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# Fatigue Strength Analysis of a Semi-Elliptical Surface Crack

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**In this paper, a computational model is proposed for estimating the crack growth behavior and fatigue life of a plate with a semi-elliptical surface crack subjected to cyclic tensile loading. In the fatigue analysis, the crack depth and surface directions were considered. The stress intensity factors were determined by applying analytical/numerical methods. The predictions are compared with available experimental data. A good agreement was obtained between the calculations and the experimental results**

*Key words*: **material fatigue, surface crack, fatigue crack, crack growth, fatigue strength, tensile strength, stress analysis.**

## Introduction

MANY investigations have shown that sudden failures of aircraft components, pressure vessels or pipeline systems might occure due to presence of surface cracks. Potential sources of these cracks are material defects or geometric discontinuities i.e. zones where stress increase happens. These zones, known as local stress concentrations, are regions where the points with an extremely high magnitude of stresses could appear. These points are areas where cracks are most often initiated and later propagate under cyclic loadings. Due to previous reasons the ability to assess the effects of these defects on structural integrity under fatigue and fracture loadings is of much practical significance.

In designing against failures, designers have to define and implement reliable computational models/procedures for fatigue life estimation. The basic parameter that should be defined when formulating computational models is a shape of the flaw i.e. initial crack. One of the most common flaws found in structural components is a part-through surface flaw. These flaws could most often be aproximated and analysed as a semi-elliptical crack. This paper investigates the fatigue behaviour of the semi-elliptical crack.

For the assessment of fracture strength and residual fatigue life for defects contained in structures, or for damage tolerance analysis recommended to be performed at the stage of structure design, the important aspect is the calculation of the stress intensity factor.

Various methods have been used to obtain stress intensity factor for semi-elliptical surface cracks in plates. Kobayashy [1], Browning and Smith [2] and then Smith and Sorensen [3] used the alternating method to obtain the stress intensity factor variations along the crack front. Raju and Newman [4,5], Kathiresan [6] and Shiratori et al. [7] analysed semi-elliptical cracks by the finite elements method. Moreover, Shen and Glinka [8] generated the weigth function for semi-elliptical surface cracks in finite thickness plates by using the Shiratori et al. [7] solutions.

The objective of this paper is to develop a computation model/procedure for the crack growth estimation of a plate with a semi-elliptical surface crack. In the crack growth analysis, both the crack depth direction and the surface direction are investigated. Numerical and/or analytical approaches were employed in the stress analysis. Additionally, the paper examines the effect of the plate thickness and initial crack length in the surface direction on the final number of loading cycles up to failure.

## Crack growth analysis

In the analysis related to fatigue behaviour of real structural components under service loading, one of the fundamental issues is the evaluation of adequate relations which could describe fatigue life. This means the calculation of the number of loading cycles required to grow a crack from a tolerable size to a critical length.

The fatigue life analysis of a semi-elliptical surface crack problem requires considering two crack growth directions. Actually, Newman and Raju [4,5] intoduced that the aspect ratio change of surface cracks should be calculated by assuming that a semi-elliptical profile is always maintained and for fatigue life estimation it is adequate to use two coupled Paris fatigue laws known as ″two-point plus semi-ellipse″ method:

$$\frac{da}{dN}=C_A\left(\Delta K_A\right)^{m_A}, \tag{1a}$$

$$\frac{db}{dN}=C_B\left(\Delta K_B\right)^{m_B}, \tag{1b}$$

where $\Delta K_A$ and $\Delta K_B$ are the ranges of the stress intensity factor at the depth and the surface points of the surface crack, $C_A$, $C_B$, $m_A$ and $m_B$ are material constants experimentally obtained.

