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Modeling of the Metal Cylinder Acceleration Under Explosive Loading

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The paper considers the motion of a cylindrical metal liner caused by expanding products of internal detonation. A new analytical model of cylinder motion is developed for the case of axial initiation of an explosive charge. The suggested model relies on the two-stage regime of cylinder motion. The first stage considers the interaction of the detonation wave and the metal liner, resulting in the initial velocity imparted to the liner. The second stage is a gas-dynamic push of the cylinder governed by detonation products expansion, similarly to the Gurney approach. The numerical simulation of the explosive propulsion of the cylinder is performed using the FEM-based software Abaqus/Explicit. The results of the analytical model are validated by a comparison with the available experimental data and the simulation results.

Key words: **detonation, detonation wave, propulsion, detonation effect, analytical model, numerical simulation, finite element method words*:***

Nomenclature

A, B, C	–parameters of the JWL equation of state, Pa
a	–acceleration of the cylinder, m/s^2
C	–mass of the explosive charge per unit length, kg/m
D	–detonation velocity, m/s
d_1-d_5	–parameters of the Johnson-Cook damage model
E	–internal energy of detonation products, J
E_0	–specific detonation heat, J/m^3=Pa
L	–Lagrange's function, J
M	–mass of metal per unit length, kg/m
p	–pressure of detonation products, Pa
Q	–detonation heat, J/kg
q	–total non-conservative generalized force, N
r_1	–inner radius of the cylinder, m
R_1, R_2, ω	–parameters of the JWL equation of state
r_2	–outer radius of the cylinder, m
T	–kinetic energy, J
t	–time, s
U	–potential of conservative forces, J
u	–velocity of detonation products, m/s
V	–expansion ratio of detonation products,
v	–velocity of the cylinder wall, m/s
v_i	–initial velocity of the cylinder wall, m/s
w	–cylinder geometry constant, m
γ	–polytropic constant of detonation products
ρ_0	–density of explosive, kg/m^3
ρ_m	–density of the metal cylinder, kg/m^3
σ_f	–flow stress of metal, N/m^2

Introduction

ACCELERATION of a cylindrical metal liner under the action of impulsive internal loading brought about by explosive detonation is of great importance primarily in the analysis of high-explosive warhead mechanisms. The detonation of an explosive charge generates gaseous detonation products of extremely high pressure (~20÷40 GPa) causing rapid expansion of the cylindrical metal liner. The goal of the present research is modeling the cylinder motion until the onset of fragmentation process.

Gurney [1] formulated the classical model based on energy balance, analyzed in detail by Kennedy [2]. This model is still in wide use as a method for calculating the final velocity of a metallic liner, i.e. the initial velocity of generated fragments. However, Gurney's model does not take into account cylinder deformation and fails to describe the evolution of the cylinder motion. Moreover, the model has certain limitations and requires the experimental determination of the Gurney energy [3].

Numerous physically based models are suggested that characterize acceleration, deformation and fragmentation of the metallic cylinder due to the action of detonation products, e.g. [4-6].

These models employ different concepts of material behavior and detonation products expansion. A new comprehensive analytical model of cylinder expansion under the action of detonation products will be briefly presented.

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Cylinder acceleration model

The following assumptions will be used for modeling the motion of a metal liner caused by explosive detonation: (i) detonation wave is one-dimensional and in steady-state, (ii) explosive material is instantaneously transformed into detonation products. In addition, two stages of the cylinder acceleration process will be considered: (i) the first stage considers the interaction of the detonation wave and the liner when the result is almost immediate impart of the initial velocity to the liner, (ii) the second stage is related to the cylinder motion under the pressure of expanding detonation products, analogously to the Gurney model.

Two types of initiation of detonation are usually analyzed: (i) axisymmetric detonation of cylindrical explosive charge that provides a simultaneous onset of motion of the entire cylinder after detonation of the complete explosive charge; cylindrical detonation wave is formed and a head-on interaction occurs between the detonation wave and the cylinder wall (Fig.1a), (ii) in the case of grazing (side-on) detonation, only the part of the cylinder traversed by the detonation wave is set into motion, while the detonation wave is orthogonal to the cylinder axis (Fig.1b).

The latter case of grazing detonation is the subject of numerous experimental, analytical and numerical investigations related to the "cylinder test", e.g. [7-9]. Measurement of the tube displacement and velocity enables the determination of detonation product equation of state, the Gurney energy of explosive used, etc. The present investigation is focused on the axial initiation and a consequent head-on interaction of the detonation wave and the cylinder.

Initial velocity of the cylinder

The analysis of the available experimental data and the results of numerical simulations (e.g. [8]) show that the cylinder has extremely high acceleration in the initial stage of motion, i.e. it reaches high velocity in a very short time interval. This fact, as well as a notable oscillatory character of liner velocity, indicates an important effect of the shock waves formed in the cylinder by the impulsive action of the detonation wave. Backofen and Weickert [10] analyzed numerous experimental data and introduced the mentioned concept of two-stage acceleration of a liner propelled by detonation products. Moreover, they suggested the empirical formula for the calculation of the initial liner velocity v_i.

