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Fatigue Life Estimation of Damaged Structural Components Under Load Spectrum

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This paper deals with fatigue behaviour of damaged aircraft structural components under cyclic loads of constant amplitudes and load spectrum. The primary attention of this investigation is to establish a computation procedure for the evaluation of the residual life of aircraft attachment lug type structural elements in the presence of initial cracks. The Strain Energy Density (SED) method is used in this investigation for a residual life estimation and a crack growth analysis. This method uses the low-cycle fatigue (LCF) properties of the material, which are also used for the lifetime evaluation until the occurrence of final failure. Therefore, the experimentally obtained dynamic properties of the material such as Forman`s constants are not required when this approach is concerned. The complete computation procedure for the crack propagation analysis using low-cycle fatigue material properties is illustrated with the cracked structural elements. To determine analytic expressions for stress intensity factors (SIF), singular finite elements are used. The results of a numerical simulations for crack propagation based on the strain density method have been compared with the author`s own experimental results.

Key words: **material fatigue, fatigue crack growth, cyclic load, low-cycle fatigue, finite elements, life time, aircraft construction.**

Introduction

IN assembling complex structures like military aircraft, riveted or bolted joints, attachment lugs are primarily used as they offer many options to the designer. To satisfy fatigue requirements, the designer can either keep the stress levels below the endurance limit or ensure that the slow crack growth life of the component is greater than the designed service goal plus a factor of safety. The latter approach is most commonly used and relies on the ability to predict fatigue crack growth at fatigue critical locations. Methods for design against fatigue failure are under constant improvement. In order to optimize constructions, the designer is often forced to use the material properties as efficiently as possible. One way to improve the fatigue life predictions may be to use the relations between the crack growth rate and the stress intensity factor range. These are fairly well established for constant amplitude loading, at least for common specimen geometries. Loading histories in engineering structures, however, often exhibit varying amplitudes. For such cases, the prediction capacity is markedly lower. Ideally, the crack advance under varying amplitude should be possible to predict using experimental data from constant amplitude testing. Numerous investigations address this problem but so far without reaching any total success. Design based on damage tolerance criteria often deals with notched components giving rise to localized stress concentrations that, in brittle materials, may generate a crack leading to a catastrophic failure or to a shortening of the assessed structural life. For a successful implementation of the damage tolerance philosophy [1] to the design and in-service operation of structures subjected to fatigue loading, it is crucial to have reliable crack growth prediction tools. Damage tolerance application to the aircraft structural components is limited to critical parts. A part, which, if it fails, alone may cause the loss of an aircraft, is classified as a critical part. This definition means that aircraft wing-fuselage attachments must comply with the damage tolerance requirements [1, 2]. The main goal is a safe life design, i.e. a slow crack growth structure not requiring any insection during its full life. The Damage Tolerance approach assumes the components have a preexisting flaw from which a crack will grow under dynamic loads. This assumption makes it possible to account for in-service or manufacturing defects in determining the dynamic life. The Damage Tolerance Methodology uses fracture mechanics to predict the fatigue crack growth in a structure. In the design analysis of a slow crack growth structure it is most important to make correct estimates for the early portion of the crack growth process, because it is where the life is. In most cases this implies that the maximum accuracy is needed for small corner cracks.

The ability to maintain aircraft airworthiness and structural integrity successfully critically depends on the application of appropriate fatigue crack growth (FCG) prediction tools. The prediction tools are required to predict FCG accurately in aircraft structures and components under flight spectrum loading, and thus reliably provide total economic lives or inspection intervals as a part of a stringent aircraft structural integrity management plan.

Fatigue crack growth in aircraft structures and components under flight spectrum loading is traditionally predicted based on FCG rates obtained from constant-

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amplitude (CA) crack growth testing using the cycle-by-cycle approaches [14, 15].

Attachment lugs are particularly critical components in crack initiation and growth because of their inherently high stress concentration levels near the lug hole. For these reasons, it is important to develop analytical/numerical as well as experimental procedures for assessing and designing damage tolerant attachment lugs to ensure the operational safety of aircraft. Over the years, several extensive studies [3-5] have been made on lug fatigue performance, involving both experimental and numerical means.

In the work of fatigue crack growth and fracture behavior of attachment lugs [6,7], an accurate calculation of the stress intensity factor is essential. Over the years, several methods have evolved to compute the stress intensity factors for structural components containing cracks. These methods include an analytical as well as an experimental approach. The experimental backtracking approach was used to derive empirically the stress intensity factors for structural components using the growth rate data of through-the-thickness cracks for simple geometry subjected to constant-amplitude loading. The finite element method is used to determine SIF`s precisely, using singular finite elements. Accurate stress-intensity factor (SIF) solutions are required to conduct thorough damage tolerance analyses of structures containing cracks. Exact closed-form SIF solutions for cracks in three-dimensional solids are often lacking complex configurations; therefore, approximate solutions must be used. Over the past two decades, considerable effort has been placed on developing computationally efficient methods which provide highly accurate SIF solutions for cracks in three-dimensional bodies.

The purpose of this investigation was to test the accuracy of the crack growth models. All necessary parameters, such as material property data, stress intensity solutions, and the load spectrum, were defined. In order to determine residual life of damaged structural components, two crack growth methods are used: the conventional Forman`s crack growth method and the crack growth model based on the strain energy density method. The last method uses the low-cycle fatigue properties in the crack growth model.

