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# TDOA BASED APPROACH FOR ACCURATE TARGET LOCALIZATION BASED ON HYBRID GENETIC ALGORITHM

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***Abstract:*** *Accurate localization of target based on time difference of arrival (TDOA) measurements is of crucial importance in a large number of different military and civil applications, especially in security systems, radars, sonars etc. This paper focuses on the determining the position of a target from a set of TDOA measurements obtained on several receivers whose positions are known. The considered target localization problem is formulated as the optimization problem, where the corresponding objective function is obtained based on least squares (LS) method. Due to the complexity of the considered problem, the resulting objective function is highly nonlinear and multimodal. Therefore, to solve this complex optimization problem this paper proposes the hybridization of Genetic Algorithm (GA) with well-known Gauss-Newton (GN) method. The performance of considered hybrid algorithm is investigated and compared to well-known conventional optimization algorithms in solving the considered TDOA based localization problem. The simulation results of the proposed optimization method indicate a significant improvement in localization accuracy compared to well-known algorithms.*

***Keywords:*** *Localization, Optimization, Least Squares, Time Difference of Arrival, Wireless Sensor Networks.*

## 1. INTRODUCTION

The problem of determining the unknown location of a target based on TDOA measurements from a set of receivers, whose positions are known, is an essential problem in many applications such as military target tracking, environmental monitoring, telecommunications, security systems, wireless sensor networks, and many others [1-2]. In each of these applications, the key requirement is determining the accurate location of a target from a set of noisy measurements.

In general, localization algorithms use various techniques such as the time of arrival (TOA), the time difference of arrival, the received signal strength (RSS), or the angle of arrival (AOA), depending on the available hardware to locate the targets [1]. This paper focuses on a target localization problem based on the TDOA measurements due to its high ranging accuracy and relatively simple required hardware structure.

The target location can be estimated based on the least squares and the maximum likelihood (ML) as a powerful methods which can be employed successfully in a practical application [3]. Hence, due to the TDOA measurement errors, the considered localization problem is formulated as an optimization problem known as least-squares problem. The LS problem is based on the minimization of the sum of squared errors between the estimated and the measured distances. Generally, the LS estimator can be divided into two classes: linear least squares (LLS), which can provide closed-form solution, and nonlinear least squares (NLS). In this paper, the NLS estimator is employed to accurately estimate the target location based on the noisy TDOA measurements. Due to the complexity of the considered problem, the objective function of the NLS estimator is highly nonlinear and multimodal [3]. Therefore, it is difficult to obtain the global optimal solution with a conventional optimization algorithm, where the convergence of these algorithms mainly depends on the appropriate choice of initial solution and thus may not always converge to the global optimal solution. In order to overcome this difficulty, in this paper the hybridization between Genetic Algorithm and a local search Gauss-Newton method is proposed, due to the individual efficiencies of these algorithms [4, 5]. The proposed hybrid GA-GN algorithm goes through two phases during the optimization process. In the first phase, the GA is employed to explore the search space, with the aim to find the region of the global optimal solution. Then, in the second phase, the solution obtained in the first phase is used as the initial solution and improved using the GN local search method.

The target location is usually obtained by the linear estimator as algebraic closed-form solutions and in this case, avoid the selection of initial solution [3, 6]. The weighted least square (WLS) algorithm for estimation of the target is presented in this paper due to its computational efficiency and the linear closed-form solution in the WLS sense [6]. This approach linearizes the nonlinear TDOA measurement equations by introducing an additional variable in order to minimize the weighted sum of the squared residuals.

The Cramer–Rao Lower Bound (CRLB) of the TDOA measurements from all receivers provides a lower bound on the variance of any unbiased estimator [7] and it is a very useful tool for evaluating the localization accuracy.

The paper is organized as follows: the target localization problem based on the TDOA measurements from a set of receivers whose positions are known is stated in Section 2. Section 3 describes target localization problem which is modeled as a least-squares estimation problem with NLS and WLS approaches. In Section 4, the hybrid GA-GN method is presented. The CRLB is given in Section 5. Section 6 gives the simulation results of the proposed hybrid optimization algorithm against the conventional approaches. Finally, conclusion and future work are presented in Section 7.

