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INFLUENCE OF THE CONTACT LINES LENGTH ON LOAD CAPACITY OF CYLINDRICAL GEAR TEETH FLANKS

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Abstract: *The load capacity of the cylindrical gears teeth is mostly limited by the surface strength of the teeth flanks. The calculation of the flanks surface strength is carried out on the basis of the maximum contact stress generated on the teeth flanks during the meshing period. Conventional analytical models for determining this contact stress, which are contained in the international ISO standard, are based on the Hertz model of contact stress for two contacting cylinders. In order to obtain the most realistic stress state on the gear teeth flanks, the original Hertz model was corrected by a large number of influencing factors. The influence of the simultaneously meshed gear teeth pairs contact lines length on the contact stress of the teeth flanks is covered by the contact ratio factor* Z_ε*. For the exact determination of this factor, appropriate mathematical models have been developed at the Faculty of Mechanical Engineering, University of Belgrade. In the international ISO standard, however, the contact ratio factor is determined on the basis of approximate models. Particular approximate models have been developed for spur and helical cylindrical gears. In certain areas of geometric and kinematic parameters of cylindrical gear pairs, the values of the contact ratio factor determined on the basis of ISO approximate models deviate significantly from the exact values. In this paper, using numerical methods, single, more precise approximate model for determining the contact ratio factor has been developed, which includes both spur and helical cylindrical gears. Based on the developed approximate model, the degree of accuracy of analytical models for determining the contact stress on the gear teeth flanks contained in the ISO standard is significantly increased.*

Keywords: *Contact stress, contact lines length, transverse contact ratio, overlap ratio, load distribution*

1. INTRODUCTION

Cylindrical gear pairs are the most commonly used mechanical power transmission drives [1]. The integrity of the entire machine structure in which the gear pairs are installed primarily depends on the surface and volume strength of gear teeth. During the gears operating period, a large number of different types of gear teeth damage [2] can occur. However, one of the most common type of gear failure is the tooth flanks contact surface destruction in the form of pitting [3]. Calculation of pitting load capacity of spur and helical gears is based on the maximum operating stress generated on the flanks of the teeth during the meshing period. Conventional analytical models for determining this contact stress are based on the Hertz model of contact stress for two contacting cylinders in static conditions. Given the significant difference between the contact of two cylindrical surfaces and two involute surfaces of gear teeth flanks, the original Hertz model needs to be corrected by a large number of influencing factors. Some of these factors are theoretical and some are empirical in nature. They are contained in the conventional pitting load capacity calculation method [4, 5]. The accuracy of determining the relevant stress for the tooth flanks load capacity largely depends on the accuracy of the applied correction factors. For this reason, permanent work is being done to increase the accuracy of correction factors [6, 7]. In this paper, a more precise approximation model is given to take into account the influence of the contact lines length on the load capacity of the cylindrical gears teeth flanks.

2. LOAD DISTRIBUTION OF SIMULTANEOUSLY MESHED GEAR TEETH PAIRS

Given the need for continuous load transfer and movement from the drive to the driven gear during the contact period, it is necessary to ensure that more than one pair of teeth is meshed at the same time. The kinematic indicator of the number of simultaneously meshed gear teeth pairs that participate in the transfer of the total load of the gear pair is called total contact ratio of the flanks ε_γ. Total contact ratio ε_γ is the sum of the transverse contact ratio ε_α and overlap ratio ε_β. The values of ε_α and ε_β depend on a large number of kinematic and geometric parameters and have a significant influence on the operating characteristics of the gear pair [8, 9].

Theoretically, the transfer of load between two simultaneously meshed gear teeth is done through their current contact line b_i, Fig. 1.

Since the total load F of the cylindrical gear pair is performed over more than one (in the general case n) simultaneously meshed teeth pairs, the following equation can be written:

$$F_1 + F_2 + \ldots + F_i + \ldots + F_n = F \quad (1)$$

where F_i (i = 1, n) denotes the loads transmitted by simultaneously meshed pair of teeth. A larger number of simultaneously meshed gear teeth pairs corresponds to a larger number of contact lines which are present in the meshing area. They represent a set of parallel lines, distant from each other by the size of the base pitch p_b, which move translationally in the meshing area, Fig. 1.