Crack growth model based on the strain energy density method

In this work, the fatigue crack growth method based on the energy concept is first considered and then it is necessary to determine the energy absorbed until failure. This energy can be calculated by using a cyclic stress-strain curve. The function between stress and strain, as recommended by Ramberg-Osgood provides a good description of elastic-plastic behavior of materials, and may be expressed as:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2k'}\right)^{\frac{1}{n'}}. \tag{1}$$

where E is the modulus of elasticity, $\Delta\varepsilon/2$ is the strain amplitude and $\Delta\sigma/2$ is the stress amplitude. Equation (1) enables the calculation of the stress-strain distribution by knowing low cyclic fatigue properties. As a result, the energy absorbed until failure becomes [10, 11]:

$$W_c = \frac{4}{1+n'}\sigma'_f \varepsilon'_f \tag{2}$$

where σ'_f is the cyclic yield strength and ε'_f - the fatigue ductility coefficient. Given the fact that the strain energy density method is considered, the energy absorbed until failure must be determined after the energy concept is based on the following fact: The energy absorbed per unit growth of crack is equal to the plastic energy dissipated within the process zone per cycle. This energy concept is expressed by:

$$W_c \delta a = \omega_p \, , \tag{3}$$

where W_c is the energy absorbed until failure, ω_p - the plastic energy and a - the crack length. In equation (2.3) it is necessary just to determine the plastic energy dissipated in the process zone ω_p. By integrating the equation for the cyclic plastic strain energy density in the units of Joule per cycle per unit volume [10] from zero to the length of the process zone ahead of the crack tip d^*, it is possible to determine the plastic energy dissipated in the process zone ω_p. After the integration, the relation of the plastic energy dissipated in the process zone becomes:

$$\omega_p = \left(\frac{1-n'}{1+n'}\right)\frac{\Delta K_I^2 \psi}{E I_{n'}} \tag{4}$$

where ΔK_I is the range of stress intensity factor, ψ - a constant depending on the strain hardening exponent n', I'_n - the non-dimensional parameter depending on the n'.

The fatigue crack growth rate can be obtained by substituting Eq. (2.2) and Eq. (2.4) in Eq. (2.3):

$$\frac{da}{dN} = \frac{(1-n')\psi}{4EI_{n'}\sigma'_f \varepsilon'_f}\left(\Delta K_I - \Delta K_{th}\right)^2 \, , \tag{5}$$

where ΔK_{th} is the range of threshold stress intensity factor and is the function of the stress ratio, i.e.

$$\Delta K_{th} = \Delta K_{th0}\left(1-R\right)^{\gamma} \, , \tag{6}$$

ΔK_{th0} is the range of the threshold stress intensity factor for the stress ratio $R=0$ and γ is the coefficient (usually $\gamma = 0.71$). Finally, the number of cycles until failure can be determined by the integration of the relation for the fatigue crack growth rate:

$$N = B\int_{a_0}^{a_c} \frac{da}{\left(\Delta K_I - \Delta K_{th}\right)^2} \, , \quad B = \frac{4EI_{n'}\sigma'_f \varepsilon'_f}{(1-n')\psi} \tag{7}$$

and

$$\Delta K_I = YS\sqrt{\pi a} \tag{8}$$

Equation (7) enables us to determine the crack growth life of different structural components. A very important fact is that equation (7) is easy for application since low cyclic material properties (n', σ'_f, ε_f) available in literature are used as parameters. The only important point is the stress intensity factor which, depending on the geometry complexity and the type of loading, could be determined by using analytical and/or numerical approaches.

Crack growth analysis using a conventional approach

Various conventional crack growth models have been used for the crack growth analysis and fatigue life estimations. Many of these models obtain correct solutions of crack growth analyses for cracked structural elements under cyclic loads of constant amplitudes. However, for constructions under cyclic loads of variable amplitudes in a form of load spectrum such as in aircraft cases, it is necessary to include the effects of the shape of load spectra and its effects on the estimation of the life of structural elements [9].

Forman, Newman and others [8] developed the equation often used to describe a crack growth. This equation describes the crack growth curve in terms of the crack length a, the number of cycles N, the stress ratio R, the stress intensity factor range ΔK, and material constants, C, n, p, q through the best fits of the da/dN - ΔK data.

$$\frac{da}{Dn} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^n \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{\max}}{K_c}\right)^q} \tag{8}$$

where: a - the crack length, N -the number of cycles, C, n, p, q – are experimentally derived material parameters , K is the stress intensity factor (SIF), K_{th} is the threshold stress intensity factor, R is the stress ratio and K_c - is the critical stress intensity factor. The Newman closure function is one of these terms and is defined as f :

$$f = \frac{K_{op}}{K_{\max}} = \begin{cases} \left(R,\ A_0 + A_1 R + A_2 R^2 + A_3 R^3\right); & R \geq 0 \\ A_0 + A_1 R; & -2 \leq R < 0 \end{cases} \tag{9}$$

and the coefficients are given by:

$$A_0 = \left(0.825 - 34\alpha + 0.05\alpha^2\right)\left[\cos\left(\frac{\pi}{2}\frac{S_{\max}}{\sigma_0}\right)\right]^{\frac{1}{2}}$$

$$A_1 = \left(0.415 - 0.071\alpha\right)\frac{S_{\max}}{\sigma_0}$$

$$A_2 = 1 - A_0 - A_1 - A_3$$

$$A_3 = 2A_0 + A_1 - 1$$

where: α - is the plane stress/strain constraint factor, (σ_{max}/σ_0) is the ratio of the maximum stress to the flow stress. The threshold stress intensity factor range is calculated by the following empirical equation:

$$\Delta K_{th} = \Delta K_{th0}\left(\frac{a}{a+1}\right)^{\frac{1}{2}} \Bigg/ \left(\frac{1-f}{(a-A_0)(1-R)}\right)^{(1+C_{th}R)} \tag{10}$$

Relation (8) represents one general crack growth model based on a conventional approach. This relation can be transformed to the conventional Forman`s crack growth model[5]. In region III, a rapid and unstable crack growth occurs, so Forman at al. proposed an equation for region III as well as for region II [9]

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_C - \Delta K} \tag{11}$$

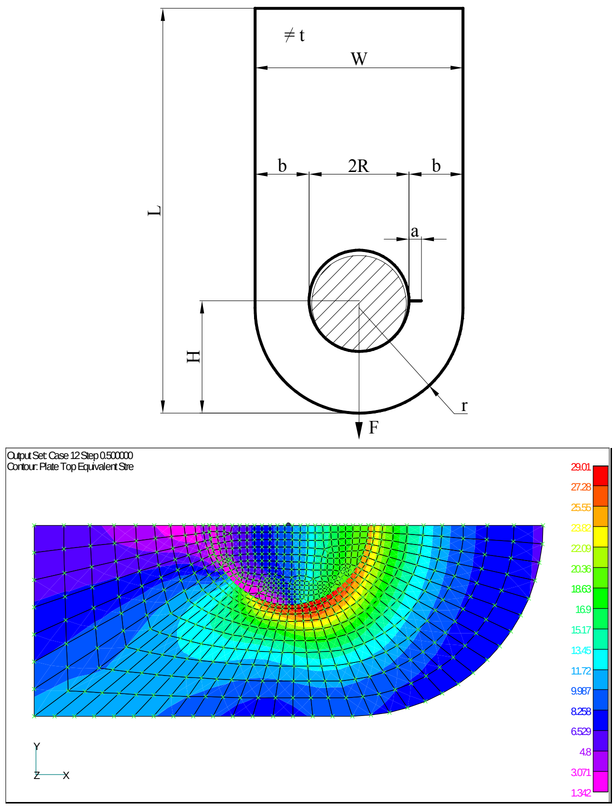
where K_C is the fracture toughness. Forman`s equation has been developed to model the unstable crack growth domain (III).

Stress intensity factor solutions of cracked lugs

Damaged attachment lugs with initial cracks through the thickness

The relations for the stress intensity factor with cracks through the thickness are given in this section.

In general, the geometry of notched structural components and loading is too complex for the stress intensity factor (SIF) to be solved analytically. The SIF calculation is further complicated because it is a function of the position along the crack front, crack size and shape, type loading and geometry of the structure. In this work, the analytic method and the FEM were used to perform a linear fracture mechanics analysis of the pin-lug assembly. The analytic results are obtained using the relations derived in this paper. A good agreement between the finite element and the analytic results is obtained. This is very important because we can use analytically derived expresions in crack growth analyses. Lugs are essential components of an aircraft for which proof of damage tolerance has to be undertaken. Since the literature does not contain the stress intensity solution for lugs which are required for proof of damage tolerance, the problems posed in the following investigation are: selection of a suitable method of determining the SIF, determination of SIF as a function of crack length for various forms of lugs and setting up a complete formula for the calculation of the SIF for lugs, allowing essential parameters. The stress intensity factors are the key parameters to estimate the characteristic of the cracked structure. Based on the stress intensity factors, fatigue crack growth and structural life predictions have been investigated. The lug dimensions are defined in Fig.1.



Contact pin/load FE model

Figure 1. Geometry and loading of lugs with cracks through the thickness

To obtain the stress intensity factor for the lugs it is possible to start with the general expression for the SIF in the next form

$$K = Y_{SUM}\sigma\sqrt{\pi a} \tag{12}$$

where: Y – the correction function, a - the crack length. This function is essential in determining the stress intensity factor. Primarily, this function depends on the stress concentration factor, k_t and the geometric ratio a/b. The correction function is defined using experimental and numerical investigations. This function can be defined in the next form [13]:

$$Y_{SUM} = \frac{1.12 \cdot k_t \cdot A}{A + \frac{a}{b}} k \cdot Q \tag{13}$$

$$k = e^{r\sqrt{a/b}} \tag{14}$$

$$b = \frac{w - 2 \cdot R}{2} \tag{15}$$

$$r = -3.22 + 10.39\left[\frac{2 \cdot R}{w}\right] - 7.67\left[\frac{2 \cdot R}{w}\right]^2 \tag{16}$$

$$Q = \frac{U\frac{a}{b} + 10^{-3}}{\frac{a}{b} + 10^{-3}} \tag{17}$$

$$U = 0.72 + 0.52\left[\frac{2 \cdot R}{H}\right] - 0.23\left[\frac{2 \cdot R}{H}\right]^2 \tag{18}$$

$$A = 0.026 \cdot e^{1.895\left(1+\frac{a}{b}\right)} \tag{19}$$

The stress concentration factor k_t is very important in the calculation of the correction function, eq.13. In this investigation, a contact finite element stress analysis was used to analyze the load transfer between the pin and the lug.

