

# Contribution to Determining the Limits of the Efficiency of Target Tracking

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The track quality measuring by the track probability existence in order to determine the limits of the possibility of effective target tracking is present in this paper. The most known algorithms for radar target tracking do not have tools to measure track quality. There is no valid procedure that guarantees the verification of the optimal algorithm for a specific application in targets tracking. Standard Integrated Track Splitting (ITS) is the efficient, fully automatic multi target tracking algorithm with initialization, maintenance and deletion tracks and is well-known single and multi target tracking algorithm, also proposed for the purpose of testing the limits of successful tracking. The paper allows the user to choose optimal parameter by the proposed parameters - thresholds of confirmed and false tracks in order to achieve better false track discrimination (FTD). In particular, a diagram of RMSE position errors is also given in the comparison of the proposed algorithm.

*Key words:* Integrated Track Splitting, PDA, multi targets tracking.

## Introduction

THE radar sensor provides measurements that can come from both the target and the background (surroundings) of the target. Target tracking algorithm must update a track quality measure in order to reject false track. Target measurements are present with some probability of detection  $P_D < 1$  [1]. The recursive Bayesian techniques like Joint Probabilistic Data Association (JPDA) [2], Joint Integrated Probabilistic Data Association (JIPDA) [3], and Linear Joint Probabilistic Data Association (LJIPDA) [4] techniques are proposed as solutions for this problem. In general, the automatic tracking methodology performs three main procedures:

- track initiation,
- track maintenance and
- track termination (or deletions).

The later techniques address automatic tracking initiation and maintenance, along with the issues of multi target data association in clutter. Also, the discrimination process, when true tracks discriminate from false, termed the false track discrimination (FTD). The automatic track initiation and maintenance for a single maneuvering target in clutter was considered and an IPDA-IMM filter was proposed in [5]. Here we extend this approach to automatic track initiation and maintenance of multiple maneuvering targets in clutter. By consistently combining LMIPDA with an IMM filter, a linearly scalable LMIPDA-IMM algorithm (the number of operations linear in the number of tracks and measurements) is derived. Also, all target measurements often come from different targets. The total number of targets in the radar surveillance area is unknown. An event in which the track follows the target is called a "true" track, while an event in which the trace does not follow the target is called a 'false' track. In order to perform the track quality measures, the

probability of the tracks are calculated by the use of the arrival measurements from the previous time interval, thus both true tracks and false tracks simultaneously exist.

The multiple hypothesis tracking (MHT) [6] is one of the first used algorithms for multi and single target tracking in clutter. At the same time, the measurement-oriented MHT [7] forms new tracks and measurement allocation hypotheses centered on global origin of measurements. The MHT uses track score methods to discriminate between false and true tracks. The most known algorithms for radar target tracking do not have tools to measure track quality. Thus, there is no valid procedure that guarantees the user the verification of the optimal algorithm for a specific application in targets tracking.

The probability of target existence obtained by utilizing the Markov chain propagation models and Bayes update is used as the track quality measure in Integrated Probabilistic Data Association (IPDA) of [8] and Integrated Track Splitting (ITS).

The incorporation of target existence into a track while scan (TWS) radar target tracking where the resulting filter was termed the integrated mixture reduction data association (IMRDA) filter. Joint integrated probabilistic data association filter (JIPDAF) has the possible presence of multiple targets in a joint PDAF (JPDAF) [9] manner. The JPDAF algorithm allows for the event that a measurement may have originated from one of a number of many tracks or from clutter. In each time interval, JPDAF partitions tracks into clusters, where tracks in each cluster have common measurements.

The single target Integrated Track Splitting (ITS) [10] and multitarget Joint ITS (JITS) and Linear Multitarget ITS (LMITS) [11] are generalizations of IPDA [11], JIPDA and LMIPDA respectively [12, 13]. They more precisely approximate the trajectory state by the Gaussian Mixture. The MHT, IPDA and ITS based algorithms are derived assuming linear trajectory propagation and linear measurement model.

