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SLIDING MODE CONTROLLER DESIGN FOR DC MOTOR DRIVEN ELECTROMECHANICAL FIN ACTUATOR

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Abstract: *In this paper modelling and control of an electromechanical actuator (EMA) system for aerofin control (AFC) with brushed DC motor driven by motor driver using pulse width modulated (PWM) control signal are investigated. Model of system has been developed and experimentally verified in actuator test bench. Model has been used as the starting point for Sliding Mode position controller synthesis.*

Keywords: *Electromechanical actuator, Aerofin, DC motor, Sliding Mode*

1. INTRODUCTION

The use of electromechanical actuation has become increasingly popular, for a while now, in the aerospace industry as more importance is placed on maintainability. Main usage of electromechanical actuators (EMAs) is for actuation of flight critical control surfaces and thrust vector control as sad in [2].

The most commonly used actuators for actuation of aerodynamic surfaces are direct current (DC) motors. For that reason, research in this paper is based on application of electromechanical actuator system driven by permanent magnet brush DC motor for aerofin position control. Aerofin position control is achieved by creating position control feedback loop using contactless magnetic position encoder. PID controllers, fuzzy controllers, different non-linear controllers, etc. are used very often for either position or speed control of DC motors. Conventional PID controller tuning relies on some tunning method such as Ziegler-Nichols or simply on hit and trial. Reasonable response is provided as long as there are no significant disturbances or parameter variations. Fuzzy controllers offer better transient response than conventional PID controller, and should be preferred. Non-linear techniques such as sliding mode control produce good quality results and offer simple to implement control laws for uncertain or complicated model dynamics.

The proposed design will be based on sliding mode controller for position control. Sliding mode control has been treated as a powerful technique to cope with complex systems with unmodelled dynamics due to its simplicity and strong robustness with respect to system parameter variations and external disturbances. The idea of sliding mode control is to employ different feedback control laws acting on opposite sides of a predetermined surface (often called sliding surface) in the system time space. Each of those control laws provides motion of the system states towards the sliding surface, and once system states reach the surface for the first time, they stay on it thereafter. The resulting motion of the system is confined to the surface, which can be interpreted as „sliding" of the system states along the surface.

2. SYSTEM DESCRIPTION

2.1. Electromechanical aerofin system

Aerofin control (AFC) system, which is considered here, is control system of the missile using two pairs of fins, for pitch and yaw control of missile. Fin configuration is given in Figure 1. Deflecting each pair of fins result in formation of moment about the center of mass of missile, which generates rotation of missile body. The resulting orientation angle generates aerodynamic force, which accelerates missile in desired direction.

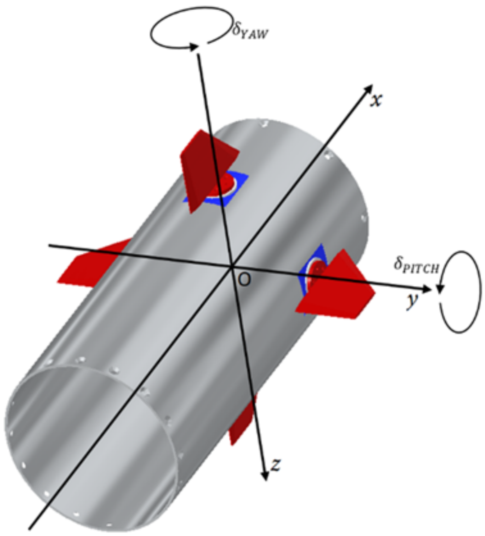

Figure 1. AFC system

2.2. Hardware and equipment

The actuator assembly, presented in Figure 2, consists of the Maxon RE 30 brushed DC motor with Maxon planetary gearhead GP 32 with reduction ratio of 1:1, which drives the screw shaft with ball nut. Output shaft with fin is connected to the ball nut through lever mechanism.

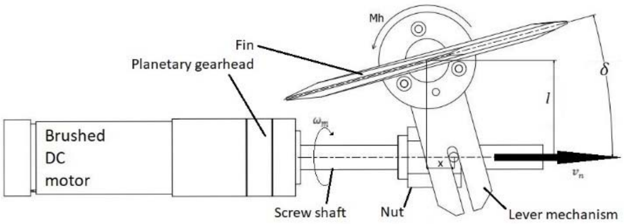

Figure 2. Actuator assembly

On the output shaft, an absolute encoder is mounted and fixed to the actuator systems body. Absolute encoder is AS5048A angle position encoder with resolutions of 14 bits. Sensor reading are forwarded to microcontroller using SPI communication protocol.