## 2. PROBLEM FORMULATION

In this section, the two-dimensional (2D) target localization problem using TDOA measurements under the line-of-sight (LOS) environment is presented. To determine the unknown position of a target, the considered localization problem requires a measurement from at least three receivers, $N \geq 3$, whose locations are known, placed at coordinates $\mathbf{x}_i := [x_i \quad y_i]^T \in \mathbb{R}^2$, $i \in \{1, 2, ..., N\}$ as shown in Fig. 1.

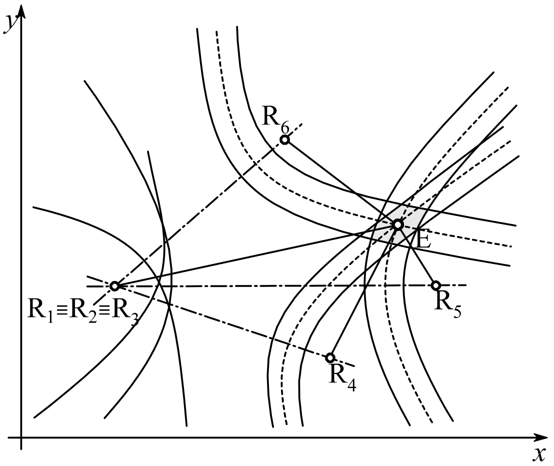


**Figure 1.** Geometrical model based on TDOA.

Here, we assume that the range difference errors $\{n_l\}$ can be modelled as independent Gaussian random variables with zero mean and variance $\sigma_l^2$, i.e, $\mathcal{N}(0, \sigma_l^2)$. We can set the first receiver $R_1$ to be the reference receiver, without loss of generality of considered localization problem.

Using geometrical relationships between a target and the receivers $R_i$ $i \in \{2, 3, ..., N\}$, the target's unknown location is determined. Unknown distances denoted by $\{r_{i,1}\}$ are produced by multiplying the calculated times with the speed of light. These lengths may be calculated using the formula

$$r_{i,1} = d_{i,1} + n_{i,1}, \quad i \in \{2, ..., N\}, \tag{1}$$

where $d_{i,1} = d_i - d_1$. Here, distances between the target and the receiver pair $R_1$ and $R_i$ can be expressed as follows

$$\begin{aligned} d_1 &= \sqrt{(x - x_1)^2 + (y - y_1)^2}, \\ d_i &= \sqrt{(x - x_i)^2 + (y - y_i)^2}. \end{aligned} \tag{2}$$

where $\mathbf{x} := [x \quad y]^T \in \mathbb{R}^2$ is the unknown position of a target.

As a result, the hyperbola is defined by the fact that, as shown in Fig. 1, the difference $d_i - d_1$, between any point on it and the two foci $R_i$ and $R_1$, respectively, is constant.

The intersection of two 2D hyperbolas, as shown in Fig. 1, provides the geometric model for finding the target's unknown actual coordinates using TDOA data in the absence of noise.

More than two hyperbolas do not meet at the same spot in real-world settings when noise is present, necessitating the use of an appropriate optimization technique to reduce the localization error.

## 3. LEAST SQUARE METHODS

This section presents the formulation of the LS method for the considered target localization model, using the obtained TDOA measurements, as described in previous section. In general, the formulation of the NLS problem comes first when formulating the LS problem. Here, the objective function $J_{NLS}(\tilde{\mathbf{x}})$ is defined as the sum of squared residuals between the estimated and the measured TDOA values, which can be written as

$$J_{NLS}(\tilde{\mathbf{x}}) = \min \sum_{i=1}^{N} R_{es,i}^2(\tilde{\mathbf{x}}), \tag{3}$$

where $\tilde{\mathbf{x}}$ denoted the vector of decision variables and residual $R_{es,i}(\tilde{\mathbf{x}})$ is calculated as

$$R_{es,i}(\tilde{\mathbf{x}}) = \tilde{r}_{i,1} - r_{i,1}. \tag{4}$$

Therefore, from the minimization problem in Eq. (3), the appropriate optimal solution $\hat{\mathbf{x}}$ can be obtained as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathrm{R}^2} J_{NLS}(\tilde{\mathbf{x}}), \tag{5}$$

