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The Stability of Linear Continous Singular and Discrete Descriptor Time Delayed Systems over the Finite Time Interval: An Overview - Part I Continuous Case

Dragutin Debeljković[1)]
Sreten Stojanović[2)]
Tamara Nestorović[3)]

This paper gives sufficient conditions for the practical and finite time stability of linear singular continuous time delay systems of the form $E\,\dot{\mathbf{x}}(t)=A_0\mathbf{x}(t)+A_1\mathbf{x}(t-\tau)$. When we consider the finite time stability concept, these new delay independent conditions are derived using an approach based on Lyapunov – like functions and their properties on the sub-space of consistent initial conditions. These functions do not need to have: a) properties of positivity in the whole state space and b) negative derivatives along the system trajectories.
Considering practical stability, the above mentioned approach is combined and supported by a classical Lyapunov technique to guarantee its attractivity

Key words: **linear system, continual system, descriptive system, singular system, time delayed system, discrete system, system stability.**

Classes of systems to be considered

IT should be noticed that in some systems we must consider their character of dynamic and static state at the same time. Singular systems (also referred to as degenerate, descriptor, generalized, differential - algebraic systems or semi – state) are those the dynamics of which is governed by a mixture of algebraic and differential equations. Recently, many scholars have paid much attention to singular systems and have obtained many good results. The complex nature of singular systems causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a need for their control.

It is well-known that singular systems have been one of the major research fields of the control theory. During the past three decades, singular systems attracted much attention due to comprehensive applications in economics as the *Leontief* dynamic model *Silva*, *Lima* (2003), in electrical *Campbell* (1980) and mechanical models *Muller* (1997), etc.

A discussion on singular systems originated in 1974 with the fundamental paper of *Campbell et al*. (1974) and later with the antological paper of *Luenberger* (1977).

The problem of investigation of time delay systems has been exploited over many years. Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause an undesirable system transient response, or even instability. Consequently, the problem of stability analysis for this class of systems has been one of the main interests for many researchers. In general, the introduction of time delay factors makes the analysis much more complicated.

We must emphasize that there are a lot of systems that have the phenomena of time delay and singular simultaneously and we call such systems *the singular differential systems with time delay*. These systems have many special characteristics. If we want to describe them more exactly, to design them more accurately and to control them more effectively, we must make tremendous efforts to investigate them, which is obviously very difficult work. In recent references, the authors have discussed such systems and got some conclusions. However, in the study of such systems, there are still many problems to be considered. When general time delay systems are considered, in the existing stability criteria, mainly two ways of approach have been adopted.

Namely, one direction is to contrive the stability condition which does not include the information on the delay, and the other is the method which takes it into account. The former case is often called the delay - independent criterion and generally provides simple algebraic conditions. In that sense the question of their stability deserves great attention.

Stability concept

In practice, there is not only an interest in system stability (e.g. in sense of Lyapunov), but also in bounds of system trajectories. A system could be stable but completely useless because it possesses undesirable transient performances.

1) University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11120 Belgrade, SERBIA
2 University of Niš, Faculty of Technological Engineering, Bulevar oslobođenja 124, 16000 Leskovac, SERBIA
3 Ruhr University of Bochum, Bochum, GERMANY

Thus, it may be useful to consider the stability of such systems with respect to certain sub-sets of state-space, which are *a priori* defined in a given problem.

Besides that, it is of particular significance to consider the behavior of dynamical systems only over a finite time interval.

These bound properties of system responses, i. e. the solution of system models, are very important from the engineering point of view. Based on this fact, numerous definitions of the so-called technical and practical stability were introduced.

Roughly speaking, these definitions are essentially based on the predefined boundaries for the perturbation of initial conditions and the allowable perturbation of a system response. In engineering applications of control systems, this fact becomes very important and sometimes crucial, for the purpose of characterizing in advance, in quantitative manner, possible deviations of the system response.

Thus, the analysis of these particular bound properties of solutions is an important step which precedes the design of control signals, when finite time or practical stability control is concerned.

Due to limited space, we will only concentrate on the countinuous case.

Motivated by the *brief discussion* on practical stability in the monograph of *La Salle and Lefschetz* (1961), *Weiss and Infante* (1965, 1967) have introduced various notations of stability over finite time interval, for continual-time systems and constant set trajectory bounds.

Further development of these results was due to many other authors, *Michel* (1970), *Grujic* (1971), *Lashirer*, *Story* (1972).

Practical stability of simple and interconnected systems with respect to time-varying subsets was considered in *Michel* (1970) and *Grujic* (1975).

A more general type of stability: "practical stability with settling time", practical exponential stability, etc., which includes many previous definitions of finite time and practical stability was introduced and considered in *Grujic* (1971, 1975.a, 1975.b).

Further results were presented by *Weiss* (1972) and *Weiss*, *Lam* (1973), and many others.

A concept of finite-time stability, called "final stability", was introduced in *Lashirer*, *Story* (1972) and further development of these results was due to *Lam*, *Weiss* (1974).

Chronological preview of the previous results

In the short overview that follows, we will familiarize only with the results achieved for *continuous linear systems* in the area of Non - Lyapunov stability.

Practical and Finite Time Stability – Singular Systems

In the context of practical stability for linear continuous singular systems, various results were first obtained in *Debeljkovic*, *Owens* (1985) and *Debeljkovic* (1986.a,1986.b)

Debeljkovic, *Owens* (1985) derived some new results in the area of practical and finite time stability for time-invariant, continuous linear singular systems.

These results represent the sufficient condition for stability of such systems and are based on Lyapunov-like functions and their properties on sub-space of consistent initial conditions. In particular, these functions do not need to have properties of positivity in the whole state space and negative derivatives along the system trajectories.