Damaged attachment lug with a partial surface crack

This part of the paper considers the stress intensity factors for a cracked lug with a partial (corner) surface crack as shown in Fig.2.

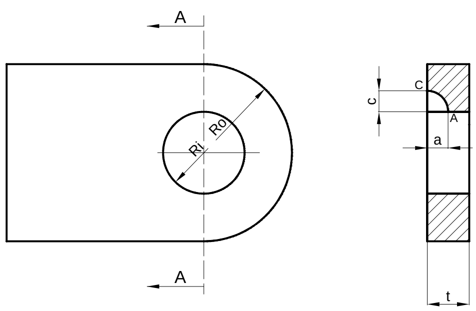


Figure 2. Lug with a partial surface crack

Computational model [16] will be used as a start point for determining the stress intensity factor of an attachment lug with a partial surface crack. Here is considered the problem of determination of SIF of a lug with a surface crack within the zone of stress concentration. This approached is extended to attachment lugs with partial surface crack [17]. This approach was used here to define analytic expressions for the determination of SIFs at the points A and B of a lug with a partial surface crack as shown in Fig.2, in the next form:

$$K_I^{(A)} = \sigma_0\sqrt{\pi c}\,\alpha_T(0)\sqrt{A_n}\,M_I^{(A)}\frac{F}{\sqrt{Q}}M_B \tag{20}$$

$$K_I^{(C)} = \sigma_0\sqrt{\pi c}\,\alpha_T\left(\frac{c}{R_i}\right)\sqrt{A_n}\frac{M_I^{(C)}}{\sqrt{Q}} \tag{21}$$

where:

$$\sigma_0 = \frac{P}{2R_0 t}$$

$$\alpha_T\left(\frac{c}{R_i}\right) = \frac{K_{IT}}{\sigma_0\sqrt{\pi c}} \tag{22}$$

where: P- force along axis of lug, K_{IT} is SIF of lug with crack through the thickness of lug c, Fig.2.

$$Q = 1.0 + 1.464\left(A_n C_n\right)^{1.65} \tag{23}$$

$$C_n = M_{in}\left(\frac{c}{a}, 1.0\right)$$

$$A_n = M_{in}\left(\frac{a}{c}, 1.0\right)$$

$$M_I^{(A)} = \left[1 + 0.025C_n + 0.0965(1 + A_n)\right]\sqrt{C_n} \tag{24}$$

$$M_I^{(A)} = \left[1 + 0.214C_n - 0.0925(1 - A_n)\right]\sqrt{A_n} \tag{25}$$

$$F = 1 - 2.09 \cdot S + 9.635 \cdot S^2 - 23.37 \cdot S^3 + 25.485 \cdot S^4 - 10.403 \cdot S^5 \tag{26}$$

$$S = \frac{c}{c + R_i} \tag{27}$$

$$M_B = \begin{cases} 1.0 \quad za \quad a > c \\ 1 + \left(\frac{a}{t}\right)^{(1.8 + A_n)}\left(0.92 + 0.82A_n\right) \quad \text{za} \quad a \le c \end{cases} \tag{28}$$

The previous analytic expressions of SIF for lugs with partial surface cracks can be used for "static" fracture mechanics and residual life estimations using crack growth models.

Numerical validation

In order to illustrate computation procedures in the damage tolerance analysis and residual life estimations of damaged structural components, some numerical examples are included.

Life estimation of damaged structural elements

The subject of this analysis is examining cracked aircraft lugs under cyclic load of constant amplitudes and spectra. The conventional Forman crack growth model and a crack growth model based on the strain energy density method are used. The material of lugs is 7075 T7351 aluminum alloy with the following material properties:

$$\sigma_{02} = 334 \text{ N/mm}^2 \Leftrightarrow \text{Tensile strength of the material}$$

$$\sigma_{02} = 334 \text{ N/mm}^2 \; K_{IC} = 2225\left[\text{N/mm}^{3/2}\right]$$

The dynamic material properties (Forman`s constants) are $C_F = 3^* \cdot 10^{-7} = 2.39$. The cyclic material properties are

$\sigma'_f = 613$ MPa, $\varepsilon'_f = 0.35$, $n' = 0.121$. The stress intensity factors (SIFs) of the cracked lugs are determined for the nominal stress levels of $\sigma_g = \sigma_{\max} = 98.1$ N/mm^2 and $\sigma_{\min} = 9.81$ N/mm^2. These stresses are determined in the net cross-section of the lug. The corresponding forces of the lugs are defined as $F_{\max} = \sigma_g (w - 2R) t = 63716$ N and $F_{\min} = 6371.16$ N, loaded on the lug. For the stress analysis, the contact pin/lug finite element model is used. For the cracked lugs defined in Table 5.1, with initial cracks a_0, the SIFs are determined using the finite elements, Table 5.2. In order to obtain high-quality results of the SIFs, the cracked lugs are modeled by singular finite elements around the crack tip.

