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Further Results on Fractional Order Control of a Mechatronic System

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This paper presents a new algorithm of the fractional order PID (FOPID) control based on genetic algorithms (GA) in the position control of a 3 DOF`s robotic system driven by DC motors. The optimal settings for a FOPID controller as well as an integer order PID controller (IOPID) are done, applying the GA tuning approach and their extension for FOPID–IOPID controllers in a comparative manner. The effectiveness of the suggested optimal FOPID control is demonstrated with a given robotic system as an illustrative example. The rest of the paper presents the design of an advanced algorithm of the FOPID control tuned by GA and the application in the control of the production of technical gases, i.e. in the cryogenic air separation process.Then, the obtained model is linearized and decoupled and consequently IOPID and FOPID controllers are applied. In the same manner, a set of optimal parameters of these controllers is achieved through the GA optimization procedure through minimizing the proposed cost function. Finally, the use of the simulation results in the time domain has shown that the FOPID controller improves a transient response and provides more robustness than a conventional IOPID.

Key words: **control algorithm, PID controller, genetic algorithms, robotics, cryogenic process, mechatronics.**

Introduction

FRACTIONAL Calculus (FC) is a generalization of classical calculus concerned with the operations of integration and differentiation of the non-integer (fractional) order. The concept of fractional operators has been introduced almost simultaneously with the development of the classical ones. This question consequently attracted the interest of many well-known mathematicians, including Euler, Liouville, Laplace, Riemann, Grünwald, Letnikov and many others. Since the 19th century, the theory of fractional calculus has developed rapidly, mostly as a foundation for a number of applied disciplines, including fractional geometry, fractional differential equations (FDE) and fractional dynamics. The applications of FC are very wide nowadays,[1-3]. It is safe to say that almost no discipline of modern engineering and science in general remains untouched by the tools and techniques of fractional calculus. For example, wide and fruitful applications can be found in rheology, viscoelasticity, acoustics, optics, chemical and statistical physics, robotics, control theory, electrical and mechanical engineering, bioengineering, etc.. The main reason for the success of FC applications is that these new fractional-order models are often more accurate than integer-order ones, i.e. there are more degrees of freedom in the fractional order model than in the corresponding classical one. All fractional operators consider the entire history of the process being considered, thus being able to model the non-local and distributed effects often encountered in natural and technical phenomena.

Fractional order dynamic systems and controllers have been arousing interest in many areas of science and engineering in the last few years. In most cases, our objective of using fractional calculus is to apply the fractional order controller to enhance the system control performance. As we know, in the classical control theory, state feedback and output feedback are two important techniques in system control. The PID controller in particular is by far the most dominating form of feedback in use today, [4]. Due to its functional simplicity and performance robustness, PID controllers are still used for many industrial applications such as process controls, motor drivers, flight control, instrumentation, etc. Similarly, a fractional-order PID (FOPID) controller is a generalization of a standard (integer) PID controller. It affords more flexibility in PID controller design due to its five controller parameters (instead of the standard three): proportional gain, integral gain, derivative gain, noninteger integral and derivative order. However, the tuning rules of FOPID controllers are much more complex than those of standard (integer) PID controllers.

Also, genetic algorithms (GA) have received much interest in recent years, [5,6] where the basic operating principles of GA are based on the principles of natural evolution. The GA technique is a stochastic global adaptive search optimization technique based on the mechanisms of natural selection. This paper proposed the FOPID control based on genetic algorithms (GA) in control of given mechatronic systems: robotic systems as well as cryogenic systems. GA can solve nonlinear multi-objective optimization problems and require little knowledge of the problem itself and need not require that the search space is differentiable or continuous. In this regard GA are used for tuning the FOPID controller i.e for finding out optimal settings for the FOPID controller in order to fulfill different design specifications for the closed-loop system, taking advantage of the fractional orders, α and β .

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Fractional calculus fundamentals with history notes

The idea of FC has been known since the development of the regular calculus, with the first reference probably being associated with Leibniz and Marquis de l'Hopital in 1695. Both Leibniz and L`Hospital, aware of ordinary calculus, raised the question of a noninteger differentiation (order $n = 1/2$) for simple functions. Following L'Hopital's and Liebniz's first inquisition, fractional calculus was primarily a study reserved for the best mathematical minds in Europe. Euler [7] wrote in 1730:

> *"When n is a positive integer and p is a function of x, $p = p(x)$, the ratio of $d^n p$ to dx^n can always be expressed algebraically. But what kind of ratio can then be made if n be a fraction?"*

Subsequent references to fractional derivatives were made by Lagrange in 1772, Laplace in 1812, Lacroix in 1819, Fourier in 1822, Riemann in 1847, Green in 1859, Holmgren in 1865, Grunwald in 1867, Letnikov in 1868, Sonini in 1869, Laurent in 1884, Nekrassov in 1888, Krug in 1890, Weyl in 1919, and others [7-10]. During the 19th century, the theory of fractional calculus was developed primarily in this way, trough insight and genius of great mathematicians. Namely, in 1819 Lacroix [10], gave the correct answer to the problem raised by Leibnitz and L'Hospital for the first time, claiming that $d^{1/2}x / dx^{1/2} = 2\sqrt{x/\pi}$. In his 700 pages long book on Calculus published in 1819, Lacroix developed the formula for *n*-th derivative of $y = x^m$, with *m* being a positive integer.

$$D_x^n y = \frac{d^n}{dx^n}\left(x^m\right) = \frac{m!}{(m-n)!}x^{m-n},\ m \ge n \tag{1}$$

Replacing the factorial symbol by Gamma function (5), he developed the formula for the fractional derivative of a power function

$$D_x^\alpha x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}x^{\beta-\alpha} \tag{2}$$

where α and β are fractional numbers and where the gamma function $\Gamma(z)$, see below.

In particular, Lacroix calculated

$$D_x^{1/2} x = \frac{\Gamma(2)}{\Gamma(3/2)}x^{1/2} = 2\sqrt{\frac{x}{\pi}} \tag{3}$$

The gamma function $\Gamma(z)$, so-called *Euler integral of the second kind* is defined for $\mathrm{Re}(z) > 0$ as:

$$\Gamma(z) = \int_0^\infty e^{-x}x^{z-1}dx \tag{4}$$

In case $z = a + ib$ is a complex number, one can obtain an absolute Gamma function (see Fig.1)

