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Analysis of Stresses in a Thin Rotating Disc With Inclusion and Edge Loading

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Seth's transition theory is applied to the problems of elastic-plastic transitional stresses in a thin rotating disc with inclusion and edge loading. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible and incompressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from the Tresca yield condition. It has been observed that, in the absence of load, a rotating disc with inclusion requires significantly higher values of angular speed for incompressible materials as compared to compressible materials. With the introduction of edge loading, lower values of angular speed are required to yield at the internal surface for incompressible/compressible materials. Radial stress has the maximum value on the internal surface of the rotating disc made of incompressible materials compared to the radial stress value of the disc made of compressible materials

Key words: **stress, stress analysis, disc, rotating disc, load, elasticity, plasticity.**

Nomenclature

a,b	– Inner and outer radii of the disc [m];
ω	– Angular velocity of rotation, [s^{-1}];
u,v,w	– displacement components, [m];
ρ	– Density of material, [kgm^{-3}];
C	– Compressibility, [-];
T_{ij}, e_{ij}	– Stress [$kgm^{-1}s^{-2}$]and Strain rate tensor;
Y	– Yield stress, [$kgm^{-1}s^{-2}$].

Greek letters

$R = r/b; R_0 = a/b$	– Radii ratio, [-],
$\sigma_0 = T_0 / Y$	– load,
$\Omega^2 = \rho_0 \omega^2 b^2 / Y$	– angular speed,
σ_r	– Radial stress component (T_{rr} / Y), [-]
σ_θ	– Circumferential stress component ($T_{\theta\theta} / Y$), [-]
A_1, A_2	– Constants of integrations, [-]

Introduction

THE accurate determination of stresses in rotating discs is important for efficient design and material usage in engineering applications such as rotors of rotating machinery, flywheels, shrink fits, turbines, compressors, high speed gear engines, computer disc drives, etc. The analysis of thin rotating discs made of isotropic material has been discussed extensively by Timoshenko and Goodier [1] for the elastic range and by Chakrabarty [2] and Heyman [3] for the plastic range. Güven [4] discussed the problem with rigid inclusion under the assumptions of the Tresca yield condition, its associated flow rule and linear strain hardening. To obtain the stress distribution, Güven matched the plastic stresses at the same radius $r = z$ of the disc. Perfect elasticity and ideal plasticity are two extreme properties of the material and the use of an ad-hoc rule such as the yield condition amounts to divide the two extreme properties by a sharp line which is not physically possible. When a material passes from one state to another, qualitatively different state transition takes place. Since this transition is non-linear in character and difficult to investigate, researchers have taken certain ad-hoc assumptions such as the yield condition, the incompressibility condition and a certain law which may or may not be valid for the problem. Seth's transition theory [5] does not require these assumptions and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and an asymptotic solution at critical points or turning points of the differential equations defining the deformed field and it has been successfully applied to a large number of problems [6-14]. Seth [6] has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ij} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], \qquad (1)$$
$$i = (1,2,3)$$

where n is the measure and $\overset{A}{e}_{ii}$ is the principal Almansi finite strain component. For n = -2, -1, 0, 1, 2 it gives Cauchy, Green Hencky, Swainger and Almansi measures, respectively.

The problem of analyzing stresses in a rotating annular disc mounted on a rigid circular shaft and edge load occurs frequently in industrial applications. In this paper, the

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plastic stresses have been derived by using Seth's transition theory. The results have been discussed numerically and depicted graphically.

Mathematical model

A thin disc of constant density with a central bore of the inner radius a and the outer radius b is considered. The annular disc is mounted on a rigid shaft with edge loading. The disc rotates with the angular speed ω about the axis perpendicular to its plane and passed through the center as shown in Fig.1. The thickness of the disc is assumed to be constant and is taken to be sufficiently small so that it is effectively in a state of plane stress, i.e. the axial stress T_{zz} is zero.

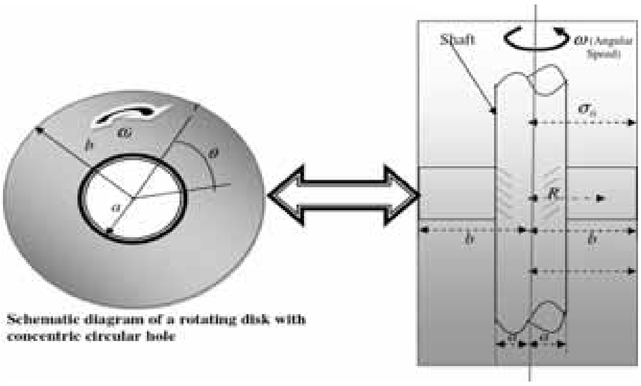


Figure 1. Geometry of the rotating disc

Boundary conditions:

The disc considered in the present study has constant density and is subjected to load. The inner surface of the disc is assumed to be fixed to a shaft. Mechanical load is applied to the outer surface of the disc. Thus, the boundary conditions of the problem are given by:

$$\begin{aligned}&(\mathrm{i})\ \ r=a,\ u=0\\&(\mathrm{ii})\ \ r=b,\ T_{rr}=T_0\end{aligned} \tag{2}$$

where u and T_{rr} denote displacement and stress along the radial direction.