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Some nonlinearity may be accommodated by measurement conversion [14] and by replacing the Kalman filter with the Extended Kalman Filter (EKF) [15, 16] or the Unscented Kalman Filter (UKF) [17] within MHT, IPDA and ITS [18]. Nonlinear measurements, e.g. Bearings Only, Time Difference of Arrival, Multistatic, have measurement likelihoods which are not Gaussian. Any distribution may be approximated by the Gaussian Mixture [19]. This is used by the Gaussian Mixture Measurement likelihood approximation ITS (GMM-ITS) [20], which approximates both the measurement likelihoods and the posterior trajectory state PDF by the Gaussian Mixtures. The GMM-ITS may be used in more difficult situations where the EKF and UKF do not deliver an adequate performance [21]. The particle filters are used for target tracking. However, the publications concentrate on trajectory estimates, without the track quality measure and false track discrimination. Here we propose the Integrated Particle Filter (IPF) solution for the target tracking in clutter. Data association is included to stochastically discriminate against clutter measurements, and the probability of target existence is recursively calculated for use in the false track discrimination [22].

Rest of the paper is organized as follows. The problem statement and models are presented in Section II. Section III derives an iteration of the Integrated Track Splitting algorithm. Section IV is dedicated to nonparametric data association, which is also proposed, followed by the results of simulations, presented from Section VI. Concluding remarks are given in Section V.

### Problem Statement

We assume the target trajectory at any time with dynamic state estimation model. In this model the state varies with any time interval. In the cluttered environment, the sensor will return measurements created by zero or more targets as well as zero or more clutter measurements at each scan. The target and the clutter measurements are referred to as the true and false measurements, respectively. Also, we use superscripts  $\tau$  to denote tracks, and also targets followed by tracks.

#### Targets model

Consider the *Markov Chain One* model [23]. In this model, at any time interval, we calculate the propagation and then a posteriori the probability of target existence ( $\psi$ ), for each target. In this model, we assume that the target exists and when it does it can be detected with a given probability of detection  $P_D$ , or it may not exist [19]. The situation in which the target is maneuvering, the motion can be changed at any time interval.

For the linear system, the target trajectory state  $x_k^\tau \in R^{nz}$  at time interval  $k$ , can be calculated by:

$$x_k^\tau = F(\sigma_k^\tau)x_{k-1}^\tau + v_k^\tau(\sigma_k^\tau) \quad (1)$$

where  $F(\sigma_k^\tau)$  is the propagation matrix, and the process noise  $v_k$  is a zero mean and white Gaussian sequence with covariance  $Q_k$ . At each time interval  $k$ , the radar returns a number of the targets and clutter measurements.

#### Measurements models

If a measurement  $z_{k,i}$  is generated by target  $\tau$  we assume that the measurement a priori probability density function

(PDF)  $p_{k,i}^\tau$  is known or can be estimated. Measurements may originate from the targets as well as from other objects.

The clutter measurements follow the Poisson distribution. We assume that the uniform intensity of the Poisson process at point  $y$  in the measurement space, termed here the clutter measurement density and denoted by  $\rho(y)$  is a priori known, or can be estimated using the sensor measurements.

At time  $k$ , one sensor delivers a set of measurements denoted by  $z_k = \{z_{k,j}\}_{j=1}^{M_k}$ . Denoted by  $Z^k$ , the sequence of selected measurement sets up to including time  $k$ ,  $Z^K = \{Z^{k-1}, z_{k,1}, \dots, z_{k,j}, \dots, z_{k,M_k}\}$ .

#### Sensors model

We assume the linear sensor in Cartesian coordinates, with an additive measurement noise covariance. At each scan the sensor returns a random number of the random target measurements and a random number of the random clutter measurements.

The measurement of the existing and detectable target is taken with a probability of detection  $P_D$  and is given by the following equation [24]:

$$y_k^\tau = Hx_k^\tau + w_k^\tau \quad (2)$$

where  $H$  is measurements matrix and the measurements noise  $w_k^\tau$  is zero mean and white Gaussian sequence with covariance matrix  $R$  [25].

