UDK: 007.52:741.5:004.6/7
COSATI: 13-06, 17-07

The Importance of Modelling an Aerial Robotic Camera

Mirjana Filipović[1]

An aerial robot (Cable-suspended Parallel Robot) has been under development for three decades. This system is now very attractive and useful to the public and is particularly interesting for engineering and scientific community from the perspective of its future intensive development. The research in this paper is directed towards the same goal, i.e. towards defining the Cable-suspended Parallel Robot that will follow and record a moving object with great accuracy wherever it is located in the workspace. The aim of this paper is a detailed analysis and development of the existing structure of aerial robots, which should enable their strong progress. This would be reflected in the implementation of highly-automated systems that would lead the camera precisely in space with the minimum participation of human labor. Setting and achieving this goal provide much wider possibilities for its future use. The sophisticated operation of this system can be provided only with the application of its high-fidelity mathematical model during the synthesis and analysis, which would further enable the development and application of modern control laws. Application possibilities are certainly much broader than it may be assumed at this moment, especially for military or police purposes.

Key words: **robots, aerial recording, cable coupling, control, mathematical modelling.**

Designations

DOF	–degree of freedom
CPR	–Cable-suspended Parallel Robot
$t(s)$	–time
$dt = 0.0001(s)$	–sample time
$g = 9.81(m/s^2)$	–gravitational acceleration
$p = [x \;\; y \;\; z]^T$	–position of the camera carrier in space of the Cartesian coordinates (external coordinate)
$i = 1,2,3.$	–total number of DOF
$\phi = [\theta_1 \;\; \theta_2 \;\; \theta_3]^T$	–vector of internal coordinates
F_1, F_2, F_3 $F_* = [F_1 \;\; F_2 \;\; F_3]^T$	–resultant force acting as a load on the motor
$F_p = [F_x \;\; F_y \;\; F_z]^T$	–acting force on the camera carrier
$P_p = [P_{px} \;\; P_{py} \;\; P_{pz}]^T$	–perturbation force acting on the camera carrier
$M_* = [M_1 \;\; M_2 \;\; M_3]^T$	–motor load moment
$\#_i$	–quantities which characterize one DOF of the observed mechanism
$\#^\circ$	–quantities that define a desired value
$\theta_i(rad)$	–rotation angle of the motor shaft after the reducer
$R = 0.15(m)$	–winch radius
$\mu = 0.05$	–friction coefficient
J_*	–Jacobi matrix
$R_{Ci} = 0.917(\Omega)$	–rotor circuit resistance
$u_i(V)$	–voltage
$G_{vi} = 0.1787$	–motor inertia characteristic
$L_{vi} = 3.4186$	–motor damping characteristic
$S_{vi} = 0.364$	–geometric characteristic of the motor
$u_i(V)$	–voltage
$i_i(A)$	–rotor current
C_{Ei}=3.3942(V/(rad/s))	–proportionality constants of the electromotive force
C_{Mi}=2.5194(Nm/A)	–proportionality constants of the moment
B_{ui}=0.0670(Nm/(rad/s))	–coefficient of viscous friction
$I_i = 1.5859(kgm^2)$	–inertia moments of the rotor and the reducer
$m = 1(kg)$	–mass of the camera carrier
$d = 3.2$(m)	–length of the recorded field
$s = 2.2$(m)	–width of the recorded field
$v = 2.0$(m)	–height of the recorded field
$\delta\theta_i(t_0) = 0$(rad), $\delta\dot{\theta}_i(t_O) = 0$(rad/s)	–initial deviation of the motor rotating angle
$K_{lpi} = 4200$, $K_{lvi} = 130$	–positional, velocity amplification for motion control
$\Diamond = 1/2$	–factor that characterizes two parallel guided ropes through the CPR

---

[1] Institute of Mihajlo Pupin, Volgina 15, 11060 Belgrade, SERBIA

Introduction

A Cable-suspended Parallel Robot (CPR) is developed to some extent and widely analyzed in the world and in various research areas as well as for different purposes. Similar system groups were analyzed and modelled as evidenced by numerous publications.

One of these groups is a parallel driven manipulator mechanism which has been the subject of research of many scientists for some years (see papers [4], [5], [9], [18], [19], [20], [23], [25], [26], [27], [29], [32], [33], and [41]).

Papers [4], [9], [18], [19], [23], [25], [26], [27], [32], [33], and [41] analyze the kinematics of different types of parallel robot mechanisms.

The general solution of the inverse dynamics of redundantly actuated parallel manipulators is given in [29]. For a special case of simple over actuation, an explicit solution is derived in terms of a single preload parameter. With this formulation, a computational efficient open-loop preload control is developed and applied to the elimination of backlash.

In [20], two solutions based on convex optimization are presented for the optimal distribution of the cables and redundant limb forces. One solution represents the minimum-norm solution for the forces in the cables and the redundant limbs and the other solution only minimizes the cable forces.

In paper [5], the motion of a 6-DOF parallel robot is studied based on the screw theory. The coupling compensation is proposed to optimize the moving track. It uses the mechanism-model combined method which takes practical moving track that considering the performance of motion controller and motor as its input to make the study.

Recently, the authors of works [1], [2], and [28] have developed a new type of a mechanism in the K. N. Toosi University of Technology, Tehran, Iran.

The cable driven redundant parallel manipulator for possible high speed and large workspace applications is under investigation. These newly developed mechanisms have several advantages compared to conventional parallel mechanisms. Its rotational motion range is relatively large, its redundancy improves safety for failure in cables, and its design is suitable for long-time high acceleration motions. The mechanism is thus designated based on such structure with 8 actuated 6 DOFs cable driven redundant parallel manipulators. The kinematic and dynamic analyses are given in the mentioned papers.

A special group of mechanisms is "Macro-Micro Parallel Manipulator", as called by the authors, developed in papers [3], [30], [37], and [38].

In these papers, the kinematic analysis of a macro-micro parallel manipulator is given in detail. The authors argue that the newly developed cable driven redundant parallel manipulators have numerous advantages compared to those of the conventional parallel mechanism. However, there are some challenging issues in over constrained mechanisms such as a cable driven redundant parallel manipulator. The manipulator architecture is a simplified planar version adopted from the structure of Large Adaptive Reflector, the Canadian design of the next generation giant radio telescopes. This structure is composed of two parallel and redundantly actuated manipulators at macro and micro levels, which are both cable-driven.

The cable driven redundant parallel manipulator for a large spherical radio telescope (LSRT), is analyzed and modelled in papers [6], [7], [21], [24], [31], [34], [35], [36], and [40].

A nonlinear dynamic analysis of the suspended cable system is carried out with some sensible results presented in [6] that could be useful to the real engineering of LSRT. Integrated mechanical, electronic, optic and automatic control technologies are employed to make considerable improvement upon the same system.