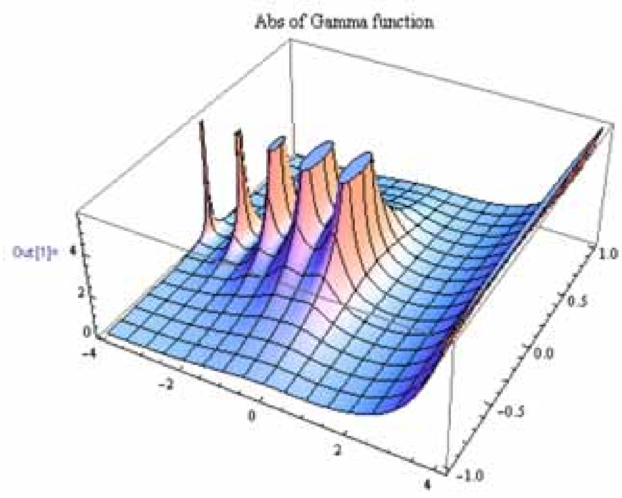

Figure 1. Gamma function of complex argument

In case that z is a real number, we can present the Gamma function as follows:

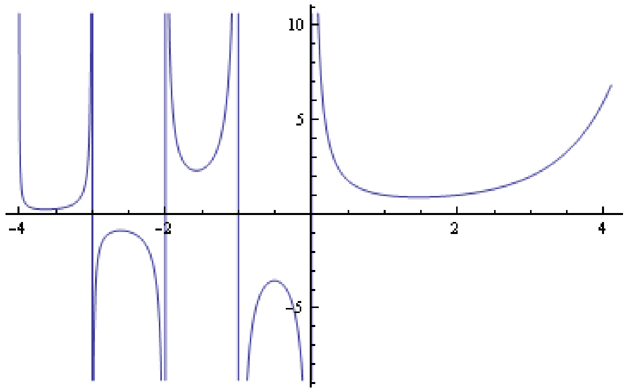

Figure 2. The real Gamma function Γ

The integral in the right side of (4) is convergent for all values of the complex argument *z* with a positive real part. However, by means of an analytic continuation it can be extended to the entire complex plane, excluding negative integers and zero. The Gamma function has several well established properties, the first of which is that it can be seen as a generalization of the factorial function. The so-called *reduction formula* holds, for $z \in C \setminus \{0,-1,-2,-3,...\}$,

$$\Gamma(z+1) = z\Gamma(z),\ \Rightarrow \Gamma(n+1) = n(n-1)! = n!\quad n \in \mathbb{N}_0 . \tag{5}$$

This reduction formula (5) can easily be proven starting from the integral (4). The analytic continuation of (4) is then conducted by the application of this formula to arguments with negative real parts. The points at which the Gamma function is not well defined, i.e. negative integers and zero, are its simple poles. Another important relationship for the Gamma function is the Legendre formula:

$$\Gamma(z)\Gamma(z+1/2) = \sqrt{\pi}2^{2z-1}\Gamma(2z),\ 2z \ne 0,-1,-2,..., \tag{6}$$

Taking $z = n + 1/2$ in the previous relation, and utilizing the fact that for integer arguments Gamma function can be evaluated by means of the factorial function, one can obtain a set of particular values of the Gamma function:

$$\Gamma(n+1/2) = \frac{\sqrt{\pi}\Gamma(2n+1)}{2^{2n}\Gamma(n+1)} = \frac{\sqrt{\pi}(2n)!}{2^{2n}n!}, \tag{7}$$

Also,the first attempts to carry out such studies were already made by J. Liouville [11-14], and N. Abel [15,16] in the solution of the tautochrone and other classical problems giving rise to the integral equations or relations representing integrals and fractional-order derivatives. However, a rigorous investigation was first carried out by Liouville in a series of papers from 1832-1837, [11-15], where he defined the first outcast of an operator of fractional integration. Liouville developed ideas on this theme and presented a generalization of the notion of incremental ratio to define a fractional derivative. This idea was discussed again by Grunwald (1867), [17] and Letnikov (1868), [18]. Later investigations and further developments by among others Riemann led to the construction of the integral-based Riemann-Liouville fractional integral operator, which has been a valuable cornerstone in fractional calculus ever since. An early attempt by Liouville was later purified by the Swedish mathematician Holmgren [19], who in 1865 made important contributions to the growing study of fractional calculus. Also, Hadamard [20], proposed a method of fractional differentiation based on the differentiation of the Taylor`s series associated with the function. The earliest work that ultimately led to what is now called the Riemann-Liouville definition appears to be the paper by N. Ya. Sonin in 1869, [21] where he used Cauchy`s integral formula as a starting point to reach differentiation with an arbitrary index. Weyl [22] and Hardy,[23,24], also examined some rather special, but natural, properties of differintegrals of functions belonging to Lebesgue and Lipschitz classes in 1917, where Weyl defined a fractional integration suitable to periodic functions, and later Marchaud (1927), [25] developed an integral version of the Grunwald-Letnikov definition of fractional derivatives. More recently, the unified formulation of integration and differentiation based on Cauchy`s integral has gained great popularity.

Among the most significant modern contributions to fractional calculus are those made by the results of M. Caputo in 1967, [26]. Caputo [26,27] reformulated the more "*classic*" definition of the Riemann-Liouville fractional derivative in order to use classical initial conditions, the same ones needed by integer order differential equations. It is interesting to note that Rabotnov [28] introduced the same differential operator into the Russian viscoelastic literature a year before Caputo's paper was published.

Also, applications to physics and engineering are not recent: application to viscosity dates back to the 1930s, namely, in the 1930s–1940s, [29] there were extensive studies of the properties of the viscoelastic materials which demonstrated that stress in fibrous polymers is representable as a convolution of the fractional power function and the deformation or its derivative. Applications of FC are very wide nowadays, in rheology, viscoelasticity, acoustics, optics, chemical physics, robotics, control theory of dynamical systems, electrical engineering, bioengineering and so on, [1-6,29]. In fact, real world processes generally or most likely are fractional order systems. Many real-world physical systems display fractional order dynamics, i.e. their behavior is governed by fractional order differential equations. The main reason for the success of FC applications is that these new fractional-order models are more accurate than integer-order models, i.e. there are more degrees of freedom in the fractional order model. Furthermore, fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes due to the existence of a "*memory*" term in a model. One of the intriguing beauties of the subject is that fractional derivatives (and integrals) are not local (or point) quantities. For example, it has been illustrated that materials with memory and hereditary effects, and dynamical processes, including gas diffusion and heat conduction, in fractal porous media can be more adequately modeled by fractional order models than by integer order models [1-4].

