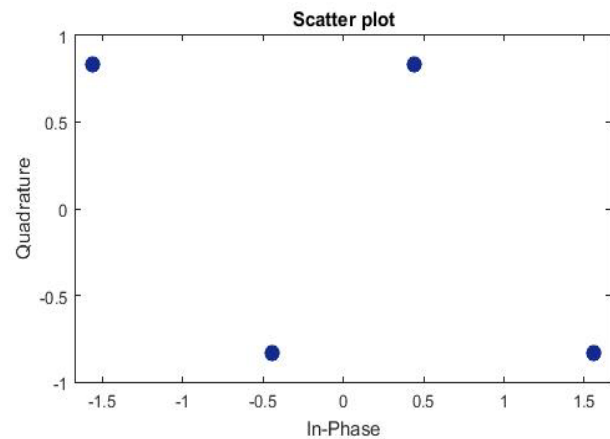
**1. Amplitude imbalance** - appears as a consequence of different amplification of signal components in the I and Q branches. In the constellation diagram, amplitude imbalance is manifested by expansion of one of the components and / or simultaneous compression of the other component of the signal. The cause of this phenomenon lies in the fact that the automatic gain control (AGC) block in the receiver keeps the mean value of the signal level constant. An example of a signal constellation diagram, in which the amplitude imbalance is expressed, corresponding to the undistorted signal from Fig. 1, is shown in Fig. 2.



**Figure 2.** QPSK signal with amplitude imbalance, constellation diagram

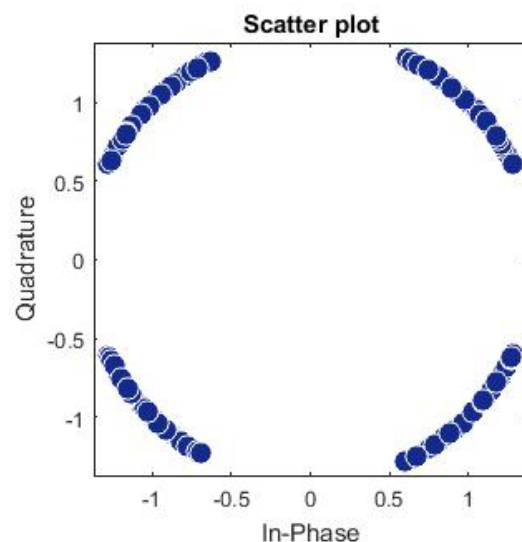
**2. Phase imbalance** - represents the deviation of the angle between the I and Q components of the signal, from the (ideal) value of 90 degrees. On the constellation diagram, it is manifested through the loss of orthogonality of the figure, i.e. it can be noticed that points of the constellation are not simultaneously parallel with both  $x$  and  $y$  axes of

the diagram. The occurrence of phase imbalance comes as a direct consequence of the imperfection of the phase shifter of the I/Q modulator. An example of signal in which a phase imbalance is present, corresponding with undistorted signal from Fig. 1, is shown in Fig. 3.



**Figure 3.** QPSK signal with phase imbalance, constellation diagram

**3. Phase jitter** - which arises as a consequence of the action of transponders present on the signal route, or within the I/Q modulators. It can also come as a result of inadequate regeneration of the carrier within the receiver. Unlike phase imbalance, this phenomenon captures the signal in both branches equally, and is manifested through the occurrence of rotation of constellation points around the center of the IQ plane. An example of this phenomenon is shown in Fig. 4.



**Figure 4.** QPSK signal with strong phase jitter, constellation diagram

In addition to the mentioned above, in practice there is also distortion phenomena like: amplitude error (free deviation of symbol amplitudes from their nominal values), phase error (rotation of all constellation points around the center for some fixed, the same angle value), DC offset (representing the translation of the diagram points for an identical, fixed vector value in the IQ plane), etc. Even more, in practice, it is realistic to expect even

the combined action of several different sources of signal degradation, simultaneously, which potentially affects the work of signal processing blocks within the receiver, thus also making an impact on AMC algorithms performances.