In the crack growth analysis, a very important aspect is to evaluate fatigue life up to failure. The final number of loading cycles for surface cracked problems can be

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calculated for both directions if equations for crack growth rate are integrated:

$$N_{depth} = \int_{a_0}^{a_f} \frac{da}{C_A \left(\Delta K_A\right)^{m_A}} , \tag{2a}$$

$$N_{surface} = \int_{b_0}^{b_f} \frac{db}{C_B \left(\Delta K_B\right)^{m_B}} . \tag{2b}$$

Eqs. (2a)-(2b) can be used to estimate the number of loading cycles required for the surface crack to grow in depth and surface directions for the corresponding increments of crack length. As it can be seen, relations under integral are complicated functions, so it is necessary to use numerical integration for the calculation of the number of loading cycles to failure.

## Stress intensity factor

In the fracture analysis of structures, the main parameter which includes geometry of the specimen or the structural element and external loading is the stress inensity factor. Only correctly determined stress intensity factors enable the exact calculation of the critical length and fatigue life up to failure of the structural element. Depending on the complexity of geometry of any type of loading for the calculation of the stress intensity factors, analytical and/or numerical approaches [9-12] could be used. Due to the fact that a semi-elliptical surface crack (see Fig.1) is examined, two stress intensity factors have to be considered i.e. for the depth direction and the surface direction.

At any point along a semi-elliptical crack in a finite plate, the stress intensity factor (for $a/b<1$, see Fig.1) can be calculated by applying the following relationship [4,5]:

$$\Delta K = \Delta S \sqrt{\frac{\pi a}{Q}} M_e , \tag{3}$$

where $\Delta S$ is the applied stress range in the vicinity of the crack. The elastic shape factor $Q$ [13] can be expressed as the square of the elliptical integral of the second kind, i.e.

$$Q = 1 + 1.47\left(\frac{a}{b}\right)^{1.64} , \left(\frac{a}{b} \le 1.0\right) , \tag{4}$$

where $a$ is the crack length in the depth direction and $b$ presents the crack length in the surface direction.

The factor $M_e$ [13] includes the front-face correction and the back-face correction and is given by

$$M_e = \left[ M_1 + \left( \sqrt{Q\frac{b}{a}} - M_1 \right) \left( \frac{a}{t} \right)^p \right] f_w \, f_\varphi \, g , \tag{5}$$

where

$$p = 2 + 8\left(\frac{a}{b}\right)^3 \tag{6}$$

and

$$M_1 = 1.13 - 0.1\frac{a}{b}, \left(0.02 \le \frac{a}{b} \le 1.0\right) . \tag{7}$$

The term $f_w$ is the finite-width correction factor and can be expresssed as follows:

$$f_w = \sqrt{\frac{1}{\cos\left(\frac{\pi b}{w}\sqrt{\frac{a}{t}}\right)}} . \tag{8}$$

The expression for $g$ is given by

$$g = 1 + \left( 0.1 + 0.35\left(\frac{a}{t}\right)^2 \right)\left(1 - \sin\varphi\right) . \tag{9}$$

Furthermore, in this paper the stress intensity factors for the plate with a semi-elliptical crack (Fig.1) are calculated by employing the finite element method and compared with the evaluations obtained by applying Eqs. (3)-(9).