The results are expressed directly in terms of the matrices E and A naturally occurring in the model thus avoiding the need to introduce algebraic transformations into the statement of the theorems. It was shown that the geometric approach gives more insight in the structural properties of singular systems and the problems of consistency of initial conditions and it also makes possible a basis-free description of dynamic properties.

Further extension of these results were presented in *Debeljkovic et al.* (1992, 1993, 1994) for both regular and irregular singular systems. Namely, these papers examine some practically important boundedness and associated unboundedness properties of response of linear singular systems. The existence of specifically bounded solutions, as well as practical instability of systems that have been considered were investigated. A development and easy application of Lyapunov's direct method for this analysis were presented. A potential (weak) domain of practical stability, consisting of the points of the phase space, which generate at least one solution with specific "practical stability" constraints, was underestimated.

In the papers *Debeljkovic et al.* (1994.a, 1994.b, 1995) and *Dihovicni et al.* (1996), some of previous results have been extended to the stability robustness problem in the context of finite time and practical stability.

In the paper of *Debeljkovic*, *Jovanovic* (1997) for the first time singular systems, operating in the forced regime, have been considered.

The finite time stability of singular systems operating under perturbing forces, has been considered in the papers of *Debeljkovic et al.* (1997), and *Kablar*, *Debeljkovic* (1998.a, 1998.b), using, for the first time, Coppel's inequality and the matrix measure approach, respectively.

The concept of finite time and practical stabilty of the class of time varying singular systems, were presented for the first time in *Kablar*, *Debeljkovic* (1998.c).

The necessary and sufficient conditions for the linear singular systems stability operating over the finite time interval were derived in the paper of *Debeljkovic*, *Kablar* (1998). Moreover, the reciprocial problem of instabilty of the same class of systems has been solved in *Kablar*, *Debeljkovic* (1999).

The Bellman-Gronwall approach was, for the first time, applied in a study of linear singular systems in the paper of *Debeljkovic*, *Kablar* (1999).

Further, a new extension of the idea of practical stability to general singular systems, was due to *Yang et al.* (2005.a). Several new concepts of practical stability were derived, based on the Lyapunov functions and the comparison principle.

A comparison system is a scalar quantity differential system which enables transfering the problem of practical stabilty of a singular system to the standard state space system.

A quite new approach in time domain, based on the fundamental matrix of singular systems, has been applied in paper *Nie*, *Debeljkovic* (2004).

In the paper of the same authors, *Yang et al* (2005.b), practical stabilization and controllability of singular systems have been examinated using the before mentioned approach.. Furthemore, the comparison principle presented can be used to analyze other properties of solutions of singular systems, for example, boundedness.This consideration is applicable to both linear and nonlinear singular systems.

The aforementioned ideas have been extended to the closed loop singular systems, in order to achieve practical stability with non impulsive motion, *Yang et al.* (2006).

The finite-time control of linear singular systems with parametric uncertainness and disturbances was considered in the paper of *Shen*, *Shen* (2006). The modified concept of finite time stability has been extended, in a particular way, to linear time invariant singular systems. Providing sufficient conditions guaranteeing finite time stability via state feedback, this problem leads to an optimization problem involving LMIs.

Practical and Finite Time Stability – Time Delay Systems

In the context of finite or practical stability for a particular class of *nonlinear singularly perturbed multiple time delay systems* various results were, *for the first time*, obtained in *Feng*, *Hunsarg* (1996). It seems that their definitions are very similar to those in *Weiss*, *Infante* (1965, 1967), clearly addopted to time delay systems.

It should be noticed that these definitions are significantly different from the definitions presented by the authors of this paper.

In the context of finite time and practical stability for linear continuous time delay systems, various results were first obtained in *Debeljkovic et al.* (1997.a, 1997.b, 1997.c, 1997.d), *Nenadic et al.* (1975, 1997).

In the paper of *Debeljkovic et al.* (1997.a) and *Nenadic et al.* (1975, 1997) some basic results from the area of finite time and practical stability were extended to the particular class of linear continuous time delay systems. Stability sufficient conditions dependent on delay, expressed in terms of a time delay fundamental system matrix, have been derived. In addition, in the circumstances when it is possible to establish a suitable connection between the fundamental matrices of linear time delay and non-delay systems, the presented results enable an efficient procedure for testing practical as well the finite time stability of time delay systems.

The matrix measure approach has been, for the first time, applied in *Debeljkovic et al.* (1997.b, 1997.c, 1997.d, 1997.e, 1998.a, 1998.b, 1998.d, 1998.d) for the analysis of practical and finite time stability of linear time delayed systems. With the Coppel's inequality and introducing the matrix measure approach, one provides very simple delay – dependent sufficient conditions of practical and finite time stability with no need for time delay fundamental matrix calculations.

In *Debeljkovic et al.* (1997.c) this problem has been solved for forced time delay systems.

Another approach, based on a very well known Bellman – Gronwall Lemma, was applied in *Debeljkovic et al.* (1998.c), to provide new, more efficient sufficient delay-dependent conditions for checking finite and practical stability of continuous systems with state delay.

An overview of all previous results and contributions was presented in paper *Debeljkovic et al.* (1999) with overall comments and a slightly modified Bellman – Gronwall approach.

Finally, a modified Bellman – Gronwall principle has been extended to the particular class of continuous *non-autonomous* time delayed systems operating over the finite time interval, *Debeljkovic et al.* (2000.a, 2000.b, 2000.c).