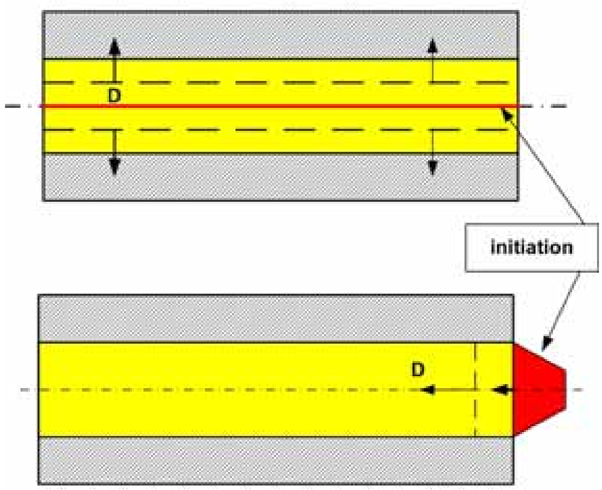

Figure 1. a) Axisymmetric detonation of the explosive charge (the head-on interaction of the detonation wave and the liner), b) grazing detonation (tangent, side-on, the interaction of the detonation wave and the liner)

An analytical approach to the problem of the interaction of the detonation wave and the metal liner, based on the impedance matching technique [11], will be pursued here.

The normal (head on) interaction of the plane detonation wave and the metallic liner is shown in Fig.2.

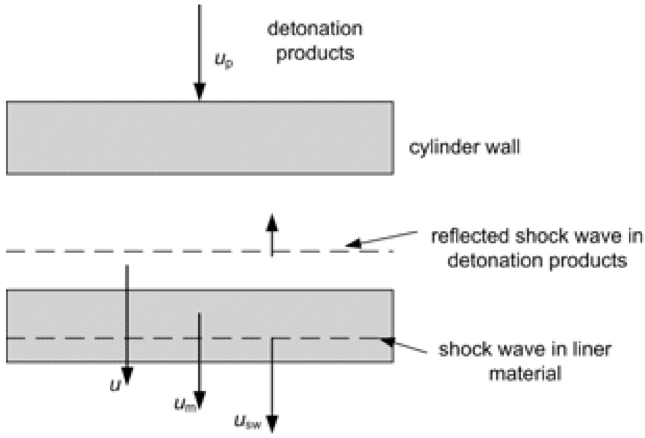

Figure 2. Normal (head on) interaction of the detonation wave and a solid obstacle; the characteristic velocities of the process are indicated

The Hugoniot shock adiabat of the detonation products in the velocity-pressure (u-p) coordinate system can be written in the form:

$$p = \frac{\gamma+1}{2}\rho_0 u^2 + (\gamma-1)\rho_0 Q \qquad (1)$$

where ρ_0 is the explosive density, γ is the polytropic coefficient of the detonation products at the Chapman-Jouget state and Q is the detonation heat.

When the detonation wave interacts with the liner, it reflects back, and the Hugoniot adiabat of the reflected shock wave has the form:

$$p = \frac{\gamma+1}{2}\rho_0 (2u_p - u)^2 + (\gamma-1)\rho_0 Q \qquad (2)$$

where u_p is the material velocity of the detonation products at the moment of the encounter with the obstacle. The shock adiabat of the cylinder material is defined by the relation:

$$p_m = \rho_m u_m (c_m + s_m u_m) \qquad (3)$$

where u_m and p_m are the velocity and the pressure in the part of the cylinder encompassed by the shock wave, while c_m and s_m are the equation of state parameters for the considered material.

The continuity condition between two considered media (gaseous detonation products and the cylindrical metal liner) implies that the velocities and pressures in the shock wave zone should be equal:

$$u = u_m, \quad p = p_m . \qquad (4)$$

Equating the right-hand sides of Eqs. (2) and (3), and using condition (4), the quadratic equation emerges, which enables the determination of the unknown liner velocity u_m. If we introduce well-known relations for the velocity of detonation products and the detonation heat:

$$u_p = \frac{D}{\gamma+1}, \quad Q = \frac{D^2}{2(\gamma^2-1)} \qquad (5)$$

the previous quadratic equation can be easily solved. The obtained velocity u_m is at the same time the initial velocity of the cylinder generated by the effect of the detonation wave:

$$(v_i)_{head\text{-}on} = u_m \tag{6}$$

In order to simplify the analytical treatment of the problem, we will assume the effect of shock waves is dominant only at the onset of the cylinder motion, i.e. the subsequent oscillatory motion produced by reverberations of shock waves in the cylinder can be neglected comparing to the motion under the action of rapidly expanding detonation products.

