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PLANETARY GEAR PAIR DESIGN USING METAHEURISTIC ALGORITHMS

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Abstract: *This paper considers the problem of formulating the non-linear optimization model for determining the optimal parameters of planetary gearbox, which is solved using a metaheuristic optimization algorithm. To determine the optimal parameters of the planetary gearbox it is necessary to formulate complex objective functions and minimize them, which is often a conflicting problem. To solve this complex optimization problem, in this paper we propose to employ metaheuristic algorithms, which characterize pseudo-randomness and the ability to find the global optimal solution to the multimodal optimization problems. The proposed metaheuristic method is based on the Genetic Algorithm (GA) which is hybridized with the local-search Nelder-Mead method. For the considered optimization problem, the appropriate software is implemented in the MATLAB software package, to verify the results. Inside the considered optimization module, we have defined the appropriate objective functions and constraints which determine the construction of the planetary gearbox. The optimal parameters of the planetary gearbox obtained using the proposed metaheuristic algorithm are compared with the results obtained using several well-known algorithms in the literature. The simulation results of the proposed optimization method indicate a significant improvement in planetary gearbox performance compared to the parameters obtained with well-known algorithms.*

Keywords: *Optimization, Planetary gear trains, Gear efficiency.*

1. INTRODUCTION

Planetary gearboxes find widespread use in a variety of mechanical systems, including industrial drives, rotorcraft, automobiles, wind turbines, and other similar applications [1,2]. In these types of mechanical systems, planetary gearboxes can offer more compact dimensions and higher power densities with less noise and higher torque-to-weight ratios, in particular when compared to standard parallel axis gear trains. However, the design of such gearboxes necessarily involves several planetary branches, which also affects efficiency [1]. This is because the total efficiency of planetary gear sets is determined by the various gear mesh areas in planetary gear sets. The primary objective is to increase the service life of the components while simultaneously decreasing the weight and power loss associated with the design of the gearbox.

Nowadays, it is necessary to accurately forecast power losses during the design stage of the gearbox. This enables the designer to make appropriate design adjustments prior to the gear box being manufactured and put through its tests. There are a great number of articles that concentrate on gear efficiency and power loss, both of which are connected to concerns about the efficiency of parallel axis gear systems [3, 4]. The bulk of research focuses on mechanical power loss. Few papers have been written on the power losses of planetary systems, and the majority of those that have concentrated on the power losses that occur in planetary gear sets, which are often the result of experiments. In most cases, such models investigate efficiency via the analysis of gear train kinematics. More specifically, they use speed and torque equations to evaluate the effectiveness of planetary gearboxes [3]. These studies do not take into account the power loss that occurs as a result of lubricant interactions when non-uniform gear normal loads are being applied. However, these studies fail to simultaneously consider the minimization of mass and the maximization of gearbox efficiency.

Many engineering optimization problems have recently been approached using natural-world-inspired metaheuristic algorithms, such as the Genetic Algorithm (GA) [5], particle swarm optimization (PSO), differential evolution (DE), ant colony optimization (ACO), firefly algorithm (FA), grey wolf optimization (GWO) [6], and others. Weight minimization, power loss reduction, and other gearbox optimization problems are only a few of the many for which the GA has been actively used as a promising optimization strategy from the group of EAs [5].

The use of evolutionary algorithms (EAs) to address optimization issues in the multiobjective optimization (MOO) of gearbox parameters has proven fruitful [6, 7]. Although evolutionary algorithms (EAs) were first developed for use in tackling unconstrained single objective optimization problems, researchers have recently found ways to include constraint management approaches into these algorithms and adjust the algorithms to be capable to tackle multiobjective problems. Limits on the choice of number of objectives, variables, different kinds of constraints, conflict between different kinds of constraints, and a close link between the constraints and the objective function all contribute to the difficulty of constrained multiobjective optimization problems [8].

There have been several EAs developed for multiobjective optimization, and as a result, in recent years there have been numerous publications in academic journals that discuss multiobjective optimization of gearboxes. The majority of the papers [6, 7] depend on the well-known non-dominant genetic algorithm (NSGA) and other enhanced forms of this algorithm to tackle the multiobjective optimization issues in the field of gearbox design. However, a number of works have recently been published in the literature that address MOO issues utilizing hybrid versions of well-known EAs [6].

As a consequence of this, the multiobjective planetary gearbox optimization problem that is being discussed in this study is tackled with the help of a hybrid strategy that is presented in this paper. The GA algorithm's capacity for global search is combined with the NM algorithm's superior ability to perform local search, resulting in the hybrid algorithm's capabilities. The outcomes of the computational simulation provide further evidence that the strategy that was suggested is preferable.