Practical and Finite Time Stability – Singular Time delay Systems

The paper, *Yang, et al*. (2006) introduces the idea of practical stability with time delays in terms of two measurements, and represents the first attempt to apply the non-Lyapunov concept to this class of control systems. Based on the Lyapunov functions and the comparison principle, a criterion, by which the problem of singular systems with time delay is reduced to that of standard (classical) state space systems without delay, is derived. By appropriate choice of the two measurements, the basic definition in this paper reduces to the practical stability defined earlier in *Debeljković*, *Owens* (1985). The paper also shows the difficulty to calculate the time derivatives along the systems trajectory using the classical aggregate (Lyapunov) function for this singular time delayed system.

Motivation

In our paper we present quite another approach to this problem and continue our investigation in a usual way.

Namely, our result is expressed directly in terms of matrices E, A_0 and A_1 naturally occurring in the system model thus avoiding the need to introduce any canonical form into the statement of the *Theorem.*

The geometric theory of consistency leads to the natural class of positive definite quadratic forms on the subspace containing all solutions. This fact enables the construction of the Lyapunov and Non- Lyapunov stability theory even for the LCSTDS in that sense that the attractivity property is equivalent to the existence of symmetric, positive definite solutions to a weak form of Lyapunov matrix equation incorporating condition which refer to boundedness of solutions.

Another approach is based on a classical approach mostly used in deriving sufficient delay independent conditions of finite time stability.

In the former case a new definition is introduced, based on the attractivity properties of system solution, which can be treated as something analogous to quasy – contractive stability, *Weiss*, *Infante* (1965,1967).

Notation and preliminaries

$\mathbb{R}$ – Real vector space

$\mathbb{C}$ – Complex vector space

I – Unit matrix

F – $=\left(f_{ij}\right)\in\mathbb{R}^{n\times n}$ real matrix

F^T – Transpose of matrix F

$F>0$ – Positive definite matrix

$F\geq 0$ – Positive semi definite matrix

$\Re(F)$ – Range of matrix F

$\aleph(F)$ – Null space (kernel) of matrix F

$\lambda(F)$ – Eigenvalue of matrix F

$\|F\|$ – Euclidean matrix norm of $F=\sqrt{\lambda_{\max}\left(A^TA\right)}$

$\Rightarrow$ – Follows

$\mapsto$ – Such that

Generally, *the singular differential control systems with time delay* can be written as:

$$\begin{gathered} E(t)\dot{\mathbf{x}}(t)=\mathbf{f}\left(t,\mathbf{x}(t),\mathbf{x}(t-\tau),\mathbf{u}(t)\right),\quad t\geq 0 \\ \mathbf{x}(t)=\boldsymbol{\varphi}(t),\ -\tau\leq t\leq 0 \end{gathered}, \tag{1}$$

where $\mathbf{x}(t)\in\mathbb{R}^n$ is a state vector, $\mathbf{u}(t)\in\mathbb{R}^m$ is a control vector, $E(t) \in \mathbb{R}^{n\times n}$ is a singular matrix, $\boldsymbol{\varphi} \in \mathrm{C} = \mathrm{C}\left([-\tau,0],\mathbb{R}^n\right)$ is an admissible initial state functional, $\mathrm{C} = \mathrm{C}\left([-\tau,0],\mathbb{R}^n\right)$ is the *Banach* space of continuous functions mapping the interval $[-\tau,0]$ into $\mathbb{R}^n$ with topology of uniform convergence.

The vector function satisfies:

$$\mathbf{f}(\;): \Im\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^m \to \mathbb{R}^n . \qquad (2)$$

and is assumed to be smooth enough to assure the existence and uniqueness of solutions over a time interval

$$\Im = \left[t_0, \left(t_0 + T\right)\right[\in \mathbb{R} , \qquad (3)$$

as well as the continuos dependence of the solutions denoted by $\mathbf{x}(t, t_0, \mathbf{x}_0)$ with respect to t and the initial data.

The quantity T may be either a positive real number or the symbol $+\infty$, so that finite time stability and practical stabilty can be treated simultaneously, respectively.

In general, it is not required that

$$\mathbf{f}(t, 0, 0) \equiv 0 , \qquad (4)$$

for an autonomous system, which means that the origin of the state space is not necessarily required to be an equilibrium state.

Let $\mathbb{R}^n$ denotes the state space of the system given by (1) and $\|(\cdot)\|$ Euclidean norm.

Let $V: \Im\times\mathbb{R}^n \to \mathbb{R}$, be a tentative aggregate function, so that $V(t,\mathbf{x}(t))$ is bounded for and for which $\|\mathbf{x}(t)\|$ is also bounded.

Define the Eulerian derivative of $V(t,\mathbf{x}(t))$ along the trajectory of system (1), with

$$\dot{V}(t,\mathbf{x}(t)) = \frac{\partial V(t,\mathbf{x}(t))}{\partial t} + \left[grad\, V(t,\mathbf{x}(t))\right]^T \mathbf{f}(\;) . \qquad (5)$$

For time invariant sets it is assumed: $\mathcal{S}_{(\;)}$ is a bounded, open set.

The closure and boundary of $\mathcal{S}_{(\;)}$ are denoted by $\bar{\mathcal{S}}_{(\;)}$ and $\partial\mathcal{S}_{(\;)}$, respectively, so: $\partial\mathcal{S}_{(\;)} = \bar{\mathcal{S}}_{(\;)} \setminus \mathcal{S}_{(\;)}$.

$\bar{\mathcal{S}}^c_{(\;)}$ denotes the complement of $\mathcal{S}_{(\;)}$.

Let $\mathcal{S}_\beta$ be a given set of all allowable states of the system for $\forall t \in \Im$.

