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Stress Intensity Factors for Elliptical Surface Cracks in Round Bars and Residual Life Estimation

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This work investigates the behavior of structural components with surface cracks[1,2] . The attention is focused on a circular bar with an elliptical surface crack under tension load. The aim of this study is to analyze the influence of the shape and size of cracks at the structural part on the fracture mechanics parameters. Stress intensity factors (SIF) are considered using the finite element method. For that purpose a straight round bar under tension is investigated. Stress intensity factors of elliptical surface cracks in tensile round bars are calculated by using three-dimensional finite element analysis (FEA) models with singular 20-node elements arranged around the crack tip.
The graphic and the table review of stress intensity factors are also determined. The stress intensity factors are calculated for various dimensions of surface cracks. These results are used to determine an analytic form of the stress intensity factors that is necessary in crack growth analyzes. Using the derived analytic formulae for the stress intensity factor based on the FEA, a crack growth analysis is carried out.

Key words: **fracture mechanics, stress analysis, tension, bar, crack, crack growth, finite element method.**

Introduction

CYLINDRICAL components have many applications in aircraft design. These structural components are subjected to cyclic stresses, which can cause damage and premature failure by fatigue crack growth. As it is well known, the design of engineering components in the past was only based on the S-N curves and did not consider the crack initiation and crack growth phases to predict life. At present, Linear Elastic Fracture Mechanics allows us to study crack growth behavior and to predict the lifetime of components by integrating the crack growth law if Stress Intensity Factor (K) solutions and material crack growth data are available. Most of mechanical failures by fatigue process on rotor shafts have origin on surface cracks that grow with a semi-elliptical shape. Surface cracks emanating from stress concentrating locations are the most common phenomena of fatigue failure. Bars with variable cross-sections are a category of cylindrical parts and components extensively used in engineering mechanisms. Surface fatigue cracks are frequently initiated in such components at the stress concentrating locations, then they propagate into the interior of the parts and can cause final fracture abruptly. In addition, smooth and notched round bars have been used as standard specimens to obtain the fatigue property of materials for safe design and assessment. An elliptical-arc surface crack in a round bar subjected to cyclic tension loading with a constant amplitude is considered. The fatigue failure of round bars often develops from surface defects, and therefore several authors have examined the stress-intensity factor variation along the front of these flaws. The assumption that an actual part-through crack can be replaced by an equivalent elliptical-arc edge flaw is experimentally supported, and therefore many analyses have been carried out related to this equivalent configuration[1-3]. The problem of fatigue propagation is very complex because the crack front can be modeled quite accurately by an elliptical arc during the whole phenomenon, but the aspect ratio of the ellipse changes under cyclic loading.

The three-dimensional elliptical arc has been used to model the crack front in cylindrical rods under axial loading. In the first section of the present work, the stress intensity factors along the crack front are computed using the FEM. The other part is reserved for fatigue crack propagation. The crack growth can be analyzed subjected to Mode I or Mixed mode loading [7]. Here Mode I loading is considered.

Determination of the stress intensity factors

Many numerical analyses, theoretical studies and experimental investigations have been conducted to obtain stress intensity factors (SIFs) for three-dimensional (3D) cracked bodies. Explicit solutions or empirical expressions have been achieved for surface and corner cracks on

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smooth strips or straight round bars and at circular holes in finite thickness plates[6].

The purpose of the present paper is to investigate the dependence of SIFs of surface cracks in round bars upon notch geometry, Fig.1. The surface crack has a semi-elliptical geometry with the semi-major axis *c* and the semi-minor axis *a* as shown in Fig.1, where *a* is also the depth of the surface crack.

The relative depth ξ=a/D of the deepest point A on the defect front and the flaw aspect ratio α=a/b define the crack configuration. The point B is the end of the flaw. The parameter ξ is made to vary from 0.05 to 0.25. The geometrical parameter α is constant and its value is 0.5. The diameter of the rod is D=30 mm.

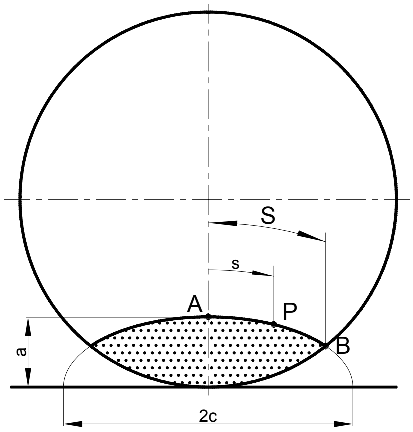

Figure 1. Elliptical-arc surface flaw in a round bar, geometrical parameters

Table 1. Axis of elliptical arc

c (mm)	a (mm)
3	1.5
4.5	2.25
6	3
7.5	3.75
9	4.5
10,5	5.25
12	6
15	7.5

The axial tensile stress has been applied to the end of the rod and the last layer of elements at the end of the rod has been constrained to move in the axial direction. The stress-intensity factors are obtained for the tension σ=100 MPa. In the analyses, a round bar made of 2024-T351 is being used. Young's modulus, E = 73174 MPa and Poisson's ratio, ν=0.33 are assumed.