### Integrated Track Splitting algorithm

The Integrated Track Splitting algorithm (ITS) approach is a multi-scan tracking algorithm. In the ITS approach each track is represented by a set of components. Also, each component corresponds to a unique measurement history over multi-scan time horizon. We assume statistics for each component with the probability of component existence and the component state estimate probability density function (PDF), which is often Gaussian. The component existence state probability implies that the target component tracking exists and the measurement history of that component consists of the target detections.

The ITS and its derivatives (joint ITS (JITS) and linear multi-target ITS (LMITS) significantly reduce the numerical complexity of the standard ITS algorithm.

#### ITS filter derivations

The three standard steps of ITS algorithms are performed during the one iteration of algorithm:

- prediction step,
- measurements selection step and
- update step.

#### Prediction step

Each component state is propagated by the Kalman filter prediction with the probability density function (PDF)  $p(x_{k-1} | \chi_{k-1}, c, Z^{k-1})$ . Probability of each component is given by the following:

$$\xi_{k-1}^c = P(c | \chi_{k-1}, Z^{k-1}) = P(c | \chi_k, Z^{k-1}) \quad (3)$$

Target existence probability is propagated by the:

$$\psi_{k|k-1} = P(\chi_k | Z^{k-1}) \quad (4)$$

At each time interval, the trajectory state PDF of targets is given by the Gauss distribution function [26]:

$$p(x_k | x_{k-1}, \chi_k) = N[x_k; Fx_{k-1}, Q] \quad (5)$$

Track state PDF is the Gauss sum:

$$\begin{aligned} p(x_{k-1} | \chi_{k-1}, Z^{k-1}) &= \\ &= \sum_{c=1}^{C_{k-1}} p(x_{k-1} | \chi_{k-1}, c, Z^{k-1}) P(c | \chi_{k-1}, Z^{k-1}) \end{aligned} \quad (6)$$

where predicted probability of each component is given by the:

$$\xi_{k-1}^c = P(c | \chi_{k-1}, Z^{k-1}) = P(c | \chi_k, Z^{k-1}), \quad c = 1, \dots, C_k \quad (7)$$

Track prediction PDF is the Gauss sum of a product of component probability and a priori PDF of each track component by the [27]:

$$p(x_{k-1} | \chi_{k-1}, Z^{k-1}) = \sum_{c=1}^{C_k} \xi_{k-1}^c \cdot p(x_{k-1} | \chi_{k-1}, c, Z^{k-1}) \quad (8)$$

Then we have tracks state PDF  $p(x_k | \chi_k, Z^{k-1})$ , given by the following equation:

$$\begin{aligned} p(x_k | \chi_k, Z^{k-1}) &= \\ &= \int_{x_{k-1}} p(x_k | x_{k-1}, \chi_k) p(x_{k-1} | \chi_k, Z^{k-1}) dx_{k-1} = \\ &= \int_{x_{k-1}} p(x_k | x_{k-1}, \chi_k) \sum_{c=1}^{C_k} \xi_{k-1}^c p(x_{k-1} | \chi_{k-1}, c, Z^{k-1}) dx_{k-1} = \\ &= \sum_{c=1}^{C_k} \xi_{k-1}^c \int_{x_{k-1}} p(x_k | x_{k-1}, \chi_k) N(x_{k-1}; \hat{x}_{k-1|k-1}^c, P_{k-1|k-1}^c) dx_{k-1} = \\ &= \sum_{c=1}^{C_k} \xi_{k-1}^c p(x_k | \chi_{k-1}, c, Z^{k-1}) \end{aligned} \quad (9)$$

$$p(x_{k-1} | \chi_{k-1}, c, Z^{k-1}) = N[x_{k-1}; \hat{x}_{k-1|k-1}^c, P_{k-1|k-1}^c] \quad (10)$$

$$p(x_k | \chi_k, c, Z^{k-1}) = N[x_k; \hat{x}_{k|k-1}^c, P_{k|k-1}^c] \quad (11)$$

$$\hat{x}_{k|k-1}^c = F \hat{x}_{k-1|k-1}^c \quad (12)$$

$$P_{k|k-1}^c = F P_{k-1|k-1}^c F^T + Q_k \quad (13)$$

A posteriori track state PDF is given by the product of track state prediction PDF and total track probability by the equation:

$$p(x_k, \chi_k | Z^k) = P(\chi_k | Z^k) p(x_k | \chi_k, Z^k) \quad (14)$$

Target existence probability is:

$$\psi_{k|k-1} = P(\chi_k | Z^{k-1}) \quad (15)$$

By the use of total probability theorem, we have a posteriori track state estimate PDF:

$$p(x_k | \chi_k, Z^k) = \sum_{c=1}^{C_k} p(x_k | \chi_k, c, Z^k) P(c | \chi_k, Z^k) \quad (16)$$