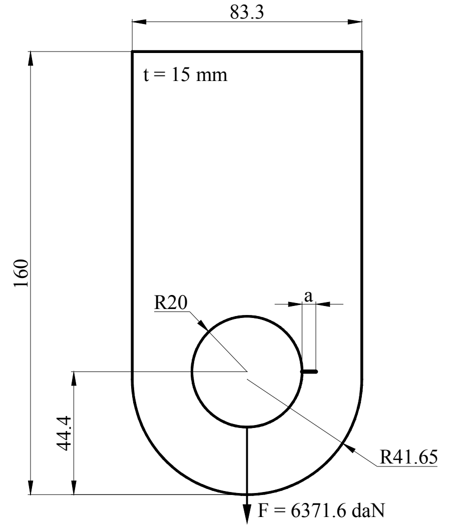


Figure 3. Geometry of cracked lug 2

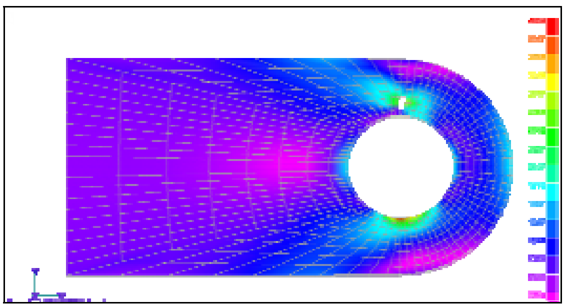


Figure 4. Finite Element Model of the cracked lug with stress distribution

Table 1. Geometric parameters of lugs [13]

Lug No.	Dimensions [mm]				
	2R	**W**	**H**	**L**	**t**
2	40	83.3	44.4	160	15
6	40	83.3	57.1	160	15
7	40	83.3	33.3	160	15

The stress intensity factors of cracked lugs are calculated under the stress level: $\sigma_g = \sigma_{\max} = 98.1$ N/mm^2, or a corresponding axial force, $F_{\max} = \sigma_g (w - 2R) t = 63716$ N. In the present finite element analysis, the cracked lug is modeled with special singular quarter-point six-node finite elements around the crack tip, Fig.4. The load of the model, a concentrated force, $F_{\max}$, was applied at the center of the pin and it reacted at the other end of the lug. Spring elements were used to connect the pin and the lug at each pairs of the nodes having identical nodal coordinates all around the periphery. The contact area was determined iteratively by assigning very high stiffness to the spring elements which were in compression and very low stiffness (essentially zero) to the spring elements which were in tension. The stress intensity factors of the lugs, the analytic and finite elements, for through-the-thickness cracks are shown in Table 2. The analytic results are obtained using the relations from the previous sections, eq. (12).

Table 2. Comparison of the analytic results with the FEM results of the SIF

Lug No.	a[mm]	$K_{I\max}^{MKE}$	$K_{I\max}^{ANAL.}$
2	5.00	68.784	65.621
6	5.33	68.124	70.246
7	4.16	94.72	93.64

It is evident from Table 2 that there is a good agreement between the analytic and finite element results for the determination of stress intensity factors. The accuracy of SIFs is very important in precise crack growth analyses and the life estimation of cracked lugs. That means that the proposed analytic model for the determination of SIFs is adequate in crack growth analyses. It is very important to know in a design process how any geometric parameters of lugs have the effects on the fracture mechanics parameters. Fig.6 shows the dependence of the SIF, K_{max}, and the height of lug head H. In this analysis, the geometric properties of the lugs are given in Table 1. It is evident from Fig.6 that there is an increase of SIFs with the increasing of the crack length and their reduction with the increasing of the lug head height. Fig.5 shows the computation and experimental results of cracked lug No.2 as defined in Fig.3 and Table 1. In this computation analysis, the Forman crack growth model is used. A good agreement between the computation and the experimental results is obtained. It is evident that the computation Forman`s crack growth model is, to a low extent, conservative for longer cracks, Fig.5.

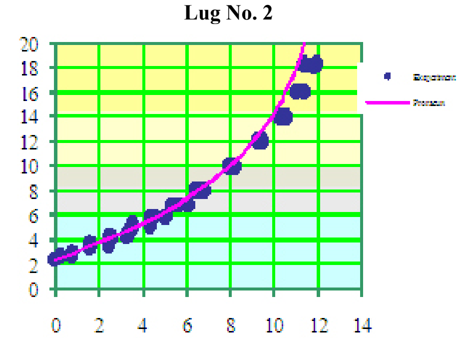


Figure 5. Comparisons of the computational with the experimental crack growth results for lug No. 2 (H=44.4 mm); k_t=2.8

