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Effects of Thermal Gradients on Fracture Mechanics Parameters

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A description of a model developed to examine thermal effects on fracture is presented. A special attention in this investigation is focused on the behavior of a thin plate with a circular hole with a radial crack, subjected to thermal loads. The finite element method is used for the determination of the temperature and stress distributions at the thin plate with a circular hole. For this structural element with a circular hole with a radial crack, the stress intensity factor (SIF) is considered as well. The finite element method is also used to determine stress intensity factors of this cracked structural element. For stress intensity factors there are two approaches used: the modified J^* integral approach and the displacement method based on singular finite elements. Good agreement between these two approaches has been obtained.

Key words: **fracture mechanics, thermal load, thermal effect, stress-strain state, aircraft structure, finite element method.**

Introduction

A large number of structural components in aerospace and electronic engineering are subjected to a wide variety of loads of mechanical and thermal origin [1,2]. The phenomenon of fatigue may be induced by a cyclic thermal gradient from the surface to the core of structures. The prediction of the fatigue durability of cracked components involves an analysis of fatigue crack growth and requires accurate stress intensity factor (SIF) solutions corresponding to the transient temperature distribution [3,4].

Thermal stresses play an important role in various fields of industries [5,6]. Mechanical load is not the only load that is considered in the design of structures and components. It is essential to determine the magnitude and influence of thermal stresses to make a realistic design of such structures. External influences such as temperature or intense exposure to radiation can lead to complex behavior of structures. In practical applications, finite element method (FEM) is used to study the stress and strain fields as well as stress intensity factor induced by fatigue thermal shock [12,13].

The investigation of stresses in structural components is very important for modern engineering. During the past few decades, a widespread attention has been given to thermal stress problems in structures. The presence of thermal stresses and the existence of cracks and defects in many structures and components can lead to disastrous consequences. Predicting the behavior of crack growth is one of the main problems in various industries. Aviation industry [7-10], gas turbines, pressure vessels and pipelines are examples where cracks growth can lead to disastrous consequences and the loss of human lives.

Fracture mechanics just studies the laws by which cracks and other defects in structures grow under the influence of a given load. This requires a comparison of the analytical expressions for crack growth and failure with experimental results. Analytical expressions describing the crack growth rate consist of determining fracture mechanics parameters such as the stress intensity factor under thermomechanical loads.

Fracture mechanics parameters

During the past three decades, extensive research on fracture mechanics has greatly enhanced the understanding of structural failures. Today, many high-performance structures and high-accuracy instruments have to consider thermal effects as a critical factor. This complex process could be realized under constant, variable and mixed conditions. A characteristic example for this is an aircraft engine running during take-off, landing and cruising [10]. During cruising, jet engines are exposed essentially to constant temperature and constant load. However, during take-off, a need for more power (thrust) will rise temperature and load levels, and it means that the corresponding damage due to fatigue would be greater. It is well known that an isotropic material plate under constant temperature and free to deform will be free of stresses as well. Meanwhile, if uniform (stress) flows in the body are disturbed by some opening, or shapes, or existence of some other material, thermal stress will occur. Some of the most damaging events arise during transition regimes when temperature, as stress, and/or deformations are changed independently. The thermomechanical fatigue problem is complex, because it is connected with changes of many physical-chemical parameters, which makes predicting safe life of structural elements more difficult. In this paper, as an example of many structural elements, a plate with a hole and an initial crack under thermal load will be considered.

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It will be treated as a two-dimensional stress/strain problem. The concepts of linear elastic fracture mechanics which lead to the plane strain toughness property, K_{IC}, have already been used in engineering applications. The use of the J-integral and its critical value J_C as a fracture criterion has also been developed into elastic-plastic and fully plastic regimes [8,9,14]. The objective of the analytical work is to provide the relationship between the stress intensity factor and thermal load conditions.

General consideration of the thermal problem

Thermal effects on structures can be grouped into three principal categories:

1. Changes in mechanical properties of material; e.g., elastic modulus, fracture toughness, and yield strength
2. Creep phenomenon associated with the time of hold,
3. Stress and strain arising from temperature change.

Neglecting working temperature changes could lead to wrong analysis results of the stress – deformation state. Because of this, two main temperature effects are most often taken into consideration:

- elastic coefficient changes in a function of temperature changes
- occurrence of thermal deformations.