The microcontroller is ESP32 with Tensillica Xtensa LX6 microprocessor. Control loop is performed on 1 kHz sampling rate, while the serial communication with acquisition program on 500 Hz. Control signal is pulse width modulated and two control signals are forwarded to the motor driver.

The motor driver is IBT-2 H-bridge high power motor driver. In order to be controlled, it needs two pulse width modulated signals, for each rotation direction of motor. The principle of operation of the H-bridge is presented on Figure 3.

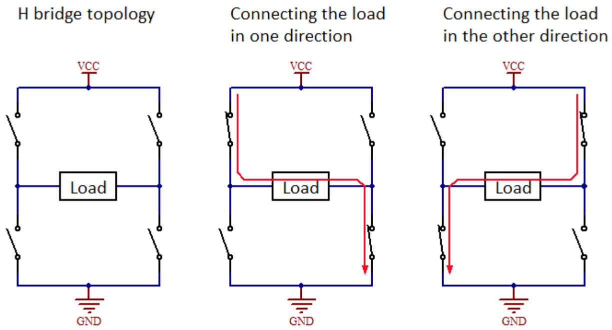

Figure 3. H-bridge operation principle

In order to simulate aerodynamic torque aerofin control system is mounted on the testing bench and equivalent torque load is applied using two stretched out springs mounted on bar which is placed instead of fin, as shown on the Figure 5.

3. SYSTEM MODELING

Mathematical model of brushed DC motor presented in this paper represents electromechanical part of the actuator assembly. Hence, torque of the motor shaft M_m is proportional to the magnetic flux, which is proportional to the armature current I_m

$$M_m = K_M I_a \tag{1}$$

where K_m is motor torque constant.

Due to rotation of motor shaft, back electromotor force E_m is induced in the motor coils. Back electromotor force is proportional to the rotor angular velocity ω_m

$$E_m = K_e \omega_m \tag{2}$$

where K_e is the motor electrical constant. Torque constant K_m and motor electrical constant K_e are equal for the ideal motor, but in case of real motor their values are similar.

Electrical equation of the rotor circuit is:

$$U_m = R_m I_m + L_m \frac{dI_m}{dt} + E_m \tag{3}$$

where R_m is electrical resistance and L_m is electrical inductance of the motor coils.

Combining equations (1), (2) and (3) yields:

$$\left(T_e \frac{dM_m}{dt} + M_m\right)\frac{R_m}{K_M} = U_m - K_e \omega_m \tag{4}$$

where T_e is electrical time constant of the motor:

$$T_e = \frac{L_m}{R_m} \tag{5}$$

It represents the time period necessary for the current to reach 63% of stall current value.

Angular velocity of the rotor depends on motor torque and load torque. The following equation describes dynamics of the motor:

$$J \frac{d\omega_m}{dt} = M_m + M_L \tag{6}$$

where J is equivalent moment of inertia $J = J_l + J_m$. Let θ_m be rotation angle of the rotor, then it yields:

$$J\frac{d\theta_m}{dt} = \omega_m \quad (7)$$

The total gear ratio of the ball screw drive is described by the following equation:

$$N = \frac{2\pi}{cos\delta}\frac{l}{h} \quad (8)$$

where l is normal distance between the ball nut and the output shaft axes, and h is the ball screw pitch. It is obvious that the gear ratio value depends on deflection angle of the fin.

Equations (1)-(8) are used to develop nonlinear simulation model of the EMA-AFC system. Still, model of the motor driver has not been modelled. Voltage motor drive can be modelled using $sign(\cdot)$ function with the gain of maximum voltage value V_{max}, that is time sampled with control frequency which was mentioned earlier. In the Figure 4. the output signal of the motor driver is shown.

