UDK: 620.178.3:629.735.3.018.4
COSATI: 11-13, 13-05, 01-03

Fatigue Life Evaluation of Damaged Aircraft Lugs

Slobodanka Boljanović[1]
Stevan Maksimović[2]
Andrea Carpinteri[3]

In the present paper, engineering procedures are proposed for estimating the strength of aircraft lugs subjected to cyclic tensile loading. The crack growth simulation examines pin loaded lugs under either a constant amplitude loading or a single overload. The fatigue life analysis tackles the stress intensity factor calculation. The residual strength of through-the-thickness damaged lugs is modeled by introducing the Walker's model. In order to investigate the single overload effect, the Wheeler's retardation parameter is employed. The estimations are compared with available experimental data. The verifications show that evaluations and experimental results are in a good agreement. Then, the effect of width, diameter and thickness on fatigue life up to failure of lugs is analysed.

Key words: **fatigue, cyclic loading, crack growth, lugs, aircraft structure, service life estimation, overload effect.**

Introduction

Some engineering structural systems are designed with load carrying members known as pin loaded lugs. During service of lug linked components, the pin bearing load together with fretting between the pin or bolt and the lug hole could endanger the strength of lug under cyclic loading. Since lug failures predominantly occur under fatigue conditions, it is very important to develop reliable computational models/procedures. The residual strength of cracked lugs can be investigated in the case of either through-the-thickness cracks or surface cracks [1-4]. This paper examines the case of a lug with a through-the-thickness crack since it is less resistant to failure than the surface crack configuration.

According to fracture mechanics and damage tolerant design philosophy, the residual strength and the fatigue life up to failure of the lug with a detected through-the-thickness crack require the stress intensity factor calculation. In the literature, various methods have been employed for the evaluation of the stress intensity factor. Schijve and Hoeymakers [5] investigated a lugs with a through-the-thickness crack, and developed an empirical relationship based on experimental data. For such a configuration, Impellizeri and Rich [6] employed the weight function, whereas Pian et al [7] computed the stress intensity factor by applying hybrid finite elements. Then, Zatz et al [8] used conventional finite elements in order to compute the stress intensity factor, and Hsu [9] applied finite elements with singularity.

Moreover, a number of methods have been developed in order to describe the crack extension under cyclic loading. Paris [10] found that the crack growth process can be analysed through the crack growth rate and the stress intensity factor range. Then, Elber [11] recognized that the stress ratio has impact on the crack extension, and proposed the relationship for crack growth rate based on the effective stress intensity factor. Further, Walker [12] suggested the two-parameter driving force model for the crack growth investigation.

The aim of the present paper is to develop mathematical procedures for the fatigue life analysis of lugs. The authors tackle two different loading conditions: a constant amplitude loading and a single overload. The strength of a lug with a through-the-thickness crack is theoretically simulated by employing the Walker's model and the Wheeler's retardation parameter. Numerical and/or analytical approaches are applied for the stress field evaluation and the stress intensity factor calculation. The validity of the estimations is discussed by comparing the present results with available experimental results. Additionally, the effects of width, diameter of a hole and thickness on the fatigue life up to failure of lugs are examined.

Residual fatigue strength modeling

Engineering structural components are usually subjected to loading with different levels, and the fatigue crack growth process can be investigated if adequate loading parameters are introduced [11, 12]. Thus, Walker [12] suggested that the stress intensity factor range can be expressed as a function of the stress ratio and the maximum stress:

$$\Delta K = (1-R)^{\gamma}\, K_{\max} \qquad (1)$$

where R is the stress ratio, $K_{\max}$ denotes the maximum stress intensity factor and γ represents a material constant.

The failure under cyclic loading can be theoretically analysed thanks to the appropriate relationships for the crack growth rate. Walker [12] modified the Paris crack growth law [10] by introducing the maximum stress

[1] College of Mechanical Engineering, TEHNIKUM TAURUNUM, Nade Dimić 4, 11080 Belgrade, SERBIA
[2] Military Technical Institute, Ratka Resanovića 1, Belgrade, SERBIA
[3] Department of Civil Engineering, Parco Area delle Scienze 181/A, 43124 Parma, ITALY

intensity factor and the stress ratio instead of the stress intensity factor range:

$$\frac{da}{dN} = C\left(\left(1-R\right)^{\gamma} K_{\max}\right)^{m} \quad (2)$$

where N is the number of loading cycles up to failure, C and m are material parameters experimentally obtained.