### 3. CUMULANT-BASED AUTOMATIC MODULATION CLASSIFICATION

The received signal sequence  $y(n)$ , corrupted by AWGN during propagation, can be represented by:

$$y(n) = x(n) + g(n), \quad (1)$$

with  $x(n)$  standing for transmitted symbols (of an unknown modulation), and  $g(n)$ , representing zero-mean AWGN with variance of  $\sigma_g^2$ . For zero-mean random variable  $x$ , associated with transmitted data sequence  $x(n)$ , the second-order cumulant is:

$$C_{21,x} = E(|x|^2). \quad (2)$$

where  $E(\cdot)$  represents a mathematical expectation, realized as an average value over observed signal samples. The fourth-order cumulant and the normalized fourth-order cumulant of the same variable are given as:

$$C_{42,x} = E(|x^4|) - |E(x^2)|^2 - 2E^2(|x^2|), \quad (3)$$

$$\hat{C}_{42,x} = C_{42,x} / (C_{21,x})^2. \quad (4)$$

The standard (classical) sixth-order cumulant and its corresponding normalized value of the same random variable  $x$  are given as:

$$C_{63,x} = E(|x|^6) - 9E(|x|^4)E(|x|^2) + 12|E(x^2)|^2 E(|x|^2) + 12E^3(|x|^2), \quad (4)$$

$$\hat{C}_{63,x} = C_{63,x} / (C_{21,x})^3. \quad (5)$$

Apart from standard, the following formulas are used for general and unbiased sixth order cumulant [7], along with its corresponding normalized value:

$$C_{63,x\_UNB} = E(|x|^6) - 9E(|x|^4)E(|x|^2) + 18|E(x^2)|^2 E(|x|^2) - 6|E(x^2)| E(x^2|x^2) + 12E^3(|x|^2), \quad (6)$$

$$\hat{C}_{63,x\_UNB} = C_{63,x\_UNB} / (C_{21,x})^3. \quad (7)$$

For random variable  $y$  associated with received sequence  $y(n)$ , normalized higher-order cumulants can be expressed in the following manner:

$$\hat{C}_{42,y} = \frac{C_{42,y}}{(C_{21,y} - \sigma_g^2)^2}, \quad (8)$$

$$\hat{C}_{63,y} = \frac{C_{63,y}}{(C_{21,y} - \sigma_g^2)^3}, \quad (9)$$

$$\hat{C}_{63,x\_UNB} = \frac{C_{63,y\_UNB}}{(C_{21,y} - \sigma_g^2)^3}. \quad (10)$$

Equations (8) - (10) describe the AMC execution within the receiver. While noise power is commonly considered to be known, it was shown that the same can be easily estimated also, without relevant loss in performance [14].

The decision-making process for modulation recognition is based on a comparison of obtained values of normalized cumulant estimates with predefined thresholds. It was shown, on the basis of intensive computer simulations, that optimal comparison threshold values are positioned at the middle of intervals between expected (theoretical) values corresponding with particular modulation formats [15].

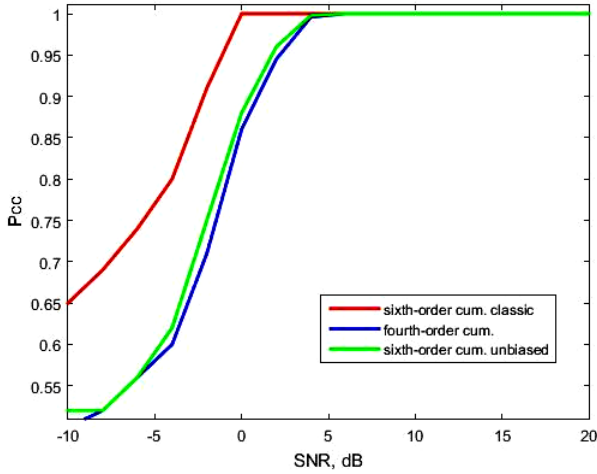
While cumulant's estimates of complex constellations are strictly unbiased, lower Signal-to-Noise (SNR) values introduce stronger bias for real signal's cumulant's estimates, for standard sixth-order cumulant structure, specially. Some theoretical values for normalized cumulants' values, for BPSK and QPSK constellations, are shown in Table 1.