Material coefficients are generally a function of temperature

$$E = E(T),\ v = v(T) \tag{1}$$

Equation (1) are valid for the steady state, i.e. in the case when the temperature field does not change as a function of time. Since temperature is not generally in the steady state, equations (1) are:

$${}^{t}E = E\left({}^{t}T\right),\ {}^{t}v = v\left({}^{t}T\right) \tag{2}$$

The prefix (‘) means that these coefficient values are values in a certain moment of time t. Equation (2) are known functions for the known temperature in a certain moment of time t

Stress – strain state

Temperature changes will produce the increase of strains, and this strain increase will result in the appearance of stresses. Thermally produced strains (deformations) could be described linearly as:

$$\varepsilon_{ii}^{t} = \alpha_{ii}\left(T - T_{ref}\right) \tag{3}$$

Double indices in this equation mean that the temperature field influences only normal components of thermal strains (deformations). The symbols in this equation are α_{ii} – the coefficient of thermal extension/contraction, or strain, the tensor T_{ref} – the referent temperature value corresponding to the condition when thermal strains are zero, and T – the chosen or working temperature. In the case when isotropic material is considered, α_{ii} must be a diagonal tensor of the second order, so equation (3) becomes:

$$\varepsilon_{ii}^{t} = \alpha\left(T - T_{ref}\right) \tag{4}$$

In addition, the assumption that the temperature field, or temperature distribution, does not depend on body strains is introduced. Therefore, the deformation field occurs because of mechanical and thermal activities and the total deformation is the sum of mechanical ε_{ii}^{M} and thermal ε_{ii}^{T} deformations:

$$\varepsilon_{ij} = \varepsilon_{ij}^{M} + \varepsilon_{ij}^{T} \tag{5}$$

In the expanded form for isotropic materials:

$$\varepsilon_{ij} = \frac{1+v}{E}\sigma_{ij} - \frac{v}{E}\sigma_{kk}\delta_{ij} + \alpha\left(T - T_{ref}\right)\delta_{ij} \tag{6}$$

Free body stress, since temperature change is zero, i.e:

$$\sigma^{T} = 0\ .$$

The inversion of expression (6) will give:

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} - (3\lambda + 2\mu)\alpha\Delta T\delta_{ij} \tag{7}$$

For $i = j$ it follows $\sigma_{ij}^{T} = 0$ and $\varepsilon_{ij}^{T} = 0$. These equations are well known as *Duhamel – Neumann*’s law for thermo elastic behavior. The plane strain state exists in long prismatic or cylindrical bodies, when temperature does not change along the lines parallel to the axes of prismatic or cylindrical bodies, but it could be variable in the cross section. In other words, temperature is independent of the coordinate z. The relations between stresses and deformations in the Cartesian coordinate system for the state of plane strain could be obtained from $\left(\varepsilon_z = y_{xz} = y_{yz} = 0\right)$ in accordance with eq.(7):

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix} = \frac{1}{(1+v)(1-2v)}\begin{bmatrix}1-v & v & 0\\ v & 1-v & 0\\ 0 & 0 & (1-2v)/2\end{bmatrix}\cdot \begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} - \frac{E\alpha\Delta T}{1-2v}\begin{Bmatrix}1\\1\\0\end{Bmatrix} \tag{8}$$

or:

$$\varepsilon_{xx} - (1+v)\alpha\Delta T = \frac{1-v^2}{E}\left(\sigma_{xx} - \frac{v}{1-v}\sigma_{yy}\right) \tag{9}$$

$$\varepsilon_{yy} - (1+v)\alpha\Delta T = \frac{1-v^2}{E}\left(\sigma_{yy} - \frac{v}{1-v}\sigma_{xx}\right),$$
$$\gamma_{xy} = \frac{2(1+v)}{E}\tau_{xy}$$

The plane stress state exists in a slender, thin, plate in the case when there are not temperature changes across the thickness. The relations between stresses and strains for the plane stress state $\left(\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0\right)$ are given:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix} = \frac{1}{1-v^2}\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} - \frac{E\alpha\Delta T}{1-v}\begin{Bmatrix}1\\1\\0\end{Bmatrix} \tag{10}$$

or:

$$\varepsilon_{xx} - (1+v)\alpha\Delta T = \frac{1}{E}\left(\sigma_{xx} - v\sigma_{yy}\right)$$

$$\varepsilon_{yy} - (1+v)\alpha\Delta T = \frac{1}{E}\left(\sigma_{yy} - v\sigma_{xx}\right),$$
$$\gamma_{xx} = \frac{2(1+v)}{E}\tau_{xy} \tag{11}$$

If strains are expressed as a function of displacements in equation (10), then:

$$\sigma_{xx} = \frac{E}{1-v^2}\left[\frac{\partial u}{\partial x} + v\frac{\partial \upsilon}{\partial y} - (1+v)\alpha T\right]$$

$$\sigma_{yy} = \frac{E}{1-v^2}\left[v\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} - (1+v)\alpha T\right], \qquad \tau_{xy} = \frac{E}{2(1+v)}\left(\frac{\partial \upsilon}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{12}$$