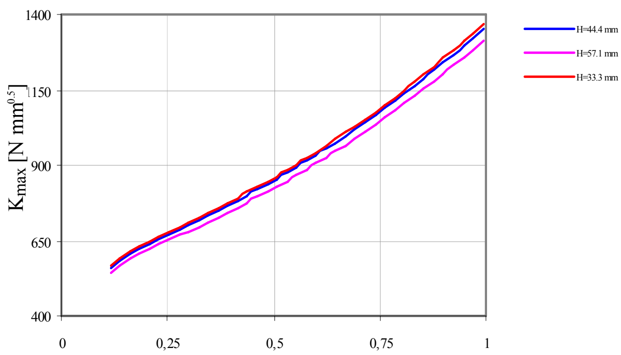


Figure 6. The effects of the head length (H) in a function of the crack length to the SIF of Lug. No: 7 *(H=33.3),* 2 *(H=44.4),* 6 (H=57.1*)*

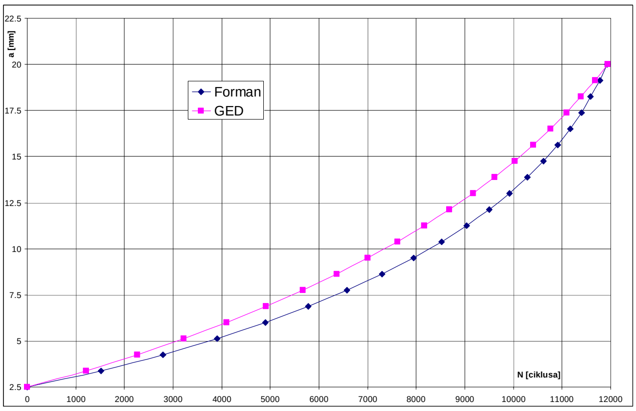


Figure 7. Comparisons of the crack growth results for the cracked lug using two crack growth models: (1) Forman`s and (2) Strain Energy Density (GED)

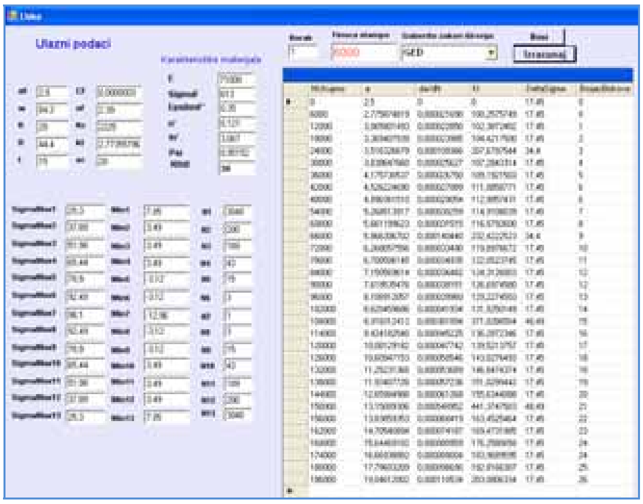
Fig.7 shows the results of crack growth for the cracked lug using two methods: (1) conventional Forman`s method and (2) strain energy density method[7] (GED).

Life estimation of the lug under general load spectrum

The residual life of cracked aircraft attachment lugs is considered here. Aircraft lug No.2, Fig.3, defined in the previous example 5.1, is under general load spectrum. The complete load spectrum and the results of the residual life of cracked lug No.2 are given in Table 3. The residual life of the cracked lug, expressed in a block form, is N_{bl}=26. Here one block corresponds to 100 flight hours of a G-2 aircraft.

A graphical illustration of the crack growth behaviour of lug No.2 under general load spectrum is given in Fig.8.

Table 3. Results of the residual life of the lug under general load spectrum



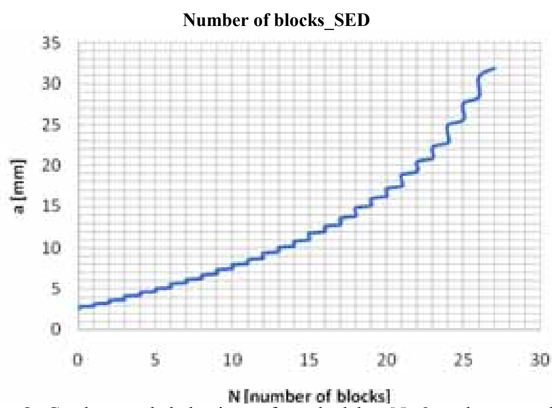


Figure 8. Crack growth behaviour of cracked lug No.2 under general load spectrum

The effects of the shape of lug surface crack on fatigue life

The analytic expressions for the determination of SIFs for the lugs with partial surface cracks are given in Section 4.2 and can be used for residual life estimation.

Fig.9 shows the results of the crack growth of cracked lugs No.2, the geometric properties of which are given in Figs.1 and 2.

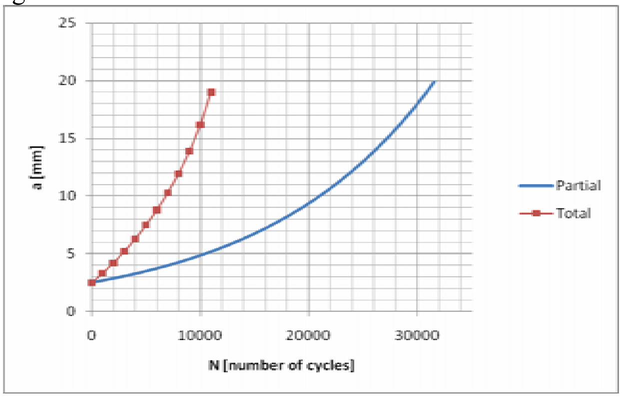


Figure 9. Comparisons of the crack growth results of the attachment lugs with cracks through the thickness and partial surface cracks

Fig.8 shows the effect of the shape of the lug surface crack on the residual fatigue life. As expected, the reduced life occurs in the lug with a crack through the thickness ("total") more than in the lug with a partial surface crack ("partial").