Target measurements overall track a priori PDF:

$$\begin{aligned} p(y_k | \chi_k, Z^{k-1}) &= \\ &= \int_{x_k} p(y_k | x_k) p(x_k | \chi_k, Z^{k-1}) dx_k \\ &= \int_{x_k} N(y_k; H \hat{x}_k, R) \sum_{c=1}^{C_k} p(x_k | \chi_{k-1}, c, Z^{k-1}) P(c | \chi_{k-1}, Z^{k-1}) dx_k \\ &= \int_{x_k} N(y_k; H \hat{x}_k, R) \cdot \sum_{c=1}^{C_k} p(x_k | \chi_{k-1}, c, Z^{k-1}) \xi_{k-1}^c dx_k \\ &= \sum_{c=1}^{C_k} \xi_{k-1}^c \int_{x_k} p(x_k | \chi_{k-1}, c, Z^{k-1}) N(y_k; H \cdot \hat{x}_k, R) dx_k \\ &= \sum_{c=1}^{C_k} \xi_{k-1}^c p(y_k | \chi_{k-1}, c, Z^{k-1}) \end{aligned} \quad (17)$$

where  $R$  is measurements noise covariance. Thus, the target measurements a priori PDF for each component is given by the:

$$p(y_k | c, \chi_k, Z^{k-1}) = N[y_k; \hat{y}_k^c, S_k^c] \quad (18)$$

where  $z_k^c$  is target measurement arrived in time interval  $k$  and selected by component  $c$ .

$$\hat{y}_k^c = H \hat{x}_{k|k-1}^c \quad (19)$$

$$S_k^c = H P_{k|k-1}^c H^T + R_k \quad (20)$$

#### Measurements selection step

Each track component has a PDF given by the [28]:

$$\begin{aligned} p_k^i(c) &= \\ &= \frac{1}{P_G} p(z_k^i | \chi_k, c, Z^{k-1}), \quad i = 1, 2, \dots, N_k, \quad c = 1, \dots, C_k \end{aligned} \quad (21)$$

where  $N_k$  is a total number of elements from selection measurements set  $z_k$ ,  $z_k^i$   $i$ -th element of set,  $C_k$  is total number of component from  $k$ -th time interval.

Otherwise, PDF of measurements, selected from the components is given by the equation:

$$p_k^i = p(z_k^i | \chi_k, Z^{k-1}) = \frac{1}{P_G} \sum_{c=1}^{C_k} p_k^i(c) P(c | \chi_k, Z^{k-1}) \quad (22)$$

where  $P_G$  indicates new measurements inside the validation gate. Next, a priori probability of the component is:

$$P(c | \chi_k, Z^{k-1}) = \frac{\xi_{k-1}^c}{\sum_{c=1}^{C_k} \xi_{k-1}^c} = \frac{P(c | \chi_{k-1}, Z^{k-1})}{\sum_{c=1}^{C_k} P(c | \chi_{k-1}, Z^{k-1})} \quad (23)$$

#### Update step

Components estimate state PDF is given by the:

$$\begin{aligned} p(x_k | \mathcal{X}_k, c, Z^k) &= \\ &= \frac{p(y_k^i | \mathcal{X}_k, x_k, c, Z^{k-1})}{p(y_k^i | \mathcal{X}_k, c, Z^{k-1})} p(x_k | \mathcal{X}_k, c, Z^{k-1}) \end{aligned} \quad (24)$$