Formulation of the Problem

The displacement components in the cylindrical polar co- ordinate (r,θ,z) are:

$$u=r(1-\beta)\,;\ v=0\,;\ w=dz \tag{3}$$

where β is the function of $r=\sqrt{x^2+y^2}$ only and d is a constant.

The finite strain components are given by Seth [6] as:

$$\begin{aligned}\overset{A}{e}_{rr}&\equiv\frac{\partial u}{\partial r}-\frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2=\frac{1}{2}\left[1-\left(r\beta'+\beta\right)^2\right],\\ \overset{A}{e}_{\theta\theta}&\equiv\frac{u}{r}-\frac{u^2}{2r^2}=\frac{1}{2}\left[1-\beta^2\right],\\ \overset{A}{e}_{zz}&\equiv\frac{\partial w}{\partial z}-\frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2=\frac{1}{2}\left[1-(1-d)^2\right],\\ \overset{A}{e}_{r\theta}&=\overset{A}{e}_{\theta z}=\overset{A}{e}_{zr}=0\end{aligned}\quad, \tag{4}$$

where $\beta'=d\beta/dr$ and the meaning of the superscripts "A" is Almansi.

By substituting eq.(4) in eq.(1), the generalized components of strain are given by:

$$\begin{aligned}&e_{rr}=\frac{1}{n}\left[1-\left(r\beta'+\beta\right)^n\right],\ e_{\theta\theta}=\frac{1}{n}\left[1-\beta^n\right],\\&e_{zz}=\frac{1}{n}\left[1-(1-d)^n\right],\ e_{r\theta}=e_{\theta z}=e_{zr}=0\end{aligned}\quad, \tag{5}$$

The stress –strain relations for isotropic material are given by [8]:

$$T_{ij}=\lambda\delta_{ij}I_1+2\mu e_{ij}\ ,\ (i,j=1,2,3\,), \tag{6}$$

where T_{ij} and e_{ij} are the stress and strain components, λ and μ are Lame's constants, $I_1=e_{kk}$ is the first strain invariant and δ_{ij} is the Kronecker delta.

Eq.(6) for this problem becomes:

$$\begin{aligned}&T_{rr}=\frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2\mu e_{rr},\\&T_{\theta\theta}=\frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2e_{\theta\theta}\quad,\\&T_{r\theta}=T_{\theta z}=T_{zr}=T_{zz}=0\end{aligned} \tag{7}$$

By substituting eq.(4) in eq.(6), the strain components in terms of stresses are obtained as [12]:

$$\begin{aligned}&e_{rr}=\frac{\partial u}{\partial r}-\frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2=\frac{1}{2}\left[1-\left(r\beta'+\beta\right)^2\right]=\\&=\frac{1}{E}\left[T_{rr}-\left(\frac{1-C}{2-C}\right)T_{\theta\theta}\right],\\&e_{\theta\theta}=\frac{u}{r}-\frac{u^2}{2r^2}=\frac{1}{2}\left[1-\beta^2\right]=\frac{1}{E}\left[T_{\theta\theta}-\left(\frac{1-C}{2-C}\right)T_{rr}\right],\\&e_{zz}=\frac{\partial w}{\partial z}-\frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2=\frac{1}{2}\left[1-(1-d)^2\right]=\\&-\frac{(1-C)}{E\left(2-C\right)}\left[T_{rr}-T_{\theta\theta}\right],\\&e_{r\theta}=e_{\theta z}=e_{zr}=0\end{aligned} \tag{8}$$

where E is Young's modulus and C is the compressibility factor of the material. In term of Lame's constant, these are given by $E=\dfrac{\mu(3\lambda+2\mu)}{(\lambda+\mu)}$ and $C=\dfrac{2\mu}{\lambda+2\mu}$.