It has been previously shown that the formulation of TDOA problem provides nonlinear equations of hyperbolas. Therefore, the following steps must be taken in order to transform the nonlinear equations into the proper set of linear equations. Firstly, the Eq. (2) is substituted into the Eq. (1), which results in the following expression

$$r_{i,1}+\sqrt{(x-x_1)^2+(y-y_1)^2}=\sqrt{(x-x_i)^2+(y-y_i)^2}+n_{i,1},\quad i\in\{2,3,\ldots,N\}. \tag{6}$$

Eq. (6) is squared on both sides, and a new variable is added

$$R_1=d_1=\sqrt{(x-x_1)^2+(y-y_1)^2}, \tag{7}$$

and using some algebraic techniques, we can demonstrate that

$$\begin{aligned}&(x_i-x_1)(x-x_1)+(y_i-y_1)(y-y_1)+r_{i,1}R_1\\&=0.5\left[(x_i-x_1)^2+(y_i-y_1)^2-r_{i,1}^2\right]+d_in_{i,1}+0.5n_{i,1}^2,\\&i\in\{2,3,\ldots,N\},\end{aligned} \tag{8}$$

Then, the obtained system of equations in Eq. (8) is linearized by

$$\begin{aligned}&(x_i-x_1)(x-x_1)+(y_i-y_1)(y-y_1)+r_{i,1}R_1\\&=0.5\left[(x_i-x_1)^2+(y_i-y_1)^2-r_{i,1}^2\right]+m_{i,1},\quad i\in\{2,3,\ldots,N\},\end{aligned} \tag{9}$$

where the second-order term $n_{i,1}^2$ is neglected and $m_{i,1}=d_in_{i,1}$. Hence, the system in Eq. (9) is linear and can be written in the following matrix form

$$\mathbf{A}\boldsymbol{\theta}=\mathbf{b}+\mathbf{m}, \tag{10}$$

in which

$$\mathbf{A}=\begin{bmatrix}x_2-x_1 & y_2-y_1 & r_{2,1}\\ x_3-x_1 & y_3-y_1 & r_{3,1}\\ \vdots & \vdots & \vdots\\ x_N-x_1 & y_N-y_1 & r_{N,1}\end{bmatrix}, \tag{11}$$

$$\boldsymbol{\theta}=\begin{bmatrix}x-x_1 & y-y_1 & R_1\end{bmatrix}^T, \tag{12}$$

$$\mathbf{b}=0.5\begin{bmatrix}(x_2-x_1)^2+(y_2-y_1)^2-r_{2,1}^2\\ (x_3-x_1)^2+(y_3-y_1)^2-r_{3,1}^2\\ \vdots\\ (x_N-x_1)^2+(y_N-y_1)^2-r_{N,1}^2\end{bmatrix}, \tag{13}$$

$$\mathbf{m}=\begin{bmatrix}m_{2,1} & m_{3,1} & \cdots & m_{N,1}\end{bmatrix}^T. \tag{14}$$

Based on the linear-matrix form, given in Eqs. (10)-(14), the following WLS objective function can be defined as follows

$$J_{WLS}(\boldsymbol{\theta})=(\mathbf{A}\boldsymbol{\theta}-\mathbf{b})^T\,\mathbf{W}(\mathbf{A}\boldsymbol{\theta}-\mathbf{b}). \tag{15}$$

where $\mathbf{W}=\left(E\left\{\mathbf{m}\mathbf{m}^T\right\}\right)^{-1}$ is the weighting matrix. Therefore, the considered localization problem can be written as the following localization problem

$$\min_{\mathbf{x}\in R^2} J_{WLS}(\mathbf{x}). \tag{16}$$

The formulated linear LS problem, given in Eq. (16) provides algebraic closed-form solution, which for WLS method is denoted as $\hat{\mathbf{x}}_{WLS}$. This solution provides the minimum of linearized objective function $J_{WLS}(\boldsymbol{\theta})$. It can be shown that $\hat{\mathbf{x}}_{WLS}$ can be obtained from Eq. (16) by the following equation [6]:

$$\hat{\mathbf{x}}_{WLS}=\left(\mathbf{A}^T\mathbf{W}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{W}\mathbf{b}. \tag{17}$$

The WLS method's main advantages are its simple implementation and high computing efficiency, however it does not achieve adequate accuracy for handling extremely nonlinear and complicated problems.