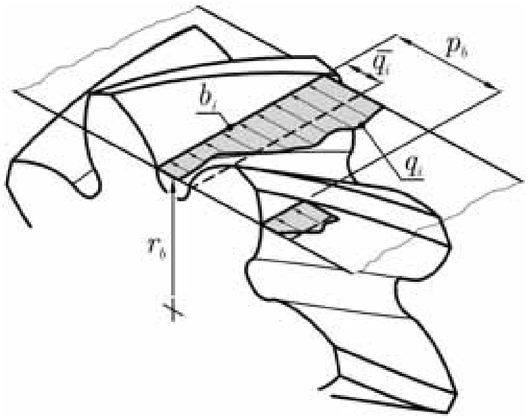

Figure 1. Contact lines in the meshing area

Equation (1) can be written in the following form

$$\frac{F_1}{F} + \frac{F_2}{F} + \ldots \frac{F_i}{F} + \ldots + \frac{F_n}{F} = 1{,}0 \,. \quad (2)$$

The share of load transmitted by each simultaneously meshed gear teeth pair is determined on the base of load distribution factor K_α [10]:

$$K_{\alpha i} = \frac{F_i}{F} \,. \quad (3)$$

This factor is used to analyze various phenomena of cylindrical gear pairs, such as: gear teeth flanks contact stress [11], gear teeth root stress [12], efficiency of gear pairs [13], etc.

The actual line load distribution q_i along the line of contact is irregular, Fig. 1, and depends on a large number of influencing factors (gear teeth accuracy degree, roughness of contact surfaces, stiffness, load intensity, etc.). If the actual line load distribution q_i along the contact lines is approximated by the corresponding uniform-mean line load $\overline{q_i}$, Fig. 1, and if the rigid body mechanics assumptions are introduced (teeth, gear bodies, shafts and supports are absolutely rigid and absolutely accurate in shape and dimensions) it is possible to obtain analytical expressions for the limiting values of this factor [10]. It was shown in [10] that, for such introduced assumptions, the values of the factor K_α can be determined only on the basis of the values of the cumulative contact lines length $\sum b_i$. The cumulative contact lines length generally changes during the contact period and oscillates between its maximum and minimum values. However, the moment when the load is transferred over the minimum cumulative contact lines length is especially important, because then the maximum value of the load distribution factor $K_{\alpha\max}$ occurs, and accordingly the maximum value of the gear teeth contact stress. Analogously, the minimum value of $K_{\alpha\min}$ occurs when the load is transmitted over the maximum cumulative contact lines length. Spur gears are a special case. For them, the limiting values of the factor K_α depend exclusively on the number of simultaneously meshed pairs of teeth, Fig. 2.

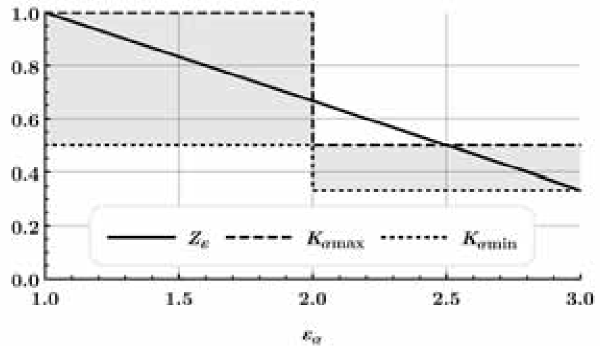

Figure 2. Load distribution factor of a spur gear pair

In conventional calculation methods [5], the influence of the contact lines on the load capacity of the tooth flanks is taken into account by the contact ratio factor Z_ε. This factor, according to [5] can be calculated as:

$$Z_\varepsilon = \begin{Bmatrix} \sqrt{\dfrac{4-\varepsilon_\alpha}{3}\left(1-\varepsilon_\beta\right)+\dfrac{\varepsilon_\beta}{\varepsilon_\alpha}}; & \varepsilon_\beta < 1 \\ \sqrt{\dfrac{1}{\varepsilon_\alpha}}; & \varepsilon_\beta \geq 1 \end{Bmatrix} \quad (4)$$

and it is also shown in Fig. 2. However, in [5] it is not emphasized that the approximate expressions (4) expressed over ε_α and ε_β are based on the approximation of the maximum value of the load distribution factor of simultaneously meshed gear teeth pairs $K_{\alpha\max}$, Fig. 2. Since these models represent an approximation, it is natural that there is a deviation between the approximate model obtained on the basis of (4) and the exact model for $K_{\alpha\max}$. This difference is best noticed when analyzing K_α change for spur gear pairs, Fig. 2.