Set $\mathcal{S}_\alpha$, $\mathcal{S}_\alpha \subset \mathcal{S}_\beta$ denotes the set of all allowable initial states and $\mathcal{S}_\varepsilon$ the corresponding set of allowable disturbances.

The sets $\mathcal{S}_\alpha$, $\mathcal{S}_\beta$ are connected and a priori known.

$\lambda(\;)$ denotes the eigenvalues of the matrix $(\;)$.

$\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues, respectively.

Some previous results

Consider a linear continuous singular system with state delay, described by

$$E\,\dot{\mathbf{x}}(t) = A_0\mathbf{x}(t) + A_1\mathbf{x}(t-\tau) , \qquad (6.a)$$

with a known compatible vector valued function of initial conditions

$$\mathbf{x}(t) = \boldsymbol{\varphi}(t), \;\; -\tau \le t \le 0 , \qquad (6.b)$$

where A_0 and A_1 are constant matrices with appropriate dimensions.

Moreover, we will asume that $rank\, E = r < n$.

Definition 1. The matrix pair (E, A_0) is said to be regular if $\det(sE - A_0)$ is not identically zero, *Xu et al*. (2002).

Definition 2. The matrix pair (E, A_0) is said to be impulsive free if $\deg ree\, \det(sE - A_0) = rank\, E$, *Xu et al*. (2002).

The linear continuous singular time delay system (2) may have an impulsive solution; however, the regularity and the absence of impulses of the matrix pair (E, A_0) ensure the existence and uniqueness of an impulse free solution to the system under consideration, which is defined in the following *Lemma*.

Lemma 1. Suppose that the matrix pair (E, A_0) is *regular* and *impulsive free*, then the solution to (6.a) *exists* and is *impulse free* and *unique* on $[0, \infty[$, *Xu et al* (2002).

A necessity for system stability investigation creates a need for establishing a proper stability definition.

So we have

Definition 3.

a) A linear *continuous singular time delay system* (6) is said to be *regular* and *impulsive free* if the matrix pair (E, A_0) is regular and imulsive free.

b) A linear *continuous singular time delay system* (6) is said to be stable if for any $\varepsilon > 0$ there exists a scalar $\delta(\varepsilon) > 0$ such that, for any compatible nitial conditions $\boldsymbol{\varphi}(t)$, satisfying the condition: $\sup\limits_{-\tau\le t\le 0} \|\boldsymbol{\varphi}(t)\| \le \delta(\varepsilon)$, the solution $\mathbf{x}(t)$ of system (2) satisfies $\|\mathbf{x}(t)\| \le \varepsilon,\ \forall t \ge 0$.

Moreover, if $\lim\limits_{t\to\infty} \|\mathbf{x}(t)\| \to 0$, the system is said to be *asymptotically stable*, *Xu et al* (2002).

Main results

Let us consider the case when the subspace of consistent initial conditions for *singular time delay* and the *singular nondelay system* coincide.

Finite time stability

Stability definition

Definition 4. A regular and impulsive free singular time delayed *s*ystem (6) is *finite time stable* with respect to $\{t_0, \Im, \mathcal{S}_\alpha, \mathcal{S}_\beta\}$, if and only if $\forall \mathbf{x}_0 \in \mathcal{W}^*_{cont}$ satisfying

$$\|\mathbf{x}(t_0)\|^2_{E^TE} = \|\mathbf{x}_0\|^2_{E^TE} < \alpha ,$$

implies

$$\|\mathbf{x}(t)\|^2_{E^TE} < \beta, \quad \forall t \in \Im ,$$

$\mathcal{W}_k^*$ being the subspace of consistent initial conditions.

Remark 1. The singularity of the matrix E will ensure that solutions to (6) exist only for a special choice of $\mathbf{x}_0$.

In *Owens*, *Debeljković* (1985) the subspace of $\mathcal{W}_k^*$ of consistent initial conditions is shown to be the limit of the nested subspace algorithm

$$\begin{aligned} &\mathcal{W}_{k,0}^* = \mathbb{R}^n \\ &\quad\vdots \\ &\mathcal{W}_{k,(j+1)}^* = A_0^{-1}\left(E\mathcal{W}_{k,(j)}^*\right)_{A_1=0}, \quad j \geq 0 \end{aligned} \tag{7}$$

Moreover, if $\mathbf{x}_0 \in \mathcal{W}_k^*$ then $\mathbf{x}(t) \in \mathcal{W}_k^*$, $\forall t \geq 0$ and $\left(\lambda E - A_0\right)_{A_1=0}$ is invertible for some $\lambda \in \mathbb{R}$ (a condition for uniqueness), then

$$\mathcal{W}_k^* \cap \aleph(E) = \{0\}. \tag{8}$$

Stability theorem

Theorem 1. Suppose that $\left(E^T E - I\right) > 0$.

A singular time delayed system (6) is *finite time stable* with respect to $\left\{t_0, \Im, \alpha, \beta, \|(\cdot)\|^2\right\}$, $\alpha < \beta$, if the following condition is satisfied:

$$e^{\bar{\lambda}_{\max}(\Psi)(t-t_0)} < \frac{\beta}{\alpha}, \quad \forall t \in \Im, \tag{9}$$

where:

$$\bar{\lambda}_{\max}(\Xi) = \\ = \bar{\lambda}_{\max}\left(\begin{matrix} \mathbf{x}^T(t)\left(A_0^T E + E^T A_0 + E^T A_1\left(E^T E - I\right)^{-1} A_1^T E + \beta I\right)\mathbf{x}(t), \\ \mathbf{x}(t) \in \mathcal{W}_k^*, \\ \mathbf{x}^T(t) E^T E\,\mathbf{x}(t) = 1 \end{matrix}\right) \tag{10}$$

Proof. The define tentative aggregation function as

$$V(\mathbf{x}(t)) = \mathbf{x}^T(t) E^T E\,\mathbf{x}(t) + \int_{t-\tau}^{t} \mathbf{x}^T(\vartheta)\mathbf{x}(\vartheta)\,d\vartheta. \tag{11}$$

Let $\mathbf{x}_0$ be an arbitrary consistent initial condition and $\mathbf{x}(t)$ the resulting system trajectory.