**Table 1.** Theoretical values of normalized cumulants

Constellation	$\hat{C}_{42}$	$\hat{C}_{63}$	$\hat{C}_{63\_UNB}$
BPSK	-2.0000	16.0000	16.0000
QPSK	-1.0000	4.0000	4.0000

### 4. SIMULATIONS

In channel with AWGN only, distinguishing BPSK from QPSK signals is reported to be superior with standard sixth-order cumulants, when compared to fourth-order and unbiased sixth-order cumulants. As previous researches showed, this comes due to a strong bias in  $\hat{C}_{63}$  values of BPSK signal, representing an interesting specific phenomena present in standard sixth-order cumulants only. For the basic reference, simulation with 2,000 Monte Carlo experiments is carried out,  $N=250$  symbols are generated from the set of signal constellations {BPSK, QPSK} in each trial, with noise variance considered to be known, and probability of correct classification values  $P_{CC}$  for all considered AMC algorithms was calculated in a wide range of SNR values, from -10dB to 20dB. Results obtained from these simulations are shown in Fig. 5.

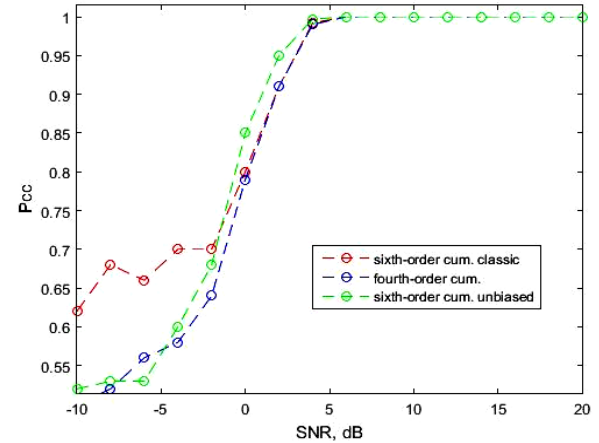
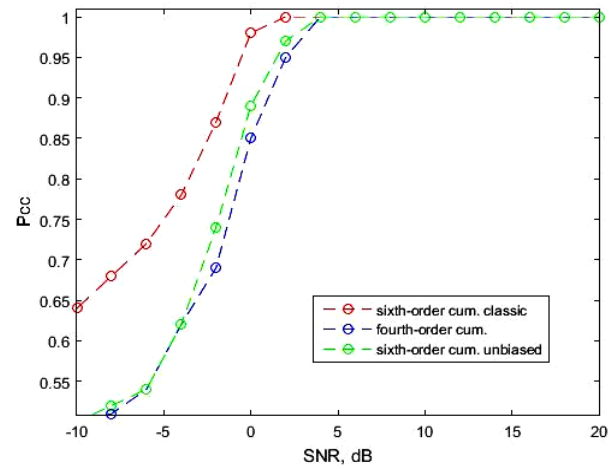
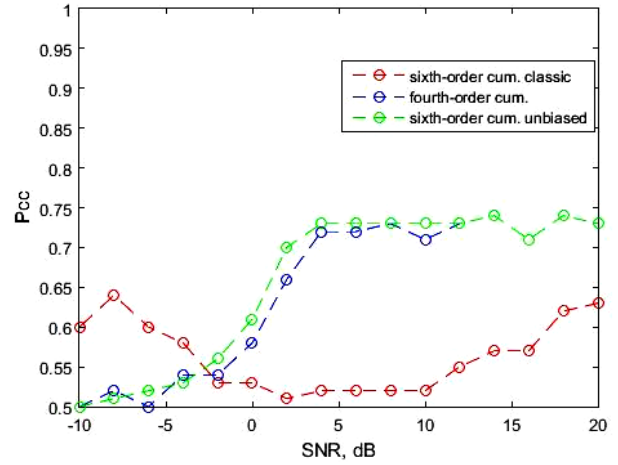


**Figure 5.** Correct classification probability in {BPSK, QPSK} scenario,  $N=250$ , AWGN channel

#### 4.1. Amplitude imbalance

In order to evaluate classification performance in the presence of amplitude imbalance, corresponding distortion was introduced into the samples of  $x(n)$  in eq. (1): like in a manner shown in Fig. 2, the I component of each sample was limited on amplitude value of  $(1-a)$ ,  $a < 1$ , while Q component preserved its unit value. As the boundary case, scenario with  $a_{\text{lim}} = 0.41485$  was recognized, when  $P_{CC} = 0.75$  for all AMC algorithms in noiseless conditions. Even the slight introduction of noise further has the impact on performance of standard sixth-order cumulants, since its specific bias in this boundary scenario reflects negatively on decision making. Simulations were carried out under the same conditions as presented at the beginning of this section, now with amplitude imbalance  $a_{\text{lim}}$  introduced, and resulting values of  $P_{CC}$  are presented in Fig. 6. For comparison of effects, scenarios with less rigid values of  $a = 0.5 * a_{\text{lim}}$  and  $a = 0.75 * a_{\text{lim}}$  are included in the same figure.