The maximum value of the factor K_α is relevant for the calculation of the load capacity of the gear teeth flanks. Based on Fig. 2, it can be concluded that the approximate model (4) in some areas has higher, and in some areas lower values than the maximum theoretically allowed. This difference is even more pronounced for helical gear pairs. The deviations that occur by approximation (4), consequently, lead to reduced accuracy of the calculation

of the maximum value of the cylindrical gear teeth flanks contact stress. Increasing the accuracy of the calculation can be achieved by using exact expressions for $K_{\alpha\max}$ values given in [10], or by an approximation that corresponds more closely to exact analytical models, which has been done in this paper.

3. NEW APPROXIMATE MODEL FOR DETERMINATION OF CONTACT RATIO FACTOR

The discretized representations of the expressions for $K_{\alpha\max}$ given in [10] are shown in Fig. 3 with an increase increment for ε_α and ε_β of 0.1. Given the previously introduced assumptions, the value of $K_{\alpha\max}$ is exclusively a function of the transverse contact ratio ε_α and overlap ratio ε_β values. Theoretically, the maximum possible value of $K_{\alpha\max}$ is 1, which corresponds to the case when the entire load of the gear pair is transmitted over only one simultaneously meshed pair of teeth. Increases in ε_α and ε_β lead to a decrease in $K_{\alpha\max}$, which is favorable from the aspect of cylindrical gear teeth flanks load capacity. Increasing the values of ε_α and ε_β can be achieved by varying the geometric and kinematic values of the gear pair (profile angle, profile shift coefficient, helix angle, addendum of the standard tool, etc.). It can also be noticed from Fig. 3 that the case $\varepsilon_\alpha + \varepsilon_\beta < 1$ is inadmissible, because under this condition there is no continuity in load transfer.

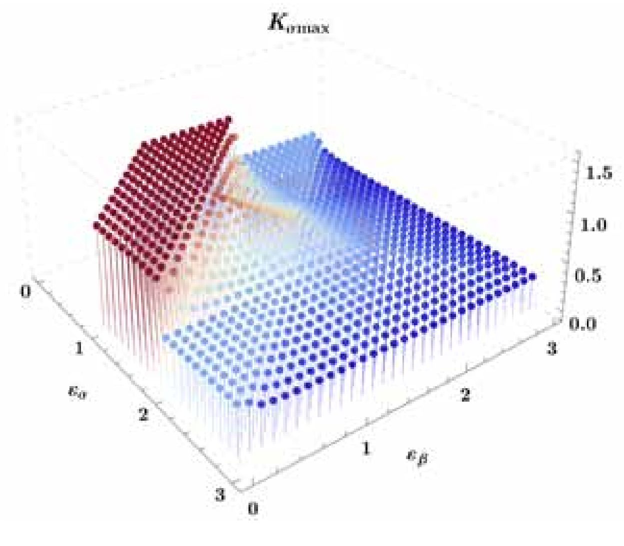

Figure 3. Discretized values of the load distribution factor $K_{\alpha\max}$

The discretized values of the load distribution factor of the simultaneously meshed gear teeth pairs from Fig. 3 can be used to form a more precise approximation model for contact ratio factor. In this paper, fitting of discrete $K_{\alpha\max}$ values is performed numerically. Considering the maximum theoretical value of the factor $K_{\alpha\max}$ as a approximation model, a polynomial of the first degree of the form:

$$Z_\varepsilon^* = 1 - c_1 \cdot \varepsilon_\alpha + c_2 \cdot \varepsilon_\beta + c_3 \cdot \varepsilon_\alpha \cdot \varepsilon_\beta \,, \qquad (5)$$

was chosen. The approximation was performed by the least squares method for the data set shown in Fig. 3, but with a ε_α and ε_β step increase of 0.01. The values of the numerically determined constants that appear in expression (5) are given in Table 1.