At the same time, a posteriori track state estimate PDF is given by:

$$\begin{aligned} p(x_k | \mathcal{X}_k, Z^k) &= \sum_{c=1}^{C_k} p(x_k | \mathcal{X}_k, c, Z^k) P(c | \mathcal{X}_k, Z^k) \\ &= \sum_{c=1}^{C_k} \xi_k^c p(x_k | \mathcal{X}_k, c, Z^k) \end{aligned} \quad (25)$$

Finally, a posteriori track existence probability, for the next iteration is:

$$\psi_{k|k} = P(\mathcal{X}_k | Z^k) = \frac{\lambda_k \psi_{k|k-1}}{1 - (1 - \lambda_k) \psi_{k|k-1}} \quad (26)$$

Target measurements likelihood ratio:

$$\lambda_k = \frac{p(y_k | \mathcal{X}_k)}{p(y_k | \bar{\mathcal{X}}_k)} = 1 - P_D P_G + P_D P_G \sum_{i=1}^{N_k} \frac{p_k^i}{\rho_k^i} \quad (27)$$

where the probability of each component is given by the:

$$\begin{aligned} \xi_k^c &= P(c | \mathcal{X}_k, Z^k) = \\ &= \frac{P(c | \mathcal{X}_k, Z^{k-1})}{\lambda_k} \begin{cases} 1 - P_D P_G, & c = 0 \\ P_D P_G \frac{p_k^i(c)}{\rho_k^i}, & c > 0 \end{cases} \end{aligned} \quad (28)$$

### Nonparametric Linear Multi Target ITS data association

In nonparametric target tracking algorithm, the clutter measurement density  $\rho_i^c(k)$  is estimated using the measurements in the current scan. Since each component has a single measurement assigned to it at each scan (or the null measurement to represent the missed target detection case), the prediction and update steps for each component can be implemented using conventional filtering techniques.

In case of Linear multi targets (LM) ITS the selection area is the selection gate of the individual track. The clutter measurement density is calculated separately for each selection area defined above. Clutter measurement density is estimated as:

$$\rho_i^c(k) = \frac{\hat{n}_k}{V_k} \quad (29)$$

where  $\hat{n}_k$  is the mean number of clutter measurements within the selection area, and  $V_k$  is the volume of the selection area. Nonparametric Linear multi-target (LM) approach is a multi-target tracking technique which reduces requests for numerical resources by eliminating joint ‘measurements to tracks’ association. It also strongly reduces algorithmic complexity. When we update the track  $\tau$ , all possible detections of targets being followed by other tracks are treated as ‘unwanted’ measurements.

In other words, the LM method modulates the clutter measurement density  $\rho_{k,i}$  of each selected measurement  $z_{k,i}$  of track  $\tau$  by considering possible contribution of other tracks. Probability density function of target state estimate is then updated using a single tracking filter. In this case we

have a priori probability that the  $i$ th measurement is originated by the  $j$ th target, by following equation:

$$p_{k,j}^{\tau} = \tilde{\psi}_{k-1}^{\tau} \frac{\tilde{p}_{k,j}^{\tau} / \rho_{k,j}}{\sum_{l=1}^{M_k} p_{k,l}^{\tau} / \rho_{k,l}} \quad (30)$$

The modified clutter density for track  $\tau$  at measurement  $z_{k,j}$  is:

$$\Theta_{k,j}^{\tau} = \rho_{k,j} + \sum_{\theta=1, (\theta \neq \tau)}^{\tau_T} \frac{p_{k,j}^{\theta}}{1 - p_{k,j}^{\theta}} \quad (31)$$

where  $\tau_T$  is the total number of tracks. In order to find the data association probabilities for target  $\tau$ , we used single-target formulae with  $\Theta_{k,j}^{\tau}$  replacing  $\rho_{k,j}$ . In case that the targets are apart  $\Theta_{k,j}^{\tau} = \rho_{k,j}$  for all  $i$  and  $j$ , and LM ITS becomes identical to *IPF*. Measurement likelihood ratio for track  $\tau$  at time  $k$  is given by the following equation:

$$\lambda_k^{\tau} = 1 - \tilde{P}_D + \sum_{j=1}^{M_k} \frac{\tilde{p}_{k,j}^{\tau}}{\Theta_{k,j}^{\tau}} \quad (32)$$

