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PREDICTION OF QUASI-STEADY FLUTTER VELOCITIES OF TAPERED COMPOSITE PLATES AT LOW MACH NUMBERS: ANALYSIS AND EXPERIMENT

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Abstract: *In the present work, based on existing quasi-steady aerodynamics theories, equations of motion for tapered composite plates are derived for a subsonic flow field. The required tapered plate frequencies of oscillation in bending and torsion are computed using the Rayleigh-Ritz principle. The material model for orthotropic materials is adopted for composites based on Ackerman and Tsai and Hanh theories for QI composite lay-ups., and initial composite material elastic coefficients are obtained based on composite micromechanics theories. The combination of these sets of equations rendered the possibility to solve the equations of motion for the lifting surface characteristic section in the closed-form and give an estimate of stability loss, and flutter velocity for tapered composite plates in the subsonic flow field.*

Using the proposed approach, calculated flutter velocities are experimentally verified by performing the tests in a subsonic wind tunnel in the flow field from 5 m/s to 30 m/s. Furthermore, the results are compared to numerical results obtained using commercial software NASTRAN/Flight Loads. Good agreement between the proposed model, numerical results, and experiment is obtained, and based on this analysis it can be concluded that the proposed model can be used with acceptable accuracy for flutter velocity estimation for tapered composite plates in a low Mach number flow fields.

Keywords: *flutter, quasi-isotropic laminates, stability loss.*

1. INTRODUCTION

The flexibility of modern aircraft structures requires the analysis of the interaction between elastic, inertial, and aerodynamic forces even in the early design stages. Aeroelastic instability, known as flutter is a dynamic instability characterized by sustained structural oscillations. It directly arises from the interaction between the inertial, elastic, and aerodynamic forces acting on the lifting surfaces and potentially may lead to complete structural failure. The flutter instability (coupled-mode flutter) arises when two eigenmodes of fluid-structure interaction coincide, leading to high-amplitude structure oscillations, hence, dynamic instability. High amplitudes induce large strains, followed by high stresses in the structure, which may lead to failure.

With the objective to determine flight conditions (flight speed, primarily), close to 80% of flutter analyses in the industry is based on a numerical approach, relying on well-established K, P, and P-k algorithms (CFA). This approach leads to very good results, confirmed by experiments and flight tests, however, it requires tedious modeling and very often relatively expensive software modules not readily available.

In the present work quasi-steady flutter theory, expressed through the NACA flutter boundary equation [1], is used to estimate the flutter velocities of tapered composite fins exposed to low Mach number axial flow. The original theory is adapted for the orthotropic materials in the form of quasi-isotropic composite laminates and is based on the Ackerman, Tsai, and Hanh theory [2], since nowadays, composites are being extensively used in airframe design.

Results obtained, for the flutter boundary velocity for the quasi-isotropic material structures (very often used nowadays in airframe construction) are compared to numerical (CFA) results and experimental results from subsonic wind tunnel testing at low Mach and small Reynolds numbers (Re).

2. NACA FLUTTER MODEL

National advisory committee for aeronautics in technical note No 4197 has defined the flutter boundary velocity, for the preliminary design of lifting surfaces on missiles in the following form. (eq.1):

$$\left(\frac{V_f}{a}\right)^2 = \frac{G_E}{\dfrac{39.3\cdot A^3}{\left(\dfrac{t}{c}\right)^3\cdot(A+2)}\left(\dfrac{\lambda+1}{2}\right)\cdot\left(\dfrac{p}{p_0}\right)} \tag{1}$$

In the previous equation (eq. 1) V_f represents flutter boundary speed, a is the speed of sound, G_e is the shear modulus of elasticity, t is the lifting surface average thickness, p/p_0 is the ratio of the fluid pressure to standard pressure and A and λ are aspect and taper ratio respectively.

Based on Akkerman, Tsai, and Hanh's quasi-isotropic (QI) theory [2], the previous relation (eq.1) is modified for the composite materials, that are manufactured in the laminate form with QI stack-up. This stack-up is of particular interest, especially in the initial phases of the design, and is often used with composite materials during design phases that include the initial sizing of the laminates [3-4]. Any stack-up that satisfies the following relation (eq. 2) is considered to be quasi-isotropic [3]:

$$\left[0^0/\frac{180^0}{m}\ldots(m-1)\frac{180^0}{m}\right]_{ns} \tag{2}$$

In equation 2, m represents the number of different orientations in the laminate (m ≥ 3) and is the number of repetition sequences. This lamination type excludes unfavorable coupling effects and composite (in-plane) stiffness is independent of composite orientation, which may be the main reason why this type of stack-up is often used as the starting point in the early stages of the design [4].