## 4. HYBRID GA-GN METHOD

The proposed hybrid GA–GN is described in this section in the context of its application to the problem of locating emitting sources using TDOA measurement. The basic purpose of hybridizing several optimization algorithms is to establish the most effective technique for solving the considered optimization problem. As a result of hybridizing the algorithms, it is possible to combine the benefits of each algorithm while avoiding their disadvantages [4, 5]. As a powerful stochastic global optimization method, the GA explores the search space by randomly generating starting solutions within boundary restrictions and finds the global or near-global optimal solution. By utilizing the neighborhood of the initial solution found by the GA, a local search algorithm is used to identify the best global optimal solution of the considered problem.

In this paper, the GA algorithm and the efficient GN local search method are merged to generate the hybrid GA–GN algorithm, which improves the efficiency and accuracy of the GA solution. As a result, the GA and GN methods are introduced in this section, and afterwards the appropriate hybridization procedure is introduced.

### 4.1. Genetic algorithm

The genetic algorithm is a widely applied metaheuristic optimization method that is based on genetics and natural selection mechanisms found in nature [4], and may be used to tackle NLS minimization problems effectively. Therefore, to apply the GA to NLS problem, the stated minimization problem in Eq. (3) may be altered by adding bound-constraints, which can be stated as

$$\min_{\mathbf{x}^l\le\mathbf{x}\le\mathbf{x}^h} J_{NLS}(\mathbf{x}), \tag{18}$$

where $\mathbf{x}$ denotes a vector of decision variables, $\mathbf{x}^l$ and $\mathbf{x}^h$ are introduced as the minimum and maximum bounds of $\mathbf{x}$ respectively. The bound-constraints are provided to prevent the objective function from being evaluated for infeasible solutions during the search process.

The optimization procedure of GA begins with a population of $N_p$ individuals chosen at random from the feasible solution space. In contrast to gradient-based optimization processes, where potential solution is directly drawn from the solution space, in GA each individual is represented by a chromosome, which is encoded as a fixed length vector of binary values. The length of this vector is determined by the number of optimization parameters employed and the required encoding precision. According to the evolutionary operators, the suggested GA procedure may be separated into selection, crossover, and mutation.

The process of selection uses the fitness values of chromosomes to determine which individuals are chosen for the mating, and eliminates the chromosomes which don't have necessary attributes for the efficient solving of the optimization problem. Chromosomes are selected according to roulette wheel selection, so individuals with lower objective function values, which is desired for minimization problems, are more likely to be selected. The probability of selecting an *i*-th individual is proportional to the quality of its original fitness, which can be formulated as a probability of selection $P_i$ formulated as follows

$$P_i = \frac{F_i}{\sum_{j=1}^{N_p} F_j}, \quad i \in \{1, \ldots, N_p\}, \tag{19}$$

where $F_i$ denotes the corresponding fitness value of the chromosome. Therefore, the appropriate cumulative probability $C_i$ of the *i*-th individual is determined according to the following expression

$$C_i = \sum_{j=1}^{N_p} F_j. \tag{20}$$

After all of the individuals in the population have been assigned a cumulative probability, they are subjected to a selection procedure in which an appropriate individual is picked based on a random number $r$ that must satisfy the following expression

$$C_{i-1} < r \le C_i, \, i \in \{1, 2, \ldots, N_p\}, \tag{21}$$

where $r$ denotes the random number generated between 0 and 1.

The crossover operator, one of the most important parts of GA, combines and exploits the available information, stored in chromosomes, to influence the search direction during the optimization process. The crossover operator combines the information of two individuals, previously chosen in the selection process, to create an offspring that shares both individuals positive traits. Such offspring could potentially have necessary abilities to successfully solve the optimization problem. As shown in Fig 2, two-point crossover is performed by selecting two random crossover points along the chromosome length, denoted as $c_1$ and $c_2$. As a result, the encoded binary values enclosed by these points are interchangeably exchanged between selected individuals.