Substituting eq. (5) in eq. (7), we get the stresses as:

$$\begin{aligned}&T_{rr}=\frac{2\mu}{n}\left[3-2C-\beta^n\left\{1-C+\left(2-C\right)\left(P+1\right)^n\right\}\right],\\&T_{\theta\theta}=\frac{2\mu}{n}\left[3-2C-\beta^n\left\{2-C+\left(1-C\right)\left(P+1\right)^n\right\}\right],\\&T_{r\theta}=T_{\theta z}=T_{zr}=T_{zz}=0\end{aligned} \tag{9}$$

Equations of equilibrium are all satisfied except

$$\frac{d}{dr}\left(rT_{rr}\right)-T_{\theta\theta}+\rho\omega^2r^2=0 \tag{10}$$

where ρ is the density of the material of the disc.

Using eq.(9) in eq.(10), we get a non-linear differential equation in β as:

$$\begin{aligned}&\left(2-C\right)n\beta^{n+1}P\left(P+1\right)^{n-1}\frac{dP}{d\beta}=\frac{n\rho\omega^2r^2}{2\mu}+\\&+\beta^n\left[1-\left(P+1\right)^n-nP\left\{1-C+\left(2-C\right)\left(P+1\right)^n\right\}\right]\end{aligned} \tag{11}$$

where $r\beta'=\beta P$ (P is the function of β and β is the function of r).

The transition or turning points of β in equation (11) are $P \to -1$ and $P \to \pm\infty$.

Solution Through the Principal Stress

For finding the plastic stress, the transition function is taken through the principal stress (see Seth's [5,6], Hulsurkar [7], Gupta [10,11] and Thakur [12-30]) at the transition point $P \to \pm\infty$. The transition function R is defined as:

$$R = T_{\theta\theta} = \left(\frac{2\mu}{n}\right)\left[(3-2C) - \beta^n\left\{2 - C + (1-C)(P+1)^n\right\}\right] \quad (12)$$

Taking the logarithmic differentiation of equation (12) with respect to *r* and using equation (11), we get

$$\frac{d(\log R)}{dr} = -\left[\frac{\beta^n\left(\frac{1-C}{2-C}\right)\left[1-(P+1)^n - n(1-C)P + \frac{n\rho\omega^2 r^2}{2\mu\beta^n}\right] + (2-C)nP\beta^n}{r\left[3-2C-\beta^n\left\{2-C+(1-C)(P+1)^n\right\}\right]}\right] \quad (13)$$

Taking the asymptotic value of equation (13) at $P \to \pm\infty$ and integrating, we get

$$R = A_1 r^{-1/(2-C)} \quad (14)$$

where A_1 is a constant of integration which can be determined by the boundary condition.

From equation (12) and (14), we have

$$T_{\theta\theta} = \left(\frac{2\mu}{n}\right) A_1 r^{-1/(2-C)} \quad (15)$$

Substituting equation (15) in equation (9) and integrating, we get

$$T_{rr} = \left\{\frac{2\mu(2-C)}{n(1-C)}\right\} A_1 r^{-1/(2-C)} - \frac{\rho\omega^2 r^2}{3} + \frac{A_2}{r} \quad (16)$$

where A_2 is a constant of integration, which can be determined by the boundary condition.

Substituting equations (15) and (16) in the second equation of (7), we get

$$\beta = \sqrt{1 - \frac{2(1-C)}{E(2-C)}\left[\frac{\rho\omega^2 r^2}{3} - \frac{A_2}{r}\right]} \quad (17)$$

Substituting equation (17) in equation (3), we get

$$u = r - r\sqrt{1 - \frac{2(1-C)}{E(2-C)}\left[\frac{\rho\omega^2 r^2}{3} - \frac{A_2}{r}\right]} \quad (18)$$

where $E = \frac{2\mu(3-2C)}{(2-C)}$ .is Young's modulus in term and is $\nu = \frac{1-C}{2-C}$ Poisson's ratio in terms of a compressibility factor.

Using boundary condition (11) in equations (16) and (18), we get

$$A_1 = \frac{n\nu}{2\mu b^\nu}\left[bT_0 + \frac{\rho\omega^2\left(b^3-a^3\right)}{3}\right], \quad A_2 = \frac{\rho\omega^2 a^3}{3} \quad (19)$$

Substituting equation (19) in equations (15), (16), and (18) respectively, we get the transitional stresses and displacement as:

$$T_{\theta\theta} = \frac{\nu}{r}\left(\frac{r}{b}\right)^\nu\left[bT_0 + \frac{\rho\omega^2\left(b^3-a^3\right)}{3}\right] \quad (20)$$

$$T_{rr} = \left(\frac{r}{b}\right)^{\nu-1} T_0 + \frac{\rho\omega^2}{3r}\left[\left(\frac{r}{b}\right)^\nu\left(b^3-a^3\right) - r^3 + a^3\right] \quad (21)$$

$$u = r - r\sqrt{1 - \frac{2\nu}{E}\frac{\rho\omega^2\left(r^3-a^3\right)}{3r}} \quad (22)$$

$$T_{rr} - T_{\theta\theta} = \left[\left(\frac{r}{b}\right)^{\nu-1}(1-\nu)T_0 + \frac{\rho\omega^2}{3r}\left\{\left(\frac{r}{b}\right)^\nu\left(b^3-a^3\right)(1-\nu) - r^3 + a^3\right\}\right] \quad (23)$$