After this step, we can calculate a posteriori state estimate probability of measurement  $j$ , by the following equations:

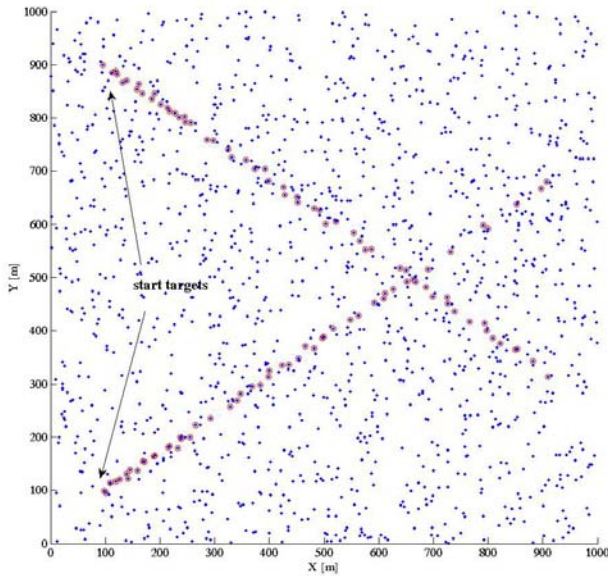
$$\beta_{k,j}^{\tau} = \frac{1}{\lambda_k^{\tau}} \begin{cases} 1 - \tilde{P}_D, & i = 0 \\ \tilde{P}_D \frac{p_{k,j}^{\tau}}{\rho_{k,j}}, & i \geq 0. \end{cases} \quad (33)$$

### Results of simulations

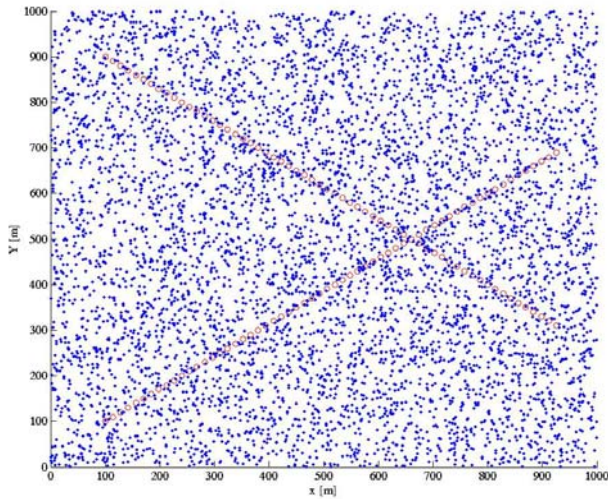
The experiments were performed and evaluated by the experimental scenarios, in which the clutter density and probability of detection changes. The scenario was used with two clutter density ( $\rho = 2 \cdot 10^{-5} [1/m^2]$  and  $\rho = 10^{-4} [1/m^2]$ ), in Fig.1 and Fig.2, respectively. Two probabilities of detection ( $P_D = 0.8$  and  $P_D = 0.6$ ) also combine with each other. The main goal of the experiment is to determine the limit of effective tracking, i.e. target detection probabilities when the CTT diagram cannot reach the value 1.

In each scenario (Fig. 1), there are two targets (red circles), which move in a straight line at a constant speed. Maximum permitted speed of movement (velocity) is  $v_{\max} = 25$  [m/s]. Dimensions of terrain surveillance are  $x=1000$  [m] and  $y=1000$  [m]. Clutter (blue dots) has uniform distribution. The sampling period of radar sensor is  $T=1s$ . Duration of the scenario is 60 scans. The following definitions of true and false tracks are used:

- Each initiated track is false with respect to all existing targets.
- False track becomes a true track with respect to a target when the state estimate is close to the true target state estimate.
- Track is true with respect to a target remaining true for as long as it selects the previous target detections.