In the crack growth rate evaluation, the geometry of the cracked structural component and the loading conditions are examined through the stress intensity factor. Since pin loaded lugs are among the most critical structural components as far as fatigue is concerned, Newman and Raju [13] investigated the lug with a through-the-thickness crack (Fig.1) and proposed the following relationship for the calculation of the maximum stress intensity factor:

$$K_{\max} = S_{\max}\sqrt{\pi a}\, f_{w1}\, f_1\, G_1 \sqrt{\frac{1}{\cos\left(\frac{\pi D}{2w}\right)}} \quad (3)$$

where w and D are width and diameter of the lug, respectively.

The interaction between the pin and a hole of lug is taken into account by the following parameter [13]:

$$G_1 = \frac{1}{2} + \frac{w}{\pi\left(D+a\right)}\sqrt{\frac{D}{D+2a}} \quad (4)$$

In order to examine the single crack lug configuration, the Bowie correction [13] can be employed in Eq. (3):

$$f_1 = 0.707 - 0.18\lambda + 6.55\lambda^2 - 10.54\lambda^3 + 6.85\lambda^4 \quad (5)$$

where

$$\lambda = \frac{1}{1+\frac{2a}{D}} \quad (6)$$

Then, the effect of the finite lug width in the fatigue process up to failure is tackled through the stress intensity factor, and can be expressed for the single crack configuration by the following correction factor:

$$f_{w1} = \sqrt{\frac{1}{\cos\left(\frac{\pi}{2}\frac{D+a}{w-a}\right)}} \quad (7)$$

The strength of the lug can be evaluated by integrating the relationship related to the crack growth rate:

$$N = \int_{a_0}^{a_f} \frac{da}{C\left(\left(1-R\right)^{\gamma} K_{\max}\right)^{m}} \quad (8)$$

where a_0 and a_f are initial and final crack length, respectively.

Due to the form of functions under integral, the here developed software is based on the Euler's algorithm for numerical integration.

Crack growth propagation under single overload

During the fatigue failure process under variable amplitude loading characterized by tensile overload, the crack growth retardation effect can occur. This phenomenon appears due to increase in magnitude and size of the compressive residual stress field in the vicinity of the crack tip. Wheeler [14] examined load retardation effects, and introduced the retardation parameter C_{pi} in the relationship for the crack growth rate, i.e. the number of loading cycles up to failure becomes as follows:

$$N = \int_{a_0}^{a_f} \frac{da}{C_{pi} C\left(\left(1-R\right)^{\gamma} K_{\max}\right)^{m}} \quad (9)$$

In the residual strength simulation, the retardation parameter describes the delay of crack extension until the current plastic zone is within the plastic zone created by the overload, and can be expressed by:

$$C_{pi} = \begin{cases} \left(\frac{r_{pi}}{a_{ol} + r_{po} - a_i}\right)^{p} & ;\ a_i + r_{pi} \le a_{ol} + r_{po} \\ 1 & ;\ a_i + r_{pi} \ge a_{ol} + r_{po} \end{cases} \quad (10)$$

where a_i is the current crack length, a_{ol} denotes the crack length at the overloading, p represents the retardation exponent experimentally obtained. Further, r_{pi} and r_{po} are the current plastic zone size and the overload plastic zone size, respectively, and can be written in the following way:

$$r_{pi} = \frac{1}{\pi}\left(\frac{\Delta K}{2\sigma_{ys}}\right)^2 \quad r_{po} = \frac{1}{\pi}\left(\frac{K_{ol}}{\sigma_{ys}}\right)^2 \quad (11)$$

where K_{ol} is the stress intensity factor generated by an overload, and σ_{ys} denotes the yield strength of the material.

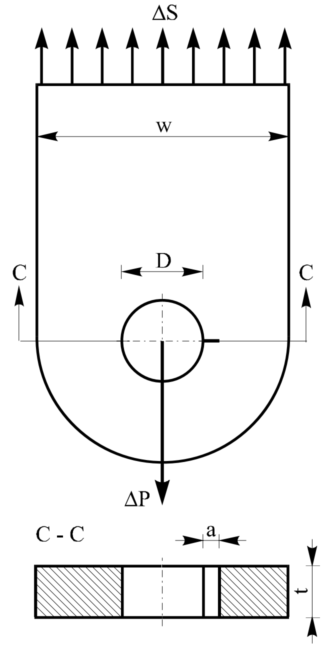


Figure 1. Geometry of a lug with through-the-thickness crack.