This paper is organized as follows. Firstly, in Section 2 the considered optimization problem is formulated and the appropriate penalty method for dealing with constraints is introduced. Next, in Section 3 the procedure of the GA and NM methods of the hybrid algorithm are outlined. In Section the corresponding simulation results are presented. Finally, the conclusions are drawn in Section 5.

2. OPTIMIZATION PROBLEM FORMULATION

Many researchers have been drawn to the problem of minimum mass design and boosting the efficiency of planetary gearboxes, due mostly to the necessity for low mass design in aerospace engineering. In this study, we will look at how to optimize the mass and efficiency of a single stage planetary gearbox, as shown in Fig. 1.

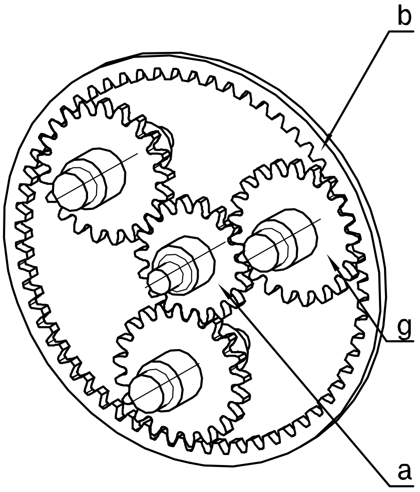

Figure 1. Illustration of a single-stage planetary gearbox considered in the optimization model

Here, (a) represents the sun gear, (b) represents the ring gear, and (g) represents the planet gear. Because weight and efficiency are mutually incompatible objectives, the described optimization problem transforms into a multiobjective optimization problem, denoted as

$$\min_{\mathbf{x}\in\mathcal{D}}\left[W(\mathbf{x}),\eta_{aH}^{b}(\mathbf{x})\right], \tag{1}$$

subject to the n equality and m inequality constrains

$$\begin{aligned} g_i(\mathbf{x}) &\le 0, \quad i=1,\ldots,m \\ h_j(\mathbf{x}) &= 0, \quad j=1,\ldots,n. \end{aligned} \tag{2}$$

The first objective is the mass of the single stage planetary gearbox, which under the assumption of the homogenous material can be defined as

$$W(\mathbf{x})=\rho\sum_{i=1}V_i\,, \tag{3}$$

where ρ is the density and V_i is the volume of the i th structural element. Therefore, the mass of the considered planetary gearbox is given as follows:

$$W(\mathbf{x})=\rho\frac{\pi}{4}b\left[d_{(a)}{}^{2}+n_w\left(d_{a(b)}^{2}-D^{2}\right)+\left(d_{(g)}^{2}-d_s^{2}\right)\right] \tag{4}$$

where $d_{(a)}$ is the pitch circle of the sun gear, $d_{a(b)}$ is tip diameter of the ring gear, d_s is outside bearing diameter, $d_{(g)}$ is the pitch circle of the planet gear, $n_w=3$ is the number of planet gears, D denotes outside diameter of a ring gear and b is the width of a gearbox.

The second objective of the considered optimization problem deals with the efficiency of the planetary gear box and can be described according to the expression

$$\eta_{aH}^{b}=\frac{1-\eta_{ag}^{H}\eta_{gb}^{H}u_{ab}^{H}}{1-u_{ab}^{H}}, \tag{5}$$

where η_{ag}^{H} and η_{gb}^{H} denote relative efficiency of the sun-planet gears and relative efficiency of planet-ring gears, respectively. These efficiencies can be determined according to the numerical procedure given in [1]. Furthermore, u_{ab}^{H} denotes the relative gear ratio.

The corresponding constraints of the considered optimization problem, take into considerations the strength of the gear, through the factor of safety, and achievable size constraints. Functional constraints in the form of inequality are given as

$$g(\mathbf{x})=\frac{[\sigma_F]}{\sigma_F}-S_F>0 \tag{6}$$

where σ_F is the working bending stress, $[\sigma_F]$ is the allowable bending stress and S_F is bending stress factor of safety. Moreover, to prevent any interference of teeth during the meshing process, the assembly requirement that must be met would be that the central sun gear and planet gears must always mesh simultaneously. Regarding this, the equality requirement is described as

$$h_1 = \frac{z_a z_b}{n_w D(z_g, z_b)} - i = 0 \quad (7)$$

Finally, taking into consideration the given optimization problem in Eq. (1) and corresponding constraints in Eqs. (6) and (7) the nonempty feasible domain is defined as follows

$$\mathcal{D} = \left\{ \mathbf{x} \in R^n \mid g(\mathbf{x}) \le 0 \wedge h(\mathbf{x}) = 0 \right\} \quad (8)$$