**Figure 1.** Simulation scenario. ( $\rho=2*10^{-5}[1/m^2]$ ), blue dot-noise measurements, red circle - targets measurements



**Figure 2.** Simulation scenario ( $\rho=10^{-4}[1/m^2]$ ), blue dot-noise measurements, red circle - target trajectory

In all simulation scenarios, the linear coordinates are assumed, with additive measurement noise covariance of  $R=2512$ . In this formula, the  $I2$  denotes the two-dimensional identity matrix. We assumed that each target has the probability of detection  $P_D=0.8$ . A number of clutter measurements is present in each time interval. All tracks are initiated from any pair of measurements [21] which satisfies the maximum speed criterion and are not selected by an existing track. In case that the target is not followed by a track, new tracks are initiated using its detections.

In each scan, a number of false tracks are initiated. The false track discrimination procedure uses the track quality measure provided by the tracker to eliminate tracks with “false” quality, as these are assumed to be tracks, and confirm the “high” quality tracks. When the trajectory state estimates converge, the tracks are merged. In the experiments, we have two possible events, with relations on the probability of target existence:

- A) If the probability rises above a confirmation threshold, the track is confirmed and
- B) If the probability falls below the termination threshold, the track is terminated.

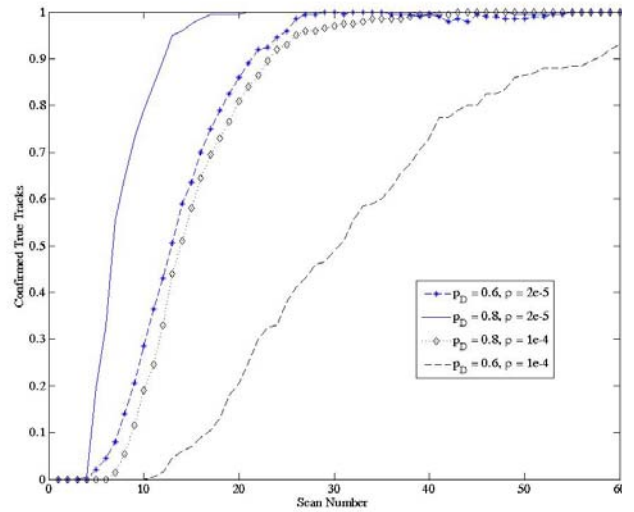
We used a criterion to compare false track discrimination.

If confirmed false tracks statistics is approximately equal across all tracks, the success rate of confirmed true tracks allows us to compare false track discrimination performance.

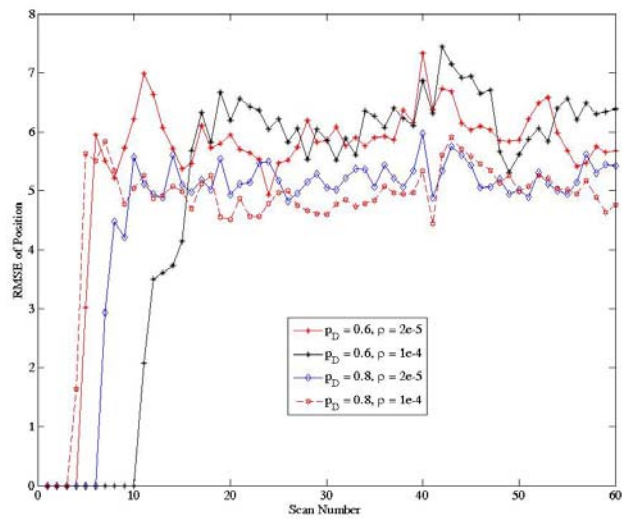
All experiments perform with 100 Monte Carlo (MC) runs over 2-dimensional test scenario. The system input is modeled as follows:  $X=[x \ \dot{x} \ y \ \dot{y}]$  is a vector state,  $x, y$  are the Cartesian coordinates of the target position,  $\dot{x}, \dot{y}$  are the appropriate velocities. Transition matrix ( $F_{CV}$  - constant velocity model and process noise matrix are given by:

$$F_{CV} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (34)$$

$$Q(k) = q \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} \quad (35)$$



**Figure 3.** Confirmed true tracks diagram



**Figure 4.** Overall RMSE of position diagram

The diagram confirmed true tracks (CTT) in Fig.3 shows that the probability of target detection  $P_D=0.6$  is the limit of the possibility of effective target tracking at a clutter density of  $10e-4 [1/m^2]$  and higher. The position RMSE error, also increases at the same values (Fig.4). At the same time, false

true tracks diagram (Fig.5) strongly deviates from the standard values before  $P_D=0.6$ , which confirms the previous claims.