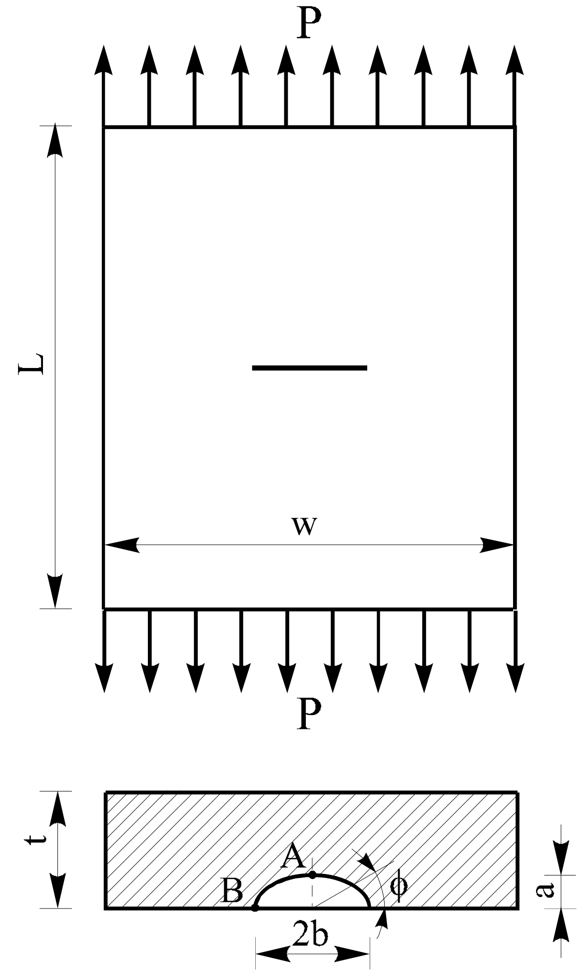


**Figure 1.** Geometry of a finite plate with a semi-elliptical surface crack.

## Numerical results

This section illustrates the efficiency of the formulated computation model for fatigue life of the plate with a semi-elliptical surface crack through a few numerical examples. These examples are the examined numerical calculation of the stress intensity factor as well as fatigue life estimation. In order to verify the validation of the presented model for surface crack growth simulation, the obtained results are compared with the experimental data.

*Semi-elliptical surface crack and fatigue growth evaluation*

This example deals with the crack growth calculation of the plate with a semi-elliptical surface crack. The geometry parameters for two different cases are as follows: a) $a_0$=3.25 mm, $a_f$=9.01 mm, $b_0$=8.51 mm, $b_f$=12.98 mm; b)

$a_0$=3.61 mm, $a_f$=7.73 mm, $b_0$=6.12 mm, $b_f$=9.67 mm and $t$=11.39 mm [14]. The plate made of 2219 T851 Al alloy is subjected to axial loading with a constant amplitude ($R$=0.1) [14]. The material characteristics of 2219 T851 Al alloy are: $\sigma_y$=331 MPa, $E$=71 GPa, $C_A$=3.90x10$^{-11}$, $m_A$=3.44, $C_B$=2.7x10$^{-11}$, $m_B$=3.44.

In the crack growth analysis, the evaluation of the stress intensity factor must be carried out first. Due to the fact that a surface crack is analysed, it is necessary to examine the calculation of the stress intensity factor for surface and depth directions as mentioned above. The stress intensity factors were calculated by applying Eq.(3) together with Eqs.(4)-(9). Furthermore, the crack growth rates (Eqs.(1a)-(1b) and Eqs.(3)-(9)) for adequate crack increments are estimated (see Fig.2). The same Fig.2 presents a comparison between the calculated values of the crack growth rates and the experimental results [14].