From Fig. 6 it can be noticed that strong amplitude imbalance  $a_{\text{lim}}$ , from the point of AMC, introduces more “BPSK-like” properties into the imbalanced QPSK signal, causing the bias of  $\hat{C}_{63}$  to start reflecting on QPSK signals also, thus significantly degrading the performance of this particular algorithm. On the other side, performance of  $\hat{C}_{42}$  and  $\hat{C}_{63\_UNB}$ , although degraded by amplitude imbalance  $a_{\text{lim}}$ , remains stable. For  $SNR$  values lower than  $-2\text{dB}$  approximately, the effect of noise becomes dominant. While small imbalance  $a = 0.5 * a_{\text{lim}}$  introduces almost no degradation in performance of algorithms, somewhere around the imbalance value of  $a = 0.75 * a_{\text{lim}}$ , performance of all three considered cumulant structures becomes approximately equal, until the point of  $SNR$  when noise effect becomes dominant.

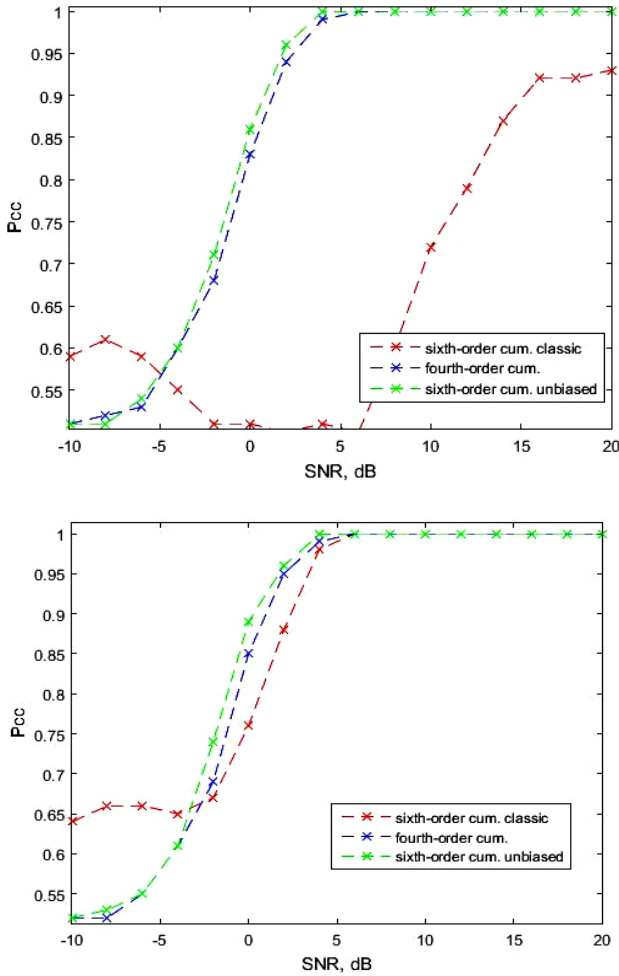


**Figure 6.** Correct classification probability in {BPSK, QPSK} scenario,  $N=250$ , AWGN channel with amplitude imbalance  $a_{\text{lim}}$  (top),  $a = 0.5 * a_{\text{lim}}$  (middle) and  $a = 0.75 * a_{\text{lim}}$  (bottom)

#### 4.2. Phase imbalance

As illustrated in Fig. 3, phase imbalance was introduced via the angle shift of  $\alpha$  between I and Q components of signal samples  $x(n)$  in eq. (1), as a measure of violation of two branches’ mutual orthogonality. As the boundary – case, scenario with  $\alpha_{\text{lim}} = 3\pi/8$  was recognized, since then  $P_{CC}$  becomes  $< 1$  for standard sixth-order cumulants

even in noiseless channel conditions. Simulations were carried out with phase imbalance  $\alpha_{lim}$  introduced, and resulting values of  $P_{CC}$  are presented in Fig. 7. For comparison, scenario with  $\alpha = 0.5 * \alpha_{lim}$  is also included.