Three-dimensional finite element (FE) models with 20-node singular finite elements arranged around the crack tip will be used to calculate the SIFs of elliptical surface cracks in round bars. The fact that the crack geometry must be explicitly meshed and that significant refinement in the vicinity of the crack fronts is needed to achieve reasonable accuracy renders finite element based methods particularly difficult to implement. This meshing difficulty is particularly acute in three dimensions. Meshing and re-meshing become, in such circumstances, a particularly large part of the computation time. High mesh density in the crack front region is required. The parameter ξ is made to vary from 0.05 to 0.25, whereas the aspect ratio is constant α=0.5. The stress-intensity factors are obtained using the FEM for all values of the geometrical parameters ξ. The finite element results of SIF`s are given in Table 2.

Table 2. Stress-intensity factor against the relative crack depth ξ at the point A and at the point B

Parameters c, a, ξ			Stress intensity factor	
c (mm)	a (mm)	ξ=a/D	K_{I-B} (MPa/m$^{1/2}$)	K_{I-A} (MPa/m$^{1/2}$)
3	1.5	0.05	5.012	6.236
4.5	2.25	0.075	6.008	7.352
6	3	0.1	6.900	8.627
7.5	3.75	0.125	7.792	9.904
9	4,5	0.15	8.823	11.005
10.5	5.25	0.175	9.721	12.140
12	6	0.2	10.891	13.389
15	7.5	0.25	14.088	16.488

As we can see, the stress-intensity factor reaches the maximum at the point **A** and the minimum at the point **B** for each value of the crack depth ξ. Therefore, the most critical part along the crack front is the deepest point. The analytical equations (1) and (2) of the stess intensity factors are determined using discrete values of stress-intensity factors from Table 2. In the next step, the crack propagation rate has been obtained using equation (1).

$$K_{I-A}(a)=\sigma\big(-31595145114.00a^5+767297129.96a^4+ +6868723.72a^3+28169.96a^2-36.41a+0.07\big) \quad (1)$$

$$K_{I-B}(a)=\sigma\big(35665174.33a^4-365703.26a^3+ +1166.38a^2+11.34a+0.03\big) \quad (2)$$

Figures 2-6 show the stress field, σ, obtained for next values of the relative crack depth ξ: 0.05, 0.1, 0.125, 0.2 and 0.25. The values for σ are given in [daN/mm^2].