It should be emphasized that the previous assumption restricts the domain of possible application of the model. The influence of reflected shock waves is dominant in the case of the liner with thin walls, i.e. for the ratio of masses of liner and explosive charge $M/C<1$. In the applications of explosive propulsion relevant to weapon system design, the metal liner mass is generally significantly higher than the explosive charge mass.

Acceleration of the cylinder by the gas push process

In the second stage of the cylinder acceleration, the metallic cylinder of known density ρ_m and geometry is considered. The cylinder is assumed to be long enough to neglect the end effects, i.e. the axial outflow of detonation products. The cylinder motion starts when the entire explosive charge is detonated. It is shown that due to the action of the detonation wave, the cylinder receives the initial velocity v_i. The one-dimensional model of the cylinder motion is considered, shock wave effects are now neglected, and Gurney's postulate of detonation products homogeneity is adopted.

Mass conservation law

Since the cylinder material is incompressible in the absence of shock waves, the mass conservation law yields:

$$\begin{aligned} & r_2^2 - r_1^2 = r_{20}^2 - r_{10}^2 = w^2 = const., \\ & r^2 - r_1^2 = r_0^2 - r_{10}^2 \end{aligned} \tag{7}$$

where r_1 and r_2 are the radial positions of the inner and outer cylinder surface, r_{10} and r_{20} are the corresponding initial cylinder dimensions, while $r\in[r_1, r_2]$ is the Lagrange coordinate of an arbitrary cylinder point, and $r_0\in[r_{10}, r_{20}]$ is its initial value. The continuity equation can be also written in the form:

$$vr = v_1 r_1 = c \tag{8}$$

where v is the cylinder velocity and $c=c(t)$ is the function of time t only. It is clear that the determination of the position r_1 and the velocity v_1 of the inner cylinder surface enables the computation of the position and velocity of any cylinder point using Eqs. (7) and (8).

Lagrange's equation of the cylinder motion

Following the idea formulated by Flis [12], the motion of the cylinder due to the rapid expansion of detonation products is modeled by the Lagrange's equation in the form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_1}\right) - \frac{\partial L}{\partial r_1} = q\ . \tag{9}$$

In the previous equation L is the Lagrange's function

$$L = T - U \tag{10}$$

where T is the total kinetic energy of the system, U – the total potential of conservative forces, q – the total non-conservative generalized force. The position of the inner surface of the cylinder r_1 is adopted as the generalized coordinate.

The total kinetic energy of the system consists of the kinetic energy of the cylinder T_M and the kinetic energy of the detonation products T_{DP}:

$$T = T_M + T_{DP}\ . \tag{11}$$

The kinetic energy of the cylinder per unit length can be written as:

$$T_M = \int_V \frac{v^2}{2} dm = \frac{M}{2} v_1^2 \frac{r_1^2}{w^2} \ln\left(1 + \frac{w^2}{r_1^2}\right). \tag{12}$$

The kinetic energy of the gaseous detonation products, having in mind the adopted assumption of their homogeneity ($\partial\rho/\partial r=0$), depends on the detonation products velocity profile. The kinetic energy of the detonation products can now be written as:

$$T_{DP} = \frac{1}{4} C v_1^2 \tag{13}$$

where C is the mass of explosive charge per unit length.

The total potential U of conservative forces is equal to the internal energy of the detonation products $U=E$. The potential derivative can be expressed in the form:

$$\frac{\partial U}{\partial r_1} = \frac{\partial E}{\partial V_{DP}} \frac{\partial V_{DP}}{\partial r_1} = -2\pi r_1 p \tag{14}$$

where V_{DP} is the volume occupied by the detonation products, and p is their current pressure.

The work of non-conservative forces is in fact the work of the forces that resist the cylinder deformation. Considering that the circular stress is dominant, the total non-conservative generalized force can be written in the form [13]:

$$q = \frac{C\sigma_f}{\rho_0}\left[\left(\frac{r_{20}}{r_{10}}\right)^2 - 1\right] \frac{1}{\sqrt{w^2 + r_1^2}}\ . \tag{15}$$

The flow stress of the cylinder material σ_f is assumed to be constant.

The substitution of the kinetic energies, Eqs. (12) and (13), the potential, Eq. (14), and the generalized force, Eq. (15), in the Lagrange equation (9) leads to the final differential equation of motion of the inner cylinder surface:

$$\begin{aligned} & \left[\frac{M}{C}\frac{r_1^2}{w^2}\ln\left(1+\frac{w^2}{r_1^2}\right) + \frac{1}{2}\right] a_1 + \\ & + \frac{M}{C}\frac{v_1^2}{r_1}\left[\frac{r_1^2}{w^2}\ln\left(1+\frac{w^2}{r_1^2}\right) - \frac{r_1^2}{w^2+r_1^2}\right] = \\ & = \frac{2r_1}{\rho_0 r_{10}^2} p(r_1) - \frac{\sigma_f}{\rho_0}\frac{w^2}{r_{10}^2}\frac{1}{\sqrt{w^2+r_1^2}} \end{aligned} \tag{16}$$

where the ratio of the cylinder mass to the explosive charge mass is defined as:

$$\frac{M}{C} = \frac{\rho_m w^2}{\rho_0 r_{10}^2}. \qquad (17)$$

Detonation products pressure

In order to solve the equation of motion (15), the detonation products pressure $p=p(r_1)$ must be defined. Two approaches are possible: (i) the polytropic expansion law for detonation products can be used, or (ii) application of an empirically based equation of state for detonation products.