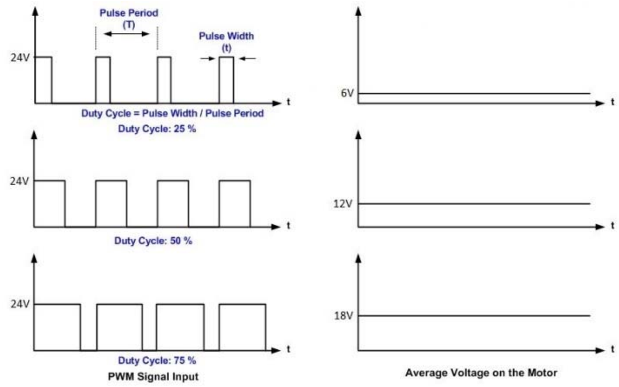

Figure 4. Motor driver output signal

The following parameters of the modelled system have been used: $J_m = 33{,}1 \times 10^{-7}\, kgm^2$, $K_e = 0{,}0398\, V/s$ $K_M = 0{,}0398\, Nm/A$, $L_m = 0{,}281 \times 10^{-3}\, H$, $R_m = 1{,}43\,\Omega$, $U_{max} = 24\, V$, $\omega_{max} = 8590\, min^{-1}$.

4. CONTROLLER DESIGN

The state space of the brushed DC motor is described by following equation:

$$\begin{bmatrix}\dot{\theta}_m \\ \dot{\omega}_m \\ i_a\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 \\ 0 & 0 & {}^{K_M}/_{J} \\ 0 & -{}^{K_e}/_{L} & -{}^{R}/_{L}\end{bmatrix}\begin{bmatrix}\theta_m \\ \omega_m \\ i_a\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ {}^{1}/_{L}\end{bmatrix} u \quad (9)$$

Let write $a_{23} = {}^{K_M}/_{J}$, $a_{32} = {}^{K_e}/_{L}$, $a_{33} = {}^{R}/_{L}$, and $b_3 = {}^{1}/_{L}$, then equation (9) can be rewritten as:

$$\begin{bmatrix}\dot{\theta}_m \\ \dot{\omega}_m \\ i_a\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 \\ 0 & 0 & a_{23} \\ 0 & -a_{32} & -a_{33}\end{bmatrix}\begin{bmatrix}\theta_m \\ \omega_m \\ i_a\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ b_3\end{bmatrix} u \quad (10)$$

As mentioned earlier sliding mode control is a powerful technique offering robust control of a system. Given below is a design of a sliding mode controller for brushed DC motor. The control law is derived from [3]:

$$u = -\frac{1}{L_g L_f^{\rho-1} h(x)}\left[L_f^{\rho} h(x) - r^{(\rho)}(t) + \sum_{i=1}^{\rho} c_i e_{i+1}\right] + v \quad (11)$$

where u represents the control law, L_f, L_g are Lie derivatives and ρ is the relative degree of the system. The reference value is given by r and the error states are given by e. Lastly, the switching function is given by v.

The state variables ca be adopted as,

$$\begin{aligned} x_1 &= \theta_m \\ x_2 &= \omega_m = \frac{d\theta_m}{dt} \\ x_3 &= i_a \end{aligned} \quad (12)$$

As it is obvious relative degree ρ of the system is 3. From state space equation 10 yields:

$$f(x) = \begin{bmatrix} x_2 \\ a_{23}x_3 \\ -a_{32}x_2 - a_{33}x_3 \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} \quad (13)$$

For designing control law using (11) it is necessary to find Lie derivatives of $h(x)$ along the $f(x)$ and its Lie derivative along $g(x)$. These derivatives are given below:

$$\begin{aligned} L_f h(x) &= \frac{\partial h}{\partial x} f(x) = [1 \quad 0 \quad 0]\begin{bmatrix} x_2 \\ a_{23}x_3 \\ -a_{32}x_2 - a_{33}x_3 \end{bmatrix} \\ &= x_2 \end{aligned} \quad (14)$$

$$\begin{aligned} L_f^2 h(x) &= \left(\frac{\partial}{\partial x} L_f h(x)\right) f(x) \\ &= [0 \quad 1 \quad 0]\begin{bmatrix} x_2 \\ a_{23}x_3 \\ -a_{32}x_2 - a_{33}x_3 \end{bmatrix} \\ &= a_{23}x_3 \end{aligned} \quad (15)$$

$$\begin{aligned} L_f^3 h(x) &= \left(\frac{\partial}{\partial x} L_f^2 h(x)\right) f(x) \\ &= [0 \quad 0 \quad a_{23}]\begin{bmatrix} x_2 \\ a_{23}x_3 \\ -a_{32}x_2 - a_{33}x_3 \end{bmatrix} \\ &= -a_{23}(a_{32}x_2 + a_{33}x_3) \end{aligned} \quad (16)$$

$$\begin{aligned} L_g L_f^2 h(x) &= \left(\frac{\partial}{\partial x} L_f^2 h(x)\right) g(x) = [0 \quad 0 \quad a_{23}]\begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} \\ &= a_{23}b_3 \end{aligned} \quad (17)$$