Equilibrium equations

The paper considers a two-dimensional body occupying the volume Ω under boundary conditions on the surface G in the field of referent temperature and in the absence of body forces. The equilibrium equations for the infinitesimal thermo mechanical deformations of the elastic body in the Cartesian coordinate system are:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \tag{13}$$

The equation of the thermal equilibrium is:

$$-\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} + Q = \rho C\frac{\partial T}{\partial t} \tag{14}$$

The symbols in this equation are q_i - the thermal flux components in the i- axes directions, Q - the inner thermal origin, c - the specific thermal capacity, ρ - the specific density, T - the temperature.

Boundary conditions

The mechanical boundary conditions (for forces and displacements) are given by the expressions:

$$u = \bar{u}\,,\ \upsilon = \bar{\upsilon} \text{ on } G_u \tag{15}$$

$$\sigma_{xx}n_x + \sigma_{xy}n_y = \bar{f}_x \text{ and } \sigma_{xy}n_x + \sigma_{yy}n_y = \bar{f}_y \text{ on } G_f \tag{16}$$

where $\bar{u}$ and $\bar{\upsilon}$ are the specified displacement components on G_u in the x and y directions, respectively; f_x and f_y are the prescribed force components on G_f in the x and y directions, respectively; and the vector $\boldsymbol{n} = n_x\boldsymbol{i} + n_y\boldsymbol{J}$ is a unit outward vector which is normal to the boundary G_f. The initial condition and the boundary conditions for the thermal equilibrium in the solid are specified as:

$$T(x,y,0) = \bar{T}_0(x,y) \quad T = \bar{T} \text{ on } G_t$$

$$-q_x n_x - q_y n_y = \bar{q} \text{ on } G_q \tag{17}$$

$$-q_x n_x - q_y n_y + h(T - T_\infty) = 0 \text{ on } G_c$$

where h is the convection coefficient, T_∞ is the prescribed temperature of the surrounding medium near the boundary G_c, $\bar{T}_0(x,y)$ is the specified temperature everywhere in the solid body when time equals zero, $\bar{q}$ is the prescribed heat flux on the boundary G_q and T is the prescribed temperature on the boundary G_t

Consideration of the thermal state

The relation between the thermal flux vector and the temperature field is given by the *Fourier* law heat conduction equation:

$$q_x = -k\frac{\partial T}{\partial x} \text{ and } q_y = -k\frac{\partial T}{\partial y} \tag{18}$$

so, when relation (18) is substituted in equation (14), and after the rearrangement, equation (14) becomes

$$\rho c\frac{\partial T}{\partial t} - \operatorname{div}(k\nabla T) = 0 \tag{19}$$

If the problem is considered as linear, equation (14) could be solved using *Galerkin's* residual method. In this case, it is necessary to introduce the virtual field T_V, so equation (14) becomes:

$$\int_\Omega T_V\left(\rho c\frac{\partial T}{\partial t} - \operatorname{div}(k\nabla T) - Q\right)d\Omega \tag{20}$$

Applying the divergence theorem using the boundary G if $G = (Gt \cup G_q \cup G_c)$ equation (20) becomes:

$$\int_\Omega T_V\rho c\frac{\partial T}{\partial t}d\Omega + \int_\Omega T_V div(k\nabla T)d\Omega = = \int_\Omega T_V Q d\Omega + \int_G T_V k\nabla T\cdot\vec{n}dG \tag{21}$$

In order to solve equation (21) numerically, it is necessary to introduce a discretization of the domain Ω and choose appropriate test functions.

Determination of the stress intensity factor under thermal loads

For the fairly complicated geometry and load conditions in the specimens of this work, the finite element technique is used to calculate both the temperature distribution and the stress intensity factor. Once a finite element solution has been obtained, the values of the stress intensity factor can be extracted from it. Three approaches to the calculation of the stress intensity factor can be used: the direct method, the indirect method and the J-integral method. The direct method is based on the use of specialized crack-tip elements that contain K_I and K_{II} (mode *I* and *II* stress intensity factors) directly as additional unknowns. This has been discussed in references [5,11]. In this work, the *J*-integral method is used.