Initial yielding: It is seen from eq. (23) that $|T_{rr} - T_{\theta\theta}|$ is maximum at the internal surface (that is at $r = a$), therefore, yielding will take at the internal surface of the disc and equation (23) gives:

$$|T_{rr} - T_{\theta\theta}|_{r=a} = \left|\left(\frac{a}{b}\right)^{\nu-1}(1-\nu)T_0 + \frac{\rho\omega^2}{3a}\left(\frac{a}{b}\right)^\nu\left(b^3-a^3\right)(1-\nu)\right| \equiv Y(say).$$

where Y is yield stress and the angular speed necessary for initial yielding is given by

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \left|\frac{3ab^2\left[1-\sigma_0(1-\nu)\left(\frac{a}{b}\right)^{\nu-1}\right]}{\left(\frac{a}{b}\right)^\nu\left(b^3-a^3\right)(1-\nu)}\right| \quad (24)$$

where $\sigma_0 = T_0/Y$ and $\omega_i = \frac{\Omega_i}{b}\sqrt{Y/\rho}.$

Fully Plastic State: The angular speed $\omega_f > \omega_i$ for which the rotating disc becomes fully plastic $(\nu \to 1/2 = 0.5)$ or $C \to 0$ at the external surface and eq.(23) becomes:

$$|T_{rr} - T_{\theta\theta}|_{r=b} = \left|\frac{T_0}{2} - \frac{\rho\omega^2}{6b}\left(b^3-a^3\right)\right| \equiv Y^*(say)$$

where Y^* is the yield stress occurring for the fully plastic state and the angular speed required for the disc to become fully plastic as given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y^*} = \left|6.\frac{(\sigma_0/2)-1)}{1-\left(a^3/b^3\right)}\right| \quad (25)$$

where $\sigma_0^* = T_0 / Y^*$ and $\omega_f = \frac{\Omega_f}{b}\sqrt{Y^* / \rho}$.

We introduce the following non-dimensional components: $R = r / b$, $R_0 = a / b$, $\sigma_r = T_{rr} / Y$, $\sigma_\theta = T_{\theta\theta} / Y$, $U = u / b$, $\Omega^2 = \rho_0 \omega^2 b^2 / Y$, $\sigma_0 = T_0 / Y$ and $H = Y / E$. Elastic-plastic transitional stresses, displacement and angular speed from equations (20), (21), (22) and (24) in a non-dimensional form become:

$$\sigma_\theta = \nu R^{\nu-1}\left[\sigma_0 + \frac{\Omega_i^2}{3}\left(1-R_0^3\right)\right],$$

$$\sigma_r = \sigma_0 R^{\nu-1} + \frac{\Omega_i^2}{3R}\left[R^\nu\left(1-R_0^3\right) - R^3 + R_0^3\right], \qquad (26)$$

$$U = R - R\sqrt{1-\frac{2\nu H\Omega_i^2}{3R}\left(R^3 - R_0^3\right)}$$

And

$$\Omega_i^2 = \frac{3\left(1-\sigma_0\left(1-\nu\right)R_0^{\nu-1}\right)}{R_0^{\nu-1}\left(1-R_0^3\right)\left(1-\nu\right)} \qquad (27)$$

Stresses, displacement and angular speed for fully plastic $\nu = 0.5$ or $C \to 0$ are obtained from eq. eqns. (26) and (25) as:

$$\sigma_\theta = \frac{1}{2\sqrt{R}}\left[\sigma_0 + \frac{\Omega_f^2}{3}\left(1-R_0^3\right)\right],$$

$$\sigma_r = \frac{\sigma_0}{\sqrt{R}} + \frac{\Omega_f^2}{3R}\left[\sqrt{R}\left(1-R_0^3\right) - R^3 + R_0^3\right],$$

$$U_f = R - R\sqrt{1-\frac{H\Omega_i^2}{3R}\left(R^3 - R_0^3\right)}, \qquad (28)$$

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y^*} = \frac{6\left(1-\frac{\sigma_0}{2}\right)}{\left(1-R_0^3\right)}$$

Equations (26), (27) and (28) are the same as those given by *Thakur* [13], when load is not applied in the rotating disc with rigid inclusion.