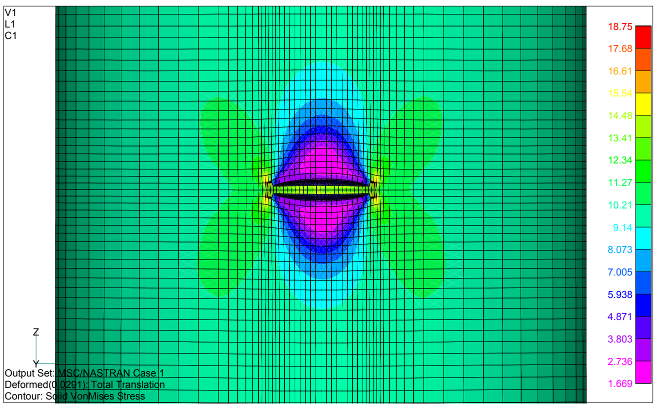

Figure 2. Stress field in the round bar with a relative crack depth ξ=0.05

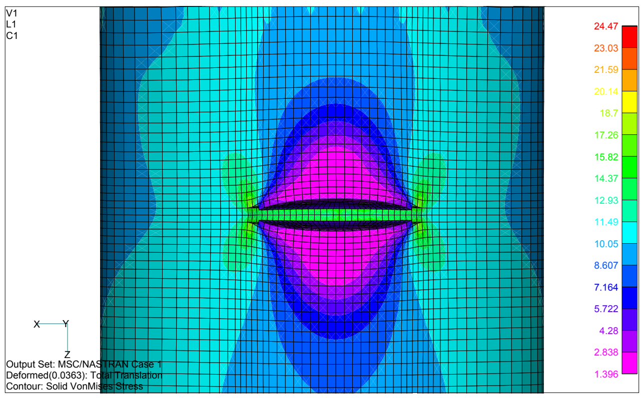

Figure 3. Stress field in the round bar with a relative crack depth ξ=0.1

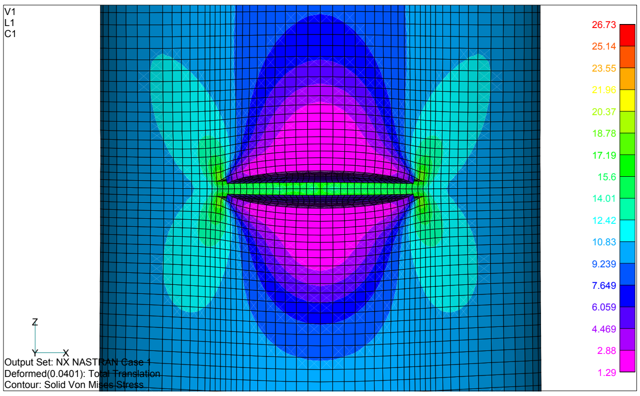

Figure 4. Stress field in the round bar with a relative crack depth ξ=0.125

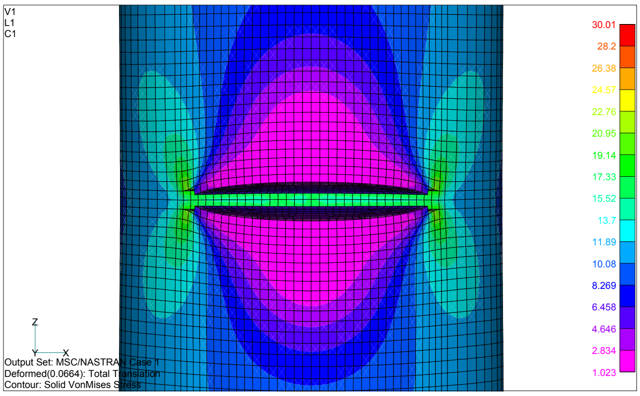

Figure 5. Stress field in the round bar with a relative crack depth ξ=0.2

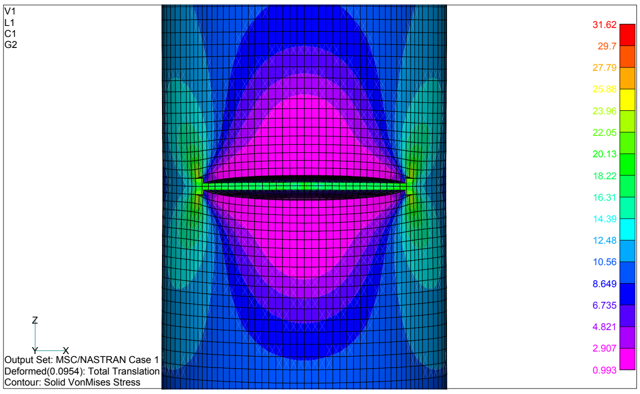

Figure 6. Stress field in the round bar with a relative crack depth ξ=0.25

The crack growth analysis

It is well known that the choice of the fatigue crack growth algorithm plays a crucial role in life prediction. Even though various algorithms are available in the literature, for the present purpose, a special approach was developed for crack propagation under constant amplitude loading.

The da/dN - ΔK curve, the so-called NASGRO equation (also called Forman–Newman–de Koning equation)[8] can be used as a general form

$$\frac{da}{dN} = C\left(K_{eff}\right)^n \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)}{\left(1-\frac{K_{\max}}{K_{crit}}\right)} \qquad (3)$$

In this equation C, n, p and q are empirical constants, ΔK_{th} is the threshold value for ΔK, K_{max} and K_{crit} are the maximum and the critical SIF values, respectively. The term ΔK_{eff} is the effective stress intensity factor range and it is described by the equation

$$\Delta K_{eff} = \left(\frac{1-f}{1-R}\right)\Delta K \qquad (4)$$

where f describes the plasticity-induced crack closure effect[9] and R is the stress ratio such as above. Eq. (3) consists of three different terms according to the three different propagation regimes: $C\,(\Delta K_{eff})^n$ represents the Paris regime, $(1-\Delta K_{th}/\Delta K)^p$ is used to describe the regime close to the fatigue threshold and $(1-K_{max}/K_{crit})^q$ describes the regime up to the critical SIF.