A multiple cable robotic crane designed in [34] to provide improved cargo handling is investigated. The equations of motion are derived for the cargo and flexible cable using Lagrange's equations and the assumed modes method. The results are compared against the desired cable lengths and the results achieved in the previous research using a rigid cable model. This is one of a few papers dealing with flexible ropes.

For the requirement of trajectory tracking of large spherical radio telescopes, a large fine tuning platform based on the Stewart platform is presented in papers [35] and [36]. The mathematical model for kinematic control is developed with coordinate transformation, and a dynamic analysis is carried out using the Jacobi matrix, which, with a singularity analysis, built a solid base for the tracking control.

Cable-suspended robots are structurally similar to parallel actuated robots but with the fundamental difference that cables can only pull the end-effectors but not push them. From a scientific point of view, this feature makes the feedback control of cable-suspended robots much more challenging than that in their counterpart parallel actuated robots. The authors of paper [31] look into the control design for non redundant cable-suspended robot under positive input constraints.

Paper [24] addresses the static analysis of cable-driven robotic manipulators with non-negligible cable mass. An approach to computing the static displacement of a homogeneous elastic cable is presented. The resulting cable-displacement expression is used to solve the inverse kinematics of general cable-driven robotic manipulators.

A cable-suspended parallel robot is analyzed in [7], in which cables are utilized to replace links to manipulate objects. It is developed from parallel and serial cable-driven robots. Compared with the parallel robot, this kind of robots has more advantages. The cooperative variation of lengths of six cables pulls the feed cabin to track the radio source with six degrees of freedom.

The cable-driven parallel manipulator can only bear tension, but not compression. Therefore, a cable system with j end-effectors DOFs requires at least (j+1) cables as in [21]. For three-translational motions of the feed in the system, a four-cable-driven parallel manipulator is developed.

For the design of the five-hundred–meter aperture spherical radio telescope, a four-cable-driven parallel manipulator, which is long in span and heavy in weight, is adopted as the first-level adjustable feed-support system. The purpose of paper [40] is to optimize the dimensions of the four-cable-driven parallel manipulator to meet the workspace requirement of constraint condition in terms of cable tension and stiffness.

In papers [8], [22], and [39], the authors analyzed the fitting accuracy of the active reflector units of a large spherical radio telescope.

The research topic of this paper comes exactly from a group of referred papers [1]-[9], [18]-[41]and it is particularly inspired by the results of papers [6], [7], [21], [24], [31], [34], [35], [36], and [40]. The Cable-suspended Parallel Robot is in papers [6], [7], [21], [24], [31], [34], [35], [36], and [40] an integral part of a larger system, the so-called Large Spherical Radio Telescopes. However, the same CPR can be used for many other purposes, and one of them is the observation of workspace.

This work was done for the suspension system in four points.

For three decades, researchers have dealt with the mechanisms that carry a camera for space observations or moving objects in space. The camera moves quietly and continuously following the observed object. The camera carrier moves in space freely allowing the capture of objects from above.

It gives a unique feeling to the event viewer of observing the action smoothly from an unusual proximity, regardless of the size of the observed space. Free motion in space opens up a completely new and unique perspective. The implementation of this system uses the latest technical knowledge. This is achieved using the latest computers, networks and new types of motors, combined with confirmed high quality camera and video components.

A workspace is an area where the camera can move. The observed space is a slightly wider area of workspace. A camera located in the workspace can capture and record the events outside the working area. The motion of the ropes carrying the camera is controlled. Ropes are unwound (or wound), thus reaching any camera position in space. The control system has the operating software, which as a result provides a three-dimensional motion of the camera. The commands for the synchronized motion of each winch are provided, with the control of motion of each motor, which ultimately provides a continuous three-dimensional camera motion. Real-time communication between the winches and the control module is achieved via a cable. The motors receive and implement the information from the control station, through the winches. The camera transmits their signal back to the control station. The carrier that carries the camera does not have only a role of mechanical motion, because through it, the lens (focus, zoom and clarity) is remotely controlled.

The gyro sensor, installed in the carrier, is stabilized towards the horizon.

The modular design system for the observation of workspace, lightweight components and a small force in the ropes allow the successful setting up of a system at almost any location, which offers wide possibilities for its use in closed or open space, regardless of its size. It may be placed in the most unlikely places, for example when film making.

Regardless of the purpose and in which areas the system is installed, top priority is definitely a high safety of participants in the recorded event. The CPR system should fly over the audience without being able to be off and fall to the ground.

Who first developed a CPR system? There are different claims. Source http://www.spidercam.net, the company Skycam claims that it was discovered in 1984 by Garrett Brown. Source http://broadcastengineering.com/infrastructure/cablecam_camera_hbo_welterweight_championships_1221/, Broadcast Engineering firm claims Jim Rodnunsky discovered it in 1985. The source http://en.wikipedia.org/wiki/ Spidercam writes that it was Jens C.Peters, a researcher from CCSytems Inc. It is certain that all of them are big names in this field because they contributed to the development of this system.

In its initial phase, the system was used in limited circumstances that the technology development allowed. Because of that, its development has been slow, due to the support of computers technique. Strong progress in the development of this system began in the middle of the 90s, inspired by the development of computer science and servo motors as well as the development of other components of the system. The use of this system has been constantly expanding since. During the transfer of sporting or other public events (football, basketball, hockey or other games, tennis or athletic competitions, Eurovision, concerts of various musical genres, etc.) the presence of CPR system used for recording can be noticed. However, in many areas this system has not yet appeared. Due to wide possibilities of application and further development of its system, meticulous research is justified and promising. The goal is to realize a CPR system that would be moving precisely in space.

Such a system is primarily very useful, expensive and demanding during the operation.

This paper aims at forming a mathematical model for a CPR system according to the original scientific principles. This paper proposes the application of theoretical methods in solving specific problems rather than the implementation of systems engineering approaches.

Due to wide possibilities of the system application as well as further development of the system, the thorough research is justified and prospective. The goal is to realize the Cable-suspended Parallel Robot that would move strictly in the space.

One constructive CPR solution is presented in Section 2. In Section 3, a detailed description of the CPR and its mathematical model are given. The samples of the system responses are analyzed for different conditions. In Section 4, the importance of forming a mathematical CPR model is emphasized, while in Section 5, the concluding remarks are presented.

Cable-suspended parallel robot

One constructive solution of the CPR is defined in Fig.1. This solution was technically implemented to some extent and abandoned for unknown reasons.

The working space of a camera carrier is in the form of a parallelepiped, so that the camera carrier hangs over the ropes connected properly on the four highest points i.e. the four upper angles of the workspace.

The suspension system is defined in four points.

A camera workspace is an area where a camera can move quietly and continuously following the observed object.

A camera carrier moves freely in the space enabling the shooting of the objects from above. It gives a unique feeling to the viewer to observe the event easily from an unusual proximity and to be very close to the action regardless of the size of the observed space.