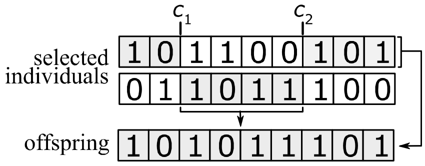


**Figure 2.** The illustration of two-point crossover operator

Mutation is a GA operator that introduces new unexplored solutions into the GA population and prevents the algorithm from becoming stuck in the local optima, allowing it to achieve better results faster. Only a small percentage of the population is mutated in order to avoid destroying valuable information for the optimization process. The mutation rate is defined as the percentage of a population's total number of genes whose values have changed. Mutation happens when a chromosome undergoes random binary changes. Each digit of the gene being mutated is changed to either 0 or 1 to express the randomness of the changes. The mutation rate must be small, for if it is large, a good chromosome might accidentally mutate into a bad one by chance.

The process of selection, crossover, and mutation is repeated, and the population evolves over successive iterations towards the global optimal solution of the given optimization problem until the termination criterion is satisfied. Although most evolutionary algorithms use the maximum number of iterations as a termination criterion, the relative error between two consecutive iterations of the average population fitness is also used. Therefore, the execution of algorithm can be stopped by comparing the population fitness in consecutive iterations when it becomes smaller than a certain threshold, which is expressed in the following expression

$$\left| \frac{f_{avg}^{(k)} - f_{avg}^{(k-1)}}{f_{avg}^{(k)}} \right| \le \varepsilon, \tag{22}$$

where $\varepsilon$ is a small positive real number, in which

$$f_{avg}^{(k)} = \frac{1}{N_p} \sum_{i=1}^{N_p} f_i, \tag{23}$$

represents the average fitness value of the entire population in *k*th iteration.

## 4.2. Gauss-Newton method

The Gauss-Newton method is widely applied method for solving the LS problems [8]. In comparison to the Newton-Raphson method, which requires the Hessian matrix to be calculated. The Gauss-Newton method requires only the first derivative of the objective function $J_{NLS}(\mathbf{x})$, making it computationally less expensive in each iteration. It is now a widely used method for

minimizing objective functions that are represented as the sum of squares of nonlinear functions.
The solution is iteratively obtained using the following expression

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \left[\mathbf{J}\left(J_{NLS}\left(\mathbf{x}^{(k)}\right)\right)^T \cdot \mathbf{J}\left(J_{NLS}\left(\mathbf{x}^{(k)}\right)\right)\right]^{-1} \cdot \mathbf{J}\left(J_{NLS}\left(\mathbf{x}^{(k)}\right)\right)^T \cdot \mathbf{e}\left(\mathbf{x}^{(k)}\right), \tag{24}$$

where $\mathbf{e}\left(\mathbf{x}^{(k)}\right)$ is a residual vector and $\mathbf{J}\left(J_{NLS}\left(\mathbf{x}^{(k)}\right)\right)$ is the Jacobian matrix evaluated at $\mathbf{x}^{(k)}$, which can be expressed as

$$\mathbf{J}\left(J_{NLS}(\mathbf{x})\right) = \begin{bmatrix} \frac{\partial\|\mathbf{x}_1-\mathbf{x}\|_2}{\partial x} & \frac{\partial\|\mathbf{x}_1-\mathbf{x}\|_2}{\partial y} \\ \vdots & \vdots \\ \frac{\partial\|\mathbf{x}_N-\mathbf{x}\|_2}{\partial x} & \frac{\partial\|\mathbf{x}_1-\mathbf{x}\|_2}{\partial y} \end{bmatrix}. \tag{25}$$

Once the gradient of the objective function $J_{NLS}(\mathbf{x})$ is sufficiently close to zero, the iteration process is stopped, e.g. when

$$\left\|\nabla J_{NLS}\left(\mathbf{x}^{(k+1)}\right)\right\| \le \varepsilon, \tag{26}$$

where the gradient is measured in a suitable norm and $\varepsilon$ is a given threshold.