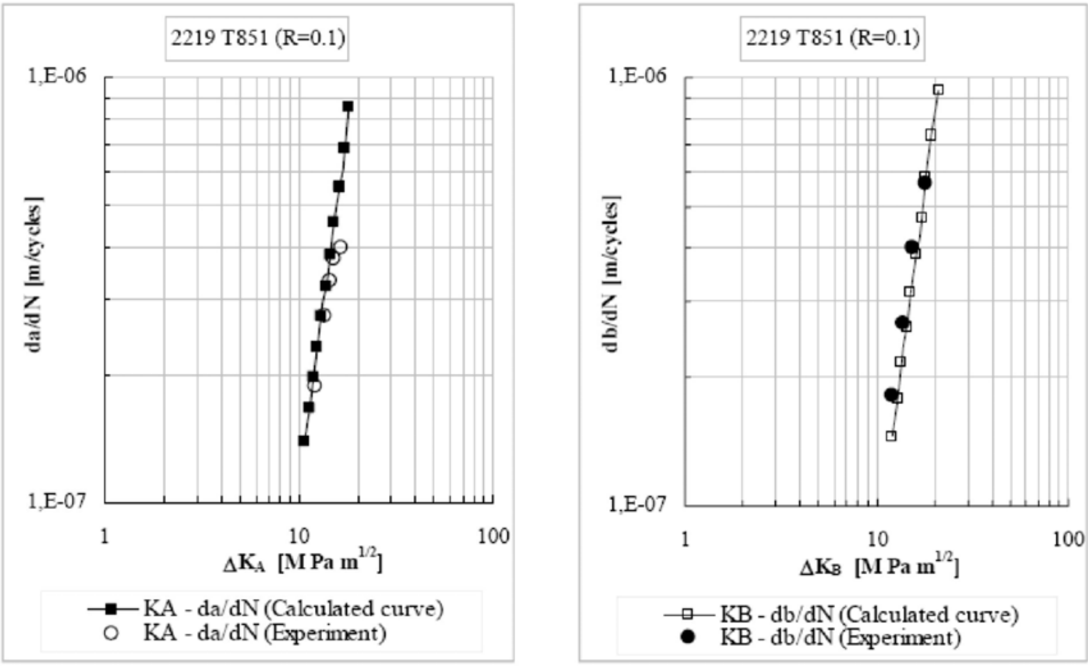


**Figure 2.** Stress intensity factor versus crack length
($a_0/t$ = 0.285, $a_0/c_0$=0.382, Experiment from Ref. [14]).

Using the fatigue performance data, in accordance with the structural geometry and the defined fatigue model in the previous section, it is possible to calculate fatigue life up to failure. The obtained results for the crack length versus the number of loading cycles up to failure are presented in Fig.3 for depth and surface directions. In the same Fig.3, all computed results for the number of loading cycles up to failure are compared with the experimental data [14].

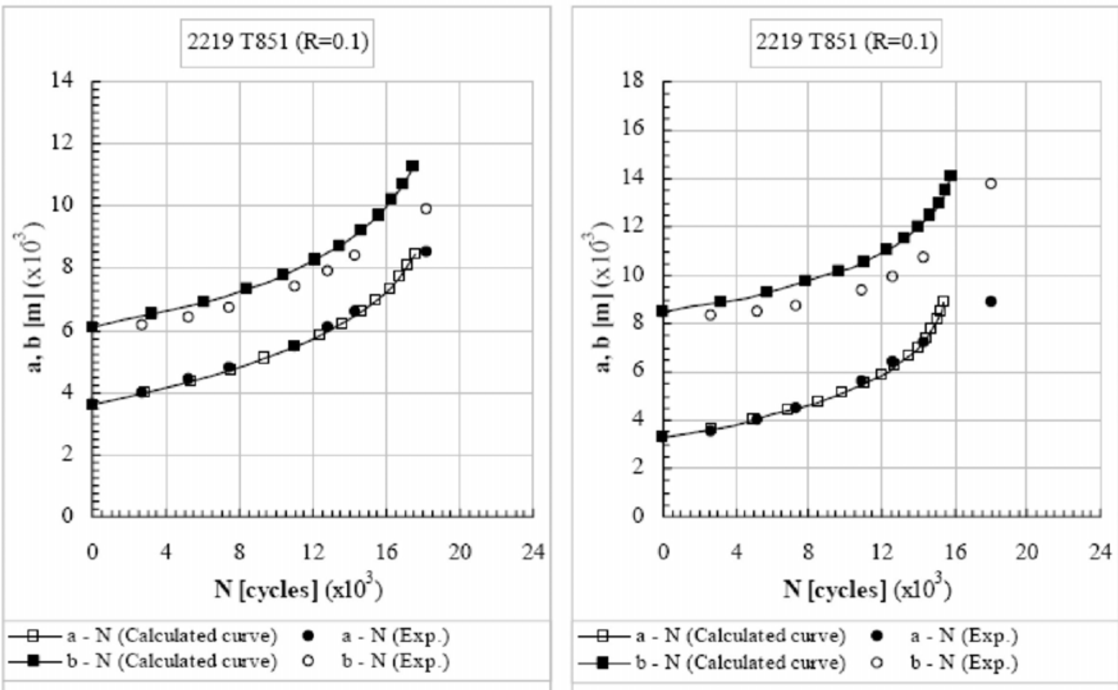


**Figure 3.** Crack length versus the number of loading cycles up to failure
(Left – $a_0/t$ = 0.317, $a_0/c_0$=0.589, right – $a_0/t$ = 0.285, $a_0/c_0$=0.382. Experiment from Ref. [14]).