This paper analyzes the mechanism which involves only the positioning of the camera carrier. It is assumed that the camera orientation can be determined using the already known, smaller mechanism with three degrees of freedom. In this case, a camera carrier has a mechanism for defining the camera orientation and a camera is installed on the top of this mechanism. For large workspaces, this should be an optimal solution.

The CPR operates as shown, using two ropes and pulleys that are fixed for up to four corners of the point space observation (see Fig.1).

By rotating the motor angles θ_1, θ_2, θ_3, the winches of radius R rotate directly which fold or unfold ropes synchronously and thus move the camera carrier in the workspace x, y, z, of the Cartesian coordinates. The desired trajectory of the motion of the camera is defined in the x, y and z Cartesian coordinates, and implemented by the motion of three motors θ_1, θ_2, θ_3.

The whole system is very reliable in the physical sense, because there is only a minimum possibility for the camera carrier to fall to the shot ground. This would happen only if both ropes broke at the same time. Two parallel guided ropes through the system are necessary for the physical functioning of the system in Fig.1.

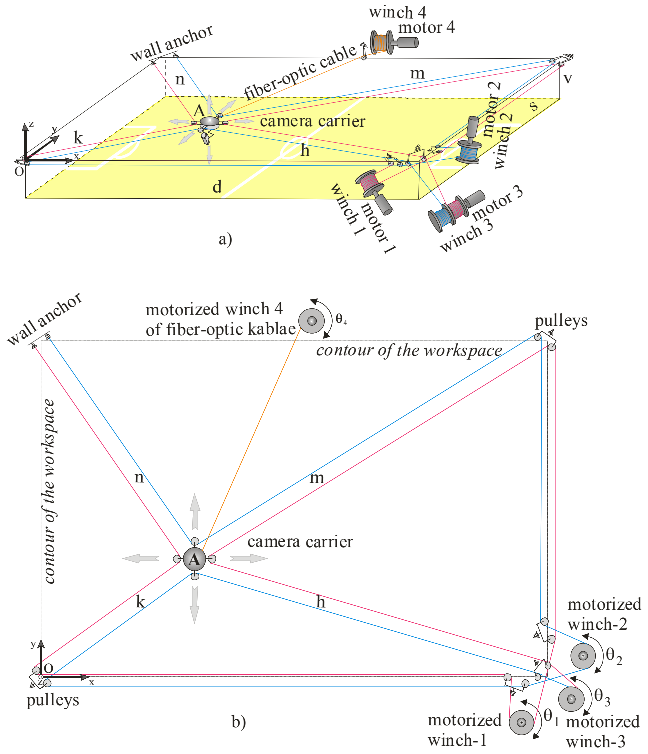

Figure 1. CPR system a) in the space and b) top view

In case that one rope breaks, the system ceases to function, but by definition the second rope will prevent the camera from falling to the ground and hurting somebody. This type of construction, by definition, guarantees the safety of participants in the workspace.

The video recording from the camera is sent to the user via a separate fiber-optic cable. Winch 4 is driven by a motor that realizes the motion θ_4 winding or unwinding fiber-optic cable, depending on the position of the camera carrier in the space.

The purpose of the motor motion θ_4 is to ensure that the fiber-optic cable is never tight or too loose to lower too much hanging above the recorded surface and thus not to obstruct the recorded participants.

This motion as well as the motion of the camera carrier was controlled by the operator using a joystick located so that at any time it followed the tension of the fiber-optic cable in relation to the camera carrier as well as to the camera motion.

This system depended in its functioning significantly on operators' concentration and responsibility. Their skill and experience played a major role. Since the implementation of task observation of an event can last for hours, it was a big responsibility for operators. Lack of concentration and their fatigue during this period indicated a significant dependence of this system on the human factor, which as a result could have a number of unavoidable inaccuracies during the operation.

Mathematical model of the Cable-suspended Parallel Robot

In this paper, the CPR is interesting from the scientific and research point of view i.e. from the aspect of defining its mathematical model (which is not known to the scientific community).

The kinetic E_k and potential E_p energy of the motion of the camera carrier with mass m are given with:

$$E_k = 1/2 \cdot m \cdot \dot{x}^2 + 1/2 \cdot m \cdot \dot{y}^2 + 1/2 \cdot m \cdot \dot{z}^2 \tag{1}$$

$$E_p = m \cdot g \cdot z \tag{2}$$

To define a dynamic model of the CPR for observation of moving objects in workspace, depicted in Fig.1, it is first necessary to define the external velocity of change of the coordinates $\dot{p} = [\dot{x} \quad \dot{y} \quad \dot{z}]^T$ and the velocity of change of the internal coordinates $\dot{\phi} = [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3]^T$. The Jacobi matrix for the CPR J_* maps the velocity vector of the external coordinates $\dot{p}$ into the velocity vector of the internal coordinates $\dot{\phi}$. This relationship is:

$$\dot{\phi} = J_* \cdot \dot{p} \tag{3}$$

where is

$$J_* = \begin{bmatrix} J_{*11} & J_{*12} & J_{*13} \\ J_{*21} & J_{*22} & J_{*23} \\ J_{*31} & J_{*32} & J_{*33} \end{bmatrix} \tag{4}$$

The elements of this matrix that are beyond the diagonal show the strong coupling between the external and internal coordinates.

The mathematical model of the system has the following form:

$$u = G_v \cdot \ddot{\phi} + L_v \cdot \dot{\phi} + S_v \cdot (1 + \mu) \cdot M_* \tag{5}$$

Where: $u = [u_1 \ u_2 \ u_3]^T$, $G_v = diag\, G_{vi}$, $L_v = diag\, L_{vi}$, $S_v = diag\, S_{vi}$. Vector equation (5) is given by applying Lagrange's equation on the generalized coordinates θ_1, θ_2, θ_3.

$$M_* = F_* \cdot R \tag{6}$$

$$F_* = \frac{\left(\left(J_{\Diamond *}\right)^T\right)^{-1}}{R} \cdot \left(F_p + P_p\right) \tag{7}$$

$$F_p = m \cdot (\ddot{p} + a_{cc}) \quad \text{where} \quad a_{cc} = [0 \quad 0 \quad -g]^T \tag{8}$$

$$J_{\Diamond *} = f(\Diamond, J_*) \tag{9}$$

The presence of the factors $\Diamond$ is a consequence of structural systems with Fig.1. Since the CPR has two parallel ropes, suspending cameras in all four directions, then equation (9) takes the form:

$$J_{\Diamond *} = \Diamond \cdot J_* \tag{10}$$

$\Diamond$ the factor which multiplies only one direction where there are two parallel ropes. In this direction, the force in each rope is half of the impact forces $F_p + P_p$ acting on the camera carrier. The considered CPR has two ropes from the camera carrier to all four-point suspensions (line k, h, m, n). The connection between the resultant forces F_* and the forces acting on the camera carrier in the Cartesian space coordinates $F_p + P_p$ is given by relation (7). This is a geometrical relationship, uniquely defined.