Based on the first approach, the pressure p is determined under the condition that the polytropic expansion starts from the Chapman-Jouget state [3, 14]. Using the results of the elementary detonation theory [3], the detonation products pressure is determined as the function of the inner cylinder surface position:

$$p(r_1) = \frac{1}{2(\gamma+1)}\rho_0 D^2 \left(\frac{r_{10}}{r_1}\right)^{2\gamma} \qquad (18)$$

The values of the parameter γ are usually in the interval [2.7, 3.0], and if experimental results lack, the most common approximation is $\gamma \approx 3$.

The second approach, based on the empirically established equation of state of detonation products, provides more reliable results. Because of simplicity and data availability, the Jones-Wilkins-Lee (JWL) equation of state [7] is frequently used in practice:

$$p(V) = A\exp(-R_1 V) + B\exp(-R_2 V) + CV^{-(1+\omega)} \qquad (19)$$

where V is the detonation products expansion factor

$$V = \frac{\rho_0}{\rho} = \left(\frac{r_1}{r_{10}}\right)^2 \qquad (20)$$

while A, B, C, R_1, R_2 and ω are the experimentally determined parameters. The handbook [15] is a comprehensive source of data for the JWL equation of state of different explosives.

Substituting the pressure, Eq. (18) or (19), in the equation of the cylinder motion (16), an ordinary differential equation of the second order emerges. This equation can be solved by numerical methods, applying the appropriate initial conditions:

$$(r_1)_{t=0} = r_{10}, \quad (\dot{r}_1)_{t=0} = v_i. \qquad (21)$$

Comparison with experimental and numerical results

In order to investigate the validity of the suggested analytical model, the computed results are compared with the available experimental data, as well as with the results of the numerical simulations.

Comparison with experimental data

The results of the presented one-dimensional model of the cylinder motion under the action of gaseous products of axisymmetric detonation are analyzed through the comparison with the experimental data from the comprehensive study by Hiroe et al. [16]. The motion of the metal cylinder after electric initiation by the bundle of copper wires is observed by a high-speed digital camera. The properties of the PETN explosive charge used are as follows: density ρ_0=950 kg/m^3, detonation velocity D=5240 m/s, and polytropic gamma γ_{CJ}=2.57. Three cylinder materials were used: stainless steel, carbon steel and aluminum alloy. The outer diameter of the cylinders was 34 mm and the length was 100 mm.

For the case of stainless steel, three thicknesses of the fully-charged cylinder are investigated: 1.65 mm, 3 mm, and 6 mm. The comparison of the measured radial positions of the outer cylinder surface and the model results is shown in Fig.3. The cylinder acceleration process is obviously impulsive and the initial acceleration is of the order of 10^8 m/s^2. A good correspondence between the experimental and model results is noted, until the moment of the onset of fragmentation, accompanied with longitudinal cracks on the outer cylinder surface. From that time, the cylinder loses structural integrity and the analyzed model cannot be applied.

The time history of the cylinder velocity is presented in Fig.4. As it can be seen from the diagram, the outer cylinder surface velocity calculated by the Gurney formula is expectedly higher than the model results.

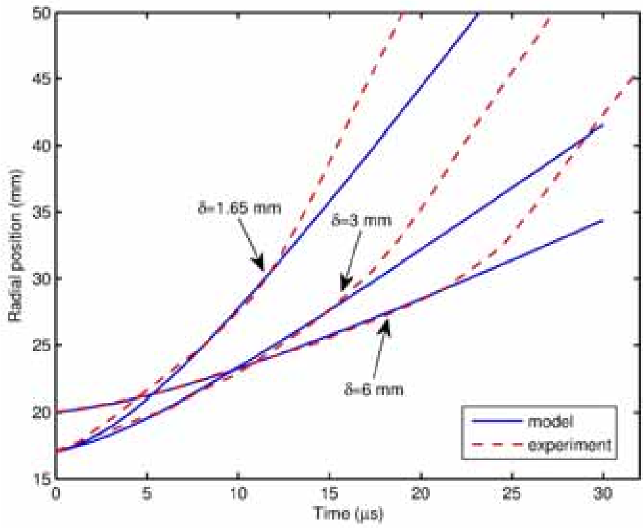

Figure 3. Radial position of the cylinder under the action of detonation products – comparison of the experimental data [16] and the model prediction for stainless steel and three different wall thicknesses