2.1. Penalty method for multiobjective optimization

Because the proposed evolutionary algorithm is incapable of dealing directly with constrained situations, a method for dealing with the constraints must be devised. This work considers the penalty approach to be a popular and straightforward constraint-handling strategy that effectively approximates the constrained optimization problem with the unconstrained optimization problem using the penalty function [8]. For each objective $f_i, \forall i = 1,2$ a pseudo-objective function F_i is defined in the following form

$$F_i(\mathbf{x}) = f_i(\mathbf{x}) + \sum_{j=1}^{m} P_j(\mathbf{x}), \quad (9)$$

where $P_j(\mathbf{x})$ is the penalty function corresponding to the j th constraint. The penalty function is used to remove any solution that violates the constraint. As a result, the limited optimization problem may be transformed into an unconstrained minimization problem that has the following form

$$\min_{\mathbf{x} \in \mathbb{R}^n} \left[F_1(\mathbf{x}), F_2(\mathbf{x}) \right]. \quad (10)$$

The penalty function can be defined as

$$P_i(\mathbf{x}) = \begin{cases} 0, & \mathbf{x} \in \mathcal{D} \\ s_i(\mathbf{x}) R_i & \mathbf{x} \notin \mathcal{D} \end{cases}, \quad i = 1, \ldots, m \quad (11)$$

where R_i is the penalty factor that measures the importance of the i th penalty function and $s_i(\mathbf{x})$ is a continuous function that considers the equality and inequality constrains, defined as

$$s_i(\mathbf{x}) = g_i^2(\mathbf{x}), \quad i = 1, \ldots, m. \quad (12)$$

According to the formulation of the penalty function in Eq. (11), the penalty method directly analyses every viable option based on their objective function values, while the penalty function is applied to the infeasible alternatives, lowering their fitness value. A significant drawback of the penalty technique is the insufficient selection of the punishment factor, which has a significant impact on the efficiency of the search process. The severity of the penalty is determined mostly by the degree of infraction and varies appropriately.

3. GENETIC ALGORITHM

As a member of the class of metaheuristic optimization algorithms, the genetic algorithm may be used to find optimum solutions to multimodal optimization problems at the global level. GA processes are inspired by natural genetics and natural selection mechanisms [5].

The process of utilizing GA to identify the global optimum solution begins with the generation of a population of initial solutions. There are two types of GA in the literature: binary GA and real-valued GA. Each solution in the Binary GA is represented by a chromosome, which is a fixed-length vector of binary variables. Binary vectors are used to encode each potential solution. The number of optimization parameters used and the desired encoding accuracy define the length of this vector. The GA consists of three evolutionary operators, selection, crossover, and mutation, which are applied to the population of solutions in each generation with the goal of guiding the population towards the global optimum.

During the selection phase, the GA uses the associated objective function value of each individual to choose chromosomes from the population that have favorable mating qualities. In this method, the chromosomes that lack the qualities required to solve the optimization problem are removed from the population. The selection method employs chromosome fitness values to decide which individuals are selected for mating and removes chromosomes that lack critical features for the effective solution of the optimization issue. The chromosomes are chosen using a roulette wheel, with individuals with lower objective function values, which is advantageous for minimization problems, being more likely to be chosen. The *i*th individual's selection probability is calculated as

$$P_i = \frac{F_i}{\sum_{j=1}^{N_p} F_j}, \quad i \in \{1, \ldots, N_p\}, \quad (13)$$

where F_i is denoting an objective function value of the chromosome. The cumulative probability C_i of the ***i***-th individual is determined according to the following expression

$$C_i = \sum_{j=1}^{N_p} F_j. \quad (14)$$

After assigning a cumulative probability to each individual in the population, they are subjected to a selection method in which a suitable individual is chosen based on a random number r that must meet the following equation

$$C_{i-1} < r \le C_i, \; i \in \{1, 2, \ldots, N_p\}, \quad (15)$$

where r denotes the random number generated between 0 and 1.