The total derivative $\dot{V}(t, \mathbf{x}(t))$ along the trajectories of the system, yields

$$\begin{aligned} &\dot{V}(t, \mathbf{x}(t)) = \frac{d}{dt}\left(\mathbf{x}^T(t) E^T E\,\mathbf{x}(t)\right) \\ &+\frac{d}{dt}\int_{t-\tau}^{t} \mathbf{x}^T(\vartheta)\mathbf{x}(\vartheta)\,d\vartheta = \mathbf{x}^T(t)\left(A_0^T E + E^T A_0\right)\mathbf{x}(t) \\ &+2\mathbf{x}^T(t) E^T A_1 \mathbf{x}(t-\tau) + \mathbf{x}^T(t)\mathbf{x}(t) + \mathbf{x}^T(t-\tau)\mathbf{x}(t-\tau) \end{aligned} \tag{12}$$

From (12) it is obvious

$$\begin{aligned} \frac{d}{dt}\left(\mathbf{x}^T(t) E^T E\,\mathbf{x}(t)\right) &= \mathbf{x}^T(t)\left(A_0^T E + E^T A_0\right)\mathbf{x}(t) \\ &+2\mathbf{x}^T(t) E^T A_1 \mathbf{x}(t-\tau) \end{aligned}, \tag{13}$$

and based on a well-known inequality[4] and with the particular choice

$$\begin{aligned} &\mathbf{x}^T(t)\Gamma\,\mathbf{x}(t) = \\ &= \mathbf{x}^T(t)\left(E^T E - I\right)\mathbf{x}(t) > 0, \ \forall \mathbf{x}(t) \in \mathcal{W}_k^* \backslash \{0\} \end{aligned} \tag{14}$$

so

$$\begin{aligned} &\frac{d}{dt}\left(\mathbf{x}^T(t) E^T E\,\mathbf{x}(t)\right) \leq \mathbf{x}^T(t)\left(A_0^T E + E^T A_0\right)\mathbf{x}(t) \\ &+\mathbf{x}^T(t) E^T A_1\left(E^T E - I\right)^{-1} A_1^T E\,\mathbf{x}(t) \\ &+\mathbf{x}^T(t-\tau)\left(E^T E - I\right)\mathbf{x}(t-\tau) \end{aligned} \tag{15}$$

and the fact that it is more than obvious, that one can adopt

$$\|\mathbf{x}(t-\tau)\|_{E^T E}^2 < \beta \|\mathbf{x}(t)\|_{E^T E}^2, \ \forall \mathbf{x}(t) \in \mathcal{W}_k^* \backslash \{0\}, \tag{16}$$

since one is looking for a real condition to avoid $\|\mathbf{x}(t)\|_{E^T E}^2 = \beta$, for any $\forall t \in \Im$.

Moreover, since $\|\mathbf{x}(t-\tau)\|_I^2 > 0$,(15) is reduced to:

$$\begin{aligned} &\frac{d}{dt}\left(\mathbf{x}^T(t) E^T E\mathbf{x}(t)\right) < \\ &< \mathbf{x}^T(t)\left(A_0^T E + E^T A_0 + E^T A_1\left(E^T E - I\right)^{-1} A_1^T E + \beta I\right)\mathbf{x}(t) \\ &< \bar{\lambda}_{\max}(\Xi)\,\mathbf{x}^T(t) E^T E\mathbf{x}(t) \end{aligned} \tag{17}$$

Remark 2. Note that *Lemma* A1[5] and *Theorem* A1 indicate that

$$V(\mathbf{x}(t)) = \mathbf{x}^T(t) E^T E\,\mathbf{x}(t), \tag{18}$$

is a *positive quadratic form* on $\mathcal{W}_k^*$, and it is obvious that all smooth solutions $\mathbf{x}(t)$ evolve in $\mathcal{W}_k^*$, so $V(\mathbf{x}(t))$ can be used as a *Lyapunov function* for the system under consideration, *Owens*, *Debeljkovic* (1985).

It will be shown that the same argument can be used to declare the same property of another quadratic form present in the sequel.

Using (10) one can get:

$$\frac{d\left(\mathbf{x}^T(t) E^T E\,\mathbf{x}(t)\right)}{\mathbf{x}^T(t) E^T E\,\mathbf{x}(t)} < \bar{\lambda}_{\max}(\Xi)\,dt, \tag{19}$$

or

$$\int_{t_0}^{t} \frac{d\left(\mathbf{x}^T(t) E^T E\,\mathbf{x}(t)\right)}{\mathbf{x}^T(t) E^T E\,\mathbf{x}(t)} < \int_{t_0}^{t} \bar{\lambda}_{\max}(\Xi)\,dt, \tag{20}$$

and

$$\mathbf{x}^T(t) E^T E\,\mathbf{x}(t) < \mathbf{x}^T(t_0) E^T E\,\mathbf{x}(t_0) e^{\bar{\lambda}_{\max}(\Xi)(t-t_0)}. \tag{21}$$