It is indicated in Fig.3 that the estimated values of the number of loading cycles up to failure are conservative when compared to experimental data. In engineering practice, the existence of conservativity in the fatigue crack growth analysis is always benefitial since in this way a safe residual service life of structural elements could be determined. Additionally, the conservativity of computed results is often a result of defined criteria which are used to formulate adequate analytical relations for the crack growth analysis.

*Stress analysis of the finite plate with a semi-elliptical crack*

In this example, a stress intensity factor calculation is carried out. The plate has a surface crack and the initial crack length in the depth direction is equal to the initial crack length in the surface direction. The geometry parameters of the plate with a semi-elliptical surface crack are as follows: $a_0$=$b_0$=3 mm, $w$=50 mm, $t$=10 mm. The material used in this example is the same as in the previous one. External cyclic loading is axial with a constant amplitude, $P_{max}$=50 kN and a stress ratio of $R$=0.1.

In addition to the analytical approach for stress intensity factor evaluation, used in the previous example, a numerical approach based on the finite element method is applied in this paper. For this purpose, singular six-node finite elements [15, 16] are employed. Actually, step-by-step, for each increment of the crack length, different meshes are modeled by applying super elements around the crack tip [16], Fig.4.

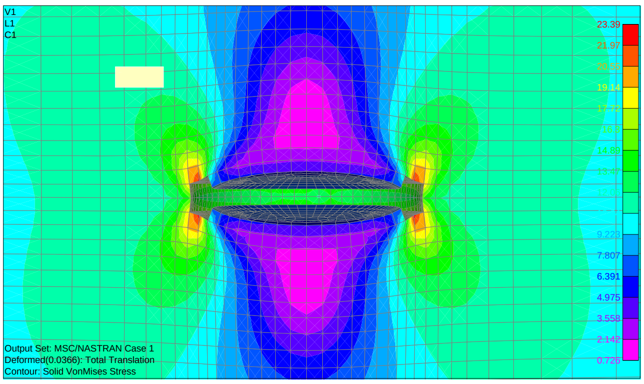


**Figure 4**. Stress distrubution for the plate with a semi-elliptical crack subjected to tensile load.

The step-by-step procedure is repeated until the computed crack growth is very close to the final failure of the plate. Fig.4 presents the stress distribution for the plate with a semi-elliptical surface crack made of 2219 T851 Al alloy (see Fig.1). The computed results are listed in Table 1 and Table 2 for the stress intensity factor for different values of the crack length in both directions (surface and depth) by applying the finite element method. The calculated stress intensity factors by using two different analytical approaches are compared with the results obtained from the finite element technique for different values of length (Table 1 and Table 2). The first listed values of the stress intensity factors ($K_A^{Anal.}$, $K_B^{Anal.}$) are calculated by applying Eqs. (3)-(9), while the values that follow for the stress intensity factors (Table 1 and Table 2) are calculated by using equations proposed in Ref. [15].

It can be observed in Table 1 and Table 2 that the analytical methods give almost the same solutions as the finite element method, so both approaches for the stress intensity factor calculation can be used in the crack growth analysis of the plate with a semi-elliptical surface crack subjected to tensile load.