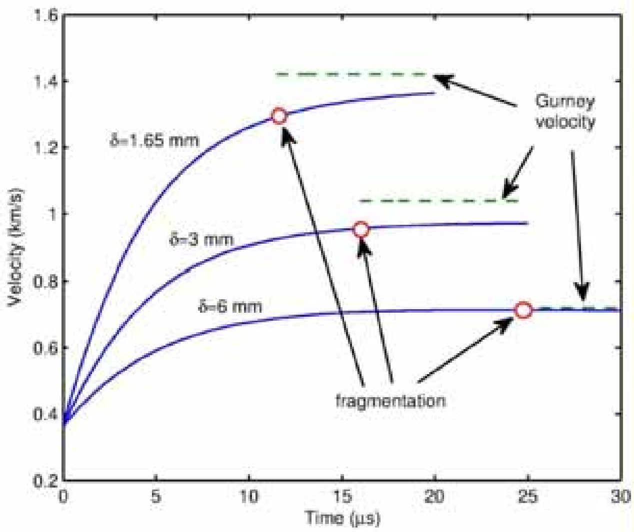

Figure 4. Time history of the cylinder velocity – model results and Gurney velocities for the cylinder of stainless steel and three different wall thicknesses

The experimentally determined positions for carbon steel and aluminum alloy cylinders are compared with the analytical model results in Fig.5. Again, a good agreement is observed until the start of the fragmentation process.

Fig.6 shows the comparison between the calculated cylinder velocities and the corresponding Gurney velocities for two materials.

Since the agreement between the theoretical and the experimental results is good overall, the suggested analytical model validity is confirmed.

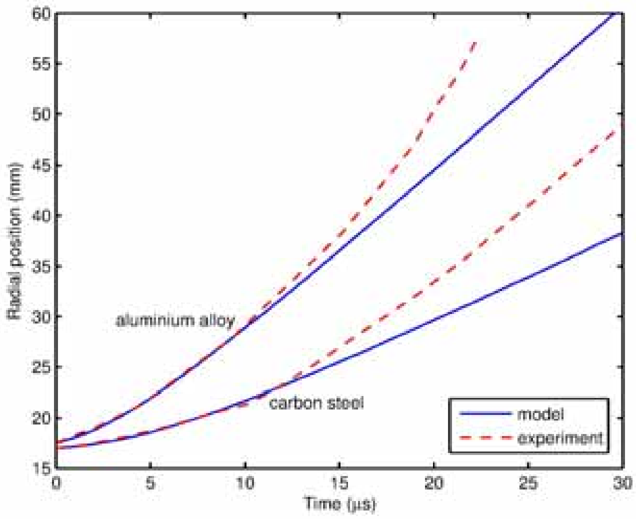

Figure 5. Radial position of the cylinder under the action of detonation products – comparison of the experimental data [16] and the model prediction for the cylinder of carbon steel and aluminum

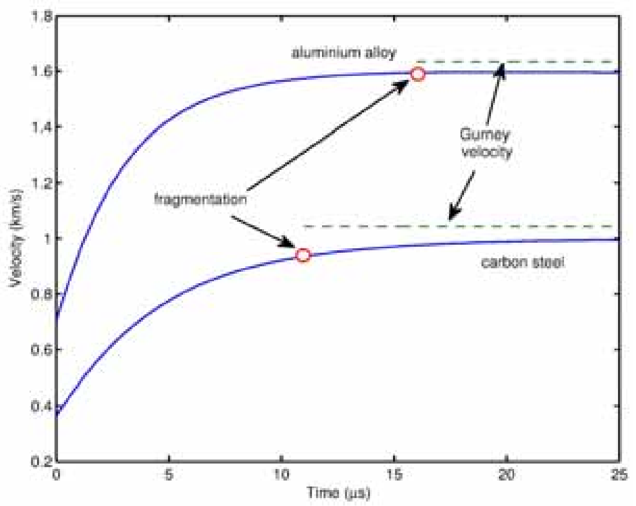

Figure 6. Time history of the cylinder velocity – model results and Gurney velocities for the cylinders of carbon steel and aluminum

Comparison with the results of numerical simulations

Additional validation of the proposed theoretical model is performed using a numerical approach. A considerable number of numerical studies have been published investigating various explosive effects using different hydrocodes, e.g. [17, 18]. A numerical analysis of the cylinder test is performed with the commercial FEM-based software Abaqus [19]. The solver Abaqus/Explicit configured for the simulation of transient nonlinear dynamic events is used. The Coupled Eulerian Lagrangian (CEL) capability of Abaqus is employed enabling the interaction between detonation products and the metal cylinder.