The modern epoch started in 1974 when a consistent formalism of the fractional calculus was developed by Oldham and Spanier, [1], Samko et al. [30] and later Podlubny, [3]. There exist today many different forms of fractional integral operators, ranging from divided-difference types to infinite-sum types, Riemann-Liouville fractional derivative, Grunwald–Letnikov fractional derivative, Caputo's, Weyl's and Erdely-Kober left and right fractional derivatives, etc. Kilbas *et al.* [30]. The three most frequently used definitions for the general fractional differintegral are: the Grunwald-Letnikov (GL) definition, the Riemann-Liouville (RL) and the Caputo definitions, [1-3]. For the expression of the Riemann-Liouville definition, we will consider the Riemann-Liouville *n*-fold integral for $n \in N, n > 0$ defined as

$$\underbrace{\int_a^t \int_a^{t_n} \int_a^{t_{n-1}} \cdots \int_a^{t_3} \int_a^{t_2}}_{n-fold} f(t_1)\, dt_1 dt_2 ... dt_{n-1} dt_n = \\ = \frac{1}{\Gamma(n)} \int_a^t (t-\tau)^{n-1} f(\tau)\, d\tau, \tag{8}$$

The distinctions of various formal definitions of fractional integrals are due to different methods of defining the limits of integration and the integrand or, more precisely, the integral kernel. The fractional Riemann-Liouville integral of order α for the function $f(t)$ for $\alpha > 0, a \in R$ i.e the *left Riemann-Liouville fractional integral* and the *right Riemann-Liouville fractional integral* are defined respectively as

$$ {}_{RL}I_a^{\alpha} f(t) \equiv {}_{RL}D_a^{-\alpha} f(t) = \\ = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau, \tag{9}$$

$$ {}_{RL}I_b^{\alpha} f(t) \equiv {}_{RL}D_b^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_t^b (\tau - t)^{\alpha-1} f(\tau)\, d\tau, \tag{10}$$

Furthermore, the left Riemann-Liouville fractional derivative is defined as

$$ {}_{RL}D_{a,t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau)\, d\tau, \tag{11}$$

and the right Riemann-Liouville fractional derivative is defined as

$$ {}_{RL}D_{t,b}^{\alpha} f(t) = \frac{(-1)^n}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_t^b (\tau - t)^{n-\alpha-1} f(\tau)\, d\tau, \tag{12}$$

where $n-1 \le \alpha < n$, *a, b* are the terminal points of the interval $[a,b]$, which can also be $-\infty, \infty$. Also, for the RL derivative, we have

$$\lim_{\alpha\to(n-1)^+} {}_a^{RL}D_t^{\alpha} f(t) = \frac{d^{n-1}f(t)}{dt^{n-1}}$$

and

$$\lim_{\alpha\to n^-} {}_a^{RL}D_t^{\alpha} f(t) = \frac{d^{n}f(t)}{dt^{n}} \tag{13}$$

The Caputo fractional derivatives are defined as follows. The *left Caputo fractional derivative* is

$${}_C D_{a,t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)}\int_a^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\,d\tau, \tag{14}$$

and the right Caputo fractional derivative is

$${}_C D_{t,b}^{\alpha} f(t) = \frac{(-1)^n}{\Gamma(n-\alpha)}\int_t^b (\tau-t)^{n-\alpha-1} f^{(n)}(\tau)\,d\tau, \tag{15}$$

where $f^{(n)}(\tau) = d^n f(\tau)/d\tau^n$ and $n-1 \le \alpha < n \in Z^+$. The previous expressions show that the fractional-order operators are *global* operators having a memory of all past events, making them adequate for modeling hereditary and memory effects in most materials and systems. Moreover, for the Caputo derivative, we have

$$\lim_{\alpha\to(n-1)^+} {}_a^{C}D_t^{\alpha} x(t) = \frac{d^{n-1}x(t)}{dt^{n-1}} - D^{(n-1)}x(a) \tag{16}$$

and

$$\lim_{\alpha\to n^-} {}_a^{C}D_t^{\alpha} x(t) = \frac{d^{n}x(t)}{dt^{n}} \tag{17}$$

where obviously, ${}_{RL}D_a^{\alpha}, n \in (-\infty,+\infty)$ varies continuously with n, but the Caputo derivative cannot do this. Obviously, the Caputo derivative is more strict than Riemann-Liouville derivative; one reason is that the *n*-th order derivative is required to exist. On the other hand, the initial conditions of fractional differential equations with the Caputo derivative have a clear physical meaning and the Caputo derivative is extensively used in real applications. The Riemann-Liouville fractional derivatives and the Caputo fractional derivatives are connected with each other by the following relations:

$$\begin{aligned}{}_{RL}D_{t,a}^{\alpha} f(t) &= \\ &= {}_C D_{t,a}^{\alpha} f(t) + \sum_{k=0}^{n-1} \frac{(-1)^k f^{(k)}(a)}{\Gamma(k-\alpha+1)}(t-a)^{k-\alpha}\end{aligned} \tag{18}$$

$$\begin{aligned}{}_{RL}D_{t,b}^{\alpha} f(t) &= \\ &= {}_C D_{t,b}^{\alpha} f(t) + \sum_{k=0}^{n-1} \frac{(-1)^k f^{(k)}(b)}{\Gamma(k-\alpha+1)}(b-t)^{k-\alpha}\end{aligned} \tag{19}$$

The Caputo and Riemann-Liouville formulations coincide when the initial conditions are zero,[1-3].

For convenience, the Laplace domain is usually used to describe the fractional integro-differential operation for solving engineering problems. The formula for the Laplace transform of the RL fractional derivative has the form:

$$\begin{aligned}&\int_0^\infty e^{-st}\, {}_{RL}D_{0,t}^{\alpha} f(t)dt = \\ &= s^\alpha F(s) - \sum_{k=0}^{n-1} s^k\, {}_{RL}D_{0,t}^{\alpha-k-1} f(t)_{|t=0}\end{aligned} \tag{20}$$

where for $\alpha < 0$ (*i.e.*, for the case of a fractional integral) the sum in the right-hand side must be omitted). Also, the Laplace transform of the Caputo fractional derivative is:

$$\begin{aligned}&\int_0^\infty e^{-st}\, {}_C D_{0,t}^{\alpha} f(t)dt = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \\ &n-1 < \alpha < n\end{aligned} \tag{21}$$

which implies that all the initial values of the considered equation are presented by a set of only classical integer-order derivatives. Also, Grunwald and Letnikov developed an approach to fractional differentiation based on the definition

$$\begin{aligned}&{}_{GL}D_x^{\alpha} f(x) = \lim_{h\to 0} \frac{\left(\Delta_h^\alpha f(x)\right)}{h^\alpha}, \\ &\Delta_h^\alpha f(x) = \sum_{0\le |j|<\infty} (-1)^{|j|}\binom{\alpha}{j} f(x-jh), h>0,\end{aligned} \tag{22}$$

which is the left Grunwald-Letnilov (GL) derivative as a limit of a fractional order backward difference. Similarly, we have the right one as

$$\begin{aligned}&{}_{GL}D_x^{\alpha} f(x) = \lim_{h\to 0} \frac{\left(\Delta_{-h}^\alpha f(x)\right)}{h^\alpha}, \\ &\Delta_{-h}^\alpha f(x) = \sum_{0\le |j|<\infty} (-1)^{|j|}\binom{\alpha}{j} f(x+jh), h<0,\end{aligned} \tag{23}$$

As indicated above, the previous definition of GL is valid for $\alpha > 0$ (fractional derivative) and for $\alpha < 0$ (fractional integral) and, commonly, these two notions are grouped into one single operator called *differintegral*. The GL derivative and the RL derivative are equivalent if the functions they act on are sufficiently smooth.