Conclusions

This investigation is focused on developing efficient and reliable computation methods for fatigue life estimation of cracked aircraft lugs under cyclic loads of constant amplitude and load spectrum. For crack growth analyses and residual lives of damaged lugs, the Strain Energy Density (SED) method and the conventional Forman`s method are used. A special attention has been focused on the determination of the fracture mechanics parameters of structural components such as stress intensity factors of aircraft cracked lugs. The effects of the shape of lug surface cracks on the residual fatigue life are investigated as well.

Predictions and experimental investigations for the fatigue life of an attachment lug under load spectrum were performed. The following can be concluded from this investigation:

- A model for the fatigue crack growth, based on the SED, is included in this investigation which incorporates the low cycle fatigue properties of the material.
- The comparisons of the predicted crack growth rate using the strain energy method with experimental data and the conventional Forman`s model point out the fact that this model could be effective when used for residual life estimations
- The stress intensity factor of the cracked lug is well defined by the analytical method since there is really a minor difference when compared with the results obtained by singular finite elements.
- The effects of the shape of the surface cracks of attachment lugs on residual fatigue life are evident, Fig.9.

In general, the presented computation method based on the SED can be effectively used in crack growth analyses and for residual life estimations.

Acknowledgments

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References

[1] JARFALL,L.: *Verification of the Damage Tolerance of Fighter Aircraft*, Fatigue, Vol. 16, No.1, 1994, pp. 67-74.

[2] BRUSSAT,T.R., KATHIRESAN,K., RUD,J.L.: *Damage tolerance assessment lugs*, Engineering Fracture Mechanics, 1986, 23, pp.1067-1084.

[3] MAKSIMOVIĆ,S.: *Fatigue Life Analysis of Aircraft Structural Components*, Scientific Technical Review, ISSN 1820-0206, 2005, Vol.LV, No.1, pp.15-22

[4] MAKSIMOVIĆ,K., NIKOLIĆ-STANOJEVIĆ,V., MAKSIMOVIĆ,S.: *Modeling of the surface cracks and fatigue life estimation*, ECF 16, 16th European Conference of Fracture, ECF 16, Alexandroupolis, Greece, 2006.

[5] POSAVLJAK,S., MAKSIMOVIĆ,K.: *Initial fatigue life estimation in aero engine discs*, Scientific Technical Review, ISSN 1820-0206, 2011, Vol.61, No. 1, pp. 25-30.

[6] ANTONI,N., GAISNE,F.: *Analytical modeling for static stress analysis of pin-loaded lugs with bush fitting*, Applied Mathematical Modeling, 2011, 35, pp.1-21.

[7] LIU,Y.Y., LIN,F.S.: *A mathematical equation relating low cycle fatigue data to fatigue crack propagation rates*, Int. J. Fatigue, Vol. 6 (1984), pp.31-36

[8] FORMAN,R.G., METTU,S.R.: *Behavior of Surface and Corner Cracks Subjected to Tensile and Bending Loads in Ti-6Al-4V Alloy,* Fracture Mechanics: Twenty-second Symposium, Vol. 1, ASTM STP 1131, H. A. Ernst, A. Saxena, and D. L., McDowell, eds., American Society for Testing and Materials, Philadelphia, 1992, pp.519-546.

[9] FORMAN,R.G., KEARNEY,V.E., ENGLE,R.M.: *Numerical analysis of crack propagation in cyclic loaded structures*, J. Bas. Engng. Trans. ASME 1967, 89, pp.459.

[10] LIU,S.B., TAN,C.L.: *Boundary element contact mechanics analysis of pin-loaded lugs with single cracks*, Engineering Fracture Mechanics, 1994, Vol. 48, No.5, pp.717-725.

[11] BOLJANOVIĆ,S., MAKSIMOVIĆ,S.: *Analysis of the crack growth propagation process under mixed-mode loading*, Engineering Fracture Mechanics, Vol. 78, Issue 8, May 2011, pp. 1565-1576.

[12] BLAZIĆ,M., MAKSIMOVIĆ,M., VASOVIĆ,I., ASSOUL.,Y.,: *Stress intensity factors for elliptical surface cracks in round bars and residual life estimation*, Scientific Technical Review, ISSN 1820-0206, 2011, Vol.61, No. 1, pp. 63-67.

[13] MAKSIMOVIĆ,K.: *Damage tolerance analysis of aircraft constructions under dynamic loading*, Master Thesis, Faculty of Mechanical Engineering, 2003.

[14] BALL,D, NORWOOD,D, MAATH,S.: *Joint strike fighter airframe durability and damage tolerance certification*, In: Proceedings of the 47th AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics, and materials conference, Newport, Rhode Island, USA; May 2006.

[15] SCHIJVE,J.: *Fatigue of structures and materials in the 20th century and the state of art*, Int J Fatigue 2003;25:679–702.