One of the most essential aspects of GA is the crossover operator, which integrates and utilizes the available information stored in chromosomes to impact the search direction throughout the optimization process. The crossover operator combines the information of two previously picked individuals in the selection process to produce offspring that share both individuals' important advantages. Such offspring may have the abilities required to solve the optimization issue successfully. As seen in Fig. 2, two-point crossover is carried out by picking two random crossing spots along the chromosomal length, indicated as c_1 and c_2. As a consequence, the encoded binary values encompassed by these points may be exchanged between individuals.

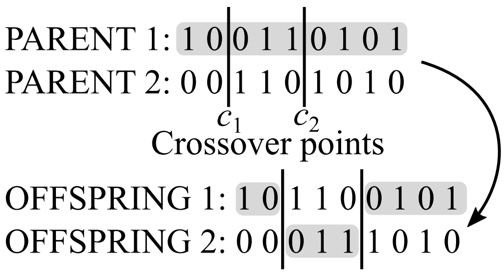

Figure 2. The illustration of two-point crossover operator

Mutation is a GA operator that adds previously unknown solutions into the GA population. To avoid damaging valuable information, just a tiny fraction of the population gets altered. Mutation occurs when random binary changes occur on a chromosome. To reflect the unpredictability of the alterations, each digit of the gene being mutated is changed to either 0 or 1. The mutation rate must be low, since if it is high, a good chromosome may accidently mutate into a poor one.

The selection, crossover, and mutation operators are repeated, and the population evolves through consecutive iterations toward the global optimal solution of the considered optimization problem until the termination condition is fulfilled. Although most evolutionary algorithms employ the maximum number of iterations as a termination criterion, the relative error of the average objective function value of the population between two successive iterations is also used. As a result, the algorithm may be stopped by comparing the objective function value of population in successive iterations when it falls below a specific threshold, which is stated in the following

$$\left|\frac{f_{avg}^{(k)} - f_{avg}^{(k-1)}}{f_{avg}^{(k)}}\right| \le \varepsilon , \tag{16}$$

where ε is a small positive real number, in which

$$f_{avg}^{(k)} = \frac{1}{N_p}\sum_{i=1}^{N_p} f_i , \tag{17}$$

represents the average objective function value of the entire population in kth iteration.

4. NELDER-MEAD METHOD

The Nelder-Mead is a direct local search method that doesn't require the knowledge of the first derivative of the objective function, which is widely applied in solving different convex optimization problem$n$s [9]. The process of NM method is based on transforming $n+1$ points during the iterations at the vertices of a simplex, for dimensional optimization problem. The main operations on a simplex, which consists of three points $S := \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$, applied in order to transform simplex and converge to optimal solution and produce new points are displayed on Fig. 3, and include: reflection $\mathbf{x}_r$, expansion $\mathbf{x}_e$, contraction $\mathbf{x}_c$ and shrinkage.

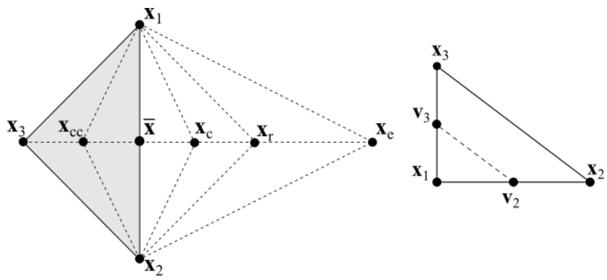

Figure 3. The illustration of four main transformations performed on simplex in Nelder-Mead method for two-dimensional optimization problem

The initial untransformed simplex is bounded by vertices ordered by the objective function value, e.g. $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$, where the last point denotes the worst point. To transform the initial simplex and provide a new point trough applying the abovementioned operations firstly we determine a centroid point $\bar{\mathbf{x}}$ which halves the line segment between points $\mathbf{x}_1$ and $\mathbf{x}_2$. During each iteration one of the transformations is applied and a new point is generated which replaces the point $\mathbf{x}_3$ in the current simplex. In this way the points on the simplex are improved until it converges towards the optimum point.

Therefore, to apply the NM method the following operations are performed.