**Table 1.** Comparison of the calculated stress intensity factors using analytical and numerical approaches (Position A - $\phi$ = 90°)

| Step | $a$ $10^{-3}$ [m] | $K_A^{Anal.}$ [MPam$^{0.5}$] | $K_A^{Anal.}$[15] [MPam$^{0.5}$] | $K_A^{FEM}$ [MPam$^{0.5}$] |
|---|---|---|---|---|
| 1 | 3 | 6.38 | 6.55 | 6.74 |
| 2 | 4 | 7.50 | 7.72 | 7.88 |
| 3 | 5 | 8.60 | 8.90 | 8.89 |
| 4 | 6 | 9.82 | 10.04 | 9.99 |

**Table 2.** Comparison of the calculated stress intensity factors using analytical and numerical approaches (Position B - $\phi$ = 0°)

| Step | $a$ $10^{-3}$ [m] | $K_B^{Anal.}$ [MPam$^{0.5}$] | $K_B^{Anal.}$[15] [MPam$^{0.5}$] | $K_B^{FEM}$ [MPam$^{0.5}$] |
|---|---|---|---|---|
| 1 | 3 | 7.22 | 7.41 | 7.92 |
| 2 | 4 | 8.66 | 8.92 | 9.24 |
| 3 | 5 | 10.21 | 10.57 | 11.18 |
| 4 | 6 | 12.04 | 12.44 | 12.48 |

A very important aspect in fracture mechanics is fatigue life estimation, so in this example, besides stress intensity factors, the number of cycles up to failure is calculated for the plate with a surface crack. Due to the fact that the verification shows a good correlation between the computed and experimental results for the crack growth rate and the number of cycles up to failure, the proposed computational method is applied. By using Eq. (3) together with Eqs. (4)-(9), it is possible to calculate stress intensity factors and after that the number of cycles up to failure for the plate (by applying Eqs.(2a)-(2b)). Since a semi-elliptical surface crack is considered, two different curves (for depth and surface directions) are calculated and presented in Fig.5.

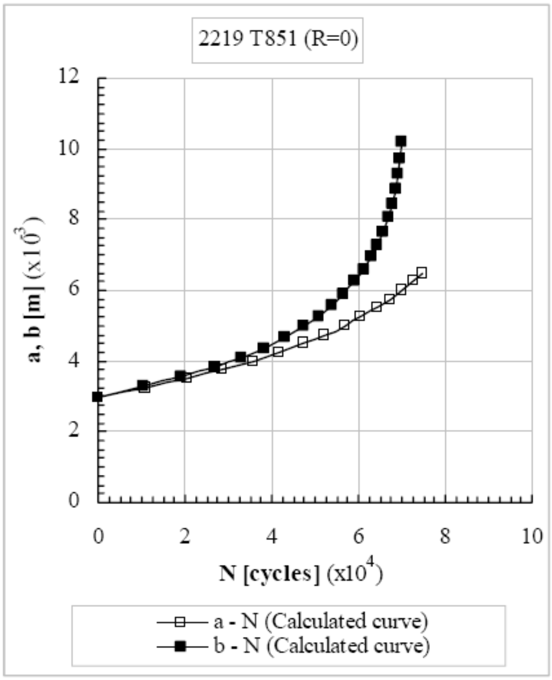


**Figure 5.** Crack length versus the number of loading cycles up to failure (a – Depth direction, b – Surface direction).

*The effect of thickness and initial surface crack length on the final number of loading cycles up to failure*

The effect of the thickness of the plate as well as the initial surface crack length in the surface direction on the number of loading cycles up to failure is analysed. The plate made of 2219 T851 Al alloy is subjected to axial loading with a constant amplitude $P_{max}$=45 kN (stress ratio $R$=0.1). The material characteristics are as follows: $\sigma_y$=331 MPa, $E$=71 GPa, $C_A$=3.90x10$^{-11}$, $m_A$=3.44, $C_B$=2.7x10$^{-11}$, $m_B$=3.44. The plate is $w$=50 mm wide and the initial crack length in the depth direction is $a_0$=3.5 mm.

During the component design process, it is important to select adequate parameters of geometry that must fulfil different requirements from the aspect of safety design. Most important requirements that should be met are definitely those relating to fatigue crack growth and failure. A parameter of geometry that can be considered in the fatigue crack growth analysis is the thickness. This example considers plates with five different thicknesses (12 mm, 13.2 mm, 13.8 mm, 14.4 and 15 mm) and their effect on the final number of loading cycles up to failure. The initial crack length in the surface direction is $b_0$=7.15 mm. By applying Eqs. (2)-(9) and with a known geometry, material and loading parameters, the number of loading cycles up to failure is estimated for both directions. All calculated values for the number of loading cycles up to failure are presented in Fig.6.