Table 1 shows the comparative values of some important system parameters:

- CPU time for one recursion
- Identification number (ID) for all confirmed tracks
- Total number of all false tracks
- Track confirmation probability

It can be seen from Table 1. that there is a rapid growth of the total number of false track for this probability of target detection (Table 1). CPU time increases mainly due to a high density of clutter.

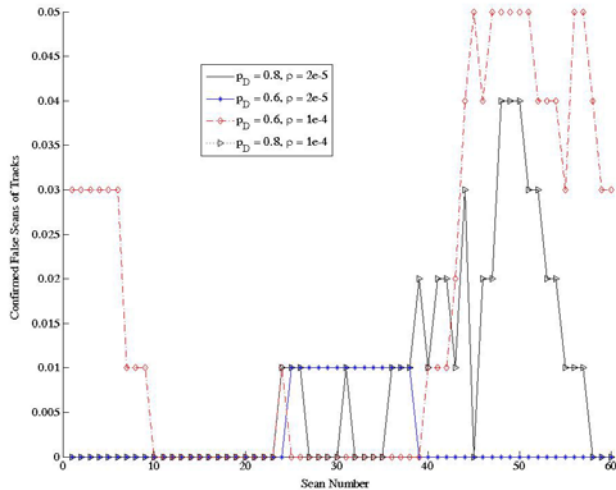


Figure 5. Confirmed false tracks diagram

Table 1. System parameters

	Execut. time[s]	Confirm tr. ID	False tr.sum	Confirm. prob.
$P_D=0.8$ $\rho=1e-4$	7.74	2	14	0.998
$P_D=0.8$ $\rho=2e-5$	0.89	5	4	0.998
$P_D=0.6$ $\rho=1e-4$	10.75	30	100	0.996
$P_D=0.6$ $\rho=2e-5$	1.21	27	47	0.997

## Conclusion

In this paper we present the examination of target tracking possibilities by the example of an efficient multi-scan algorithm. In case with variable parameters (clutter measurements and probability of target detection), the limit of probability of target detection is determined, up to which it makes sense to continue tracking the target.

The good performance of this approach primarily provides the reduction in numerical complexity. Other benefits include a much simpler algorithm structure, which translates into faster development software implementation. Whole target tracking procedure is performed and tested by the extensive simulation. Simulations showed the reader how to choose the optimal algorithm for known system parameters. The justification of the further improved procedure for rejecting false clues was also confirmed.

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## Doprinos određivanju granica efikasnog praćenja ciljeva

U ovom radu je prikazano merenje kvaliteta traga uvođenjem verovatnoće postojanja traga u cilju određivanja granica mogućnosti efikasnog praćenja ciljeva. Najpoznatiji algoritmi za radarsko praćenje ciljeva nemaju alate za merenje kvaliteta traga. Ne postoji validna procedura koja garantuje verifikaciju optimalnog algoritma za konkretnu primenu u praćenju ciljeva. Standardni algoritam Integriranog razdvajanja tragova (Integrated Track Splitting-ITS) je efikasan, potpuno automatski algoritam za praćenje više ciljeva sa inicijalizacijom, održavanjem i brisanjem tragova, dobro je poznat kao algoritam za praćenje jednog i više ciljeva, i predložen u svrhu testiranja granica uspešnog praćenja. Rad omogućava korisniku izbor optimalnog parametra po predloženim parametrima – pragovima potvrđenih i lažnih tragova kako bi se postigla bolja diskriminacija lažnih tragova (False Track Discrimination -FTD). Posebno je dat dijagram srednje kvadratne greške pozicije cilja (Root mean Square Error -RMSE) u svrhu verifikacije kvaliteta predloženog algoritma.

*Ključne reči:* Algoritam integralnog razdvajanja tragova, algoritam pridruživanja podataka po verovanoci, praćenje više ciljeva.