Brief introduction to fractional order controls

PID (proportional integral derivative) controllers are the most popular controllers used in industry because of their simplicity, performance robustness, and the availability of many effective and simple tuning methods based on minimum plant model knowledge [4,31]. A survey has shown that 90% of control loops are of PI or PID structures [4,31]. In control engineering, a dynamic field of research and practice, better performance is constantly demanded. On the other hand, there is a remarkable increase in a number of studies related to the application of fractional controllers in many areas of science and engineering, where specially fractional-order systems are of interest for both modeling and controller design purposes. The fractional-order systems are controlled only by the fractional controllers because in this case they are much superior to the integer-order controllers, [3,32]. Thanks to the widespread industrial use of PID controllers, even a small improvement in PID features, achieved by using fractional order PID controllers (FOPID), could have a relevant impact. Clearly, for closed-loop control systems, there are four situations: (a) integer order (IO) plant with IO controller; (b) IO plant with fractional-order (FO) controller; (c) FO plant with IO controller, and (d) FO plant with FO controller. Here, our objective is to apply the fractional-order control (FOC) to enhance the (integer order) dynamic system control performance.

Several design methodologies of FOPID controllers have been introduced to facilitate their use. Maybe the first sign of the potential of FOC, though without using the term "fractional," emerged with Bode [33], i.e *non-integer integral* and its application to control, where Bode suggested an ideal shape of the loop transfer function in his work on design of feedback amplifiers. A key problem in the design of a feedback amplifier was to devise a feedback loop so that the performance of the closed-loop is invariant to changes in the amplifier gain. Bode presented an elegant solution to this robust design problem, which he called the *ideal cutoff characteristic*, nowadays known as *Bode's ideal loop transfer function*, with the transfer function $G(s)=(\omega_{CG}/s)^{\alpha}$ where ω_{CG} is the gain crossover frequency and the constant phase margin is $\phi_m=\pi-\alpha\pi/2$, whose Nyquist plot is a straight line through the origin giving a phase margin invariant to gain changes. This frequency characteristic allows robustness of the system to parameter changes or uncertainties, and several design methods have made use of it. We can say that the pioneering applications of fractions calculus in the control theory date back to the sixties. The frequency response and the transient response of the non-integer-order integral (Bode's loop ideal transfer function) and its application to control systems was introduced by Manabe [34].

In the nineties, Oustaloup and his group [33] proposed a non-integer robust control strategy named CRONE (*Commande Robuste d'Ordre Non-Entier*) based on non-integer derivatives and demonstrated significant improvement of CRONE controllers over integer PID controllers. There are three generations of CRONE controllers [35-38], where CRONE controllers are obtained using a rational form and the major differences between the three generations lie in the design of the open-loop, the slope of which depends on consideration of plant uncertainty. In the same time, the TID scheme proposed by [39], the proportional compensating unit of a classical PID device is replaced by an element referred to as a "tilt" compensator with the transfer function equal to $s^{-(1/n)}$ where *n* is a positive integer. The introduction of the $s^{-(1/n)}$ into the PID structure helps realize a better approximation of a theoretically optimal loop transfer function and hence improves the performance of the feedback control system as a whole. Another well-known fractional control algorithm is the fractional-order PID (FOPID, or $PI^{\lambda}D^{\mu}$) controller introduced by Podlubny in time domain, [40] where the FPID controller involves an integrator of the fractional order λ and a differentiator of fractional order μ. Also, the $PI^{\lambda}D^{\mu}$ controller is studied by [41] in the frequency domain. Recently, the subclass of the fractional-order controllers is still analyzed in the frequency domain to take advantage of the fractional order λ in process compensation, [42].

Further research activities run in order to define new effective tuning techniques for non-integer order controllers by an extension of the classical control theory, [43]. In this respect, in [44], the extension of derivative and integration orders from integer to non-integer numbers provides a more flexible tuning strategy. An optimal fractional-order PID controller based on specified gain and phase margins with a minimum integral squared error (ISE) criterion is suggested and discussed in [45].The tuning of integer-order PID controllers is addressed in [46-48] by minimizing an objective function that reflects how far the behavior of the PID is from that of some desired fractional-order transfer function (FOTF). Generally, tuning methods for fractional PIDs can be divided into types: analytical, numerical and rule-based. Numerical tuning of fractional PIDs relies on the numerical evaluation of an objective function which measures the extent to which several design specifications are fulfilled, weighting them as the control designer finds it appropriate.

GA-based optimal FOPID control

Tuning of the FOPID controller-problem statement

The fractional order PID controller (FOPID) is the generalization of a standard (integer-order) PID (IOPID) controller, whereas its output is a linear combination of the input and the fractional integer/derivative of the input. Thanks to the widespread industrial use of PID controllers, even a small improvement in PID features, achieved by using $PI^{\beta}D^{\alpha}$, could have a relevant impact. One of the most important advantages of the $PI^{\beta}D^{\alpha}$ controller is the better control of dynamical systems which are described by fractional order mathematical models. Recently published results, [50,51] indicate that the use of a fractional-order PID controller can improve both the stability and performance robustness of feedback control systems. Another advantage lies in the fact that the $PI^{\beta}D^{\alpha}$ controllers are less sensitive to changes of the parameters of a controlled system [51,52]. In fact, it affords more flexibility in PID controller design due to the selection of five controller parameters that involve the proportional gain, the integral gain, the derivative gain, the integral order, and the derivative order.

In order to pose the same ease of use of standard PID controllers, many different methodologies for the design of FOPID controllers have been introduced in the literature. Some of these techniques are based on an extension of the classical PID control theory. The time equation of the FOPID controller ($PI^{\beta}D^{\alpha}$), is given by:

$$u(t)=K_p e(t)+K_d\,{}_0D_t^{\alpha}e(t)+K_i\,{}_0D_t^{-\beta}e(t) \qquad (24)$$

The continuous transfer function of the controller is obtained through the Laplace transform as $PI^{\beta}D^{\alpha}$:

$$G_{FOPID}(s)=\frac{K_p s^{\beta}+K_i+K_d s^{\beta+\alpha}}{s^{\beta}},\quad \left(\alpha,\beta\in R^{+}\right) \qquad (25)$$

However, in theory, $PI^{\beta}D^{\alpha}$ itself is an infinite dimensional linear filter due to the fractional order in the differentiator or the integrator. Furthermore, the fractional effect has to be band-limited when it is implemented. Therefore, the fractional integrator must be implemented as $1/s^{\beta}=(1/s)s^{1-\beta}$, ensuring this way the effect of an integer integrator $1/s$ at a very low frequency. Similarly to the fractional integrator, the fractional differentiator s^{α} has also to be band-limited when implemented, ensuring this way a finite control effort and noise rejection at high frequencies. For practical digital realization, the derivative part has to be complemented by the first order filter

$$G_{FOPID}(s) = K_p \left(1 + \frac{1}{s^\beta T_i} + \frac{T_d s^\alpha}{\frac{T_d}{N} s + 1} \right), \quad (26)$$

The controller parameters are the proportional gain K_p, the derivative gain K_d, the integral gain K_i, the noninteger order of the derivative α and the integrator β, as well as the integral time constant, $T_i = K_p / K_i$, and the derivative time constant $T_d = K_d / K_p$.