Numericaly Illustration and Discussion

For calculating the stresses, angular speed and displacement based on the above analysis, the following values have been taken as C = 0.00, 0.25, 0.5, E/Y = 0.2 and σ_0 = 0, 0.125 and 0.2 respectively.

Table 1: Angular speed required for initial yielding and a fully plastic state for different load values

	Load σ_0	Compressibility of Material C		Angular Speed required for initial yielding Ω_i^2	Angular Speed required for a fully plastic state Ω_f^2	Percentage increase in Angular speed $\left(\sqrt{\frac{\Omega_f^2}{\Omega_i^2}}-1\right)\times 100$
0.5 < R < 1.0	0	0	Incompressible materials	4.848732	6.8571431	18.92071 %
	0.125	0		4.420161	6.4285714	20.59747 %
	0.2	0		4.163018	6.1714286	21.75553 %
	0	0.25	Compressible materials	4.037701	6.8571431	30.31803 %
	0.125	0.25		3.609129	6.4285714	33.46151 %
	0.2	0.25		3.351986	6.1714286	35.6881 %
	0	0.5	Compressible materials	3.239797	6.8571431	45.48315 %
	0.125	0.5		2.811226	6.4285714	51.22004 %
	0.2	0.5		2.554083	6.1714286	55.4445 %

It can also be seen from Table 1 that incompressible materials require higher percentage increase in angular speed to become fully plastic as compared to a rotating disc made of compressible materials.

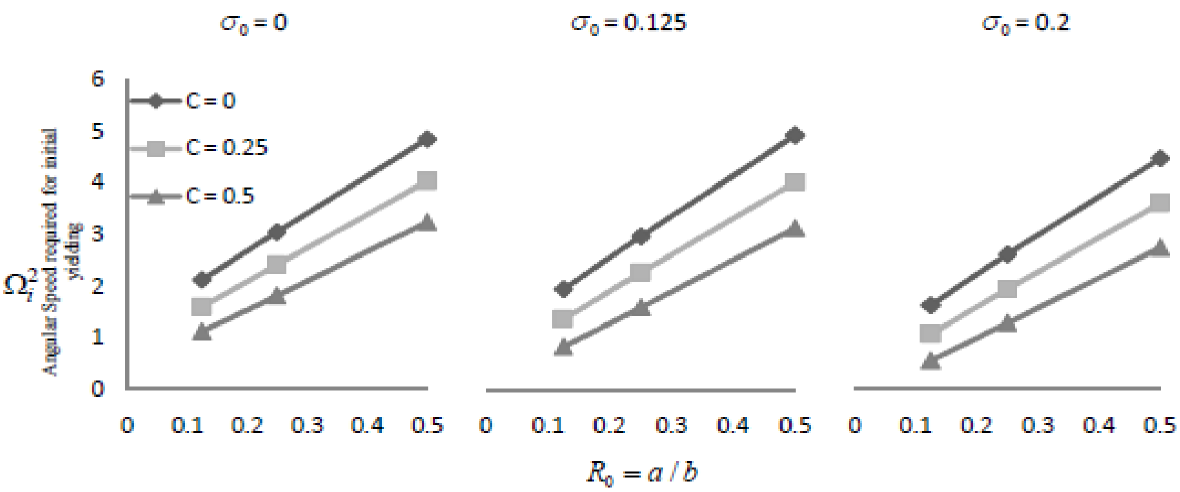


Figure 2. Angular speed required for initial yielding at the internal surface of the rotating disc with inclusion for C = 0, 0.25, 0.5 at different values of load along the radii ratio $R_0 = a / b$

The curves have been drawn in Fig.2 between the angular speed required for initial yielding and various radii ratio $R_0 = a/b$ for C = 0, 0.25, 0.5 and σ_0 = 0, 0.125. 0.2 of the disc with rigid inclusion. It has been observed that, in the absence of load, the rotating disc with inclusion requires significantly higher values of angular speed for incompressible materials as compared to compressible materials. With the introduction of edge loading, lower

Meaning: sigma theta σ_θ (*circumferential stress*); sigma r- σ_r (*Radial stress*) and displacement-*U*, Load- σ_0