The objective is to express the elastic coefficient Ge (shear modulus) given in equation 1 in terms of principal lamina properties (Young's moduli E_1 and E_2, in-plane lamina shear modulus G_{12}, and major Poisson's ratio ν_{12}) and further by means of equivalent laminate properties based on above-mentioned theory [2].

Using the laws of composite lamina micromechanics, composite lamina elastic coefficient required (principal lamina props.), and based on constituent (fiber and matrix) properties, moduli can be obtained from the following system of equations (eq. 3) [3, 5]:

$$\begin{aligned}
E_1 &= V_F\cdot E_f + V_m\cdot E_m,\\
\nu_{12} &= V_F\cdot \nu_f + V_m\cdot \nu_m,\\
E_2 &= \frac{E_m}{1-\sqrt{V_F}\left(1-\dfrac{E_m}{E_2^f}\right)},\\
G_{12} &= \frac{G_m}{1-\sqrt{V_F}\left(1-\dfrac{G_m}{G_{12}^f}\right)}
\end{aligned} \tag{3}$$

Composite constituent properties (E_f, E_m, and G_m) are usually obtained from material OEM, data found in the literature [5], or by experiment. Phases volume fractions (V_F and V_m) are determined by the composite design and are usually within the 55 – 65 [%] range for Vf, for fly-worthy aerospace structural components.

Based on these data the effective in-plane shear modulus required by the NACA flutter boundary equation (eq.1), in terms of principal lamina elastic coefficients, for QI stack-up can be expressed in the following form [2].

$$G_{xy} = \frac{1}{2}\cdot G_{12} + \frac{1}{8}\frac{E_1\cdot(E_1+E_2-2\upsilon_{12}E_2)}{E_1-2\upsilon_{12}^2E_2} \tag{4}$$

Substituting equations 3 – 4 in the initial flutter equation (eq.1), for QI laminates reads:

$$V_f = a\cdot\left[\frac{(A+2)}{78.6\cdot A^3}\cdot\left(\frac{t}{\dfrac{2}{S}\cdot\displaystyle\int_0^{b/2}c(y)\cdot dy}\right)^3\cdot\frac{\left(G_{12}+\dfrac{1}{4}\dfrac{E_1\cdot(E_1+E_2-2\upsilon_{12}E_2)}{E_1-2\upsilon_{12}^2E_2}\right)}{\left(\dfrac{c_t/c_r+1}{2}\right)\cdot\left(\dfrac{p}{p_0}\right)}\right]^{1/2} \tag{5}$$

In equation 5 fin chord (denoted with c in equation 5) is taken as the mean aerodynamic chord, for tapered plates, and is calculated using the known relation, given here for completeness.

$$C_{mac} = \frac{2}{S}\cdot\int_0^{b/2}c(y)\cdot dy \tag{6}$$

In the previous equation (eq.6), S is fin planform area, b is fin's total span, and $c(y)$ chord length at y span coordinate

3. NUMERICAL ANALYSIS

Using Commercial software MSC Nastran/Flight Loads, flutter speed, for the e-glass composite fin, of QI stack up $[0^0/45^0/-45^0/90^0]_s$, thickness 0.65 [mm] is calculated. Geometric dimensions of the fin analyzed are as follows: Root chord, Cr=180 [mm], tip chord Ct=90 and span b/2 = 262 [mm]. The numerical model is presented in the following figure (Figure 1):

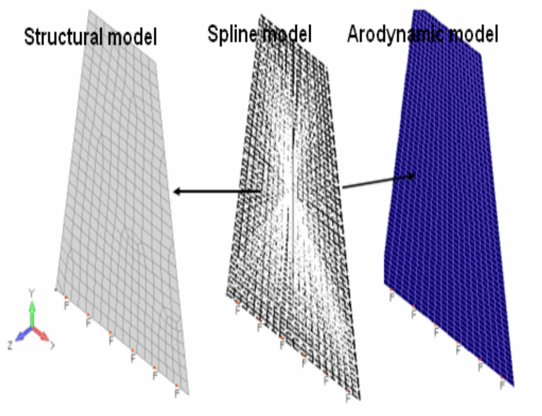

Figure 1. Flutter numerical model

The complete fin flutter model consists of a structural model and the aerodynamic model. The structural model consists of 172 laminate finite type finite elements (Classic Lamination Theory), that discretize the complete missile fin domain. The aerodynamic model computes the aerodynamic forces, based on the vortex panel method. To ensure the transfer of aerodynamic loading onto fin structure both models are mutually interconnected with beam-type splines. This kind of flutter model, enables analysts to obtain the flutter velocities, hence the fin dynamic stability loss. The algorithm used for this coupled flutter numerical model was a very well-known P-k algorithm [6].