Thus, the strength of a cracked lug subjected to overload is theoretically investigated thanks to the developed software based on Eqs.(9-11) together with Eqs.(3-7) by also employing the Euler's algorithm for numerical integration.

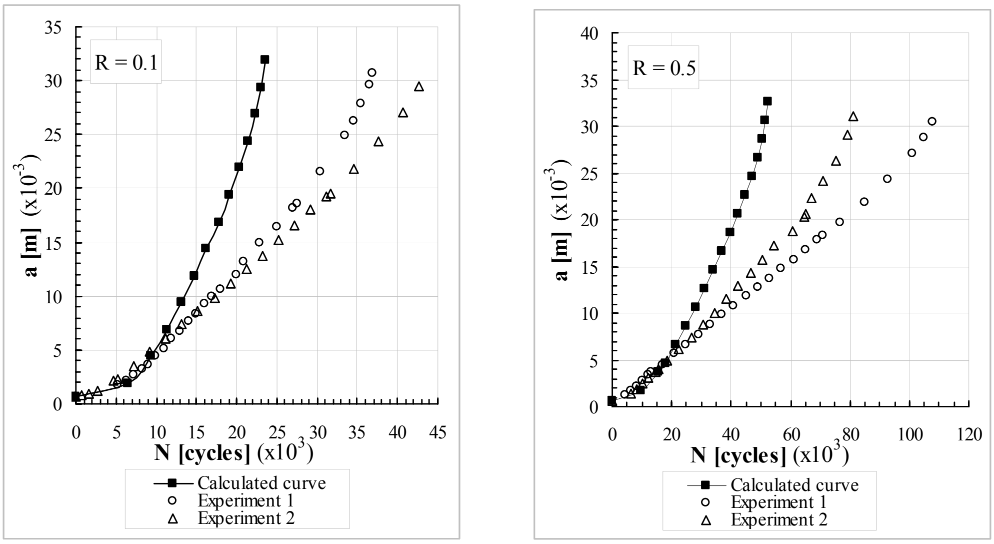
Numerical results

The reliability of the engineering procedure for the fatigue life evaluation of the lug with through-the-thickness crack is now examined. Through the following numerical examples, the residual strength of lug subjected to either a constant amplitude loading or a single overload is investigated. Such examples are analysed for both the numerical calculation of the stress intensity factor and the fatigue life estimation. In order to verify the validation of present model for surface crack growth simulation, the obtained results are compared with experimental data.

Fatigue strength estimation of through-the-thickness damaged lug

In this example, the crack growth analysis of the attachment lug (Fig.1) is carried out. The lug ($a_0 = 0.635$ mm , $D = 38.1$ mm , $t = 12.7$ mm) subjected to cyclic axial loading is made of 7075 T651. Geometry and loading parameters for two examined lugs are as follows: (a) $w = 114.3$ mm, $S_{\max} = 103.5$ MPa ($R = 0.1$, $R = 0.5$); (b) $w = 85.76$ mm, $S_{\max} = 103.5$ MPa ($R = 0.1$) [15]. The material characteristics of 7075 T651 Al alloy are: $E = 70$ GPa, $\gamma = 0.425$ [12], $C = 8.65 \times 10^{-11}$, $m = 3.49$ (for $R = 0.1$) [16], $c = 2.55 \times 10^{-10}$, $m = 3.06$ (for $R = 0.5$) [17].

In the context of fracture mechanics, the stress intensity factors are computed by applying Eq. (3) together with Eqs. (4-7) for adequate crack increments. Then, the crack growth rates are evaluated through Eqs. (2-7). Thereafter, the final number of loading cycles for the lug can be calculated by integrating the crack growth rate equation (Eq.(8)). The fatigue life evaluations are presented in Figures 2 and 3 for the lug with width $w = 114.3$ mm ($R = 0.1$, $R = 0.5$) and $w = 85.76$ mm ($R = 0.1$), respectively.



(a) 1–ABPLC85, 2–ABPLC89. (b) 1–ABPLC84, 2– ABPLC91.

Figure 2. Crack length against number of loading cycles up to failure (Experiments from Ref. [15]).