To obtain the relationship between the internal and the external forces, the virtual work principle is applied. Equation (7) is particularly important because it participates in the formation of a dynamic CPR model.

It is assumed that ropes are rigid. Substituting (6)-(10) in equation (5) gives a dynamic CPR model:

$$u = G_v \cdot \ddot{\phi} + L_v \cdot \dot{\phi} + S_v \cdot (1+\mu) \cdot \left((J_{\Diamond *})^T \right)^{-1} \cdot \left(m \cdot (\ddot{p} + a_{cc}) + P_p \right) \tag{11}$$

The matrix $\left((J_{\Diamond *})^T \right)^{-1}$ characterizes a strong coupling between the presented motors.

Control law is selected by the local feedback loop for the position and velocity of the motor shaft in the following form:

$$u_i = K_{lpi} \cdot (\theta_i^o - \theta_i) + K_{lvi} \cdot (\dot{\theta}_i^o - \dot{\theta}_i) \tag{12}$$

Example 1

The camera carrier has the starting point $p_p^o = [0.1 \;\; 1.0 \;\; -0.5](\mathrm{m})$, and the end point $p_{end}^o = [2.9 \;\; 1.0 \;\; -0.5](\mathrm{m})$.

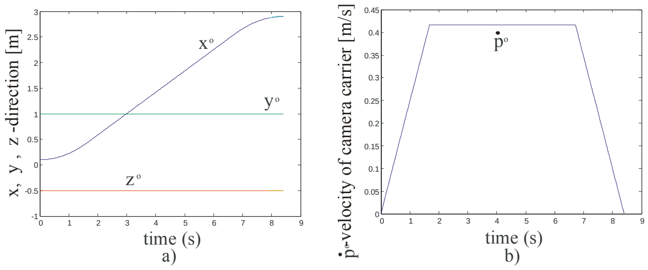

Figure 2. The reference trajectory motion of a) position x^o, y^o, z^o, b) velocity $\dot{p}_{\max}^o = 0.417(m/s)$ of camera carrier

(see Fig.2a). The camera moves in the x direction, while the coordinates y and z are constant. The camera motion velocity has a trapezoid form and $\dot{p}_{\max}^o = 0.417(\mathrm{m/s})$, as shown in Fig.2b. The motors are of Heinzman SL100F type and the gears are HFUC14-50-2A-GR+belt.

To emphasize the presence of the coupling properties, selected motion trajectories of the camera carrier are particularly appropriate. At the reference level, the mathematical model of the system is defined by (11). Despite the fact that the camera moves only in the x direction, while the directions x and y are constant (see Fig.3a), the results show that all three engines θ_1, θ_2, and θ_3 must participate in the realization of this task coordinated. This is especially evident in Fig.3b).

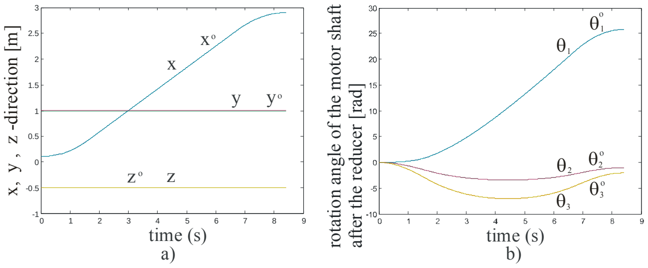

Figure 3. Reference and real trajectories of the motion position a) x, y, z camera carrier, b) θ_1, θ_2, θ_3 motor shaft (Example 1)

Namely, in order to realize the desired motion in the x direction, the motion of the motor θ_1 is not enough, also in order to realize the desired motion in the y direction, the motion of the motor θ_2 is not enough, and finally to realize the desired motion in the z direction, the motion of the motor θ_3 is not enough.

This is a confirmation that all these motions are mutually coupled. There has been a good track of the desired trajectory at the level of motor motions (about $10^{-3}(\mathrm{rad})$) and at the level of motion of the camera carrier (about $10^{-4}(\mathrm{m})$) (see Fig.4).

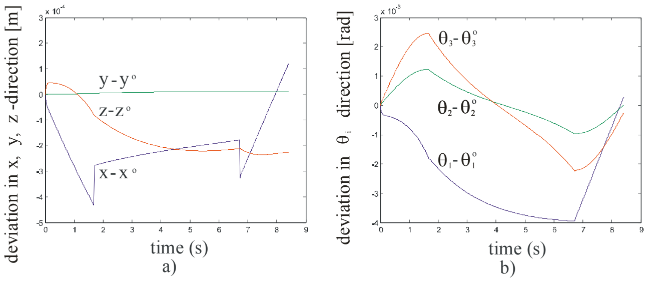

Figure 4. Real deviation from the reference values of the motion trajectory a) the camera carrier b) the motor shaft (Example 1)

The level of the control signals is given in Fig.5a) and does not exceed the limits of $\pm 24(V)$. Fig.5b) gives the resultant forces that are orders of magnitude to $22(N)$.

Example 2

The analyzed example is from Fig.1.

In order to make results mutually comparable, the desired trajectory and all other parameters of the system are the same. All system and control parameters are the same as in Example 1. The camera carrier has the starting point $p_{start}^o = [0.1 \;\; 1.0 \;\; -0.215](\mathrm{m})$, and the end point $p_{end}^o = [2.9 \;\; 1.0 \;\; -0.215](\mathrm{m})$.

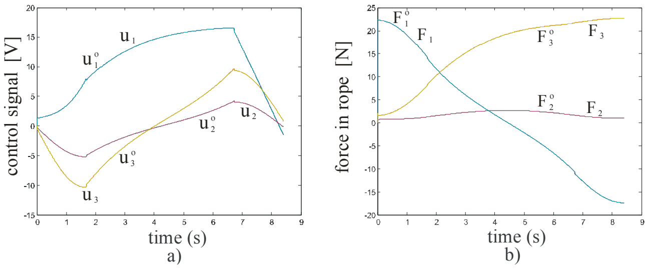

Figure 5. Real and reference a) control signal u_i, b) resultant forces F_i (Example 1)

Fig.6a) shows the results for the motion of the camera carrier in all three directions of the Cartesian coordinates at the real x, y, z and the reference x^o, y^o, z^o level.

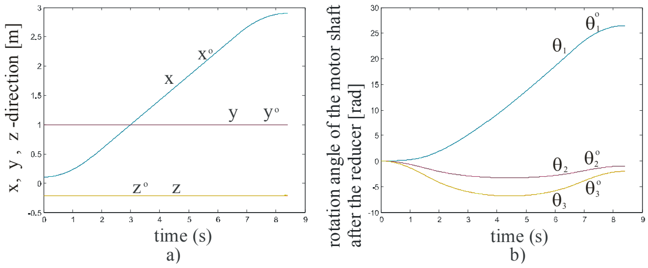

Figure 6. Reference and real trajectories of the motion position a) x, y, z camera carrier, b) θ_1, θ_2, θ_3 motor shaft (Example 2)

The trajectory of the motor rotation angles at the real θ_1, θ_2, θ_3 and the reference θ_1^o, θ_2^o, θ_3^o level is given in Fig.6b).