The tuning rules of fractional-order PID (FOPID) controllers are much more complex compared with standard PID (IOPID) controllers having only three parameters. Unlike conventional PID controllers, there is no systematic and rigor design or a tuning method existing for $PI^\beta D^\alpha$ controllers. Many of the tuning methods proposed so far for the design of FOPID controllers have been based on mathematical optimization, where the design of fractional PID controllers could be treated as a multi-objective optimization problem, which is to compromise the rapidity, stability and accuracy of system control.

For the most applications, load disturbances are typically low frequency signals and their attenuation is a very important characteristic of a controller. It is shown [4] that by maximizing the integral gain K_i, the effect of load disturbance at output will be minimum. Some works use performance indices as the objective functions as follows: integral of the absolute value of the error (IAE), mean of the squared error (MSE), integral of time multiplied by the absolute error (ITAE), integral of the absolute magnitude of the error (IAE), and integral of the squared error (ISE)

$$\begin{aligned} IAE &= \int |e(t)| dt, \quad MSE = \frac{1}{T} \int \left(e(t) \right)^2 dt, \\ ITAE &= \int t |e(t)| dt, \quad ISE = \int e(t)^2 dt, \\ ITSE &= \int te(t)^2 dt \end{aligned} \quad (27)$$

The authors [49] use a set of frequency-domain specifications to formulate the PI^λ and $PI^\lambda D^\mu$ tunings as a nonlinear optimization problem, in which the best solution to a constrained nonlinear equation has to be found. Particularly, there is a need for an effective and efficient global approach to optimize these parameters automatically. In that way, several other authors exploit well-known intelligent and evolutionary search algorithms to find the optimal set of parameters for their fractional controllers [53–56].

Optimal tuning FOPID using genetic algorithms (GA)

Using *genetic algorithms* for determining the optimal parameters of fractional order PID controllers, [57,58] is proposed here. Recently, GA have been recognized as an effective and efficient technique to solve optimization problems, [4,5,57,58]. As a mathematical means for optimization, genetic algorithms (GA) can naturally be applied to the optimal-tuning of fractional order PID controllers. GA is one of the optimization methods based on the natural selection such as inheritance and mutation. GAs are simulated in a computing system, and consist in a population of representations of candidate solutions for an optimization problem, that evolve towards better solutions. GA is a search technique that manipulates the coding representation of a parameter set to search a near optimal global solution through cooperation and competition among potential solutions. This algorithm is highly relevant for the industrial application, because it is capable of handling a problem with constraints, objectives and dynamic components. This paper thus describes the application of GA to the fine-tuning of the parameters for fractional PID controllers. In real coding implementation, each chromosome is encoded as a vector of real numbers, of the same lengths as the solution vector. According to control objectives, five parameters $K_p, K_d, K_i, \alpha, \beta$ of a fractional PID controller are required to be designed in these settings. This study introduces a next optimality criterion which involves besides the steady state error *e*, i.e IAE, the overshoot P_0, as well as the settling time T_s.

$$J = |P_0| + T_s + \int |e| dt \rightarrow min \quad (28)$$

The fitness function is designed as:

$$f_g = J_{\max} + J_{\min} - J_g \quad (29)$$

where are $J_{\max}, J_{\min}$ the largest value and the smallest value of J, respectively, observed thus far, as well as J_g value of the criterion for the current population. All the GA parameters are arranged as follows:

- population size: $N = 100$;
- crossover probability: $p_c = 0.75$;
- mutation probability: $p_m = p_{m0} \min(1, l/g)$
- $p_{m0} = 0.1$ - initial mutation probability,
- $l = 25$ - generation threshold,
- g - current number of the generation
- generation gap $gr = 0.35$

Here*,* remainder stochastic sampling with replacement is used as a selection method*.* In our case, the stopping conditions for GA are: the GA stops when the maximum number of generations (2.5N) has been reached or the first 50% of individuals reaches approximately the same value of the fitness function.

Simulations and discussion

FOPID control for a robotic system

Simulation studies have been carried out to verify the effectiveness of the proposed fractional PID controller tuned by genetic algorithms for robot control. Both the FOPID and the integer order IOPID controllers are designed based on the proposed GA. For the calculation of fractional derivatives and integrals, the Crone approximation of the second order was used, [59,60].

A robotic system with 3 DOF's is used here, Fig.3a, driven by 3 DC motors, where Rodriguez` method [58,61,62], is proposed for modeling the kinematics and dynamics of the robotic system. Equations of motion of the robotic system can be expressed in the identical covariant form as follows

$$\sum_{\alpha=1}^{n} a_{\alpha i}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \Gamma_{\alpha\beta,i}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_i, \quad i = 1,2,\ldots,n. \tag{30}$$

where the coefficients $a_{\alpha\beta}$ are the covariant coordinates of the basic metric tensor $[a_{\alpha\beta}] \in R^{n\times n}$ and $\Gamma_{\alpha\beta,\gamma}$ $\alpha,\beta,\gamma = 1,2,\ldots,n$ presents Christoffel symbols of the first kind as well as Q_i are generalized forces. The equivalent circuit of a DC motor is represented in Fig.3.b.