Several authors have analytically and experimentally deduced that the front of a surface flaw in a metallic round bar can be modeled quite accurately by an elliptical arc during the whole fatigue growth[4,5].

A round bar of the previously mentioned geometry has been subjected to cyclic axial loading with a constant amplitude. The stress range is expressed as follows :

$$\Delta\sigma = \left(\sigma_{\max} - \sigma_{\min}\right) = \sigma_{\max}\left(1-R\right) \qquad (5)$$

$\sigma_{\max}$ – the maximum stress applied in a loading cycle
$\sigma_{\min}$ – the minimum stress applied in a loading cycle

$R = \frac{\sigma_{\max}}{\sigma_{\min}}$ – the loading ratio

The fatigue life prediction is examined for three different stages of $\sigma_{\max}$: 50MPa, 60MPa, 70MPa

The value of $\sigma_{\min}$ is always zero, $\sigma_{\min} = 0$

This problem could be analyzed by means of a two-parameter model based on the Paris-Erdogan law.

$$\frac{da}{dN} = C\left(\Delta K\right)^m \qquad (6)$$

$\frac{da}{dN}$ - the crack propagation rate, $\left[\frac{\text{m}}{\text{cycle}}\right]$

$\Delta K = K_{I\max} - K_{I\min}$ - the stress-intensity factor range, $\left[MPa\sqrt{m}\right]$ (7)

The values C and m depend of the material. For the material 2024-T351 they are: C=1.51*10^{-10}, m=4

The stress intensity factor at the point A could be shown using equations (7) and (1) as :

$$\Delta K_A = K_{IA\max} - K_{IA\min} = \Delta\sigma\left(-31595145114.00a^5 + 767297129.96a^4 - 6868723.72a^3 + 28169.96a^2 - 36.41a + 0.07\right)$$

$$= \left(\sigma_{\max} - \sigma_{\min}\right)\left(-31595145114.00a^5 + 767297129.96a^4 - 6868723.72a^3 + 28169.96a^2 - 36.41a + 0.07\right) \qquad (8)$$

$$= \sigma_{\max}\left(1-R\right)\left(-31595145114.00a^5 + 767297129.96a^4 - 6868723.72a^3 + 28169.96a^2 - 36.41a + 0.07\right)$$

For $\sigma_{\min} = 0$, equation (8) can be writen in the form:

$$\Delta K_A = \sigma_{\max}\left(-31595145114.00a^5 + 767297129.96a^4 - 6868723.72a^3 + 28169.96a^2 - 36.41a + 0.07\right) \qquad (9)$$

The number of loading cycles to reach certain crack depth, a, is obtained using the Paris-Erdogan law and equation (9) for the stress intensity factor and is given in Table 3. The initial crack size is a_0= 0.015m

Table 3. Number of loading cycles, *N* to reach crack depth a

a	*N*(*σ*=50MPa)	*N*(*σ*=60MPa)	*N*(*σ*=70MPa)
0.0015	0	0	0
0.0015	17218.94	8303.89	4482.23
0.0017	31815.21	15342.98	8281.76
0.0019	43969.46	21204.41	11445.61
0.0021	54004.68	26043.92	14057.86
0.0023	62277.38	30033.46	16211.31
0.0025	69119.74	33333.20	17992.43
0.0027	74816.11	36080.30	19475.24
0.0029	79598.75	38386.74	20720.20
0.0031	83652.33	40341.59	21775.38
0.0033	87121.50	42014.61	22678.44
0.0035	90118.74	43460.04	23458.64
0.0037	92731.24	44719.92	24138.70
0.0039	95026.61	45826.87	24736.20
0.0041	97057.36	46806.21	25264.82
0.0043	98864.41	47677.66	25735.21
0.0045	100479.81	48456.69	26155.71
0.0047	101928.80	49155.48	26532.90
0.0049	103231.49	49783.70	26872.00
0.0051	104404.03	50349.17	27177.22
0.0053	105459.71	50858.27	27452.02
0.0055	106409.64	51316.38	27699.30
0.0057	107263.41	51728.11	27921.54
0.0059	108029.48	52097.55	28120.96
0.0061	108715.60	52428.43	28299.56
0.0063	109328.96	52724.23	28459.22
0.0065	109876.37	52988.21	28601.72
0.0067	110364.32	53223.53	28728.74
0.0069	110799.06	53433.19	28841.90
0.0071	111186.53	53620.04	28942.76
0.0073	111532.39	53786.83	29032.79

Relation (7) represents the analytic formulae of the stress intensity factors for elliptical surface cracks in round bars. This relation is derived using discrete values of SIF`s that are determined by singular finite elements.