In this example, the camera carrier also moves only in the x direction while the coordinates y and z direction are constant. The coordinate y^o during the whole motion is $y^o = 1.0(\mathrm{m})$, while the coordinate z^o is no longer at the level $z^o = -0.5(\mathrm{m})$, as in Example 1, but during the implementation of this example it is at the level $z^o = -0.215(\mathrm{m})$ (see Fig. 6a).

The motor motion dynamics on both real and reference level is given in Fig. 6b). In Figs. 7a) and 7b) one can see the presence of singularities at the end of the motion.

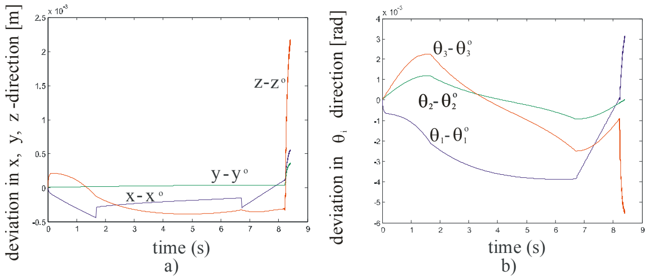

Figure 7. Real deviation from the reference values of a motion trajectory a) the camera carrier b) the motor shaft (Example 2)

This is reflected in the sudden instability of the system without the influence of the outside disturbance. At the moment when instability occurs, the accuracy of the trajectory tracking decreases. This is reflected in a sudden jump of the deviation of the camera carrier real positions on the reference position in relation to the reference positions in the space of the Cartesian coordinates $x - x^o$, $y - y^o$, $z - z^o$, as well as the deviation of the real position of each motor in relation to the reference $\theta_1 - \theta_1{}^o$, $\theta_2 - \theta_2{}^o$, $\theta_3 - \theta_3{}^o$.

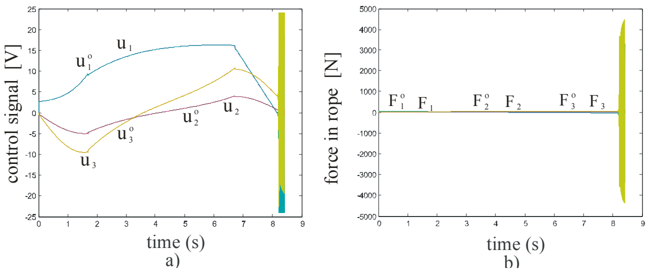

Figure 8. Real and reference a) control signal u_i, b) resultant forces F_i (Example 2)

In Fig.8a), one can clearly observe the presence of singularities because at that moment all three motors enter into saturation to the level $\pm 24(\mathrm{V})$. The motors do not come out of saturation by the end of the motion. The resultant forces spiral to the scale $10^3(\mathrm{N})$, as seen in Fig.8b).

It is presented that the edges of the CPR workspace are the singular space and that part should be avoided as a part of the workspace. It is not profitable to use more powerful engines as the singular space is thus slightly narrowed. The solution is simple: the singular part of the workspace should be avoided.

Example 3

The analyzed example is from Fig.1. All system and control parameters are the same as in Example 1. This example has one illogical assumption, and this is the case when the system user assumes that the system is uncoupled at the reference level. In that case, the Jacobi matrix has the diagonal form:

$$\dot{\phi} = J_{*\oplus} \cdot \dot{p} \tag{13}$$

where

$$J_{*\oplus} = \begin{bmatrix} J_{11} & 0 & 0 \\ 0 & J_{22} & 0 \\ 0 & 0 & J_{33} \end{bmatrix} \tag{14}$$

$$u = G_v \cdot \ddot{\phi} + L_v \cdot \dot{\phi} + S_v \cdot (1+\mu) \cdot M_{*\oplus} \tag{15}$$

$$M_{*\oplus} = F_{*\oplus} \cdot R \tag{16}$$

The resultant force is defined as:

$$F_{*\oplus} = \frac{\left(\left(J_{\Diamond * \oplus}\right)^T\right)^{-1}}{R} \cdot \left(F_p + P_p\right) \tag{17}$$

$$J_{\Diamond * \oplus} = f(\Diamond, J_{*\oplus}) \tag{18}$$

The considered CPR has two ropes from the camera carrier to all four-point suspensions and (18) takes the form:

$$J_{\Diamond * \oplus} = \Diamond \cdot J_{*\oplus} \tag{19}$$

It follows directly that the matrix $\left(\left(J_{\Diamond * \oplus}\right)^T\right)^{-1}$ is diagonal like the matrix $J_{*\oplus}$.

$$\begin{gathered} u = G_v \cdot \ddot{\phi} + L_v \cdot \dot{\phi} + S_v \cdot (1+\mu) \cdot \left(\left(J_{\Diamond * \oplus}\right)^T\right)^{-1} \\ \cdot \left(m \cdot (\ddot{p} + a_{cc}) + P_p\right) \end{gathered} \tag{20}$$

Unlike the previous example, the mathematical model of the system at the reference level in this example is defined by equations (1), (2), (13), (14), (15), (16), (17), (8), (18), (19) and (20). At the real level, the system is coupled and its kinematic and dynamic model is defined by equations (1)-(11).

The camera carrier has the starting point $p_{start}^o = [0.1 \;\; 1.0 \;\; -0.215](\mathrm{m})$, and the end point $p_{end}^o = [2.9 \;\; 1.0 \;\; -0.215](\mathrm{m})$, the same as in Example 2.

Fig.9a) shows the results for the motion of the camera carrier in all three directions of the Cartesian coordinates at the real x, y, z and the reference x^o, y^o, z^o level. The trajectory of the motor rotation angles at the real θ_1, θ_2, θ_3 and the reference θ_1^o, θ_2^o, θ_3^o level is given in Fig.9b).

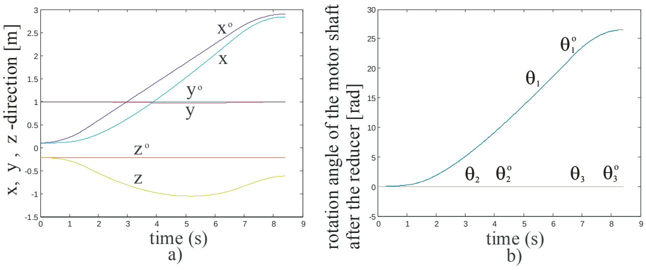

Figure 9. Reference and real trajectories of the motion position a) x, y, z camera carrier, b) θ_1, θ_2, θ_3 motor shaft (Example 3)

Since the positional control law is applied (with local feedback on position and velocity) for each motor rotation angle, defined in equation (12), the control of the motor motion is solid (see Fig.10b).