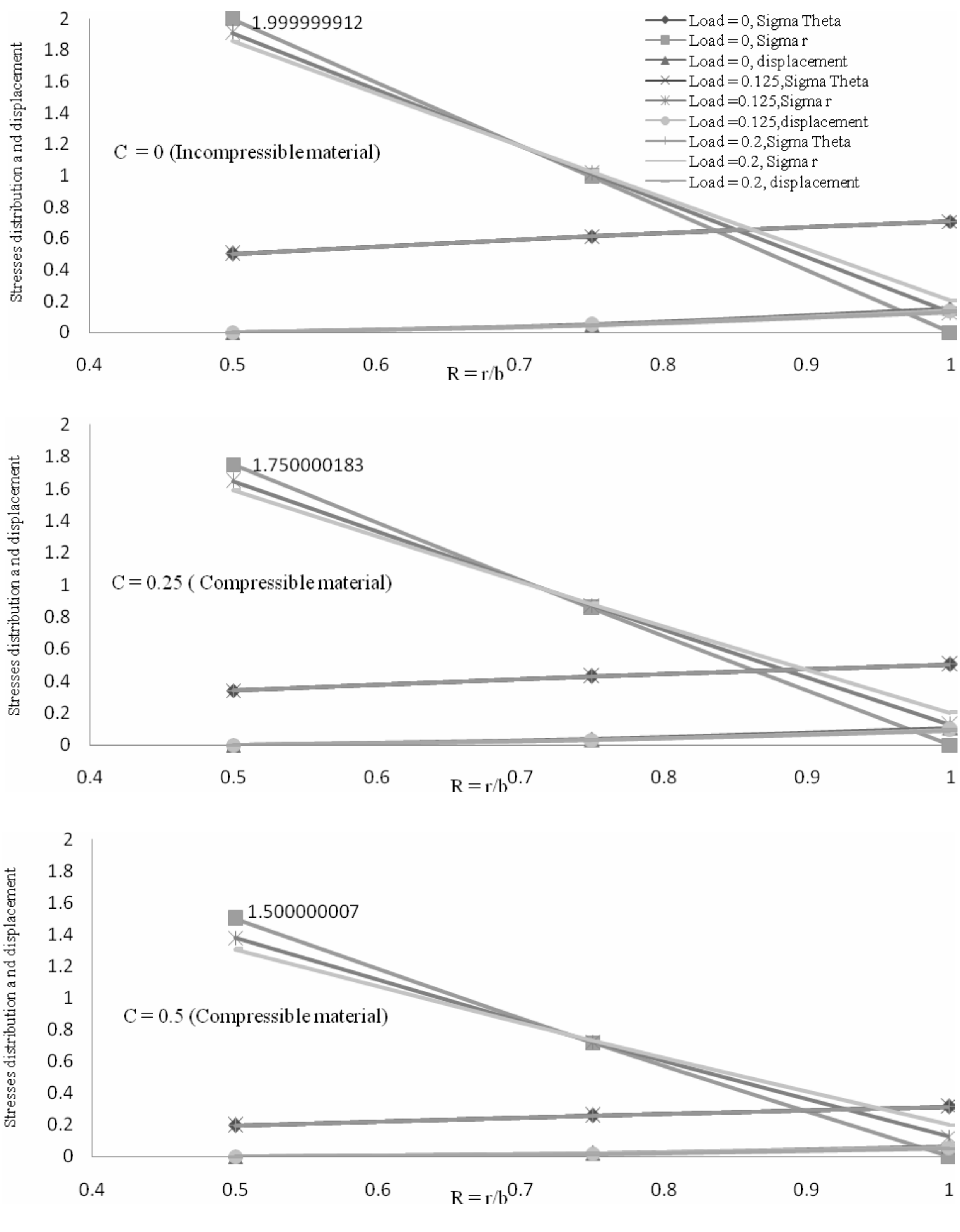
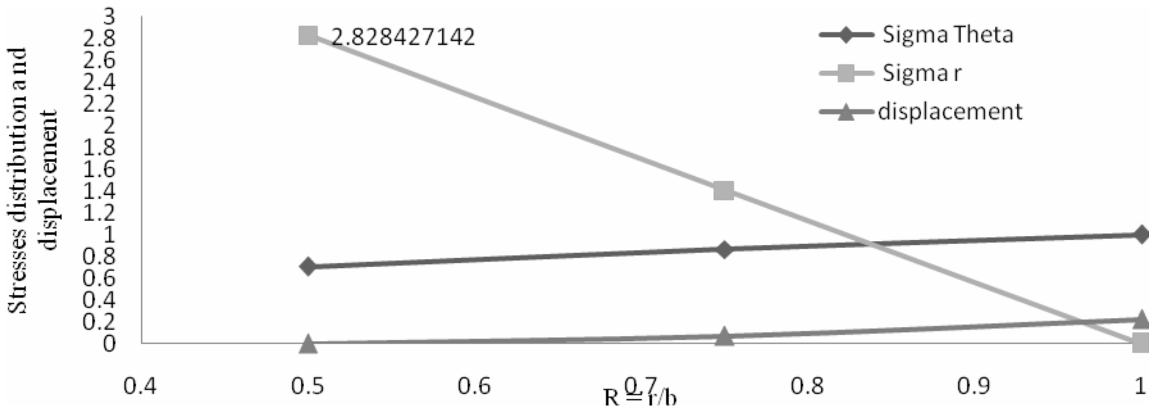