Wilson and Ainsworth [3] proposed the two-dimensional thermal J^*-integral for linear thermo elastic materials which can be expressed in a simple form:

$$J^* = \int_\Gamma\left(W^*dX_2 - \sigma_{ij}\frac{\partial u_i}{\partial X_1}n_j dS\right) + \frac{E\alpha}{1-2\nu}\int \varepsilon_{ii}\frac{\partial\theta}{\partial X_1}dA \tag{22}$$

$$W^* = W - \frac{E\alpha\theta}{2(1-2\nu)}\varepsilon_{ii} \tag{23}$$

$$\sigma_{ij} = \lambda\varepsilon_{ii}\delta_{ij} + 2\mu\varepsilon_{ij} - \frac{E\alpha}{1-2\nu}\theta\delta_{ij} \tag{24}$$

$$W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} \tag{25}$$

where μ and λ are the Lame constants, θ is the temperature and α is the coefficient of thermal expansion. The physical interpretation of J is the energy release rate, and hence in the case of a linear thermo-elastic material

$$J^* = \left(1-\nu^2\right) K_I^2 / E \tag{26}$$

in which K_I is the stress intensity factor (SIF). The FEM applications together with the J^*-integral approach in thermo-elastic fracture mechanics, including the behavior of a cracked structure subjected to transient thermal singularities, are illustrated in references [5-10]. The values of K_I for various examples were calculated in accordance with equation (26).

Numerical validation

To illustrate the thermal stress analysis, a plate with a hole is considered here, Fig.1. This case is selected to examine the multi-connected effects of a hole when a thermal gradient crosses the width of the plate. Since loads and geometries are symmetrical with respect to the x-axis, only the upper half of the model is analyzed. The Finite Element Method (FEM) is used to determine temperature distribution. The finite element models for two different hole sizes, 0.25W and 0.35W, are considered. The results of the temperature distribution along the x-axis and the boundary of the holes are shown in Figures 2 and 3.

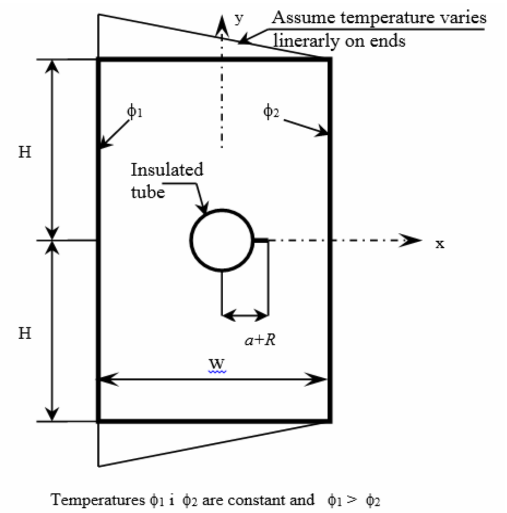

Figure 1. Sheet with the traction-free boundaries

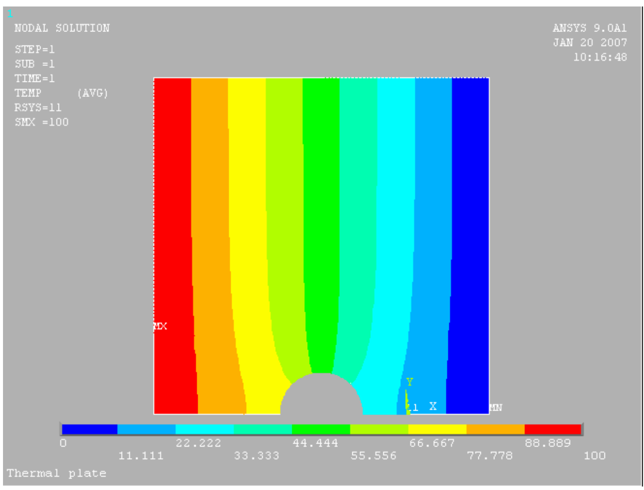

Figure 2. Temperature distribution along the x-axis and the boundary of the hole

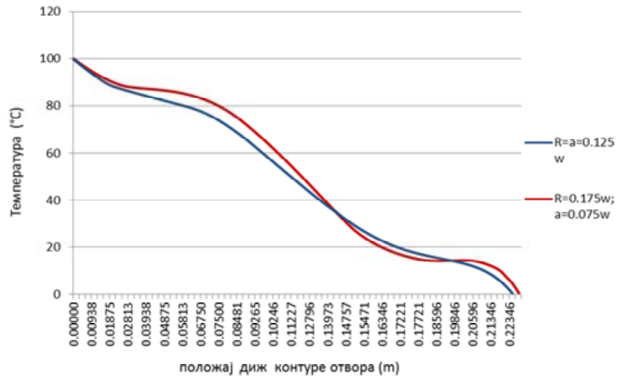

Figure 3. Temperature distribution along the x-axis and the boundary of the hole

(Case 1: R=a=0.125w and Case 2: R=0.175w; a=0.075w)