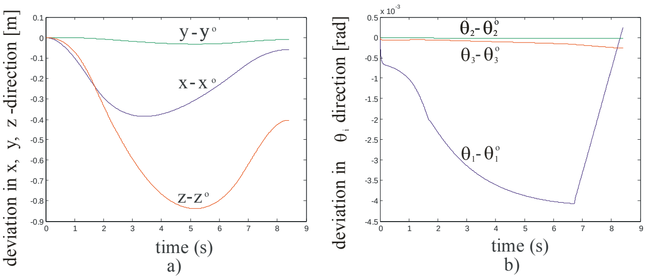

Figure 10. Real deviation from the reference values of the motion trajectory a) the camera carrier b) the motor shaft (Example 3)

However, since the characteristics of coupling are not taken into consideration at the reference level, it is obtained the bad tracking real trajectories compared to the reference in the space of the Cartesian coordinates (see Fig.10a). Not knowing the coupling characteristics significantly affects the accuracy of tracking the trajectory, the value of which reached $\left|z-z^o\right|=0.84(\mathrm{m})$ in this example. The level of control signals is given in Fig.11a) and does not exceed the limits of $\pm 24(\mathrm{V})$. There are resultant forces in Fig.11b).

With such a rough assumption that the mathematical model of the system at the reference level in this example is defined by equations (1), (2), (13), (14), (15), (16), (17), (8), (18), (19) and (20) loses the information about the presence of singularities.

The CPR is modelled and analyzed by the AIRCAM software package.

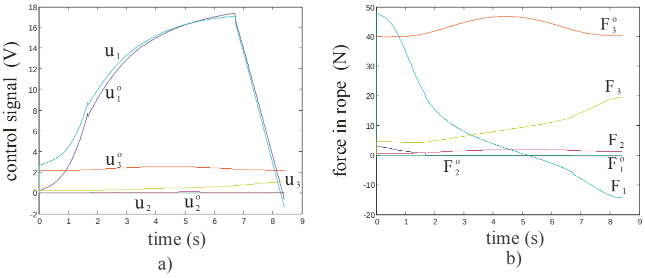

Figure 11. Real and reference a) control signal u_i, b) resultant forces F_i (Example3)

The CPR system being developed in the Mihajlo Pupin Institute, which observes space, is a part of a more complex system (see Fig.12). The CPR system observes the area in which a humanoid robot moves as well as a vehicle that carries a robotic arm.

Reasons for CPR development

The aim of this paper is to highlight the importance of forming a mathematical model of any CPR system since only in that way the CPR system provides precise guidance in the area.

The control of this system is an important component of its operation. The control system can be significantly enhanced if a good dynamic model of the CPR system is defined. For this purpose, special cameras and control systems that can recognize and follow the marked object (e.g. a ball, a robot or a marked person, e.g., a player or soldier) are used.

Furthermore, the control system evaluates the desired camera motion in relation to the current motion of the followed object, and the absolute camera motion in relation to the coordinate system x, y, z. The information or the control signal is constantly sent to the motors that provide such a motion of the motors so that the camera continuously monitor and record the marked object.

Only a highly intelligent control system based on high-fidelity mathematical model of the system can provide it. The established mathematical model provides an opportunity for the CPR system to be significantly modernized and for its application to become much wider.

The aim of this paper is to ensure future accurate and highly automated guidance of the camera in space with minimum involvement of the human factor in the realization of the task for several hours.

From the visionary aspect, this mathematical model provides a realistic possibility of further development of this system, e.g. in order to participate in the planned match between the robots in the not-too-distant 2050. This camera does not keep track of people who, by their decision, move through the observed space, but of humanoid robots who receive the image of the observed space and make decisions about their further actions accordingly.

Figure 12. Organized workspace with the CPR system

Conclusion

The mathematical model of the CPR is defined in the first phase of the study in this paper. A very important feature of the coupling between the motion of the camera carrier in the space of the Cartesian coordinates and the motion of motors is discovered. The results show how significant it is to know the properties of the coupling and its effects on the accuracy of the camera carrier motion. This means that it is very important to define the relationship between the camera motion and the coordinated motion of the motors. The selected characteristics of the CPR motors are satisfactory in the considered examples. This allowed a very precise motion of the camera carrier in the Cartesian space which opens horizons for much wider development and implementation of this system.

The AIRCAM software package was defined before the CPR implementation. It was used for the analysis and selection of all parameters of the CPR. If any size of workspace is chosen as well as any weight of the camera carrier, the choice of the motor type can be checked through the implementation of any trajectory. In addition, the choice of the control law with the analysis of the motion coordination in relation to the desired one can be analyzed and verified. The AIRCAM software package can also be used for the confirmation of the validity of defined mathematical models. The possibilities are broader than the aforementioned, because this research can be expanded with various aspects.

This work demonstrated the presence of the space singular part to which the border space parts belong in the x direction as well as in the y and z directions. The size of the singular space depends on the total size of the workspace, camera carrier weight, camera carrier speed, engine type selection and other parameters. This problem has special significance because it indicates the need for the workspace analysis in order to avoid singularities.

The CPR modular design and lightweight components allow the successful setting up of a system to almost any location, which offers wide possibilities for its use in closed or open space irrespective of its size. It may be placed in the most unlikely places, e.g. when monitoring and recording various military and police exercises in order to monitor the participants in the action.

Future research goes a step further in the implementation of the features of elastic ropes (as defined in [10]-[17]) in the mathematical model of the CPR. The aim of this study is to define the CPR characteristics for the purpose of its modernization and wider application.

Acknowledgment

This research was supported by the Ministry of Science and Technological Development of Serbia for financing the national research project “Ambientally intelligent service robots of anthropomorphic characteristics” TR-35003.

References

[1] AREF,M.M., GHOLAMI,P., TAGHIRAD,H.D.: *Dynamic and Sensitivity Analysis of KNTU CDRPM: A Cable Driven Redundant Parallel Manipulator*, 4th IEEE/ASME International Conference on Mechatronic and Embedded Systems and Applications, Beijing, China, Oct 12-15, 2008, pp.528-533.

[2] AREF,M.M., TAGHIRAD,H.D.: *Geometrical Workspace Analysis of a Cable Driven Redundant Parallel Manipulator: KNTU CDRPM*, IEEE/RSJ International Conference on Intelligent Robots and Systems, Nice, France, September 22-26 2008, pp.1958-1963.

[3] BEDOUSTANI,Y.B., TAGHIRAD,H.D.: *Iterative-Analytic Redundancy Resolution Scheme for A Cable-Driven Redundant Parallel Manipulator*, International Conference on Advance Intelligent Mechatronics, Montreal, Canada, July 6-9, 2010, pp.219-224.