## 5. CRAMER-RAO LOWER BOUND

The Cramer-Rao Lower Bound is a theoretical lower bound on the covariance matrix, which is obtained from the Fisher information matrix (FIM) of the unbiased estimator [7]. As a result, the connection between the CRLB and the variance is as follows:

$$E\left[(\hat{\mathbf{x}}-\mathbf{x})(\hat{\mathbf{x}}-\mathbf{x})^T\right] \ge \mathrm{CRLB}(\mathbf{x}) = \mathrm{trace}\left\{\mathbf{I}(\mathbf{x})^{-1}\right\} \tag{27}$$

where $E[\cdot]$ denoted the expectation operator and $\mathbf{I}(\mathbf{x})$ is FIM given by

$$\mathbf{I}(\mathbf{x}) = -E\left[\frac{\partial^2 \ln\left(f(\mathbf{r}|\mathbf{x})\right)}{\partial\mathbf{x}\partial\mathbf{x}^T}\right], \tag{28}$$

The probability density function $f(\mathbf{r}|\mathbf{x})$ can be defined as

$$f(\mathbf{r}|\mathbf{x}) = \frac{1}{(2\pi)^{(N-1)/2}|\mathbf{C}|^{1/2}} \cdot \exp\left(-\frac{1}{2}(\mathbf{r}-\mathbf{d}(\mathbf{x}))\right)^T \mathbf{C}^{-1}\left(-\frac{1}{2}(\mathbf{r}-\mathbf{d}(\mathbf{x}))\right), \tag{29}$$

where $\mathbf{C}$ is covariance matrix is given as

$$\mathbf{C} = \begin{bmatrix} \sigma_1^2+\sigma_2^2 & \sigma_1^2 & \cdots & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2+\sigma_3^2 & & \sigma_1^2 \\ \vdots & & \ddots & \vdots \\ \sigma_1^2 & \sigma_1^2 & \cdots & \sigma_1^2+\sigma_N^2 \end{bmatrix}. \tag{30}$$

After performing differentiation on the natural logarithm of (29) with respect to $\mathbf{x}$, the FIM can be obtained as

$$\mathbf{I}(\mathbf{x}) = \left[\frac{\partial\mathbf{d}(\mathbf{x})}{\partial\mathbf{x}}\right]^T \cdot \mathbf{C}^{-1} \cdot \left[\frac{\partial\mathbf{d}(\mathbf{x})}{\partial\mathbf{x}}\right] \tag{31}$$

where $\mathbf{d}(\mathbf{x}) = [d_1, d_2, \ldots, d_N]^T$ the true distance vector.

## 5.SIMULATION RESULTS

This section presents the results of the numerical simulation performed in order to compare the localization performance of the proposed hybrid GA-GN method, with the well-known WLS method and derived CRLB. The considered simulation environment includes, five receivers which are position at known coordinated $[200,300]^T$ m, $[1500,100]^T$ m, $[120,1500]^T$ m, $[1510,1500]^T$ m and $[700,900]^T$ m

It is assumed, that for simulation purposed the true position of the target is placed at $[300,500]^T$ m. To evaluate and compare the localization performance of different considered algorithms the root mean square error (RMSE) measure is employed, which can be defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\|\hat{\mathbf{x}}(n)-\mathbf{x}\|_2^2}. \tag{32}$$

where $\mathbf{x}$ and $\hat{\mathbf{x}}(n)$ are the true and estimated positions of the target, respectively, and $N = 1000$ is a number of Monte Carlo simulation runs.

Firstly, the accuracy of the localization of different considered algorithms is evaluated depending on the level of TDOA measurement noise. Therefore, on Figure 3 the RMSE of the GA-GN and WLS methods is plotted as a function of SNR, with the appropriate calculated value of CRLB.

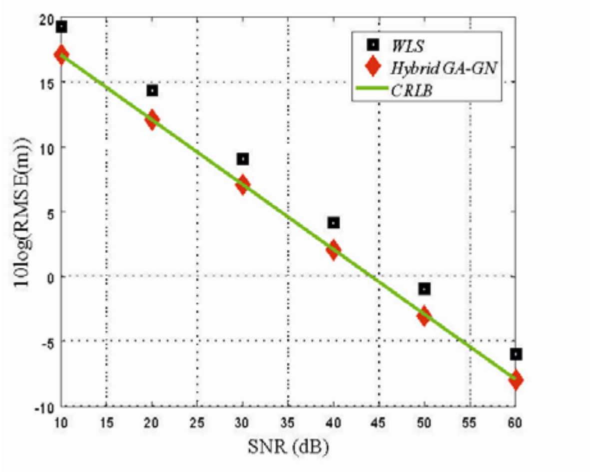


**Figure 3.** Comparison of RMSE versus SNR levels for different considered algorithms

According to the findings in Fig. 3, the proposed hybrid GA-GN technique can achieve the CRLB for a wide range of SNR, demonstrating the robustness of the hybrid GA-GN algorithm in various noisy measurement situations. Furthermore, the suggested hybrid GA-GN algorithm consistently outperforms the current WLS method.