**Figure 7.** Correct classification probability in {BPSK, QPSK} scenario,  $N=250$ , AWGN channel with phase imbalance  $\alpha_{lim}$  (top) and  $\alpha = 0.5 * \alpha_{lim}$  (bottom)

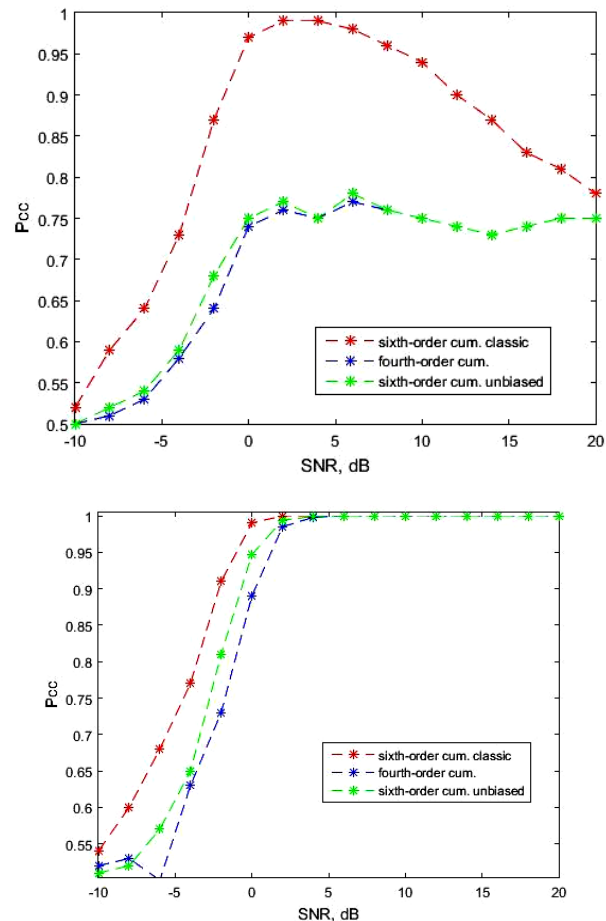
From Fig. 7 it can be noted that strong phase imbalance  $\alpha_{lim}$ , similarly like strong amplitude imbalance, makes significant impact on properties of the imbalanced QPSK signal, causing the bias of  $\hat{C}_{63}$  to start reflecting on QPSK signals and violating the decision making whose decision thresholds are derived from theoretical cumlants' values, thus significantly degrading the performance of standard sixth-order cumulants algorithm. On the other side, it is very much interesting to note that performance of  $\hat{C}_{42}$  and  $\hat{C}_{63\_UNB}$  seems not affected by phase imbalance at all, and keeps the value of  $P_{CC} \approx 1$  for higher values of SNR. Again, for SNR values lower than -2dB approximately, the effect of noise becomes dominant, in comparison with effects of phase imbalance. Lighter phase imbalance  $\alpha = 0.5 * \alpha_{lim}$ , as expected, introduces almost no degradation in performance of two unbiased algorithms, while this phase imbalance value approximately brings the AMC algorithm based on standard (i.e. biased) sixth-

order cumulants to the same order of  $P_{CC}$  values, making the performances of all three considered cumulant structures (more or less) equal, down to the point when noise effect becomes dominant.

Obtained results under amplitude and phase imbalance are potentially important since, to the best of our knowledge, they represent the first examples published so far where classification in {BPSK, QPSK} scenario shows as being not superior with standard (biased) sixth-order cumulants.

### 4.3. Phase jitter

The presence of phase jitter was introduced into the  $x(n)$  samples via the random values of phase offset per each particular sample, generated with equal probability from the interval of angles  $[-\varphi, \varphi]$ . Here as the boundary case, value of  $\varphi_{lim} = 2\pi/9$  was selected, when  $P_{CC} = 0.75$  for all observed AMC algorithms in noiseless conditions. Simulations were executed for phase jitter under limits of  $\varphi_{lim}$ , with resulting values of  $P_{CC}$  presented in Fig. 8, along with results achieved in scenario with  $\varphi = 0.5 * \varphi_{lim}$  included in the same figure.