For the geometry given, composite material characteristics, and subsonic flow conditions, the result obtained is that flutter occurs at 18.5 m/s.

Using the same geometry and material characteristics, using the analytic NACA model, described in previous sections (eq.6 at flight altitude corresponding to p/p_0 =1 [-]), obtained value for the flutter speed is 16.7 m/s.

Fin mode shapes in bending and twisting (bending frequency f_1=7.98 [Hz], twisting frequency f_2=28.68 [Hz]) are presented in Figure 2. These frequencies are of great importance for the problem analyzed (composite fin bending-torsion flutter). The algorithm for modal extraction used in this example was the complex Lancosz algorithm since it was found that this algorithm for problems of similar size does not miss any modes, it is fairly fast and does not require fine mesh, which can be of importance when several case scenarios have to be analyzed. This was concluded based on the sensitivity analysis performed and results obtained from the experimental modal analysis.

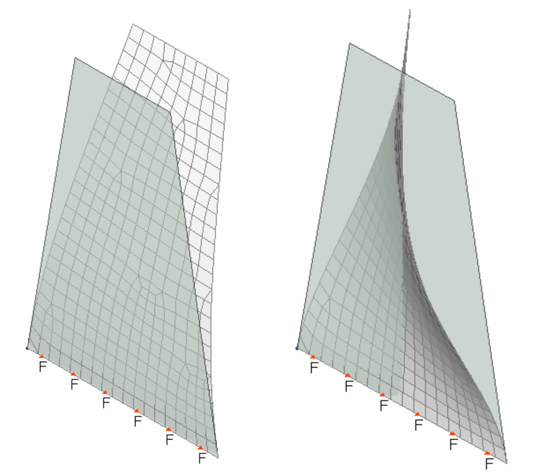

Figure 2. Mode shapes in bending and twisting

Frequency values obtained by Lanczo's method are compared to results obtained by the Rayleigh-Ritz method for natural frequencies calculations for cantilevered beams. The natural frequency of the cantilever beam is:

$$\omega_n = \frac{\lambda_n}{b^2}\sqrt{\frac{D}{\rho \cdot t}} \tag{7}$$

In equation 7 ρ is material density and D is plate flexural rigidity computed from the following relation:

$$D = \frac{E_{eq} \cdot t^3}{12\left(1-\upsilon_{eq}^2\right)} \tag{8}$$

Using the same theory as for the shear modulus of QI laminates [2], Young's modulus of elasticity for quasi-isotropic laminates is obtained using the following equation:

$$E_{eq} = 2 \cdot \left(1+\upsilon_{xy}\right) \cdot G_{xy} \tag{9}$$

Poisson ratio (ν_{xy}) in this analysis for e-glass laminates is set to 0.3. Coefficients λ_n in equation 7 are the function of boundary condition, material type, and analyzed plate taper ratio. Coefficients λ_n for isotropic materials as a function of taper ratio (tip chord/root chord) are given in the following table (Table 1):

Table 1. Coefficients λ_n for isotropic materials (frequency parameters Ritz Method) [7]

C_{tip} / C_{root}	Mode 1 λ	Mode 2 λ
2	3.51	5.37
1	3.49	8.55
0.5	3.47	14.90

Ritz coefficients presented in table 1are further modified for quasi-isotropic materials and plate aspect ratio. For the first three modes of vibrations of cantilevered, tapered thin plates λ_n coefficients are presented in the following figure (Figure 3.)