[4] BRIOT,S., ARAKELIAN,V., BONEV,I.A, CHABLAT,D., WENGER,P.: *Self-Motions of General 3- RPR Planar Parallel Robots*, The International Journal of Robotics Research, July 2008, Vol.27, No.7, pp.855-866.

[5] CONG,M., WU,Y., LIU,D., WEN,H., YU,J.: *Coupling compensation of 6-DOF Parallel Robot Based on Screw Theory*, World Academy of Science, Engineering and Technology, 2010, Vol.72, pp.423-428.

[6] DUAN,B.Y.: *A new design project of the line feed structure for large spherical radio telescope and its nonlinear dynamic analysis*, Mechatronics, 1998, Vol.9, pp.53-64.

[7] DUAN,B.Y., DU,J.L. BAO,H.: *Dynamic modeling and active control of a cable-suspended parallel robot*, Mechatronics, 2008, Vol.18, pp.1-12.

[8] DUAN,B.Y., DU,J.L.: *On Analysis and Optimization of an Active Cable-Mesh Main Reflector for a Giant Arecibo-Type Antenna Structural System,* IEEE Transaction on Antennas and Propagation, May 2007, Vol.55, No.5, pp.1222-1229.

[9] FANG,J., HUANG,Z.: *Kinematics of a three-degree-of-freedom in-parallel actuated manipulator mechanism*, Mech. Mach. Theory, 1997, Vol.32, No.7, pp.789-796.

[10] FILIPOVIĆ,M.: *Dynamic of Biped Movement on a Mobile Platform in the Presence Elasticity Elements*, Scientific Technical Review, ISSN 1820-0206, Military Technical Institute, Belgrade, 2008, Vol.58, No.1, pp.15-24.

[11] FILIPOVIĆ,M.: *New View of the Euler-Bernoulli Equation*, Scientific Technical Review, ISSN 1820-0206, Military Technical Institute, Belgrade, 2009, Vol.59, No.1, pp.41-51.

[12] FILIPOVIĆ,M.: *Analogue Between New Formulation of Euler-Bernoulli Equation and Algorithm of Forming Mathematical Models of Robot Motion*, Scientific Technical Review, ISSN 1820-0206, Military Technical Institute, Belgrade, 2010, Vol.60, No.1, pp.19-29.

[13] FILIPOVIĆ,M.: *Relation between Euler-Bernoulli Equation and Contemporary Knowledge in Robotics*, Robotica, 2011, Vol.30, pp.1-13.

[14] FILIPOVIĆ,M., POTKONJAK,V., VUKOBRATOVIĆ,M.: *Elasticity in Humanoid Robotics*, Scientific Technical Review, ISSN 1820-0206 Military Technical Institute, Belgrade, 2007, Vol.57, No.1, pp.24-33.

[15] FILIPOVIĆ,M., VUKOBRATOVIĆ,M.: *Contribution to modeling of elastic robotic systems*, Engineering & Automation Problems, International Journal, September 23, 2006, Vol.5, No.1, pp.22-35.

[16] FILIPOVIĆ,M., VUKOBRATOVIĆ,M.: *Complement of Source Equation of Elastic Line*, Journal of Intelligent & Robotic Systems, International Journal, June 2008, Vol.52, No.2, pp.233-261.

[17] FILIPOVIĆ,M., VUKOBRATOVIĆ,M.: *Expansion of source equation of elastic line*, Robotica, International Journal, November 2008, Vol.26, No.6, pp.739-751.

[18] GOSSELIN,C., ANGELES,J.: *The Optimum Kinematic Design of a Planar Three-Degree-of-Freedom Parallel Manipulator*, Journal of Mechanisms, Transmissions, and Automation in Design, March 1988, Vol.110, No.35, pp.35-41.

[19] HAN,C., KIM,J., KIM,J., PARK,F.C.: *Kinematic sensitivity analysis of the 3-UPU parallel mechanism*, Mech. Mach. Theory, 2002, Vol.37, pp.787-798.

[20] HASSAN,M., KHAJEPOUR,A.: *Optimization of Actuator Forces in Cable-Based Parallel Manipulators Using Convex Analysis*, IEEE Transactions on Robotics, June 2008, Vol.24, No.3, pp.736-740.

[21] HILLER,M., FANG,S.Q.: *Design, analysis and realization of tendon-base parallel manipulators*, Mech. Mach. Theory, 2005, Vol.40, pp.429-445.

[22] HUANG,P., WANG,L., GUAN,L., YAO,R.: *Three-Dimensional Fitting Accuracy study of Active Reflector for Large Spherical Radio Telescope*, ASME/IFToMM International Conference on Reconfigurable Mechanisms and Robots, London, UK, 2009, pp.267-273.

[23] KIM,J., PARK,F.C., RYU, S.J., KIM,J., HWANG,J.C., PARK,C., IURASCU,C.C.: *Design and Analysis of a Redundantly Actuated Parallel Mechanism for Rapid Machining*, IEEE Transaction on Robotics and Automation, August 2001, Vol. 17, No 4, pp. 423-434.

[24] KOZAK,K., ZHOU,Q., WANG, J.S.: *Static Analysis of Cable-Driven Manipulators With Non-Negligible Cable Mass*, IEEE Transaction on Robotics, June 2006, Vol.22, No.2, pp.425-433.

[25] KURTZ,R., HAYWARD,V.: *Multiple-Goal Kinematic Optimization of a Parallel Spherical Mechanism with Actuator Redundancy*, IEEE Transaction on Robotics and Automation, October 1992, Vol.8, No.5, pp.644-651.

[26] LEE, K.-M., SHAH,D.K.: *Kinematic Analysis of a Three-Degrees-of-Freedom In-Parallel Actuated Manipulator*, IEEE Journal of Robotics and Automation, June 1988, Vol.4, No 3, pp.354-360.

[27] LIU, X-J., WANG,J., GAO,F., WANG,L.P.: *Mechanism design of a simplified 6-DOF 6-RUS parallel manipulator*, Robotica, Cambridge University Press, 2002, Vol.20, pp.81-91.

[28] LOLOEI,A.Z., AREF,M.M., TAGHIRAD,H.D.: *Wrench Feasible Workspace Analysis of Cable-Driven Parallel Manipulators Using LMI Approach*, International Conference on Advanced Intelligent Mechatronics, Suntec Convention and Exhibition Center, Singapore, July 14/17, 2009, pp.1034-1039.

[29] MULER,A.: *Internal Preload Control of Redundantly Actuated Parallel Manipulators-Its Application to Backlash Avoiding Control*, IEEE Transaction on Robotics, August 2005, Vol.21, No 4, pp.668-677.

[30] OFTADEH,R., AREF,M.M., TAGHIRAD,H.D.: *Forward Kinematic Analysis of A Planar Cable Driven Redundant Parallel Manipulator Using Force Sensors*, IEEE/RSJ International Conference on Intelligent Robots and Systems, Taipei, Taiwan, October 18-22, 2010, pp.2295-2300.