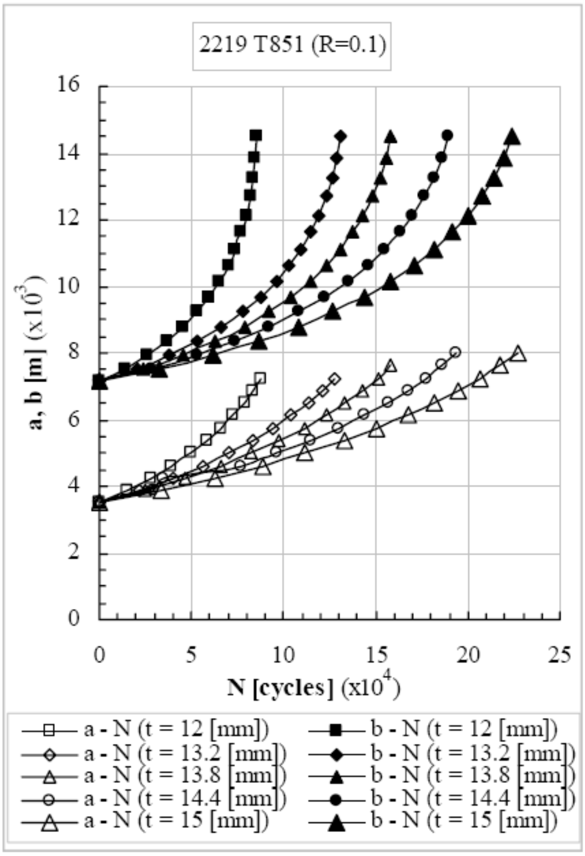


**Figure 6.** Crack length versus the number of loading cycles up to failure for different plate thicknesses ($b_0$=7.15 mm).

Additionally, the proposed computational model has been used for the analysis of an impact of the initial crack length in the surface direction on the number of loading cycles up to failure. Four different initial crack lengths $b_0$ are considered (5.5 mm, 6.6 mm, 7.7 mm and 8.8 mm). In the crack growth analysis, the thickness of the plate was $t$=12 mm.

Based on the known geometry, material and loading characteristics, it is possible to define a stress intensity factor range, a crack growth rate and the number of loading cycles up to failure using Eqs. (1a)-(1b) and (2a)-(2b) together with Eqs. (3)-(9). The calculated results for the crack length (depth and surface directions) as a function of number of cycles are shown in Fig.7 for different initial crack lengths in the surface direction.

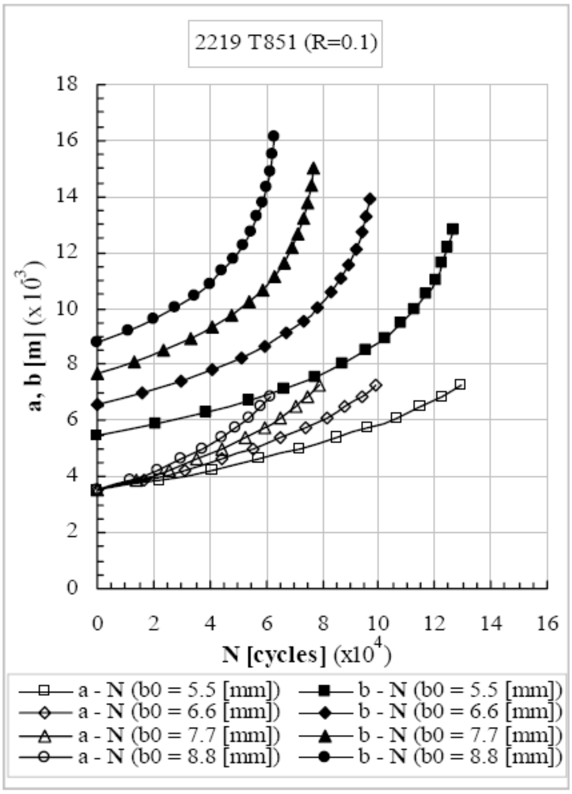


**Figure 7.** Crack length versus the number of loading cycles up to failure for different initial crack lengths in the surface direction ($t$=12 mm).