Table 1. Values of numerically determined constants

	Constant		
	c_1	c_2	c_3
Value	- 0.155269	-0.155380	0.019983

Increase of the new model accuracy can be achieved by increasing the number of discretized points, increasing the degree of the polynomial, changing the shape of the approximation model or method of approximation, etc. In a simplified form, suitable for practical use, the new approximation model with included numerically determined coefficients can be presented in the form:

$$Z_\varepsilon^* = 1 + \frac{\varepsilon_\alpha \cdot \varepsilon_\beta - 8\left(\varepsilon_\alpha + \varepsilon_\beta\right)}{50} \,. \qquad (6)$$

Unlike model (4), the presented model (6) is applicable in the shown form for both spur and helical types of cylindrical gear pairs.

4. COMPARATIVE ANALYSIS

The discretized $K_{\alpha\max}$ values from Fig. 3 were used for comparative analysis with the conventional Z_ε model (expression 4, Figure 4a) and the new Z_ε^* model (expression 6, Figure 4b).

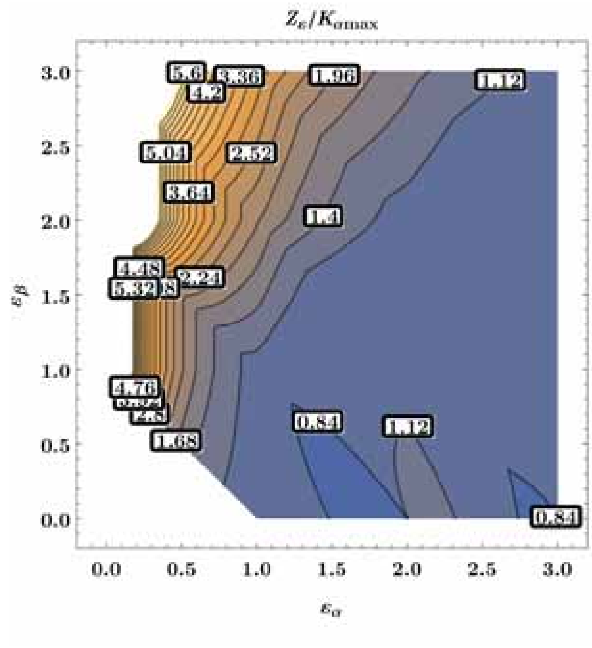

Figure 4a. Comparative contour diagrams of the conventional model Z_ε / $K_{\alpha\max}$

From the comparative contour diagram in Figure 4a, a significant deviation can be observed between the conventional model Z_ε and the theoretical exact model $K_{\alpha\max}$.

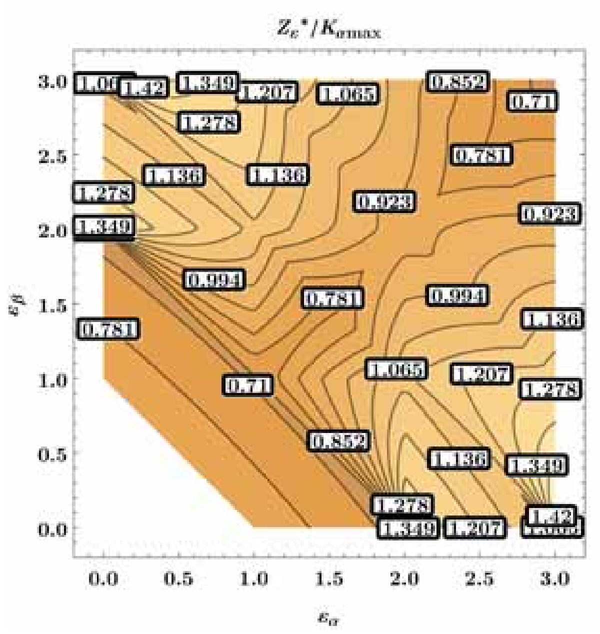

Figure 4b. Comparative contour diagrams of the new model Z_ε^* / $K_{\alpha\max}$

This deviation varies over a wide range, in extreme cases from 0.84 to 5.6. In the area bounded by the contours $\varepsilon_\alpha + \varepsilon_\beta = 1$and $0 < \varepsilon_\alpha < 1$, the conventional model Z_ε takes values that are up to several times larger than the maximum theoretically allowed. In contrast to this case, the comparative contour diagram of the new model Z_ε^* and the theoretical model $K_{\alpha\max}$ (Figure 4b) has a much more uniform flow of change, which is symmetric with respect to the line $\varepsilon_\alpha = \varepsilon_\beta$. The new proposed approximation model also has some deviations, but they are less pronounced (in extreme cases from 0.71 to 1.349), for all values of ε_α and ε_β.