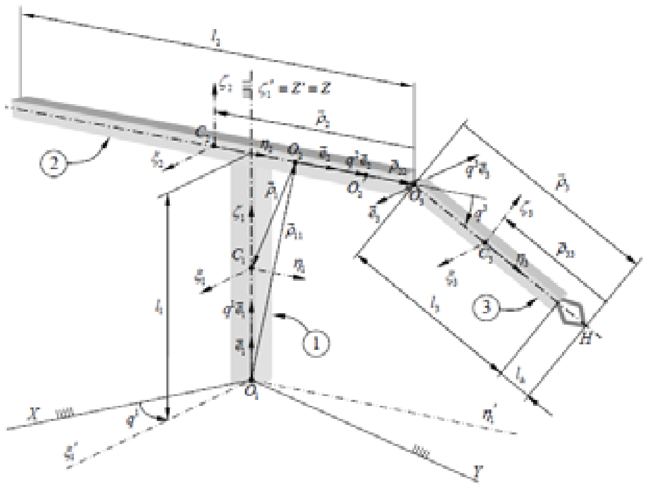

Figure 3.a) Robot with 3 DOF's

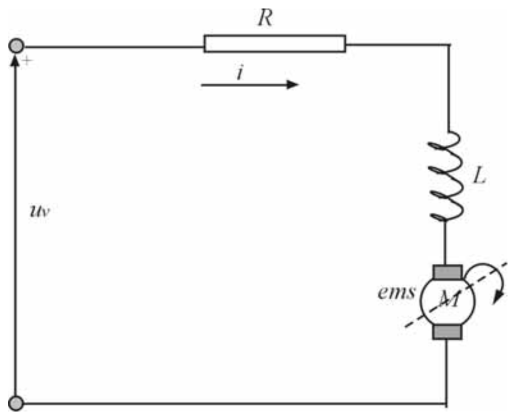

Figure 3.b) The equivalent circuit of a DC motor

The next equation describes the given circuit of a DC motor

$$R_i i_i(t) + L_i \frac{di_i(t)}{dt} + ems_i(t) = u_{vi}(t), \; i = 1,2,3 \tag{31}$$

where R_i, L_i, i_i and u_{vi} are, respectively, resistance, inductivity, electrical current and voltage. Electromotive force is $ems_i(t) = k_e dq_m / dt$ where $k_e = const$ and $q_m(t)$ is a generalized coordinate of a DC motor. If there is a reductor with a degree of reduction N_i than is $q_{mi}(t) = N_i q_i(t), \; i = 1,2,3$. It can be assumed that

$$Q_i^u(t) = N_i k_m i_i(t) \tag{32}$$

where $k_m = const$ is the torque constant as well as that $L \approx 0$. If the equation of a robotic system (30) is combined with (32), and (31) becomes

$$R[NK_m]^{-1}\left(A(q)\ddot{q} + C(q,\dot{q})\right) + K_e N\dot{q} = u_v(t) \tag{33}$$

or in state space, $x_p = [q_1\, q_2\, q_3]^T$, $x_v = [\dot{q}_1\, \dot{q}_2\, \dot{q}_3]^T$, as follows

$$\dot{x} = \begin{bmatrix} \dot{x}_p \\ \dot{x}_v \end{bmatrix} = \begin{bmatrix} x_v \\ -A^{*-1}(x_p)C(x) + Fx_v \end{bmatrix} + \begin{bmatrix} 0_{3\times 3} \\ NK_m R^{-1} \end{bmatrix} u_v(t) \tag{34}$$

$$y = h(x) = x_p \tag{35}$$

where $A^*(x_p) = R[NK_m]^{-1} A(x_p)$, $F = NK_m R^{-1} K_e N$. In order to obtain a step response, a simulation model has been developed using the Simulink Library of MATLAB by using a special toolbox for non-integer control. In this case, each individual vector has the FOPID parameters (five parameters) where for reducing the time of optimization, the ranges of *FOPID* parameters are selected as, [58]:

$$K_p \in [10,200], \; K_i \in [0,100], \; K_d \in [10,200], \; \alpha \in (0.2,1], \; \beta \in [0,1], \tag{36}$$

Table 1. The optimal parameters of the FOPID controller and the IOPID controller based on GA

controller		K_p	K_i	K_d	β	α	J_{opt}
PID	1.	199	2	24	1	1	0.98651
	2.	212	2	26	1	1	0.84875
	3.	246	1	28	1	1	0.68718
FOPID	1.	199	2	24	0.020	0.965	0.69887
	2.	212	2	26	0.145	0.933	0.72954
	3.	246	1	28	0.135	0.932	0.56187

In Table 1, the optimal parameters of the FOPID and IOPID controllers are presented using GA. The step responses of these two optimal FOPID/IOPID controllers, presented in Figs.4-6, are compared in simulations.

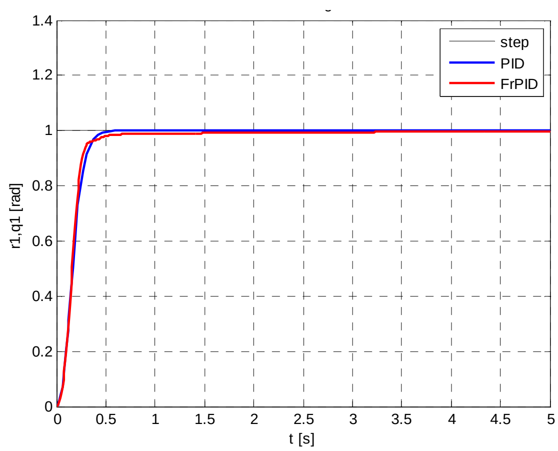

Figure 4. The step responses of the $q_1 q_1(t)$[rad] - using the optimized FOPID and IOPID controller

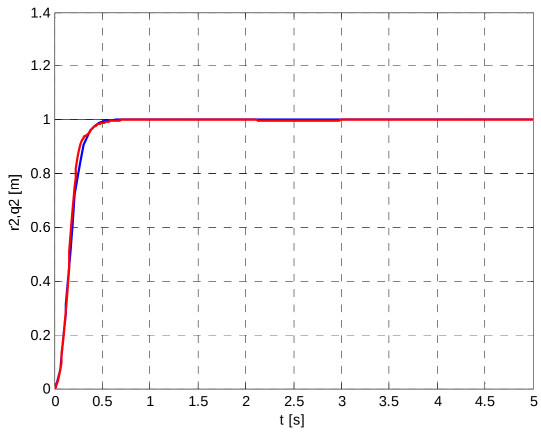

Figure 5. The step responses of the $q_2(t)[\mathrm{m}]$ - using the optimized FOPID and IOPID controller

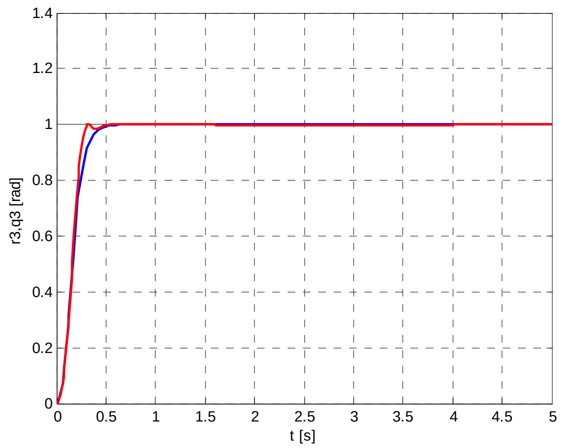

Figure 6. The step responses of the $q_3(t)[\mathrm{rad}]$ - using the optimized FOPID and IOPID controller

As it can be seen from Figs.4-6 and Table1, a better performance for robot control can be achieved using FOPID. We can conclude from this comparison that the optimal FOPID controller gives a better performance for robot control as compared to the optimal IOPID controller method.