Finally, if one uses the first condition of *Definition* 4, then

$$\mathbf{x}^T(t) E^T E\,\mathbf{x}(t) < \alpha \cdot e^{\bar{\lambda}_{\max}(\Xi)(t-t_0)}, \tag{22}$$

and finally by (9) yields to

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[4] $2\mathbf{u}^T(t)\mathbf{v}(t-\tau) \leq \mathbf{u}^T(t)\Gamma^{-1}\mathbf{u}(t) + \mathbf{v}^T(t-\tau)\Gamma\mathbf{v}(t-\tau), \ \Gamma > 0$

[5] See *Appendix* A .

$$\mathbf{x}^T(t)E^T E\,\mathbf{x}(t) < \alpha\cdot\frac{\beta}{\alpha} < \beta, \quad \forall t\in\Im, \tag{23}$$

Q.E.D.

Remark 3. In the case of non-delay systems, e.g. $A_1 \equiv 0$, (9) and (10) reduce to the basic result given in *Debeljkovic*, *Owens* (1985).

Practical stability

Stability definitions

Definition 5. A regular and impulsive free singular time delayed *s*ystem (6) is *attractive practically stable* with respect to $\{t_0,\Im,\mathcal{S}_\alpha,\mathcal{S}_\beta\}$, if and only if $\forall\mathbf{x}_0\in\mathcal{W}_k^*$ satisfying

$$\|\mathbf{x}(t_0)\|^2_{G=E^TPE} = \|\mathbf{x}_0\|^2_{G=E^TPE} < \alpha,$$

implies:

$$\|\mathbf{x}(t)\|^2_{G=E^TPE} < \beta, \quad \forall t\in\Im,$$

with a property that:

$$\lim_{k\to\infty}\|\mathbf{x}(t)\|^2_{G=E^TPE}\to 0,$$

$\mathcal{W}_k^*$ being the subspace of consistent initial conditions.

Stability theorems

Theorem 2. Suppose that $\left(E^TE-I\right)>0$.

A singular time delayed *s*ystem (6), with the system matrix A_0 being nonsingular, is *attractive practically stable* with respect to $\left\{t_0,\Im,\alpha,\beta,\|(\,\cdot\,)\|^2_{G=E^TPE}\right\}$, $\alpha<\beta$, if there exists a matrix $P=P^T>0$, being the solution of:

$$A_0^TPE+E^TPA_0=-Q, \tag{24}$$

with the matrice $Q=Q^T>0$, such that

$$\mathbf{x}^T(t)Q\,\mathbf{x}(t)>0, \quad \forall\mathbf{x}(t)\in\mathcal{W}_k^*\backslash\{0\}, \tag{25}$$

is a positive definite quadratic form on $\mathcal{W}_k^*\backslash\{0\}$, $\mathcal{W}_k^*$ being the subspace of consistent initial conditions, and if the following condition is satisfied

$$e^{\bar{\lambda}_{\max}(\Psi)(t-t_0)} < \frac{\beta}{\alpha}, \quad \forall t\in\Im, \tag{26}$$

where:

$$\bar{\lambda}_{\max}(\Psi) = \max\left(\begin{array}{c}\mathbf{x}^T(t)\left(\left(E^TPA_1\left(E^TPE-Q\right)^{-1}A_1^TPE+\beta I\right)+\beta I\right)\mathbf{x}(t),\\ \mathbf{x}(t)\in\mathcal{W}_k^*,\\ \mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)=1\end{array}\right) \tag{27}$$

Proof. Define the tentative aggregation function as

$$V(\mathbf{x}(t)) = \mathbf{x}^T(t)E^TPE\,\mathbf{x}(t) + \int_{t-\tau}^{t}\mathbf{x}^T(\vartheta)Q\,\mathbf{x}(\vartheta)\,d\vartheta. \tag{28}$$

The total derivative $\dot V(t,\mathbf{x}(t))$ along the trajectories of the system, yields

$$\begin{aligned}\dot V(t,\mathbf{x}(t)) &= \frac{d}{dt}\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)\\ &+\frac{d}{dt}\int_{t-\tau}^{t}\mathbf{x}^T(\vartheta)Q\mathbf{x}(\vartheta)\,d\vartheta = \mathbf{x}^T(t)\left(A_0^TPE+E^TPA_0\right)\mathbf{x}(t)\\ &+2\mathbf{x}^T(t)E^TPA_1\mathbf{x}(t-\tau)+\mathbf{x}^T(t)Q\,\mathbf{x}(t)\\ &+\mathbf{x}^T(t-\tau)Q\,\mathbf{x}(t-\tau)\end{aligned} \tag{29}$$

From (29), it is obvious

$$\begin{aligned}&\frac{d}{dt}\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)=\\ &=\mathbf{x}^T(t)\left(A_0^TPE+E^TPA_0\right)\mathbf{x}(t)\\ &+2\mathbf{x}^T(t)E^TPA_1\mathbf{x}(t-\tau)\end{aligned} \tag{30}$$

or

$$\begin{aligned}&\frac{d}{dt}\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)=\\ &=\mathbf{x}^T(t)\left(A_0^TPE+E^TPA_0+Q\right)\mathbf{x}(t)\\ &+2\mathbf{x}^T(t)E^TPA_1\mathbf{x}(t-\tau)-\mathbf{x}^T(t)Q\,\mathbf{x}(t)\end{aligned}. \tag{31}$$