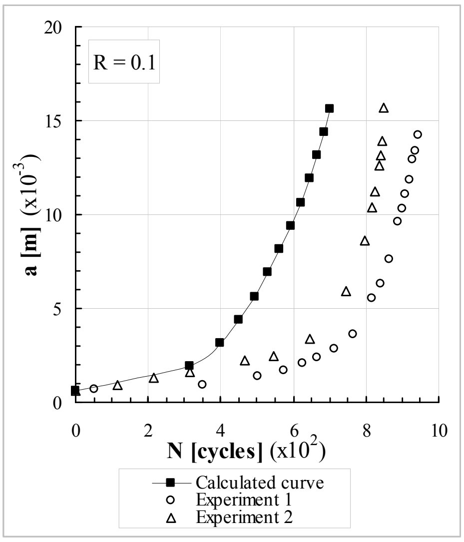
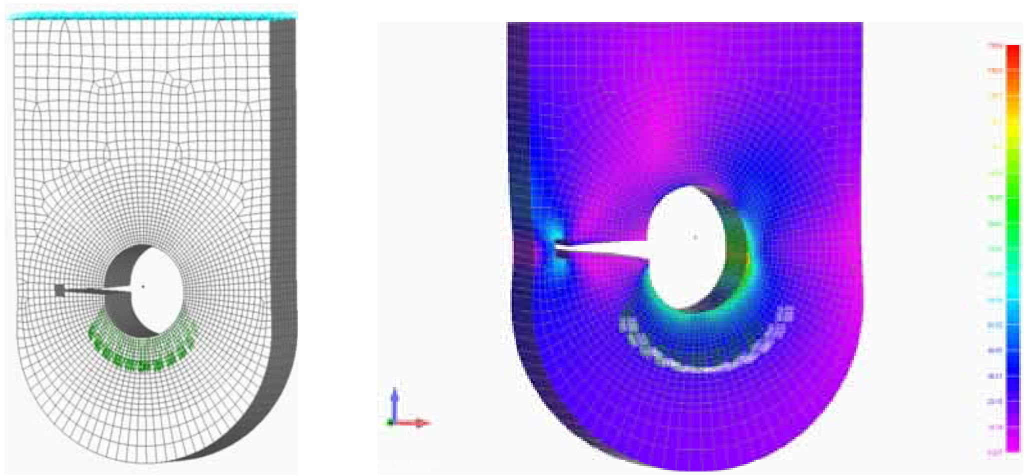


Figure 3. Crack length against number of loading cycles up to failure (1–ABPLC48, 2–ABPLC92. Experiments from Ref. [15]).

Further, in Figures 2 and 3, the calculated number of loading cycles up to failure is compared with experimental results available in the literature [15]. It can be observed that the estimations are in a good correlation with experimental results. Hence, by employing the proposed engineering procedure during service operation under cyclic loading, the safety residual fatigue life of a lug with a through-the-thickness crack can be estimated.

The stress field simulation and the stress intensity factor calculation

Consider the stress analysis of the lug with a through-the-thickness crack (Fig.1). The lug ($w = 50$ mm, $D = 16$ mm, $t = 16$ mm) is subjected to cyclic axial loading with a constant amplitude (a far field stress $S_{\max} = 126.5$ MPa). For the lug examined here, made of 7075 T7351 Al Alloy, the following material characteristics are assumed: $S_{0.2} = 334$ MPa, $\nu = 0.33$.



(a) The finite element mesh (a=8.83 mm) (b) The stress distribution (a=12.81 mm)

Figure 4. The lug with through-the-thickness crack subjected to cyclic axial loading.

In engineering practice, the contact problem related to the aircraft lug has to be analysed with special attention due to the fact that the stress field around the crack tip is complex. For the known lug geometry, material and loading parameters, both the stress distribution and the stress intensity factor are here simulated by performing a finite element analysis. In the numerical investigation, the two-dimensional 4-node shell finite elements together with super elements [18] are employed. Thus, for adequate crack growth increments, different meshes are modeled, and the stress distributions are evaluated. In Figures 4a and 4b, the finite element mesh and the stress distribution are presented, respectively.

Further, thanks to the stress analysis by applying FEM, the stress intensity factors are calculated for different crack lengths. The stress intensity factor results computed by employing FEM are compared with those obtained by applying the analytical approach (Eqs.(3-7)). The stress intensity factor results are listed in Table 1.