5. RELATIVE ERROR ESTIMATION

Since both analyzed models from Sections 2 and 3 represent an approximation of the maximum limiting value of the load distribution factor of simultaneously meshed gear teeth pairs $K_{\alpha\max}$, it is natural for an error to occur. This error is qualitatively expressed by forming the ratio Z_ε / $K_{\alpha\max}$ and Z_ε^* / $K_{\alpha\max}$ and shown in the form of contour diagrams in Figure 4. In order to perform a quantitative analysis of the error that occurs using both models, a relative error was introduced in the form:

$$E(i,j) = \frac{K_{\alpha\max}(i,j) - Z_\varepsilon(i,j)}{K_{\alpha\max}(i,j)}, \qquad E^*(i,j) = \frac{K_{\alpha\max}(i,j) - Z_\varepsilon^*(i,j)}{K_{\alpha\max}(i,j)} \tag{7}$$

wherein i and j are the coordinates of the points within the variation intervals for ε_α and ε_β used for analysis. The relative error estimation was performed at all points in the range $\varepsilon_\alpha = 1 \ldots 3$ and $\varepsilon_\beta = 0 \ldots 3$. The area $\varepsilon_\alpha = 0 \ldots 1$ was intentionally rejected due to unacceptably large deviations that occur in it, when the conventional model of Z_ε is applied.

After calculating the value of the relative error at a large number of points within the defined interval, the relative error was represented by a two-parameter Gaussian distribution function in the form:

$$f(x) = \frac{1}{S\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{S}\right)^2}, \tag{8}$$

wherein:

S - standard deviation of relative error,

m - mean value of relative error.

The parameters of the Gaussian distribution function for the relative errors $E\,(i, j)$ and $E^*(i, j)$ are shown in Table 2.

	Mean value	**Standard deviation**
f(E)	0.185522	0.30008
f(E*)	0.138057	0.089929

A graphical representation of these functions is given in Fig. 5.

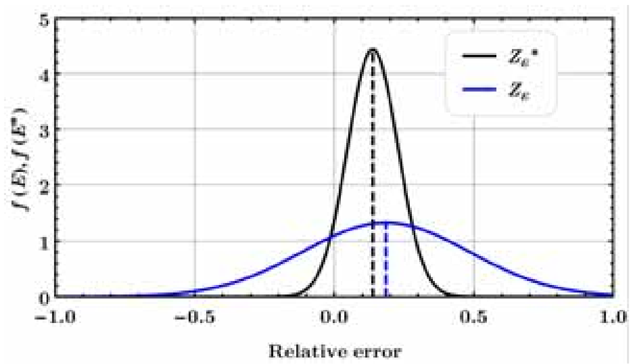

Figure 5. Gaussian distribution function of relative error for analyzed models

From the diagram in Figure 5, it can be concluded that the new proposed model is much more acceptable even in a narrowed variation interval of $\varepsilon_\alpha = 1 \ldots 3$. The mean value of the relative error that occurs by applying the new model is closer to the ideal theoretical zero value. More importantly, however, the relative error distribution function of the new model has a much sharper character, i.e. much less scattering around its mean value. This means that the application of the new model avoids the possibility that the contact ratio factor Z_ε in some areas takes several times higher values than the maximum theoretical allowed.

6. CONCLUSION

In this paper, the influence of the contact lines length through which the load of simultaneously meshed gear teeth pairs is transfered on the relevant stress, ie the load capacity of the cylindrical gear teeth flanks, is considered. In conventional calculation methods this influence is included by the contact ratio factor Z_ε, which is approximately determined only on the basis of ε_α and ε_β. This conventional approximate model has different forms

for spur and helical gears. However, a comparative analysis of the model thus formed with the exact maximum values of the load distribution factor of simultaneously meshed gear teeth pairs that the Z_ε factor should approximate showed certain deviations that can be multiple in certain areas of geometric and kinematic parameters of cylindrical gear pairs. In order to more precisely include the influence of the contact lines length on the calculation of the load capacity of the cylindrical gear teeth flanks, a new, more precise approximate model has been proposed. This model has smaller deviations from the exact, theoretical model. It is obtained numerically and in a unique form is applicable to cylindrical spur and helical gears. Its application can increase the accuracy degree of analytical models used for determining the contact stress relevant for calculation of the cylindrical gear teeth flanks load capacity.

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