After a convergence study, an optimal quarter symmetry finite element model was adopted (Fig.7). The Johnson-Cook plasticity model [20], as well as the Johnson-Cook damage model [21] have been used for the copper cylinder. A linear U_s-U_p equation of state is employed. The geometry of the cylinder corresponds to the standard cylinder test and the material properties of copper used in the Abaqus model are taken from [20, 21]. The geometry and the material properties of the copper cylinder are presented in Table 1.

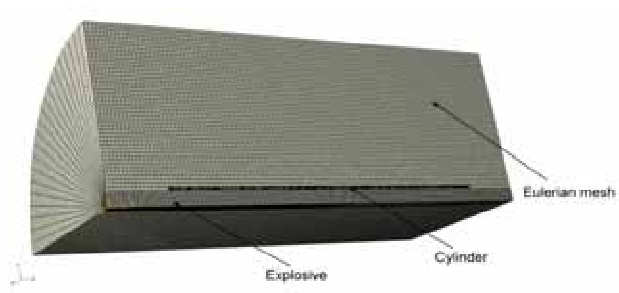

Figure 7. Model of the explosively driven cylinder in Abaqus/CAE

Table 1. Properties of the cylinder – parameters of the Abaqus model

Cylinder (copper)			
Dimensions	Internal radius (mm)		12.70
	External radius (mm)		15.30
	Length (mm)		300
Density (kg/m^3)			8940
Specific heat (J/kgK)			383.5
Johnson-Cook plasticity model	Yield stress (MPa)		89.63
	Hardening coefficient (MPa)		291.6
	Hardening exponent		0.31
	Strain-rate coefficient		0.025
	Thermal softening coefficient		1.09
	Initial temperature (K)		294
	Melt temperature (K)		1356
Johnson-Cook damage model	parameters	d_1	0.3
		d_2	0.28
		d_3	-3.03
		d_4	0.014
		d_5	1.12
EOS model	Sound speed (m/s)		3940
	Slope		1.49
	Gruneisen coefficient		1.96

The explosive material is modeled by the JWL equation of state. Explosive is detonated using a programmed burn, and pressure and energy are calculated from the equation of state, without the consideration of intermediate species and non-equilibrium processes in the reaction zone. Four explosives are analyzed: TNT, PBX-9404 (HMX/NC/CEF - 94/3/3), HMX, and phlegmatized hexogen FH-5 (RDX/wax - 95/5). The values for the explosive density, the detonation properties, and the JWL equation of state parameters are taken from [13, 15] and listed in Table 2.

Table 2. Detonation properties and the JWL parameters for four explosives

	TNT	PBX-9404	HMX	FH-5
Density, ρ_0 (kg/m^3)	1630	1840	1891	1600
Detonation velocity, D (m/s)	6930	8800	9110	7930
Pressure at CJ state, p_{CJ} (GPa)	21.0	37.0	42.0	24.96
Specific detonation heat, E_0 (GPa)	7.0	10.2	10.5	8.7
R_1	4.15	4.60	4.2	4.2750
R_2	0.95	1.30	1.0	0.3175
ω	0.3	0.38	0.30	0.2178
A (GPa)	371.21	852.40	778.28	573.43
B (GPa)	3.2306	18.020	7.0714	0.96006
C (GPa)	1.0453	1.2070	0.6430	0.82373

A representative example of the simulation results showing the evolution of the cylinder acceleration process including the velocity field is shown in Fig.8.