Table 1. Comparison of the calculated stress intensity factors for the lug employing analytical and numerical approaches.

Step	a 10^{-3} [m]	$K_{\max}^{Anal.}$ [MPam$^{0.5}$]	$K_{\max}^{FEM}$ [MPam$^{0.5}$]
1	5.33	33.14	36.85
2	8.82	37.91	40.95
3	12.81	52.47	52.34

The comparison in Table 1 implies that the analytical approach gives us almost the same stress intensity factor solutions as the finite element method. Thus, both approaches for the stress intensity factor calculation can be used in the strength analysis of the lug with through-the-thickness crack subjected to cyclic axial loading.

The effect of width, diameter and thickness of lug on the final number of loading cycles up to failure

In engineering practice, reliable exploitation of lugs could be achieved only if lugs are designed in such a way that they can stand different loading conditions for both crack initiation phase and crack growth phase. As far as the lugs represent an essential type of joint, the lug geometry parameters demand a careful analysis, especially for the crack growth phase. Therefore, the effect of width, diameter and thickness of the lug on the number of loading cycles up to failure is now examined. The lug made of 7075 T651 Al alloy is subjected to cyclic axial loading with either constant amplitude $P_{\max} = 40$ kN (stress ratio $R = 0.1$) or a single overload. The material characteristics are the same as those mentioned in example 4.1. The length of initial through-the-thickness crack (Fig.1) is $a_0 = 0.7$ mm for all considered lugs.

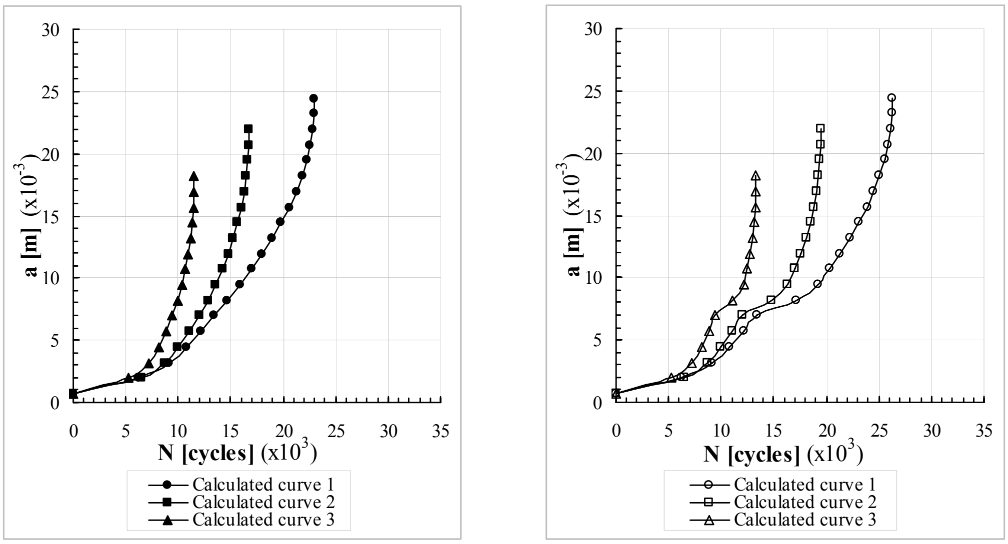
Under constant amplitude loading as well as overload, the following geometries of lugs are tackled: (a) $w = 80$ mm, $t = 12$ mm together with three different diameters of lugs (30 mm, 36 mm and 42 mm); (b) $w = 80$ mm, $D = 35.55$ mm and three different thicknesses (10 mm, 12.5 mm and 15 mm); (c) $D = 35$ mm, $t = 12$ mm with three different widths of lug (70 mm, 80.5 mm and 91 mm).

According to the mentioned geometries, material and loading parameters, fatigue life up to failure under constant amplitude loading is estimated by applying Eqs.(3-7) together with Eq.(8). Figures 5a, 6a and 7a present the calculated number of loading cycles against crack length for different values of diameter, thickness and width, respectively.

Then, the effect of different parameters (diameter, thickness and width) on the fatigue strength of lugs subjected to a single overload is examined. If the lug is overloaded, the external force is $P_{\max ol} = 80$ kN (i.e. the stress ratio under overload is $R_{ol} = 2$). All geometries, material and loading parameters mentioned for the lugs subjected to constant amplitude loading are used in the failure analysis of overloaded lugs. Note that the effects of diameter, thickness and width are investigated here for the crack length at overloading equal to a_{ol}=6.95 mm, a_{ol}=5.70 mm and a_{ol}=8.20 mm, respectively.