FOPID control for the expansion turbine in the cryogenic air separation process

Another application of FOPID is also presented here, [63]. Namely, tuning FOPID based on GA as well as classical *PID* controllers is proposed. It is applied to the control of a cryogenic process of mixing of two gaseous airs flows at different temperatures before the entrance of the expansion turbine. The cryogenic air separation process is operated at extremely low temperatures (-170 to -190^{o}C) to separate air components according to their different boiling temperatures and it takes place in air separation units (ASUs) which present cryogenic distillation systems. Due to high demand for these commodity materials, the ASU has become a crucial technology in many processes including the next generation of power plants. The cryogenic air separation process is an energy- intensive process that consumes a tremendous amount of electrical energy. Therefore, an ASU must automatically and rapidly respond to the changing product demand from customers. As expected, there is a significant economic interest in reducing the operating costs of ASUs through advanced process control technology. So far, the dominating control practice in ASU processes has been to adapt traditional regulatory controllers to maintain good performance. A number of studies on the process control and optimization of the cryogenic air separation process have been published, [64-66]. Also, numerous uncertainties make effective operation of these complex processes difficult. The cryogenic distillation process can be very complex in practice, but here a simplified model that approximates the actual process is used for the analysis.

Improving the implementation of traditional PID controllers by applying FOPID controllers tuned by GA is proposed here.

Fig.7 presents the diagram of the process and a symbolic-functional scheme with relevant variables (G_{56} gas air flow at the entrance to the expansion turbine, the position control valve $TV946A$ - $Y_A[\mathrm{mm}]$ as well as $TV946B$ - $Y_B[\mathrm{mm}]$, $\theta_5 = T_5[K]$ temperature gas air environment with exchangers, $\theta_3 = T_3[K]$, temperature air with the end of the cold heat exchangers, $\theta_1 = T_1[K]$ - heat)

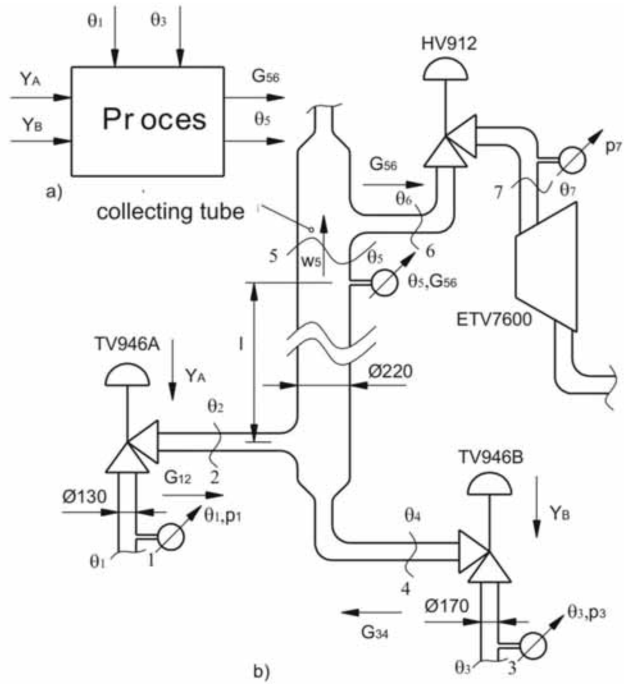

Figure 7. Diagram of the cryogenic process (a) symbolic-functional scheme (b)

After the linearization of the model that describes the cryogenic process of mixing two gaseous air flows at different temperatures before the entrance of the expansion turbine, one can obtain a simplified model as an MIMO system

$$\dot{x}(t) = \begin{bmatrix} -0,2 & 0 \\ 0 & -0,2 \end{bmatrix} x(t) + \begin{bmatrix} 45,736 & 28,07 \\ 0,174 & -0,085 \end{bmatrix} u(t) + \begin{bmatrix} 0 & 0 \\ 0,088 & 0,112 \end{bmatrix} z(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) \tag{37}$$

where the corresponding vectors are, $u(t) = [y_A(t)\ y_B(t)]^T$, $z(t) = [z_1(t)\ z_2(t)]^T$. To decouple the system, applying a new input $u(t)$ by means of feedback, $u(t) = -K_c x(t) + F_c v(t)$, it yields

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} v(t) + \begin{bmatrix} 0 & 0 \\ 0.088 & 0.112 \end{bmatrix} z(t) \tag{38}$$

The proposed approach based on the fractional PID control tuned by GA is also used for the control of a cryogenic process of mixing two gaseous airs flows at different temperatures before the entrance of the expansion turbine. In this case, each individual vector has the FOPID parameters (five parameters) where, for reducing the time of optimization, the ranges of *FOPID* parameters are selected as

$$\begin{aligned} &K_p \in [0,20],\ K_i \in [0,20], \\ &K_d \in [0,20],\ \alpha \in (0,1], \beta \in [0,1], \end{aligned} \tag{39}$$

Table 2. presents the optimal parameters of FOPID and classical PID controllers using GA.

controller		K_p	K_i	A	β	α	J
PID	1.	15	1	0	1	1	0.81
	2.	15	7	0	1	1	19.15
FOPID	1.	13	3	11	0.034	0.073	0.24
	2.	14	8	11	0.98	0.069	13.22

Table 2. The optimal parameters of the FOPID controller and the conventional IOPID controller based on the proposed GA-cryogenic process

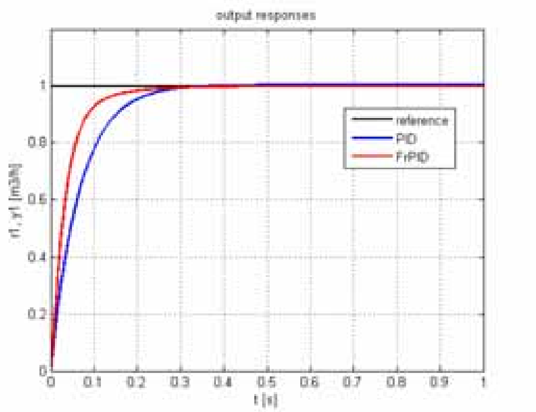

Figure 8. The step responses of the $y_{i1}(t) = x_{i1}(t)\left[\mathrm{m_N^3/h}\right]$ - gas air flow at the entrance to the expansion turbine using the optimized FOPID and conventional PID controller.