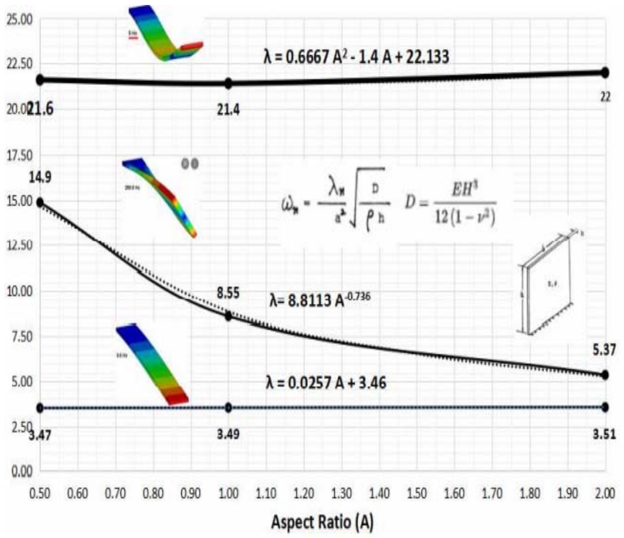

Figure 3. Mode shapes coefficients

4. WIND TUNNEL EXPERIMENT

To ensure and verify the validity of the proposed methodology, for the aeroelastic stability of the QI tapered e-glass plates, tests in a subsonic wind tunnel are performed at relatively low Mach number flows (up to 30 m/s). A specially designed support structure is used to support the test samples (e-glass tapered fins) and is placed in the test section of the tunnel. Test samples are presented in figure 4. and the complete test setup (wind tunnel working section) in figure 5. The wind tunnel used for this experiment is a subsonic wind tunnel at the University of Belgrade, faculty of mechanical engineering.

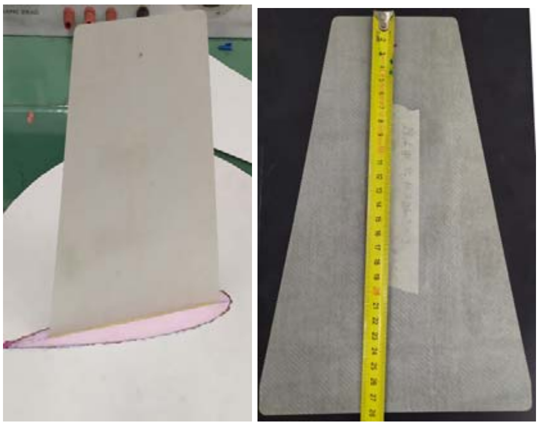

Figure 4 E-glass tapered plate for wind tunnel flutter test

Figure 5 Wind tunnel flutter test setup

The support structure with a clamped sample at the root chord, positioned in the test section of the wind tunnel, is presented in figure 4. In order to monitor the magnitude of the amplitudes during an oscillation cycle, accelerometers connected to the DAQ system were mounted on the test plate at the location of the root chord. The PCE PFM2 micro manometer with pitot tube is used to determine airflow velocity. The system can increase or decrease air-flow velocities in the wind tunnel by 0.5 m/s increments, with acceptable air-flow stabilization times.

Table 2. Samples Geometric and material data

Charact.	
Root chord	180 [mm]
Tip chord	90 [mm]
Span	262 [mm]
thickness	0.65 [mm]
QI	$[0^0/ 45^0/ -45^0/ 90^0]_s$
E_1	41 000 [MPa] / ref [5]
E_2	10 400 [MPa] / ref [5]
G_{12}	4300 [MPa] / ref [5]
G_E	7962.59 [MPa] / equation 4

The experimental results for flutter velocities versus calculated values based on equation 5 and the numerical model (section 3) for the fin geometry (Table 1) are presented in table 2 Experimental velocity data is obtained based on 5 fin samples, with 10 runs in the wind tunnel. The flow velocity was gradually increased until the flutter was observed. The flow speed was then reduced to 10 m/s and ramped close to the flow speed where the flutter occurred in the previous run. The loss of stability of the test sample is presented in figure 5.

Table 3. Flutter velocity comparative results

Model	Flutter velocity [m/s]
NACA	16.7
Experiment	19.6 – 21.5 / (5 test samples / 10 runs)
Numerical P-K	*18.2*

Figure 6 wind tunnel flutter test

5. CONCLUSION

In the present work, the NACA boundary flutter equation was adopted for tapered composite thin plates, based on Akkerman, Tsai, and Hanh's quasi-isotropic (QI) theory. Flutter velocities for the e-glass tapered fin are calculated and results obtained compared with numerical analysis (CFA) based on the P-k algorithm and experimentally obtained results in the wind tunnel at subsonic flow speeds in the range of 10 – 30 m/s. Based on the results obtained and the analysis performed the following can be concluded:

1. Modified NACA boundary equation tends to underestimate flutter speeds by 15 -25 % when compared to experimentally obtained data
2. Bearing in mind that flutter is a very complex phenomenon, this approach is advisable especially in the early stages of the design

(preliminary sizing) since it leads to conservative, hence safe designs.

3. CFA approach based on the P-k algorithm renders results very close to experimentally obtained velocities, however, it requires tedious modeling and very often expensive software modules as well as extensive CPU resources and long computing time.

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