With $\rho = 3$ the control law defined in (11) becomes:

$$\begin{aligned} u &= -\frac{1}{L_g L_f^2 h(x)}\left[L_f^3 h(x) - \dddot{r}(t) + c_1 e_2 + c_2 e_3\right] + v \\ &= -\frac{1}{a_{23}b_3}[c_1 e_2 + c_2 e_3 - a_{23}a_{32}x_2 \\ &\quad - a_{23}a_{33}x_3 - \dddot{r}(t)] + v \end{aligned} \quad (18)$$

Variable v represents switching function here given by $v = -\beta(x)sgn(s)$ where $\beta(x)$ is found by finding the upper bound of the uncertainties. The rest of parameters are defined earlier.

Tracking error is defined as:

$$\begin{aligned} e_1 &= r - x_1 \\ e_2 &= \dot{r} - x_2 \\ e_3 &= \ddot{r} - x_3 \end{aligned} \tag{19}$$

$$\begin{aligned} \frac{de_1}{dt} &= e_2 \\ \frac{de_2}{dt} &= a_{23}x_3 - \ddot{r} \\ \frac{de_3}{dt} &= -a_{32}x_2 - a_{33}x_3 + b_3u - \dddot{r} \end{aligned} \tag{20}$$

The sliding surface is $s = c_1x_1 + c_2x_2 + x_3$, taking its derivative we have:

$$\begin{aligned} \dot{s} &= c_1\dot{e}_1 + c_2\dot{e}_2 + \dot{e}_3 \\ &= c_1e_2 + c_2a_{23}x_3 - c_2\ddot{r} \\ &\quad - a_{32}x_2 - a_{33}x_3 + b_3u - \dddot{r} \end{aligned} \tag{21}$$

To find $\beta(x)$ it is necessary to find upper bound, as shown in [1] and [3]. The upper bound is:

$$\left|\frac{c_1e_2 + c_2a_{23}x_3 - a_{32}x_2 - a_{33}x_3 - c_2\ddot{r} - \dddot{r}}{b_3}\right| \le \rho(x) \tag{22}$$

$$\left|\frac{-(c_1 + a_{32})x_2 + (c_2a_{23} - a_{33})x_3 + c_1\dot{r} - c_2\ddot{r} - \dddot{r}}{b_3}\right| \le \rho(x) \tag{23}$$

therefore

$$\beta(x) = \left|\frac{-(c_1 + a_{32})x_2 + (c_2a_{23} - a_{33})x_3 + c_1\dot{r} - c_2\ddot{r} - \dddot{r}}{b_3}\right| + 1 \tag{24}$$

From (23) the final control law becomes:

$$u = -\frac{1}{a_{23}b_3}[c_1e_2 + c_2e_3 - a_{23}a_{32}x_2 - a_{23}a_{33}x_3 - \dddot{r}(t)] + \left(\left|\frac{-(c_1 + a_{32})x_2 + (c_2a_{23} - a_{33})x_3 + c_1\dot{r} - c_2\ddot{r} - \dddot{r}}{b_3}\right| + 1\right) sgn(s) \tag{25}$$

5. RESULTS

To verify design of control law system was simulated in MATLAB-Simulink environment and experimentally evaluated on testing bench with spring load that is shown in Figure 5.

Both in simulation and experimental evaluation of EMA-AFC system, as reference value, has been used step signal with amplitude of $\pm 5°$. Reference signal characteristics are given in following equation:

$$r(t) = \begin{cases} 0° & for \quad t \in [0,5[\cup]7,12[\cup]14,\infty[\\ \in [0°, 5°] & for \quad t = 5 \\ 5° & for \quad t \in]5,7[\\ \in [0°, -5°] & for \quad t = 12 \\ -5° & for \quad t \in]12,14[\end{cases} \tag{26}$$