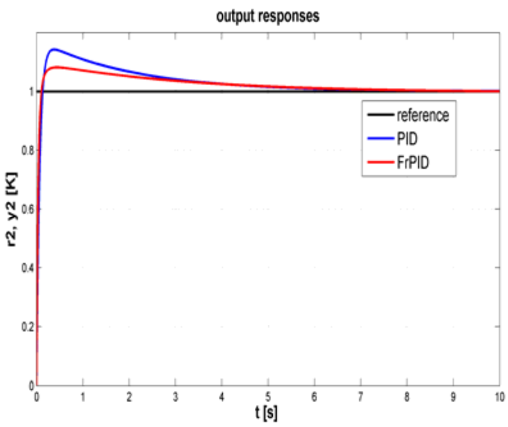

Figure 9. The step responses of the $y_{i2}(t) = x_{i2}(t)\ [\mathrm{K}]$ - gas air temperature at the entrance to the expansion turbine using the optimized FOPID and conventional PID controller.

As it can be seen in Fig.8, the step response of the $y_1(t)\left[m_N^3/h\right]$ - gas air flow at the entrance to the expansion turbine (disturbances are $z_1 = 10[K]$, $z_2 = 10[K]$) using the optimized FOPID controller and the conventional PID controller has a better transient response in the case of the FOPID controller. Also, we have obtained that the overshoot in the FOPID controller is 0.006% and the rising time is 0.184. On the other hand, in the classical *PID,* the overshoot is 0.43% and the rising time is 0.248. Also, Fig.9 represents the step response of the the $y_2(t)[K]$ - gas air temperature at the entrance to the expansion turbine, applying the optimized FOPID and the conventional PID controller. In a similar way, we obtained in the robust FOPID controller that the overshoot is 8.19% and the rising time is 4.520; while in the robust classical *PID* the overshoot is 14.25% and the rising time is 4.609.

Conclusion

This paper proposes and studies an advanced algorithm of the FOPID control based on genetic algorithms in the position control of a 3 DOF`s robotic system driven by DC motors. The present work tries to focus on those characteristics of GA tuning approach taking into account the optimality criterion and their extension for FOPID – IOPID controllers in a comparative manner. From this comparison, we can conclude that the optimal FOPID controller gives better performance in the position control of the robotic system as compared to the optimal IOPID controller method. We also proposed a robust FOPID controller as well as an IOPID controller tuned by GA in the control of the cryogenic process of mixing two gaseous air flows at different temperatures before the entrance of the expansion turbine. This method allows the optimal design of all major parameters of FOPID and IOPIOD controllers where the step responses of the two proposed optimal controllers are compared. A time-domain simulation confirms better the performance of the FOPID controller with respect to a traditional optimized IOPID controller.

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Novi rezultati upravljanja necelobrojnog reda datim mehatroničkim sistemom

Ovaj rad predstavlja jedan novi algoritam PID upravljanja necelobrojnog reda zasnovani na genetskim algoritmima (GA) u zadatku pozicioniranja robotskog sistema sa tri stepena slobode pogonjen jednosmernim motorima. Urađena su optimalna podešavanja parametara FOPID kontrolera kao i IOPID kontrolera, primenom GA pristupa za date FOPID/IOPID kontrolere na uporedni način. Efektivnost predloženog optimalnog FOPID upravljanja je demonstrirano na datom robotskom sistemu kao jednim ilustrativnim primerom. Takođe, u preostalom delu rada prezentovano je projektovanje naprednog algoritma FOPID upravljanja podešavanog primenom GA i primena u upravljanju proizvodnjom tehničkih gasova, tj. kriogenog procesa separacije vazduha. Zatim je izvedeni model linearizovan i raspregnut i gde su zatim primenjeni IOPID i FOPID kontroleri. Na sličan način, skup optimalnih parametara datih kontrolera su dobijeni primenom GA optimizacione procedure minimizujući predloženi kriterijum optimalnosti. Konačno, koristeći rezultate simulacije u vremenskom domenu pokazano je da FOPID kontroler poboljšava odgovor sistema u prelaznom režimu i obezbeđuje više robusnosti u poređenju sa klasičnim IOPID kontrolerom.

Ključne reči: **algoritam upravljanja, PID kontroler, genetski algoritmi, robotika, kriogeni proces, mehatronika.**

Новые результаты управления дробными порядками данной мехатронной системой

Эта статья представляет собой новый алгоритм ПИД управления дробных порядок, основанный на генетических алгоритмах (ГА) в задаче позиционирования роботизированной системы с тремя степенями свободы с приводом от электродвигателя постоянного тока. Исполнены оптимальные настройки параметров ФОПИД контроллера и ИОПИД контроллера с помощью ГА подхода на сегодняшний день ФОПИД/ИОПИД контроллеров на сопоставимой основе. Эффективность предложенного оптимального ФОПИД управления показано на данной роботизированной системе в качестве иллюстративного примера. Кроме того, на оставшуюся часть работы представленна разработка усовершенствованных алгоритмов управления ФОПИД приспособленного использованием ГА, а в том числе и применение в управлении производством технических газов, то есть криогенного процесса разделения воздуха. Затем полученная модель сделана линейной и развязаной, где затем применены ИОПИД и ФОПИД контроллеров. Аналогично, совокупность множества данных оптимальных параметров контроллера получаются применением ГА оптимизации процедуры при минимизации предложенного критерия оптимальности. Наконец, используя результаты моделирования во временной области, как было показано ФОПИД контроллер улучшает реакцию системы в переходном режиме и обеспечивает более надежности по сравнению с обычным класическим ИОПИД контроллером.

Ключевые слова: **алгоритм управления, ПИД-регулятор, генетические алгоритмы, робототехника, криогенные процессы, мехатроника.**

Nouveaux résultats dans le contrôle de l'ordre fractionnel par le système mécatronique donné

Ce papier présente un nouveau algorithme du contrôle PID de l'ordre fractionnel basé sur les algorithmes génétiques (AG) dans la position du système robotique à trois degrés de liberté actionné par les moteurs à courant continu. On a effectué les ajustages optimales pour les paramètres des contrôleurs FOPID ainsi que pour les contrôleurs IOPID par l'approche AG pour les contrôleurs FOPID/IOPID donnés de façon comparative. L'efficacité du contrôle optimale du FOPID proposé a été démontrée sur le système robotique donné en exemple illustratif. Ensuite on a présenté la conception de l'algorithme avancé du contrôle FOPID ajusté pour l'utilisation du AG et par l'application dans la production des gaz techniques c'est-à-dire le processus cryogène de la séparation de l'air. Le modèle dérivé a été ensuite linéarisé et découplé et on y a utilisé les contrôleurs FOPID et IOPID. De même façon on a obtenu l'ensemble des paramètres optimales des contrôleurs donnés par l'emploi de la procédure d'optimisation AG et en minimisant le critère proposé d'optimisation. Enfin en utilisant les résultats de la simulation dans le domaine temporel on a démontré que le contrôleur FOPID améliore la réponse du système dans le régime transitoire et assure plus de robustesse que le contrôleur IOPID conventionnel.

Mots clés: **algorithme de contrôle, contrôleur PID, algorithme génétique, robotique, processus cryogène, mécatronique.**