Initialization. Firstly, to start with the optimization process, the initial simplex is generated around the provided initial solution by adding to this point the scaled value of unit vector in each direction, e.g.

$$\mathbf{x}_{i+1} := \mathbf{x}_1 + \lambda \mathbf{e}_i, \; i = 1,\ldots,n, \tag{18}$$

where $\lambda \in R$ is represents the unit vector scale factor, usually set as $\lambda = 1$, and $\mathbf{e}_i$ denotes the unit vector of the ith axis. To form the initial simplex, the generated point are sorted in ascending order based on the objective function value, such that the following is true

$$f(\mathbf{x}_1) \le f(\mathbf{x}_2) \le \ldots \le f(\mathbf{x}_{n+1}). \tag{19}$$

Simplex generation. To transform the initial simplex the four transformations are applied. Firstly, we determine the centroid $\bar{\mathbf{x}}$ is the centroid of the n best vertices which can be calculated as

$$\bar{\mathbf{x}} := \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_i. \tag{20}$$

Reflection. Then, the reflection point is the first one that is created, by reflecting the worst point $\mathbf{x}_{n+1}$ around the axis which contains the points $\mathbf{x}_n$ and $\mathbf{x}_{n-1}$, as follows

$$\mathbf{x}_r := \bar{\mathbf{x}} + \alpha\left(\bar{\mathbf{x}} - \mathbf{x}_{n+1}\right), \tag{21}$$

The newly generated point lies on the line that contains points $\mathbf{x}_{n+1}$ and $\bar{\mathbf{x}}$. In order to check if the point $\mathbf{x}_r$ is kept in the simplex the expression $f(\mathbf{x}_r) < f(\mathbf{x}_1)$ must be satisfied. Then, we try to further explore the solution space with expansion. Otherwise, is $f(\mathbf{x}_1) \le f(\mathbf{x}_r) < f(\mathbf{x}_n)$, the worst point is replaced with generated point $\mathbf{x}_r$ and no additional transformations are performed.

Expansion. This operation is performed in order to further extend the search area if the reflection vertex points to the right direction. The expansion point is generated along the line which contains the points $\bar{\mathbf{x}}$ and $\mathbf{x}_r$ according to the expression:

$$\mathbf{x}_e = \bar{\mathbf{x}} + \beta\left(\mathbf{x}_r - \bar{\mathbf{x}}\right). \tag{22}$$

If $f(\mathbf{x}_e) < f(\mathbf{x}_r)$, the new expansion point is pointing to the optimal solution and vertex $\mathbf{x}_{n+1}$ is replaced by $\mathbf{x}_e$ and iteration terminated. Otherwise, replace $\mathbf{x}_{n+1}$ with $\mathbf{x}_r$ and terminate iteration.

Contraction. After the reflection point is generated, and expansion is not the desired direction in the movement of the simplex, the contraction is applied. Depending on whether contraction point lies inside or outside of the current simplex we recognize two types of contraction which can be performed, outside and inside contraction. Outside contraction is performed when $f(\mathbf{x}_n) \le f(\mathbf{x}_r) < f(\mathbf{x}_{n+1})$, and the point $\mathbf{x}_c$ is determined as follows:

$$\mathbf{x}_c := \bar{\mathbf{x}} + \gamma\left(\mathbf{x}_r - \bar{\mathbf{x}}\right). \tag{23}$$

On the other hand, if $f(\mathbf{x}_n) \le f(\mathbf{x}_r) < f(\mathbf{x}_{n+1})$, inside contraction is performed to generate a point $\mathbf{x}_{cc}$ as follows

$$\mathbf{x}_{cc} := \bar{\mathbf{x}} - \gamma\left(\bar{\mathbf{x}} - \mathbf{x}_{n+1}\right). \tag{24}$$

If $f(\mathbf{x}_{cc}) < f(\mathbf{x}_{n+1})$, replace $\mathbf{x}_{n+1}$ by $\mathbf{x}_{cc}$ and terminate the iteration; otherwise, perform shrinkage.

Shrinkage. The last transformation which is applied on the simplex is aimed to reduce the size of a simplex as it converges towards the global optimal solution. In this regard, shrinkage is performed on all vertices except $\mathbf{x}_1$, according to the expression

$$\mathbf{v}_i := \mathbf{x}_1 + \delta\left(\mathbf{x}_i - \mathbf{x}_1\right). \tag{25}$$

In Eqs. (21) – (25) the following real valued parameters are provided α, β, γ and δ which control each respecting operation. For 2D optimization problem these parameters can be set as $\alpha = 1$, $\beta = 2$, $\gamma = 0.5$, and $\delta = 0.5$ [6].

The iterative process of applying the above transformations is repeated until the points of the simplex become close to the optimal solution, i.e.

$$\frac{1}{n+1}\sum_{i=1}^{n+1}\left\|\mathbf{x}_i^{(k)} - \mathbf{x}_i^{(k+1)}\right\|^2 \le \varepsilon, \tag{26}$$

where $\mathbf{x}_i^{(k)}$ and $\mathbf{x}_i^{(k+1)}$ are the vertices in iteration k and $k+1$, respectively and ε is an arbitrarily small positive number.