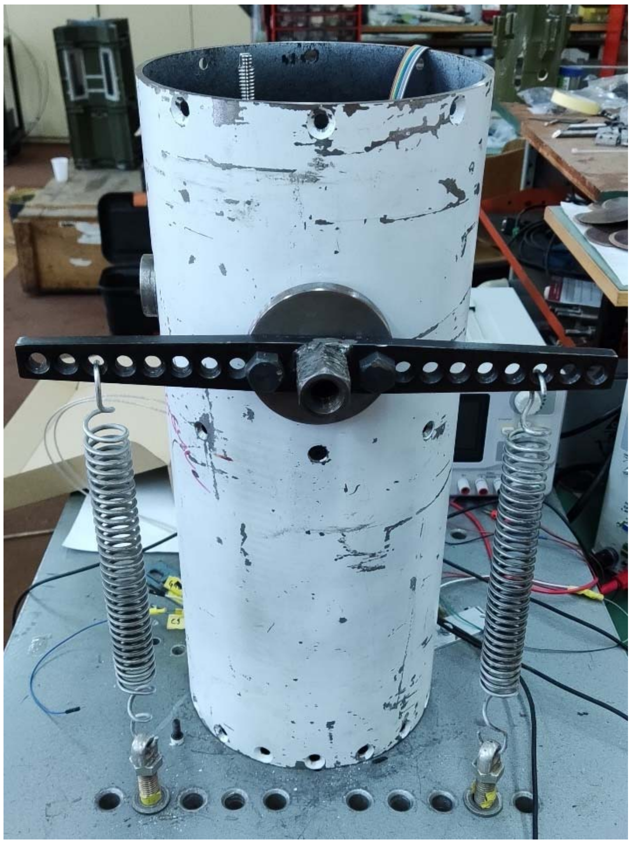

Figure 5. EMA-AFC system on the testing bench

A sliding mode controller based on the control law proposed in (11) and determined for EMA-AFC system in (22) has been implemented in simulation and microcontroller using parameters given in table 1.

Table 1. Sliding mode control law parameters

Parameter	Value
c_1	−141,597
c_2	0,4232282
β	150

Using hit and trial approach for finding best values of parameters given in Table 1. it has come to optimal output response for simulated and real system. Output signals of simulated and experimentally tested system are shown in Figure 6.

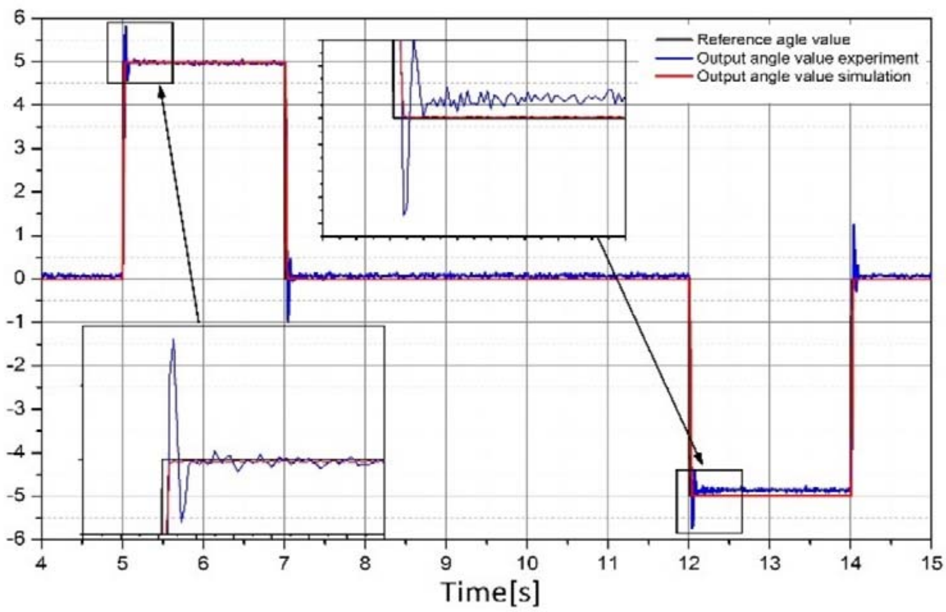

Figure 6. Reference and output signals from simulation and experiment

5. CONCLUSION

This paper presents solution for control electromechanical aerofin actuator system using sliding mode control. For this type of system robustness, disturbance rejection and low sensitivity to parameter variation is necessary which is provided using sliding mode control.

Modelling of dynamics of the system and design of the control law was given in this paper both in simulation and experimental environment. Given research could be used for obtaining robust control of EMA-AFC systems.

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