Next, in order to assess the localization performance, the CRLB is compared with the cumulative distribution functions (CDFs) of the hybrid GA-GN and WLS localization methods, at different SNR levels. Here, the SNR is used in simulations at two distinct levels, 10 dB and 50 dB, respectively.

The proposed hybrid GA-GN method and the WLS CDFs are shown in figure 4, with the SNR level set to 10 dB.

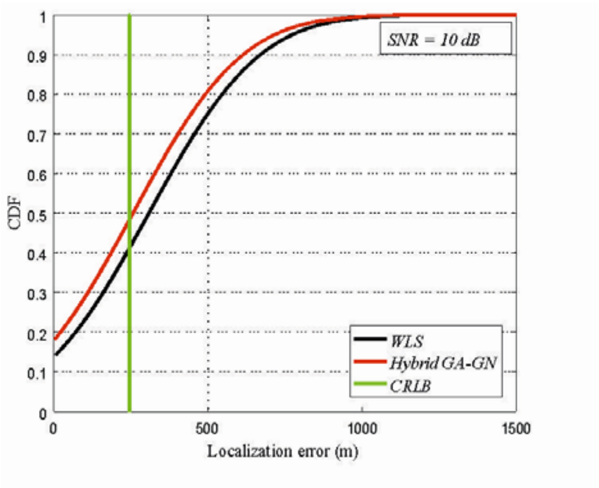


**Figure 4.** CDFs of the localization error of different localization algorithms for SNR = 10 dB.

According to Fig. 4, the hybrid GA-GN technique outperforms the WLS estimator with localization errors in more than 50% of the simulated runs near to the CRLB.

Fig. 5 depicts the CDFs for the case when the SNR = 50 dB.

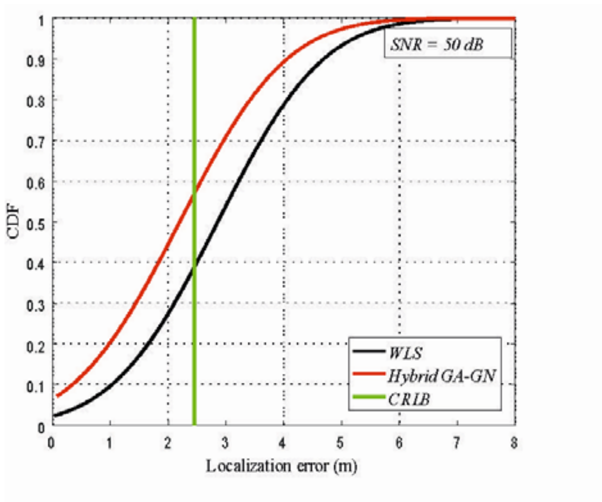


**Figure 5.** CDFs of the localization error of different localization algorithms for SNR = 50 dB.

When comparing the numerical results from Figures 4 and 5, it is clear that the hybrid GA-GN method performs better in terms of localization accuracy than the WLS estimator, particularly at high SNR values. These, findings are in correlation with the conclusions drawn from the Figure 3.

## 6. CONCLUSION

This paper considers the localization problem based on a set of TDOA measurements obtained on several receivers whose positions are known. The paper provides the definition of the TDOA localization problem, which is formulated as an optimization problem, where the corresponding objective function is obtained based on the least squares (LS) method. To solve the complex optimization problem, this paper proposes a hybridization between genetic algorithms and the conventional gradient-based Gauss Newton method. Furthermore, in order to compare the localization performance, the CRLB is derived for the considered localization problem. Simulation results show that the proposed nonlinear optimization methods outperform the WLS method and can achieve higher localization accuracy over a wide range of SNR values.

In future work, the developed optimization models can be further extended with the additional energy efficiency criterion, which can be solved by multi-objective optimization.

## AKNOWLEGMENT

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