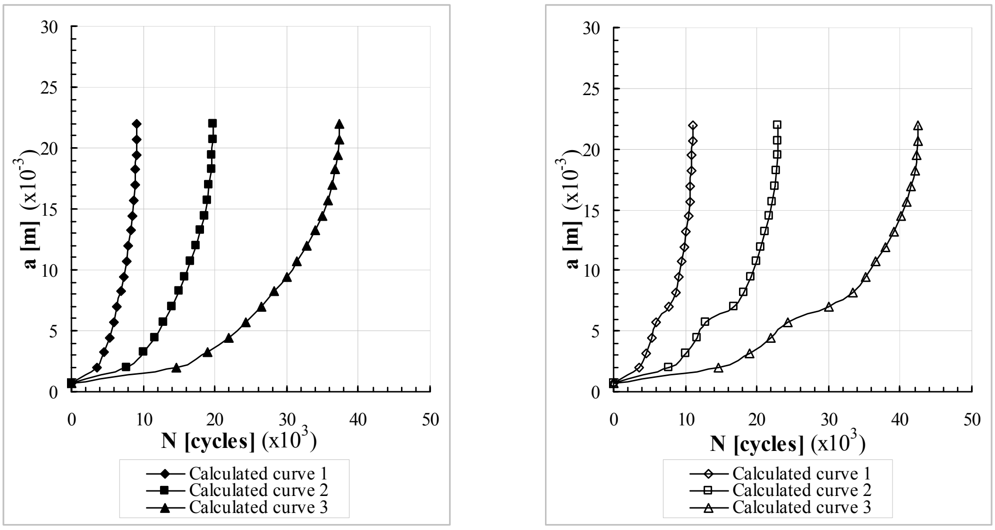
For the known lug geometries, material, overloading and constant amplitude loading parameters, the stress intensity factor, the crack growth rate and fatigue life up to failure under single overload are evaluated by applying Eqs.(9-11) together with Eqs.(3-7). The effects of diameter, thickness and width are shown (as numbers of loading cycles up to failure against crack length) in Figures 5b, 6b and Fig.7b, respectively.

From Figures 5b, 6b and 7b, it can be deduced that diameter, thickness and width have significant impact on the residual strength of the overloaded lug. Further, by taking into account overloading, the fatigue life up to failure increases.



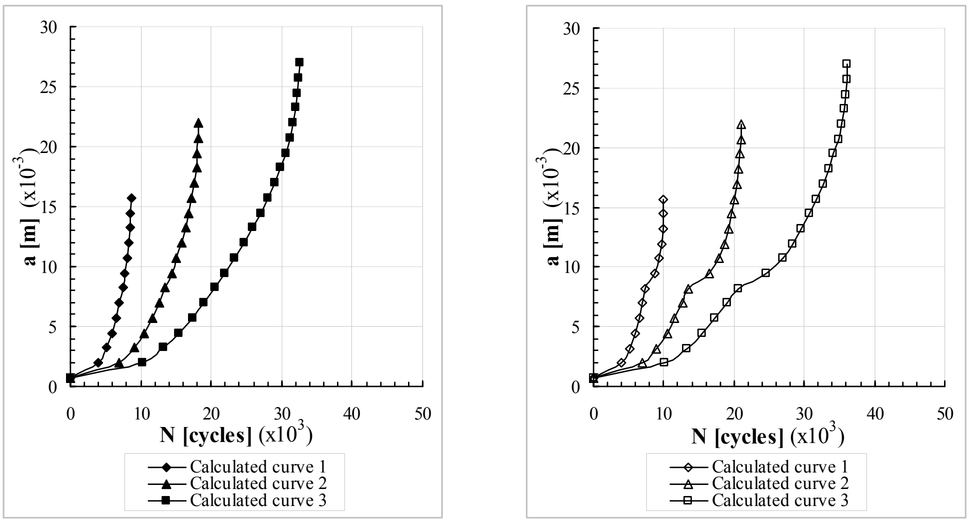
(a) Constant amplitude loading (b) Overload

Figure 5. Crack length against number of loading cycles up to failure for different diameters of lug (1 – *D*=30 mm, 2 – *D*=36 mm, 3 – *D*=42 mm).