From (24) it follows

$$\begin{aligned}&\frac{d}{dt}\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)=-\mathbf{x}^T(t)Q\,\mathbf{x}(t)\\ &+2\mathbf{x}^T(t)E^TPA_1\mathbf{x}(t-\tau)\end{aligned}, \tag{32}$$

as well, using the before mentioned inequality, with a particular choice

$$\begin{aligned}\mathbf{x}^T(t)\Gamma\,\mathbf{x}^T(t)=\mathbf{x}^T(t)\left(E^TPE-Q\right)\mathbf{x}^T(t)>0,\\ \forall\mathbf{x}(t)\in\mathcal{W}_k^*\backslash\{0\}\end{aligned} \tag{32}$$

And the fact that

$$\mathbf{x}^T(t)Q\,\mathbf{x}(t)>0, \quad \forall\mathbf{x}(t)\in\mathcal{W}_k^*\backslash\{0\}, \tag{33}$$

is a positive definite quadratic form on $\mathcal{W}_k^*\backslash\{0\}$, one can get

$$\begin{aligned}&\frac{d}{dt}\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)=2\mathbf{x}^T(t)E^TPA_1\mathbf{x}(t-\tau)\\ &\le\mathbf{x}^T(t)E^TPA_1\left(E^TPE-Q\right)^{-1}A_1^TPE\,\mathbf{x}(t)\\ &+\mathbf{x}^T(t-\tau)\left(E^TPE-Q\right)\mathbf{x}(t-\tau)\end{aligned} \tag{34}$$

And the fact that it is more than obvious, that one can adopt

$$\|\mathbf{x}(t-\tau)\|^2_{E^TPE} < \beta\cdot\|\mathbf{x}(t)\|^2_{E^TPE}, \ \forall\mathbf{x}(t)\in\mathcal{W}_k^*\backslash\{0\} \tag{35}$$

since we look, in real conditions, to avoid $\|\mathbf{x}(t)\|^2_{E^TPE}=\beta$ for any $\forall t\in\Im$, so (34) is reduced to

$$\begin{aligned}&\frac{d}{dt}\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)<\\ &<\mathbf{x}^T(t)\left(E^TPA_1\left(E^TPE-Q\right)^{-1}A_1^TPE+\beta I\right)\mathbf{x}(t)\end{aligned}, \tag{36}$$

or using (27), one can get

$$\begin{aligned}&\frac{d}{dt}\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)<\\ &<\mathbf{x}^T(t)\left(E^TPA_1\left(E^TPE-Q\right)^{-1}A_1^TPE+\beta I\right)\mathbf{x}(t)<\\ &<\bar{\lambda}_{\max}(\Psi)\,\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\end{aligned} \tag{37}$$

or

$$\frac{d\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)}{\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)} < \bar{\lambda}_{\max}(\Psi)\,dt, \tag{38}$$

or

$$\int_{t_0}^{t}\frac{d\left(\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)\right)}{\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t)} < \int_{t_0}^{t}\bar{\lambda}_{\max}(\Psi)\,dt, \tag{39}$$

and

$$\mathbf{x}^T(t)E^TPE\mathbf{x}(t) < \mathbf{x}^T(t_0)E^TPE\mathbf{x}(t_0)e^{\bar{\lambda}_{\max}(\Psi)(t-t_0)}. \tag{40}$$

Finally, if we use the first condition, given by *Definition* 5, then

$$\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t) < \alpha\cdot e^{\bar{\lambda}_{\max}(\Psi)(t-t_0)}, \tag{41}$$

and basic condition (9) of *Theorem* 2, we can get

$$\mathbf{x}^T(t)E^TPE\,\mathbf{x}(t) < \alpha\cdot\frac{\beta}{\alpha} < \beta, \quad \forall t\in\Im, \tag{42}$$

Q.E.D.

Remark 3. This *Remark c*oincides with *Remark* 2, with a particular choice

$$V(\mathbf{x}(t)) = \mathbf{x}^T(t)E^TPE\,\mathbf{x}(t), \tag{43}$$

Remark 4. (24) is, in its original form, taken from *Owens*, *Debeljkovic* (1985).

Remark 3. Let us discuss first the case when the time delay *is absent.*

Then the *singular* (weak) Lyapunov matrix equation (24) is a natural generalization of the classical Lyapunov theory.

In particular

a) If E is a *nonsingular matrix*, then the system is asymptotically stable if and only if $A = E^{-1}A_0$ Hurwitz matrix. Equation (24) can be written in the form

$$A^TE^TPE + E^TPEA = -Q, \tag{44}$$

with the matrix Q being symmetric and positive definite, in the whole state space, since then $\mathcal{W}^*_k = \Re\left(E^{k^*}\right) = \mathbb{R}^n$.

In these circumstances, E^TPE is a Lyapunov function for the system.

b) The matrix A_0 by necessity is nonsingular and hence the system has the form

$$E_0\dot{\mathbf{x}}(t) = \mathbf{x}(t), \quad \mathbf{x}(0) = \mathbf{x}_0. \tag{45}$$

Then for this system to be stable, a familiar Lyapunov structure must also hold

$$E_0^TP + PE_0 = -Q, \tag{46}$$

where Q is a symmetric matrix but only required to be positive deffinite on $\mathcal{W}^*_k$.

Conclusion

Generally, this paper extends some of the basic results in the area of Non-Lyapunov to *linear*, *continuous singular time invariant time-delay systems* (LCSTDS).

In that sense, the part of this result is hence a geometric counterpart of the algebraic theory of *Campbell* (1980) charged with appropriate criteria to cover a need for system stability under the presence of actual time delay term.

A quite new sufficient delay–independent criterion for the finite and attractive practical stability of LCSTDS is presented.