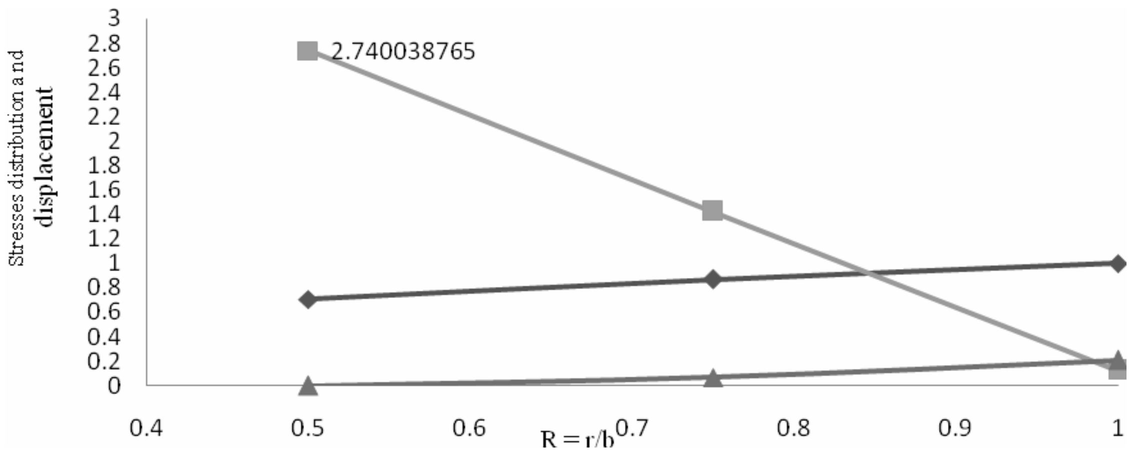
Figure 3. Stresses distribution and displacement at the elastic-plastic transitional state of the rotating disc with inclusion and different values of load along the radii ratio $R = r/b$.

Meaning: sigma theta σ_θ (*circumferential stress*); sigma r- σ_r (*Radial stress*) and displacement-U_f, Load- σ_0

$\sigma_0 = 0$



$\sigma_0 = 0.125$



$\sigma_0 = 0.2$

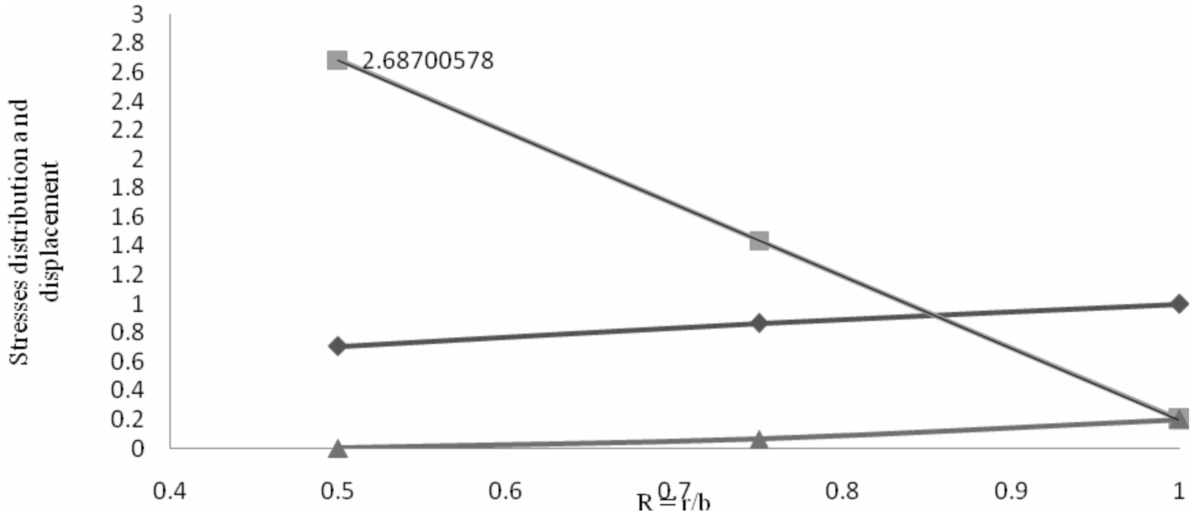


Figure 4. Stresses distribution and displacement for the fully plastic state of the rotating disc with inclusion and different values of load along the radii ratio $R = r / b$.

values of angular speed are required to yield at the internal surface for incompressible/compressible materials. In Figs.3-4, the curves have been drawn for stresses distribution and displacement with respect to the radius R = r/b for the elastic-plastic transitional state and a fully plastic state respectively. It has been seen that the radial stress has the maximum value at the internal surface of the rotating disc made of incompressible materials as compared to compressible materials. With the effect of edge loading, radial stresses must be decreased with the increased values of edge load at the elastic-plastic and fully plastic state.