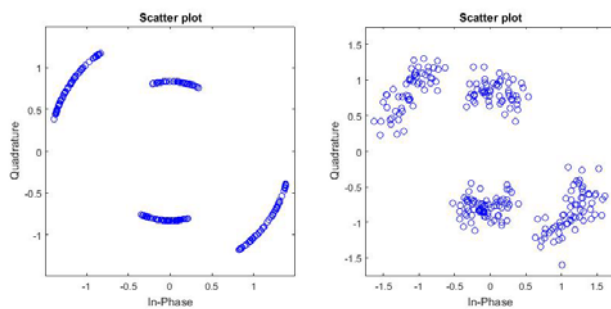
**Figure 8.** Correct classification probability in {BPSK, QPSK} scenario,  $N=250$ , AWGN channel with phase jitter under  $\varphi_{lim}$  (top) and  $\varphi = 0.5 * \varphi_{lim}$  (bottom)

In contrary to effects of amplitude imbalance, from Fig. 8 it can be noted that strong phase jitter  $\varphi_{lim}$  introduces

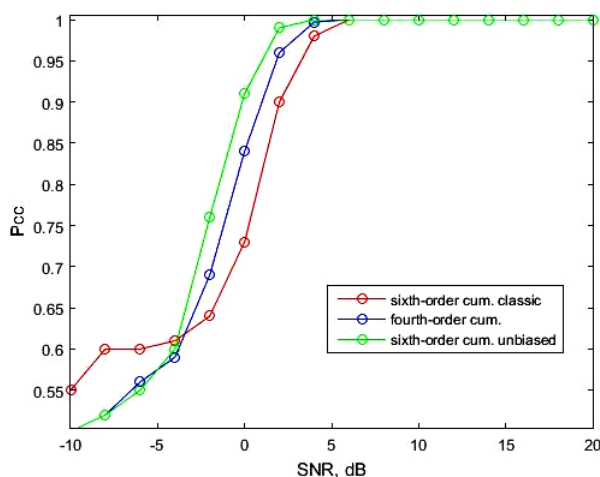
more “QPSK-like” properties to the imbalanced BPSK signal, which then gets compensated by the bias of  $\hat{C}_{63}$  with moderate SNR, in manner contributing to successful classification. Performance of  $\hat{C}_{42}$  and  $\hat{C}_{63\_UNB}$ , both degraded by presence of phase jitter, remains stabile, while smaller jitter limits of  $\varphi = 0.5 * \varphi_{lim}$  introduce only slight loss in performance of all considered AMC algorithms, down to the point when noise effect becomes dominant.

#### 4.4. Combined distortion

Finally, simulations have been repeated for scenario of combined joint effect of all previously discussed sources of signal degradation: with samples  $x(n)$  corrupted by amplitude imbalance with parameter  $a = 0.75 * a_{lim}$ , phase imbalance of  $\alpha = 0.5 * \alpha_{lim}$  and phase jitter within the boundaries  $[-\varphi, \varphi], \varphi = 0.5 * \varphi_{lim}$ , simultaneously. As an illustration of this joint degradation effect, resulting QPSK signal is presented in Fig. 9, while classification performance of considered cumulants-based AMC algorithms is given in Fig. 10.



**Figure 9.** QPSK signal constellation diagram in combined presence of phase jitter, amplitude and phase imbalance: without noise (left) and at SNR=20dB (right)



**Figure 10.** Correct classification probability in {BPSK, QPSK} scenario,  $N=250$ , AWGN channel with amplitude imbalance  $a = 0.75 * a_{lim}$ , phase imbalance  $\alpha = 0.5 * \alpha_{lim}$  and phase jitter under  $\varphi = 0.5 * \varphi_{lim}$

Simulations show that effects of amplitude imbalance

and, especially, phase imbalance, dominate over the impact of phase jitter in overall performance, for combined presence of different sources of signal degradation. Bias introduced for BPSK signals by classical sixth-order cumulants-based AMC algorithm no longer brings outstanding success in classification. Even in contrary: it accelerates the effects of imbalance, resulting with lower performances in terms of AMC for standard sixth-order cumulants, when compared with fourth-order and unbiased sixth-order cumulants used as features of interest.