[16] KATHRIRESAN,K., HSU,T.M., BRUSSAT,T.R.: *Advanced life analysis methods-crack growth analysis methods for attachment lugs*, AFWAL-TR-84-3080 II, 1984.

[17] MAKSIMOVIĆ,K.: *Strength and residual life estimation of structural elements under general load spectrum*, Doctoral Thesis, Faculty of Mechanical Engineering, 2009 (In Serbian).

[18] JANKOVIC,M.: *Low-Cyclic Fatigue*, Faculty of Mechanical Engineering, Belgrade, 2001. (in Serbian)

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Procena preostalog veka elemenata konstrukcija sa inicijalnim oštećenjima pri opštem spektru opterećenja

Rad se bavi ponašanjem na zamor elemenata avionskih konstrukcija pri cikličnim opterećenjima konstantne amplitude i spektra. Primarna pažnja ovog istraživanja je usmerena na uspostavljanje proračunske procedure za procenu preostalog veka elemenata avionskih konstrukcija sa inicijalnim oštećenjima u obliku prskotina. U ovom istraživanju za procenu preostalog veka se koriste malociklusne karakteristike materijala na zamor odnosno iste kao i za procenu veka do pojave inicijalnog oštećenja. Prema tome u ovoj analizi širenja prskotine nisu neophodne eksperimentalno određene dinamičke karakteristeristike materijala. Kompletna proračunska procedura za analizu širenja prskotina i procene preostalog veka na bazi korišćenja malociklusnih karakteristika na zamor ilustrovana je sa strukturalnim elementima sa inicijalnim oštećenjima. Da bi smo odredili faktore intenziteta napona (FIN), koji su neophodni za analizu širenja prskotine, korišćeni su singularni konačni elementi. Rezultati numeričke simulacije širenja prskotine zasnovani na gustini energije deformacije su upoređeni sa sopstvenim eksperimentalnim rezultatima.

Ključne reči: **zamor materijala, zamorna prskotina, širenje prskotine, ciklično opterećenje, niskociklični zamor, konačni element, vek trajanja, konstrukcija letelice.**

Оценка остаточного срока службы элементов конструкции с начальними повреждениями под общим спектром нагрузки

В настоящей работе рассматривается поведение структурных компонентов самолёта на усталость, с циклическими нагрузками постоянной амплитуды и нагрузки спектра. Основное внимание этого расследования направлено на установление процедуры вычисления для оценки остаточного ресурса авиационных конструкций с элементами первоначального повреждения в виде трещин. В данном исследовании для оценки остаточного ресурса использовались низкоциклические усталостные свойства материалов т.е. те же самые как для оценки ресурса до появления начального повреждения (анализ при помощи метода деформации плотности энергии (SED - Strain Energy Density). Этот метод использует малоциклическую усталость (LCF - low-cycle fatigue) свойства материала, которые также используются для оценки ресурса до наступления начального повреждения. Поэтому в данном анализе расширения трещин не нужны экспериментально определённые динамические характеристики материала, такие как константы Формана. Полная процедура вычисления для анализа распространения трещин и оценки остаточного ресурса, основанная на использовании низкоциклических усталостных характеристик материала,

иллюстрируется с повреждёнными структурными элементами с исходными повреждениями. Чтобы определить аналитические выражения для коэффициентов интенсивности напряжений (SIF), которые необходимы для анализа распространения трещин, мы использовали особых конечных элементов. Результаты численного моделирования распространения трещин на основе штамма плотности энергии были сравниваны с нашими собственными экспериментальными результатами.

Ключевые слова: **усталость материала, усталостные трещины, циклические нагрузки, низкоциклическая усталость, конечный элемент, срок службы, конструкция самолёта.**

Estimation de la vie de fatigue chez les éléments de construction aux endommagements initiaux sous le spectre général de charge

Ce papier considère le comportement des éléments constructifs des aéronefs quant à la fatigue sous les charges cycliques à l'amplitude et au spectre constants. Ces recherches sont orientées principalement sur l'établissement du processus de computation pour l'évaluation de la vie de fatigue chez les éléments constructifs des aéronefs aux endommagements initiaux en forme de fissure. Dans ce but on a utilisé les caractéristiques des matériaux à faible cycle qui ont été utilisées déjà pour l'estimation de la durée de vie jusqu'à l'apparition de l'endommagement initial . Par conséquent dans l'analyse de la croissance de la fissure les caractéristiques des matériaux dynamiques déterminées par voie expérimentale ne sont pas nécessaires. La procédure complète de computation pour analyser la croissance de la fissure et estimer la durée de vie en utilisant les caractéristiques à faible cycle est illustrée par les éléments structuraux contenant les endommagements initiaux. Pour déterminer les facteurs de l'intensité de la charge (FIC) qui sont nécessaires dans l'analyse de la croissance de la fissure on a employé les éléments singuliers finis. Les résultats de la simulation numérique de la croissance de la fissure, basés sur la densité de l'énergie de la déformation, ont été comparés avec nos propres résultats expérimentaux.

Mots clés: **fatigue du matériel, fissure de fatigue, charge cyclique, fatigue faible cyclique, élément fini, durée de vie, construction de l'aéronef**