APPENDIX – A

The fundamental geometric tool in the characterization of the subspace of consistent initial conditions, for *linear singular systems without delay*, is the subspace sequence

$$\mathcal{W}^*_0 = \mathbb{R}^n, \tag{A1}$$

$$\vdots$$

$$\mathcal{W}^*_{j+1} = A_0^{-1}\left(E\mathcal{W}^*_j\right), \; j\geq 0, \tag{A2}$$

where $A_0^{-1}(\cdot)$ denotes the inverse image of ( ) under the operator A_0.

Lemma A1. The subsequence $\{\mathcal{W}_0, \mathcal{W}_1, \mathcal{W}_2 .....\}$ is nested in the sense that

$$\mathcal{W}_0 \supset \mathcal{W}_1 \supset \mathcal{W}_2 \supset \mathcal{W}_3 \ldots \supset \tag{A3}$$

Moreover,

$$\aleph(A) \subset \mathcal{W}_j, \quad \forall j\geq 0, \tag{A4}$$

and there exists an integer $k\geq 0$, such that

$$\mathcal{W}_{k+1} = \mathcal{W}_k, \quad \forall j\geq 1. \tag{A5}$$

Then, it is obvious that

$$\mathcal{W}_{k+j} = \mathcal{W}_k, \quad \forall j\geq 1. \tag{A6}$$

If k^* is the smallest such integer with this property, then

$$\mathcal{W}_{k^*} \cap \aleph(E) = \{0\}, \quad k\geq k^*, \tag{A7}$$

provide that $(\lambda E - A_0)$ is invertible for some $\lambda\in\mathbb{R}$.

Proof. See *Owens*, *Debeljkovic* (1985).

Theorem A.1. Under the conditions of *Lemma* A1, $\mathbf{x}_0$ is a consistent initial condition for the system under consideration if and only if $\mathbf{x}_0 \in W_{k^*}$.

Moreover, $\mathbf{x}_0$ generates a unique solution $\mathbf{x}(t)\in\mathcal{W}_{k^*}$, $t \geq 0$, that is real analytic on $\{t : t\geq 0\}$.

Proof. See *Owens*, *Debeljkovic* (1985).

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Stabilnost linearnih vremenski kontinualnih singularnih i vremenski diskretnih deskriptivnih sistema sa čistim vremenskim kašnjenjem na konačnom vremenskom intervalu: Pregled rezultata - I deo kontinualni slučaj

U radu se daje pregled dovoljnih uslova praktične stabilnosti i stabilnosti na konačnom vremenskom intervalu linearnih vremenski kontinualnih singularnih sistema sa čistim vremenskim kašnjenjem. Kadase razmatra concept stabilnosti na konačnom vremenskom intervalu, ovi novi uslovi nezavisni od kašnjenja, su izvedeni korišćenjem prilaza koji se osnova na tzv. Kvazi-Ljapunovljevim funkcijama I njihovim osobinama na potprostoru konzistentnih početnih uslova. U opštem slučaju ove funkcije ne moraju: a) da budu pozitivno određene u celom prostoru stanja, b) niti njihovi izvodi duž trajektorije sistema negativno određeni.

Kada se razmatra praktična stabilnost, ranije pomenuti prilaz, kombinuje se i podržan je klasičnom teorijom Ljapunova a sa ciljem da bi se obezbedila atraktivna praktična stabilnost.

Ključne reči: **linearni sistem, kontinualni sistem, deskriptivni sistem, singularni sistem, sistem sa kašnjenjem, diskretni sistem, stabilnost sistema.**

Устойчивость линейных сингулярных непрерывных и дискретных дескриптивных систем с чистым временем задержки на конечном интервале времени: резюме результатов непрепывного случая - I - часть

В настоящей работе представлен подробный обзор достаточных условий практической устойчивости и стабильности в конечное время линейных непрерывных сингулярных систем с чистим временем задержки. Когда рассматривается концепция стабильности в конечное время, эти новые условия независимые от временной задержки, получены с помощью подходов, которые лежат в основе так называемых квази-Ляпуновских функций и их свойств в подпространстве соответствующих начальным условиям.

В целом, эти функции не должны быть: а) положительно определены во всём пространстве состояния, б) а в том числе ни отрицательно определены их выводы вдоль траектории системы. При рассмотрении вопроса о практической стабильности, ранее указанный подход находится в сочетании и поддерживается классической теорией Ляпунова, с целью обеспечения привлекательной практической стабильности.

Ключевые слова: **линейная система, непрерывная система, дескриптивная система, сингулярная система, система с временной задержкой, дискретная система, устойчивость системы.**

Stabilité des systèmes linéaires continus singuliers et des systmes discrets temporels descriptifs à délai temporel fini: Tableau des résultats – Première partie, continus

Dans ce papier on a donné les conditions suffisantes pour la stabilité pratique et pour la stabilité sur l'intervalle temporelle finie chez les systèmes linéaires singuliers continus à délai temporel pur. Lorsque le concept de la stabilité sur l'intervalle temporelle finie est considéré, ces nouvelles conditions, indépendantes du délai, ont été dérivées à l'aide de l'approche basée sur les fonctions quasi de Lyapunov et leurs caractéristiques dans le sous espace des conditions initiales consistantes. Dans le cas général ces fonctions: a) ne doivent pas être positives dans l'espace entier de l'état b)leurs dérivées ne doivent pas être négatives le long de la trajectoire. Considérant la stabilité pratique , l'approche citée ci-dessus est combinée et appuyée par la théorie classique de Lyapunov dans le but d'assurer la stabilité pratique attrayante.

Mots clés: **système linéaire, système continu, système descriptif, système singulier, système à délai, système discret, stabilité de système**