Conclusion

It has been observed that, in the absence of load, the rotating disc with inclusion requires significantly higher values of angular speed for incompressible materials as compared to compressible materials. With the introduction of edge loading, lower values of angular speed are required to yield at the internal surface for incompressible/compressible materials. It has been seen that the radial stress has the maximum value at the internal surface of the rotating disc made of incompressible materials as compared to compressible materials. With the effect of edge loading, radial stresses must be decreased with increased values of edge load at the elastic-plastic and fully plastic state.

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Analiza napona u tankom rotirajućem disku sa uključivanjem opterećenja na ivicama

Setovi prelazne teorije su primenjeni na problem elastoplastičnih prelaznih napona kod problema tankog rotirajućeg diska sa opterećenim ivicama. U ovom razmatranju nije pretpostavljen kriterijum tečenja niti drugi konvencionalni zakon. Prezentovani rezultati su primenljivi na stišljive i nestišljive materijale.

Ako je dodatni uslov nekompresibilnosti postavljen, onda izraz za naponsko stanje proističe iz Treskinog uslova tečenja. Uočeno je da je u prisustvu opterećenja kod diska koji se obrće u području visokih vrednosti ugaonih brzina neophodno da se razmatra uticaj kompresibilnosti materijala. Sa uvođenjem ivičnog opterećenja, niže vrednosti ugaone brzine su potrebne za nekompresione/kompresione materijale.

Radijalni naponi imaju maksimalne vrednosti na unutrašnjoj površini rotirajućeg diska napravljenog od nekompresibilnog materijala kao i poređeni sa kompresibilnim materijalom.

Ključne reči: **naponsko stanje, analiza napona, disk, rotacioni disk, opterećenje, elastičnoplastičnost.**

Анализ напряжений в тонком вращающемся диске с включением нагрузки на краях

Представленная переходная теория была применена на проблеме упругопластических переходных напряжений у проблем тонкого вращающегося диска с кромками под нагрузкой. В этой статье не предусмотрен и не рассматриван критерий потока, но даже ни другие обычные конвенциональные законы. Представленные результаты применимы к сжимаемым и несжимаемым материалам. Если установлено дополнительное условие несжимаемости, то выражение для напряженного состояния происходит в результате потока Трески. Было обнаружено, что в присутствии нагрузки у вращающегося диска в области высоких значений угловой скорости необходимо учитывать влияние сжимаемости материала. С введением нагрузки на кромках, низкие значения угловой скорости необходимы для несжимаемых / сжимаемых материалов. Радиальные напряжения имеют максимальные значения на внутренней поверхности вращающегося диска, изготовленного из несжимаемого материала, а они сравниваны и с значениями на дисках из сжимаемого материала.

Ключевые слова: **напряженное состояние, анализ напряжений, диск, вращающийся диск, нагрузка, упругопластичность.**

Analyse de la tension dans le mince disque rotatif avec inclusion de la charge sur les bords

La théorie transitoire de Seth a été appliquée au problème des tensions transitoires chez le disque mince rotatif ayant les bords chargés. Dans cette considération le critère du courant ainsi que d'autres lois conventionnelles n'ont pas été supposés. Les résultats présentés peuvent s'appliquer aux matériaux compressibles et incompressibles. Si la condition additionnelle de la non compressibilité est posée, l'expression pour l'état de tension provient de la condition du courant de Tesca. On a remarqué que dans la présence de charge chez le disque rotatif tournant dans le domaine de hautes valeurs de la vitesse d'angle il était nécessaire de prendre en considération l'influence de la compressibilité des matériaux. Si l'on introduit la charge de bord, la plus petite vitesse d'angle est nécessaire pour les matériaux incompressibles ou compressibles. Les tensions radiales ont les valeurs maximales sur la surface interne du disque rotatif produit en matière incompressible ainsi comparé avec le matériau compressible.

Mots clés: **état de tension, analyse de tension, disque, disque rotatif, charge, élasticité plastique.**