5. SIMULATION RESULTS

The numerical simulation results are shown in this chapter to validate the improvements in the optimum design solution utilizing the proposed GA-NM technique.

The optimization procedure that takes into considerations the minimization of gearbox mass and maximization of efficiency is applied to an example planetary gear set. The planetary gear set in this example is made up of a floating sun gear and three evenly spaced planetary gears, with the internal gear permanently attached to the housing. Table 1 shows the essential design characteristics of the planetary gearbox under consideration.

Table 1. Parameters of the planetary gear set used in this paper

Parameters	Symbol	Value
Input Power [kW]	P_a	200
Input speed [rpm]	n_a	2750
Pressure angle [degree]	α_n	20
Gear material	18CrNi8	
Gear surface Roughness [μm]	R_a	0.8
Factor of safety against bending	$S_{F\min}$	1.2
Factor of safety against pitting	$S_{H\min}$	1.25
Number of planet gears	n_{pg}	3

Due to conflicting objectives, the solution to the MOO problem is a Pareto set. To compare a single solution to the reference values, the ideal solution must be identified. Ideal solutions are best for each unique objective, independent of other objective functions. Because the ideal solution doesn't exist in the Pareto set, it's estimated from the Pareto frontier's objective values. Compromise solutions to the MOO issue are found as the Pareto curve cossets to the ideal solution, where Euclidian distance is used to measure proximity.

Furthermore, in aircraft applications, the minimizing of the planetary gearbox's volume, and hence its mass, is critical. The mentioned MOO issue has been carried out in this respect, with the objectives being the volume of the gears in the gearbox and the power loss of the planetary gear set. The obtained Pareto curves for this scenario are shown in Figure 4.

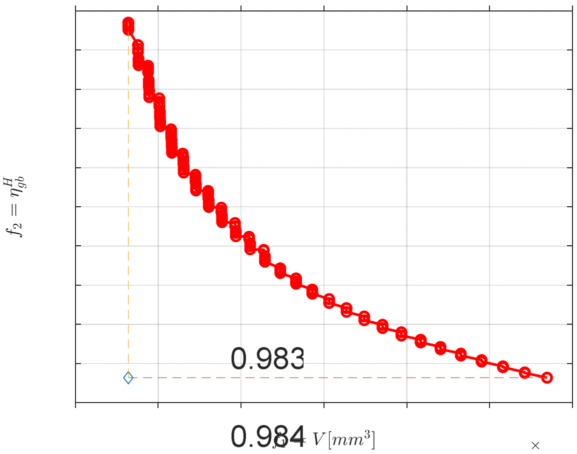

Figure 4. 2D illustration of the Pareto frontier for planetary gearbox volume and efficiency

From the results depicted in Fig. 4 it can be observed that the considered objective functions are conflicting. Analyzing the obtained Pareto set, starting from the left to right and moving along the Pareto set it can be observed that with the increase in the value of gearbox volume, leads to simultaneous decrease in the value of the planetary gearbox efficiency, and vice versa. Therefore, the ideal solution is determined as $[5.175\times10^{10} \quad 0.9924]$ while the compromise solution is $[5.314\times10^{10} \quad 0.9892]$. Compared to the industrial gearbox reference [10] it leads to the 10% reduction in gearbox weight and improvement of 0,25% in efficiency. The observations made on the numerical simulations show that the proposed gear optimization procedure based on hybrid GA-NM algorithm can achieve better design solutions, compared to the traditional algorithms, however with the cost of higher number of iterations.

6. CONCLUSION

This paper considers the topic of developing a non-linear optimization model for determining the optimal parameters of a planetary gearbox, which is solved using a metaheuristic optimization technique. To identify appropriate planetary gearbox parameters, complex multimodal objective functions are minimized, which are conflicting problems. In the paper, a hybrid metaheuristic algorithm is proposed to address this challenging optimization problem. The proposed metaheuristic method combines the GA with local-search Nelder-Mead. In the considered multiobjective optimization problem, the appropriate objectives are developed, such as weight minimization of the gearbox and maximization of gearbox efficiency. The ideal planetary gearbox settings obtained using the proposed metaheuristic method are compared with results from well-known algorithms. The proposed optimization method improves planetary gearbox performance compared to well-known algorithms.

In the future, work can be done on making more objective functions and putting them into more complicated multi-objective models to add to the optimization problem and make the solution more accurate.

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