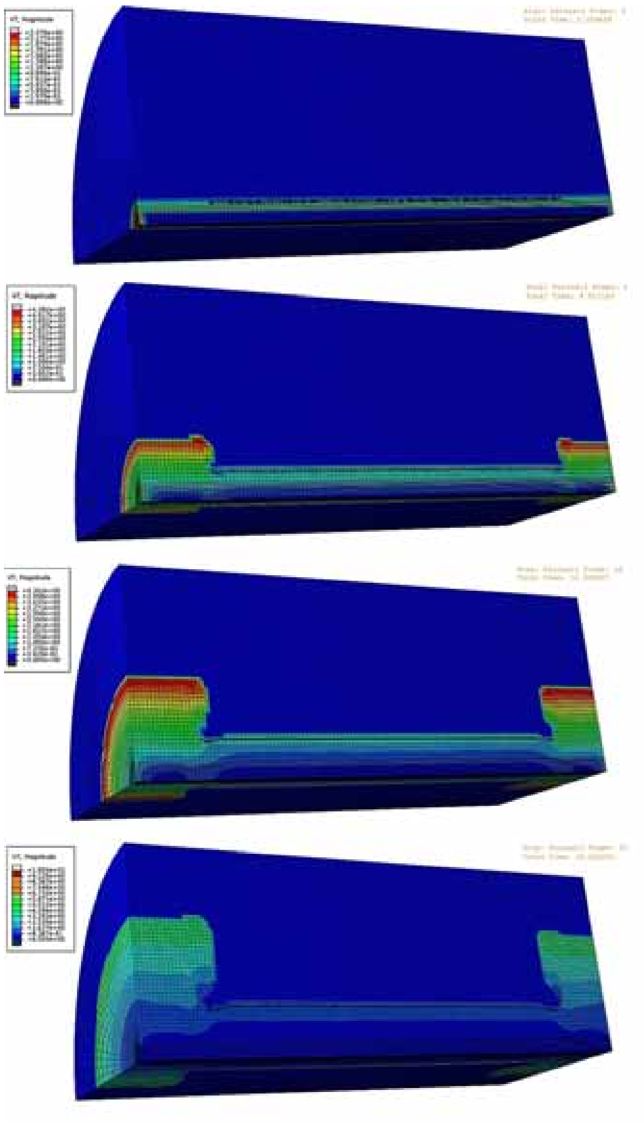

Figure 8. Evolution of the cylinder motion under the action of detonation products; velocity fields from the top to the bottom corresponds to times 2 µs, 9 µs, 16 µs and 23 µs after the explosive initiation

The calculated time histories of both the cylinder displacement and the velocity are compared with the Abaqus simulation results for four explosives in Figures 9 – 12. Although there are some discrepancies between the velocities, the displacement history curves are almost indistinguishable. This is the additional confirmation of the validity of the suggested analytical model.

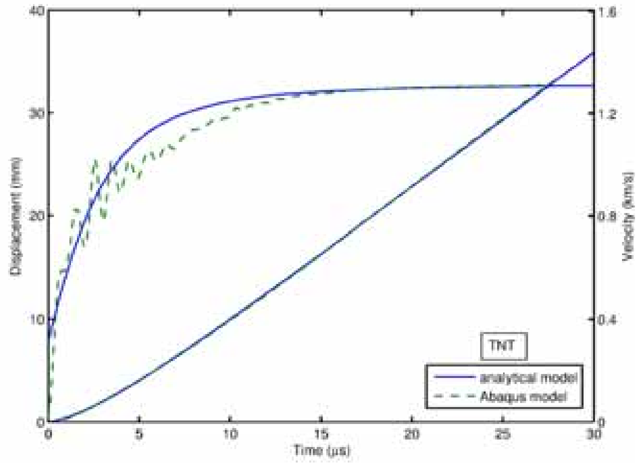

Figure 9. Time histories of the cylinder displacement and the velocity - comparison between the model calculations and the Abaqus simulation results for TNT

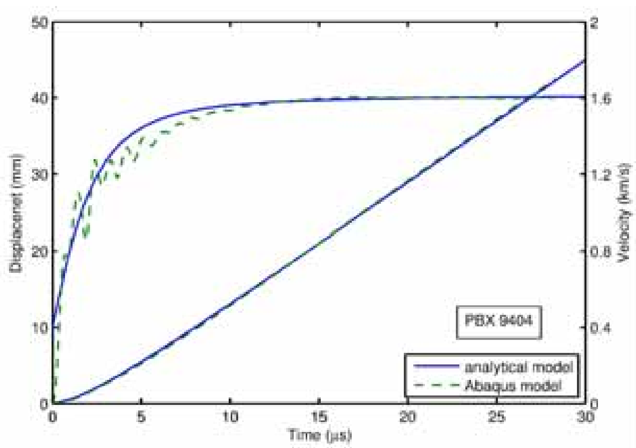

Figure 10. Time histories of the cylinder displacement and the velocity - comparison between the model calculations and the Abaqus simulation results for PBX-9404

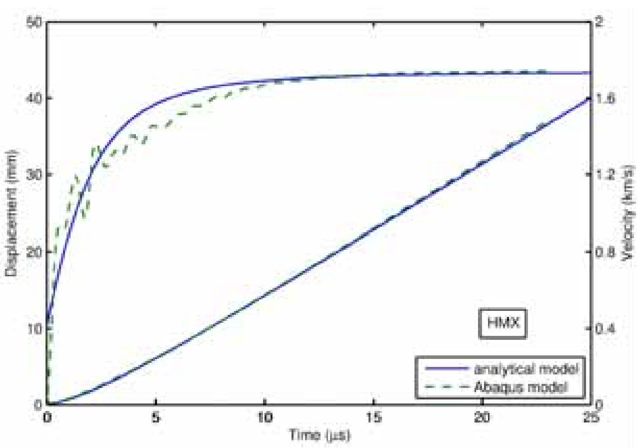

Figure 11. Time histories of the cylinder displacement and the velocity - comparison between the model calculations and the Abaqus simulation results for HMX