(a) Constant amplitude loading (b) Overload

Figure 6. Crack length against number of loading cycles up to failure for different lug thicknesses (1 – *t*=10 mm, 2 – *t*=12.5 mm, 3 – *t*=15 mm).



(a) Constant amplitude loading (b) Overload

Figure 7. Crack length against number of loading cycles up to failure for different lug widths (1 – *w*=70 mm, 2 – *w*=80.5 mm, 3 – *w*=91 mm).

Conclusion

In the present paper, a computational procedure has been developed for the strength evaluation of a pin loaded lug with a through-the-thickness crack. The lug is subjected to cyclic loading with either a constant amplitude or an overload. The formulated procedure examines both the stress intensity factor calculation and residual strength evaluation. The stresses are examined by using analytical and numerical approaches. The fatigue life up to failure is estimated by employing the Walker's two-parameter driving force model and the Wheeler's retardation model for a constant amplitude cyclic loadings and overload, respectively.

The proposed crack growth procedure is assessed by comparing the estimations with experimental results available in the literature. A good agreement between different results implies that the developed procedure provides the reliable strength estimation for pin loaded lug with through-the-thickness crack under cyclic loading.

Moreover, such a procedure is implemented in order to investigate the differences between the residual fatigue life under a constant amplitude loading and that under an overload. Finally, the influence of width, diameter of a hole and thickness on the strength of the lug is analysed.

Acknowledgment

This research work was funded by the Mathematical Institute of the Serbian Academy of Science and Arts and the Ministry of Science and Technological Development, Serbia through the project No. OI 174001.

References

[1] CARPINTERI,A.: *Propagation of surface cracks under cyclic loading*, In. Carpinteri A. Editior Handbook of Fatigue Crack Propagation in Metallic Structures, Amsterdam, Elsevier Science B.V., 1994.

[2] CARPINTERI,A., BRIGHENTI,R., VANTADORI,S.: *Notched shells with surface cracks under complex loading*, International Journal of Mechanical Sciences, 2006, Vol.48, pp.638-649.

[3] BOLJANOVIĆ,S., MAKSIMOVIĆ,S.: *Fatigue crack growth modeling of attachment lugs*, International Journal of Fatigue, 2014, Vol.58, No.1, pp.66-74.

[4] MAKSIMOVIĆ,S., VASOVIĆ,I., MAKSIMOVIĆ,M., DJURIĆ,M.: *Some Aspects of Damage Tolerance Analysis of LASTA Training Aircraft Structures*, Scientific Technical Review, ISSN 1820-0206, 2013, Vol. 63, No.2, pp.70-74.

[5] SCHIJVE,J., HOEYMAKERS,A.H.W.: *Fatigue crack growth in lugs*, Fatigue of Engineering Materials and Structures, 1979, Vol.1, pp.185-201.

[6] IMPELLIZERI,L.F., RICH,D.L.: *Spectrum fatigue crack growth in lugs, in fatigue crack growth under spectrum loads*, ASTM STP, 1976, 595:320-36.

[7] PIAN,T.H.H., MAR,J.W., ORRINGER,O., STALK,G.: *Numerical computation of stress intensity factors for aircraft structural details by the finite element method*, AFFDL-TR-76-12 May 1976.

[8] ZATZ,I.J., EIDINOFF,H.L., ARMEN,H.Jr.: *An application of the energy release rate concept to crack growth in attachment lugs*, In: Proceedings of the 22nd AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conf. Atlanta, Georgia, 6-8 April, 1981.

[9] HSU,T.M.: *Analysis of cracks at attachment lugs*, In: Proceedings of the 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conf. Seattle, Washington,12-14 May, 1980.

[10] PARIS,P.C., ERDOGAN,F.A.: *A critical analysis of crack propagation laws*, J Basic Engng. Trans. SME, Series D, 55, 1963, pp.528-534.

[11] ELBER,W.: *The significance of fatigue crack closure*. In: Damage tolerance in aircraft structure, ASTM STR 486. American Society for Testing and Materials.1971, pp.230-242.

[12] WALKER,E.K.: *The effect of stress ratio during crack propagation and fatigue for 2024-T3 and 7076-T6 aluminum*, In: Effect of environment and complex load history on fatigue life, ASTM STR 462 Philadelphia. American Society for Testing and Materials, 1970, pp.1-14.