[31] OH,S.R., AGRAWAL,S.K.: *A Reference Governor-Based Controller for a Cable Robot Under Input Constraints*, IEEE Transaction on Control Systems Technology, July 2005, Vol.13, No.4, pp.639-645.

[32] PICCIN,O., BAYLE,B., MAURIN,B., MATHELIN,M.de.: *Kinematic modeling of a 5-DOF parallel mechanism for semi spherical workspace*, Mech. Mach. Theory, 2009, Vol.44, pp.1485-1496.

[33] PRAJAPATI,J.M.: *Synthesis of parallel mechanism based robot manipulators from structural point of view*, International Journal of Engineering Science and Technology, Feb 2011, Vol.3, No.2, pp.1228-1232.

[34] SHIANG,W-J., CANNON,D., GORMAN,J.: *Optimal Force Distribution Applied to a Robotic Crane with Flexible Cables*, Proceedings of the 2000 IEEE International Conference on Robotics & Automation, San Francisco, Ca, April 2000, pp.1948-1954.

[35] SU,X.Y., DUAN, B.Y.: *The Application of the Stewart Platform in Large Spherical Radio Telescopes*, Journal of Robotic Systems by John Wiley & Sons, 2000, Vol.17, No.7, pp.375-383.

[36] SU,X.Y., DUAN, B.Y.: *The mathematical design and kinematics accuracy analysis of a fine tuning stable platform for the large spherical radio telescope*, Mechatronics, 2000, Vol.10, pp.819-834.

[37] TAGHIRAD,H.D., NAHON,M.A.: *Jacobian Analysis of A Macro-Micro Parallel Manipulator*, IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Zurich, Switzerland, Sep, 2007, pp 1-6.

[38] TAGHIRAD,H.D., NAHON,M.A.: *Forward Kinematics of A Macro-Micro Parallel Manipulator*, IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Zurich, Switzerland, Sep, 2007, pp.189-195.

[39] TANG,X.Q., WANG, J.S., WANG, Q.M.: *A Study of the Active Reflector for a Large Spherical Radio Telescope*, Chin. J. Astron. Astrophysics, 2003, Vol.3, No.5, pp.478-490.

[40] YAO,R., TANG,X., WANG,J., HUANG,P.: *Dimensional Optimization Design of the Four-Cable Driven Parallel Manipulator in FAST*, IEEE/ASME Transaction on Mechatronics, 2010, Vol.15, No.6, pp.932-941.

[41] YU,D., LI,H., CHEN,W.: *Kinematic Calibration of Parallel Robots for Dcking Mechanism Motion Simulation*, Int. J. Adv Robotic Sys, 2011, Vol.8, No 4, pp.158-165.

Received: 24.01.2012.

Značaj modeliranja vazdušnih robotizovanih kamera

Već treću deceniju razvija se vazdušni robot (paralelni robot nošen kablovima). Ovaj sistem je sada veoma atraktivan i zanimljiv inženjerskoj i naučnoj javnosti sa stanovišta njegovog budućeg intenzivnog razvoja. Istraživanja u ovom radu su usmerena ka istom cilju, a to je definisati paralelni robot nošen kablovima koji će sa velikom preciznošću pratiti i snimati pokretni objekat ma gde da se on kretao u radnom prostoru. Cilj ovog rada je da se detaljno analizira i razvije postojeća struktura vazdušnog robota, što bi omogućilo njegov snažni progres. To će se ogledati u primeni visoko-automatizovanog sistema koji će precizno voditi kameru u prostoru sa minimalnim učešćem ljudskog rada. Postavljanje i postizanje ovog cilja obezbeđuje mnogo šire mogućnosti za njegovu buduću upotrebu. Savremenije funkcionisanje ovog sistema može da se obezbedi jedino formulacijom i primenom njegovog visoko verodostojnog matematičkog modela tokom sinteze i analize, što bi dalje omogućilo razvoj i primenu savremenih zakona upravljanja. Mogućnosti su svakako mnogo šire nego što se može pretpostaviti u ovom trenutku, posebno za vojne ili policijske svrhe.

Ključne reči: **roboti, snimanje iz vazduha, kablovska veza, upravljanje, matematičko modeliranje.**

Вазможность моделирования воздушных роботов камеры

Уже течёт третья декада развития воздушных роботов (параллельный робот осуществляется кабельной связью). Эта система в настоящее время является очень привлекательной и интересной для инженерных и научных кругов с точки зрения её будущего интенсивного развития. Исследования в этой статье, направлены на одну цель, а именно для определения параллельных роботов осуществляюшихся кабельной связью, которые с большой точностью могут следовать за движущимся объектом и делать записи, куда бы он не двигался в рабочем пространстве. Целью данной работы является тщательно проанализировать и развить существующую структуру воздушного робота, которая позволила бы его мощный прогресс. Это будет отражено в применении высокоавтоматизированной системы, которая будет точно держать камеру в пространстве, с минимальным участием человеческого труда. Постановка и достижение этой цели обеспечивает более широкие возможности для использования робота в будущем. Современное функционирование этой системы возможно обеспечить только разработкой и осуществлением его в высшей степени надёжных математических моделей в процессе синтеза и анализа, что ещё больше способствует развитию и применению современных прав управления. Возможности, конечно, гораздо больше, чем можно себе представить в данный момент, особенно для военных или полицейских целей.

Ключевые слова: **роботы, запись из воздуха, кабельная связь, управление, математическое моделирование.**

L’importance de la modélisation de la caméra de l’air robotiques

Le robot aérien (robot parallèle porté par câbles) est en développement depuis trois décades. Maintenant ce système est très attrayant et intéresse les milieux d’ingénieurs et de scientifiques de point de vue son futur développement intense. Dans ce travail les recherches sont orientées vers le même but c’est-à-dire vers la définition d’un robot parallèle porté par câbles qui suivra et photographiera l’objet mobile avec grande précision partout où cet objet se déplace dans l’espace de travail. Le but de ce travail est d’analyser en détail et de développer la structure actuelle du robot aérien ce qui permettra son grand progrès. Cela se reflètera dans l’utilisation d’un système hautement automatisé qui guidera la caméra avec précision dans l’espace avec la participation minimale du travail humain. L’implémentation et la réalisation de ce but assure des possibilités plus grandes pour son utilisation dans le futur. Le fonctionnement moderne de ce système peut être assuré uniquement par la formulation et l’application de son modèle mathématique de grande fiabilité au cours de l’analyse et de synthèse, ce qui permettrait le développement et l’emploi des lois de contrôle modernes. Les possibilités sont certainement beaucoup plus grandes qu’on peut imaginer en ce moment, particulièrement pour les fins militaires ou policiers.

Mots clés: **robots, photographie aérienne, connexion par câble, commande, modélisation mathématique.**