#### 5. CONCLUSION

In this paper well known AMC algorithms based on higher-order cumulants are observed in context of various effects of signal degradation, and their performances were evaluated in intensive computer simulations. Several specific effects of degradation were explained, with boundary – value parameters thereof reported, in an AMC application. It was shown that presence of amplitude and phase imbalance cannot be successfully compensated with bias introduced by standard sixth-order cumulants, which still works well under the terms of phase jitter. Nevertheless, in scenario with combined presence of various signal degradation sources, unbiased (fourth and sixth-order) cumulants-based AMC algorithms show as more reliable and better performed, which makes them a solution of choice, for any application where presence of discussed signal degradation sources seems unavoidable.

#### References

- [1] DOBRE,O.A.: *Signal identification for emerging intelligent radios: Classical problems and new challenges*, IEEE Instrumentation & Measurement Magazine, 18(2) (2015) 11-18.
- [2] ELDERMERDASH,Y.A., DOBRE,O.A., ONER,M.: *Signal identification for multiple-antenna wireless systems: Achievements and challenges*, IEEE Commun. Surv. Tutor., 18(3) (2016) 1524–1551.
- [3] PAJIC,M.S., VEINOVIC,M., PERIC,M. et al.: *Modulation order reduction method for improving the performance of AMC algorithm based on sixth order cumulants*, IEEE Access, 8 (2020) 106386-106394.
- [4] PENNACCHIO,A.A, LUCTOSA da COSTA, J.P.C., BORDINI, V. M. et al.: *Eigenfilter-based automatic modulation classification with offsets for distributed antenna systems*, Simposio Brasileiro de telecomunicacoes e processamento, Sinais, Brasil, (2016) 260-261.
- [5] SWAMI,A., SADLER,B.M.: *Hierarchical digital modulation classification using cumulants*, IEEE Trans. Commun., 48(3) (2000) 416-429.
- [6] WU,H., SAQUIB,M., YUN,Z.: *Novel automatic modulation classification using cumulant features for communications via multipath channels*, IEEE Trans. Wirel. Commun., 7(8) (2008) 3098-3105.
- [7] SIMIC,M., STANKOVIC,M., ORLIC,V.D.: *Automatic modulation classification of real signals in AWGN channel based on sixth order cumulants*,

- Radioengineering, 30(1) (2021) 204-214.
- [8] PAJIC,M.S., VEINOVIC,M., ORLIC, V.D.: *Complex signal constellations in cumulants-based AMC: Statistics and performance*, Telfor Journal, 13(2 ) (2021) 63-68.
- [9] BOZOVIC,R.R., ORLIC,V.D.: *Estimation of bias in numerical values of normalized sixth-order cumulants' structures for various signal constellations*, 2021 International Conference on Computational Performance Evaluation (ComPE), (2021) 410-414.
- [10] LI,X., DONG,F., ZHANG,S. et al.: *A survey on deep learning techniques in wireless signal recognition*, Wireless Comm. and Mobile Computing, (2019).
- [11] MENG,F., CHEN,P., WU,L., et al.: *Automatic modulation classification: A deep learning enabled approach*, IEEE Transactions on Vehicular Technology, 67(11) (2018) 10760-10772.
- [12] SIMROCK,S., GENG,Z.: *RF Detection and Actuation*. In: *Low-Level Radio Frequency Systems. Particle Acceleration and Detection*, Springer, Cham, (2022) 149-182.
- [13] ERGEN,M.: *Basics of cellular communication*. In *Mobile Broadband*, Springer, Boston, MA, (2009) 19-65.
- [14] NERANDZIC,M., BOZOVIC,R.R., ORLIC,V.D.: *Impact of AWGN estimation on classification performance of AMC algorithms based on higher order cumulants*, 2021 29th Telecommunications Forum (TELFOR), (2021) 1-4.
- [15] ORLIC,V.D., DUKIC,M.L.: *Setting the optimal decision threshold and analysis of impact of sample size on automatic modulation classification based on sixth-order cumulants*. In Proc. Int. Sci. Conf. Defensive Technol. OTEH, (2012) 511-515.