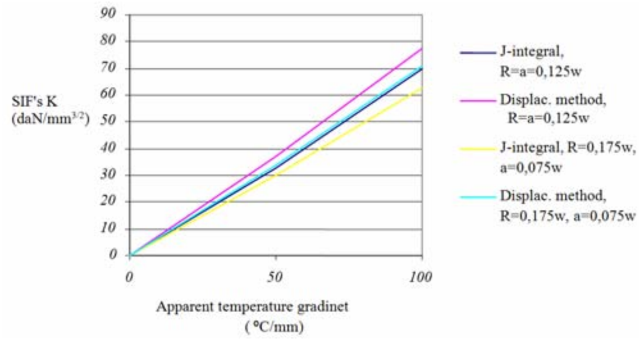

Figure 4. SIFs versus temperature gradients for the traction-free boundary conditions

Fig.4 shows the results of the stress intensity factors versus the temperature gradients for two different sizes of the hole. The stress intensity factors are derived from the values of the J-integral approach (Eq.22) and the crack opening displacement method. The values of the stress intensity factors evaluated by two different approaches agree well. Therefore, for the condition a+R=0.25w=cons, the SIF values are lower for the higher values of the hole size.

Conclusions

This paper considers the problem of thermal loads founded on theoretical equations, primarily for the plane state. The paper discusses the steps needed to study the problem of thermo mechanical loading to the numerical one. The Finite Element Method (FEM) is used to determine thermally induced stresses in cracked structural components. Two computation approaches are used to determine stress intensity factors of damaged structural elements under thermal loads: the J-integral approach and the crack opening displacement method. The values of the stress intensity factors evaluated by two different approaches and the FEM agree well.

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Uticaj temperature na parametre mehanike loma

Razvijen je kompletan model za određivanje uticaja termičkih opterećenja na vrednosti faktora intenziteta napona. Posebna pažnja bila je usmerena na ponašanje polja oplate sa kružnim otvorom i inicijalnom prskotinom pod dejstvom termičkih opterećenja. Za određivanje raspodele temperature i naponskog stanja u polju oplate sa otvorom korišćen je metod konačnih elemenata. Za ovaj strukturalni element sa kružnim otvorom i radijalnom prskotinom razmatran je i faktor intenziteta napona (FIN). Za određivanje faktora inteziteta napona korišćena su dva pristupa i to: metod konačnih elemenata u sprezi sa modifikovanim J^* integral pristupom i metod konačnih elemenata sa singularnim konačnim elementima oko vrha prskotine.

Ključne reči: **mehanika loma, termičko opterećenje, uticaj temperature, naponsko stanje, struktura letelice, metoda konačnih elemenata.**

Влияние температуры на параметры механики разрушения

Здесь развита полная модель для определения влияния тепловой нагрузки на значение коэффициентов интенсивности напряжений. Особое внимание было сосредоточено на поведении полей опалубки с круглым отверстием и с начальной трещиной под действием термической нагрузки. Чтобы определить распределение нагрузки и напряжённое состояние в области опалубки с отверстием, использован метод конечных элементов. Для этого структурного элемента с круглым отверстием и радиальными трещинами Комитет рассмотрел и коэффициент интенсивности напряжений (КИН). Для определения коэффициентов интенсивности напряжений использован метод конечных элементов в сочетании с модифицированным J* интегральным подходом и метод конечных элементов с применением особых сингулярных конечных элементов.

Ключевые слова: **механика разрушения, термические нагрузки, влияние температуры, напряжённое состояние, структура самолёта, метод конечных элементов.**

Les effets de la température sur les paramètres de la mécanique de la fracture

Le modèle complet, développé pour la détermination des effets des charges thermiques sur les valeurs du facteur de l'intensité de tension, est présenté dans cet article. L'attention particulière a été prêtée au comportement du champ de revêtement à l'ouverture circulaire et à la rupture initiale sous l'effet des charges thermiques. Pour déterminer la distribution de la charge et l'état de tension dans le champ de revêtement à l'ouverture on a utilisé la méthode des éléments finis. On a considéré aussi le facteur de l'intensité de tension (FIT) pour cet élément structural à l'ouverture circulaire et à la rupture radiale. La méthode des éléments finis a été utilisée pour déterminer le facteur d'intensité couplé à l'intégrale J* modifiée. Cette méthode a été appliquée en utilisant les éléments singuliers finis.

Mots clés: **mécanique de fracture, charge thermique, effet de température, état de tension, structure d'aéronef, méthode des éléments finis.**