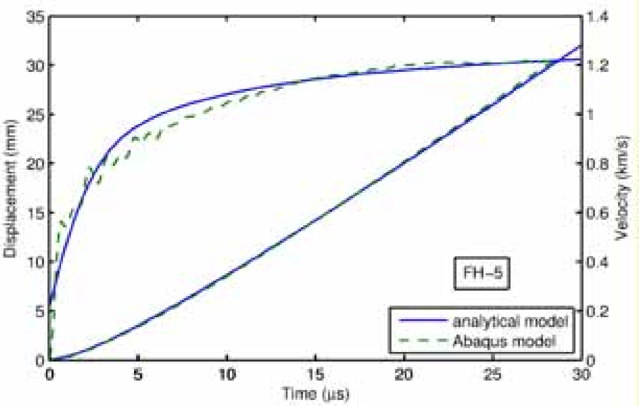

Figure 12. Time histories of the cylinder displacement and the velocity - comparison between the model calculations and the Abaqus simulation results for FH-5

Conclusions

Cylindrical liner motion under the action of expanding detonation products has been considered. A new cylinder acceleration model is suggested for the case of axisymmetric detonation of an explosive charge. The model implies a two-stage character of the liner motion: (i) the early interaction of the detonation wave and the metallic liner results in the impart of the initial velocity to the cylindrical liner, and (ii) the gas-push stage of rapid cylinder motion, similarly to the Gurney's model. The initial liner velocity is determined by the impedance matching technique. The cylinder motion in the second stage is modeled by the Lagrange's equation.

It is shown that the computational results using the suggested model provide a very good agreement with the experimental data. The model results are also shown to be in accordance with the numerical simulations performed in Abaqus/Explicit. Therefore, the proposed analytical model can be used as a design tool for various applications in the domain of explosive propulsion.

Acknowledgement

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Modeliranje ubrzavanja metalnog cilindra pod dejstvom produkata eksplozije

U radu se razmatra kretanje cilindrične metalne obloge usled ekspanzije produkata unutrašnje detonacije. Razvijen je nov analitički model kretanja cilindra za slučaj inicijacije eksplozivnog punjenja duž njegove ose. Predloženi model zasniva se na dvostepenom režimu kretanja cilindra. U prvoj fazi razmatra se interakcija detonacionog talasa i metalne obloge koja rezultira saopštavanjem početne brzine oblozi. Druga faza podrazumeva gasodinamičku propulziju cilindra usled ekspanzije produkata detonacije, slično Garnijevom pristupu. Numerička simulacija procesa ubrzavanja cilindra realizovana je primenom programa Abaqus/Explicit koji je zasnovan na metodi konačnih elemenata. Valjanost analitičkog modela utvrđena je poređenjem sa dostupnim eksperimentalnim podacima, kao i sa rezultatima simulacija.

Ključne reči: **detonacija, detonacioni talas, propulzija, uticaj detonacije, analitički model, numerička simulacija, metoda konačnih elemenata.**

Моделирование ускорения металлического циллиндра под действием продуктов взрыва

В данной статье обсуждается движение циллиндрической подкладке металла за счёт расширения продуктов внутренней детонации. Разработана новая аналитическая модель движения цилиндра для случая начала заряда взрывчатого вещества вдоль его оси. Предлагаемая модель основана на двухступенчатом режиме движения циллиндра. В первой фазе рассматривается взаимодействие детонационных волн и металлической оболочки, что приводит объявляющей их первоначальной скорости обертывания оболочке. Второй этап предполагает газодинамическое движение цилиндра двигателей за счёт расширения продуктов детонации, как и подход Гарние. Численное моделирование процесса ускорения цилиндра проводили с использованием програмы ABAQUS / Explicit, которая основана на методе конечных элементов. Срок действия аналитической модели определялся путём сравнения с доступными экспериментальными данными и результатами моделирования.

Ключевые слова: **детонация, детонационная волна, движения, влияние детонации, аналитическая модель, численное моделирование, метод конечных элементов.**

Modélisation de l'accélération du cylindre métallique sous l'action des produits de l'explosion

Le mouvement du revêtement cylindrique en métal, causé par l'expansion des produits de détonation interne, est considéré dans ce travail. On a développé un nouveau modèle analytique de mouvement de cylindre pour le cas de l'initiation de la charge explosive le long de son axe. Le modèle proposé est basé sur le régime à deux degrés du mouvement de cylindre. Dans la première phase on considère l'interaction de l'onde de détonation et du revêtement de métal qui résulte par la transmission de la vitesse initiale au revêtement. La seconde phase comprend la propulsion gaz dynamique du cylindre provoquée par l'expansion des produits de la détonation ce qui ressemble à l'approche de Garnier. La simulation numérique du processus de l'accélération du cylindre a été effectuée au moyen de logiciel Abaqus /Explicit qui se base sur la méthode des éléments finis. La validité du modèle analytique a été confirmée par la comparaison avec les données expérimentales disponibles ainsi que par les résultats des simulations.

Mots clés: **détonation, onde de détonation, propulsion, effet de détonation, modèle analytique, simulation numérique, méthode des éléments finis.**