[13] RAJU,I.S., NEWMAN,J.C.Jr.: *Stress intensity factors for two symmetric corner cracks*, Fracture Mechanics (Edited by C.W. Smith), ASTM STR, 1979, pp.411-430.

[14] WHEELER,O.E.: *Spectrum loading and crack growth*, J Bas Engineering, 1972, Vol.94, pp.181-186.

[15] KATHIRESAN,K., BRUSSAT,T.R.: *Advanced life analysis methods*, AFWAL-TR-84-3080, Ohio; 1984.

[16] CEN.ENV 1999-2: Eurocode9: *Design of aluminium structures – Part 2: Structures susceptible to fatigue*, European Committee for Standardisation, 1998.

[17] FLECH,W.G., ANDERSON,R.B.: *A mechanical model of fatigue crack propagation*, In: Proceedings of the 2nd International Conference on Fracture (Edited by Pratt P.L), Brighton, London, Chapman & Hall; 1969.

[18] Msc/NASTRAN, Theoretical Manuels.

Received: 01.12.2013.

Procena veka pri zamoru avionskih uški sa oštećenjem

U ovom radu su predložene praktične inženjerske procedure za procenu čvrstoće avionskih uški pri dejstvu cikličnog opterećenja na zatezanje. Simulacija širenja prskotine kod uški opterećenih preko osovinice je razmatrana za ciklična opterećenja konstantne amplitude ili pri preopterećenju sa pojedinačnim pikom. Analiza veka pri zamoru podrazumeva određivanje faktora intenziteta napona. Prilikom modeliranja preostale čvrstoće uški oštećenih po debljini razmatran je Walker-ov model. Efekat pika preopterećenja je istražen uvođenjem Wheeler-ovog parametra kašnjenja. Proračunate vrednosti su upoređene sa raspoloživim eksperimentalnim rezultatima. Dobijena su dobra slaganja numeričkih i eksperimentalnih rezultata. Zatim je analiziran uticaj pojedinih parametara geometrije na preostali vek uški pri zamoru.

Ključne reči: **zamor, ciklično opterećenje, rast prskotine, uška, avionska konstrukcija, procena veka trajanja, uticaj preopterećenja.**

Оценка жизненного цикла на усталость ушко самолёта с повреждениями

В этой статье предложены практические инженерные процедуры для оценки прочности ушка самолёта на эффект циклического нагружения при растяжении. Моделирование распространения трещины ушко загруженного через штифт рассматиривано для циклических нагружений с постоянной амплитудой или под перенагрузкой с одним пиком. Анализ жизненного цикла на усталость подразумевает определение фактора интенсивности напряжений. При расчёте оставшееся прочности ушко поврежденного по толщине рассматривана Walker-модель. Эффект пика перенагрузки был исследован путём введения Wheeler-параметра задержки. Оцененые значения были сопоставлены с имеющимися экспериментальными результатами. В результате получены хорошие согласия цифровых и экспериментальных данных. Влияние некоторых геометрических параметров ушко анализировали на оставшийся жизненный цикл при усталости.

Ключевые слова: **усталость, циклическая нагрузка, рост трещины, ушко, структура самолёта, оценка жизненного цикла, воздействие перенагрузки.**

Estimation de la durée de vie à la fatigue chez l’épinglette d’avion endommagée

Dans ce papier on propose les procédures pratiques d’ingénieurs pour l’estimation de la force d’épinglette d’avion sous l’effet de la charge cyclique à la résistance. La simulation de la croissance de fracture chez l’épinglette chargée par l’axe est considérée pour les charges cycliques de l’amplitude constante ou à la surcharge au pic particulier. L’analyse de la durée de vie à la fatigue sous-entend la détermination du facteur d’intensité de la charge. Pendant le calcul de la force résiduelle de l’épinglette endommagée en épaisseur on a examiné le modèle Walker. L’effet du pic de la surcharge a été étudié en introduisant le paramètre de retard de Weeler. Les valeurs estimées ont été comparées avec les résultats expérimentaux disponibles. On a obtenu bon accord entre les résultats numériques et ceux expérimentaux. On a analysé aussi l’effet de certains paramètres géométriques de l’épinglette quant à la durée de vie à la fatigue.

Mots clés: **fatigue, charge cyclique, croissance de fracture, épinglette, construction d’avion , estimation de la duré de vie, effet de surcharge.**