From Fig.6 and Fig.7, it can be deduced that the plate thickness as well as the initial crack length in the surface direction have a significant impact on the number of loading cycles up to failure.

## Conclusion

An engineering procedure is proposed for estimating the crack growth behavior of a finite plate with a semi-elliptical crack subjected to cyclic tensile loading. The proposed model considers the stress analysis as well as the fatigue life estimation. Analytical and numerical approaches were used for the stress analysis. The numerical approach is based on singular six-node finite elements. In the stress analysis, two different analytical approaches are compared with the finite element method. The developed crack growth model is compared with the experimental data available in the literature. All comparisons between numerical and experimental results are in a good agreement.

Furthermore, a computational model for fatigue life estimation is presented in order to analyse the influence of the plate thickness and the initial surface crack length on the final number of loading cycles up to failure. The fatigue crack growth analysis shows that the plate thickness as well as the initial surface crack length have significant effect on fatigue life up to failure under cyclic loading.

The presented model offers a reliable fatigue crack growth prediction for the finite plate with a semi-elliptical surface crack subjected to cyclic tensile loading.

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# Analiza čvrstoće na zamor strukturalnih elemenata sa površinskom prslinom polueliptičnog oblika

**U ovo radu je predložen proračunski model za procenu širenja prsline i veka pri zamoru ploče sa polueliptičnom površinskom prslinom u uslovima dejstva zatežućeg opterećenja. U okviru analize pri zamoru razmatrani su pravac po dubini i pravac po površini. Faktor intenziteta napona je određen primenom analitićkih/numeričkih metoda. Rezultati dobijeni primenom formulisanog proračunskog modela su upoređeni sa dostupnim eksperimentalnim podacima. Dobijeno je dobro slaganje između rezultata dobijenih putem proračuna i eksperimentalnih rezultata.**

*Ključne reči*: **zamor materijala, površinska prskotina, zamorna prskotina, rast prskotine, čvrstoća na zamor, čvrstoća na zatezanje, analiza napona.**

# Анализ усталостной прочности элементов конструкций, содержащих поверхностные трещины полуэллиптической формы

**В этой работе предлагается разработка моделей для оценки распространения трещины усталости и срока службы при усталости пластины с полуэллиптической поверхностной трещиной в условиях действия натянутой нагрузки. В рамках анализа при усталости подробно обсуждались направления по глубине и направления по поверхности. Коэффициент интенсивности напряжений определяли с помощью аналитических / численных методов. Предусмотреные результаты сопоставлены с доступными экспериментальными данными. Получено хорошее согласие между результатами расчётов и экспериментальными результатами.**

*Ключевые слова*: **усталость материала, поверхностная трещина, усталостная трещина, рост трещины, усталостная прочность, предел прочности, анализ напряжений.**

# Analyse de la résistance à la fatigue des éléments structuraux contenant une fissure superficielle semi elliptique

**Cet article propose un modèle de computation pour l'estimation de la croissance de fissure et de la durée de vie de fatigue de la plaque avec une fissure superficielle semi elliptique exposées aux effets de la charge de tension. Dans le cadre de l'analyse à la fatigue on a considéré les directions de profondeur et de surface. Le facteur de l'intensité de tension a été déterminé par l'emploi des méthodes analytiques et numériques. Les résultats prévus ont été comparés avec les données expérimentales disponibles. On a obtenu bon accord entre les résultats réalisés par les calculs et les résultats expérimentaux.**

*Mots clés*: **fatigue de matériel, fissure superficielle, fissure de fatigue, croissance de fissure, résistance à la fatigue, résistance à la tension, analyse de tension.**