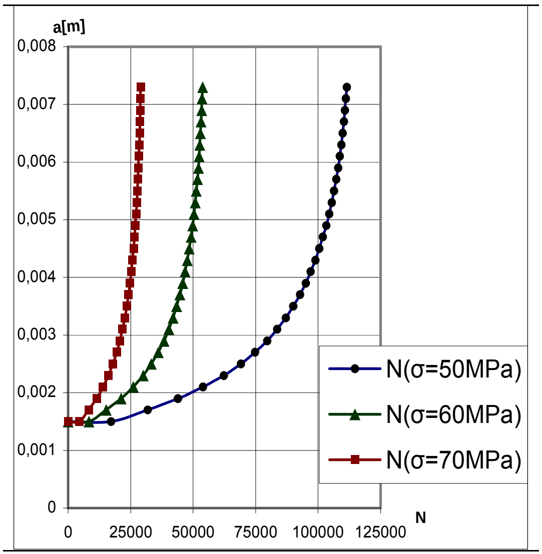

Figure 7. The crack depth *a* during the growth of the number of cycles *N* for three cases of the stress range

Conclusions

In this work the stress intensity factors for elliptical surface cracks in round bars and residual life estimation are considered. The attention focused on a circular bar with an elliptical surface crack under tension load. The SIFs of surface cracks in round bars are studied systemically by using the 3D FE method with 20-node singular elements arranged around the crack border. From the numerical results, an empirical formula of engineering interest for the SIF of surface cracks in notched bars is obtained.

The stress intensity factors are determined by singular finite elements for various crack depths. Using these discrete values of the stress intensity factors, a general analytic expression of stress intensity factors is derived. An empirical expression for the SIFs as a function of crack geometry is obtained by fitting the numerical results. These analytic expressions are used in the crack growth analysis of a cracked structural component. Therefore, the empirical expression can be used conveniently in the life prediction of notched bars with various notch geometries and stress concentration coefficients at least within the range of parameters studied in this work.

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Određivanje faktora intenziteta napona kod kružnog štapa sa eliptičnom površinskom prskotinom i procena preostalog veka

Rad se bavi istraživanjem ponašanja strukturalnih elemenata sa površinskim prskotinama[1,2]. Pažnja u radu je usmerena na kružni štap sa eliptičnom površinskom prskotinom pri opterećenju štapa na istezanje. Primarna pažnja u radu je usmerena na analizu uticaja oblika i veličine površinske prskotine na parametre mehanike loma. Faktori intenziteta napona (FIN) su određeni primenom metode konačnih elemenata (MKE). Za tu svrhu strukturalni element tipa okruglog štapa opterećenog na istezanje je razmatran. Faktori intenziteta napona kod strukturalnog elementa tipa okruglog štapa sa površinskom prskotinom su sračunati koristeći trodimenzionu analizu konačnih elemenata sa singularnim 20-čvornim konačnim elementima. Dat je pregled rezultata za FIN određene primenom MKE. Sračunati su FIN za različite dimenzije površinskih prskotina. Ovi rezultati su korišćeni za određivanje analitičkog izraza za FIN kakav je neophodan za analizu širenja prskotine i procenu preostalog veka. Koristeći izvedeni analitički izraz za FIN na bazi MKE izvršena je analiza širenja prskotine.

Ključne reči: **mehanika loma, analiza napona, istezanje, štap, prskotina, rast prskotine, metoda konačnih elemenata.**

Определение коэффициента интенсивности напряжения для круглого стержня с эллиптической поверхностной трещиной и оценки остаточного ресурса

В работе исследуется поведение структурных элементов с поверхностными трещинами. Внимание направлено на работы по круговому стержню с эллиптической поверхностной трещиной при нагрузке стержня на растяжение. Основное внимание в этой работе направлено на анализ влияния формы и размера поверхностных трещин на параметры механики разрушения. Коэффициентов интенсивности напряжений (КИН) определяли с помощью метода конечных элементов (МКЭ). Для этой цели рассматриван структурный элемент типа круглого стержня нагруженного на разрыв. Коэффициенты интенсивности напряжений (КИН) у структурного элемента типа круглого стержня с поверхностными трещинами рассчитываются с использованием трёхмерного анализа методом конечных элементов (МКЭ) с особыми 20-узловыми конечными элементами. Обзор результатов стресс-факторов интенсивности напряжений определяется с использованием метода конечных элементов. Коэффициенты интенсивности напряжений рассчитаны на различные аспекты и размеры поверхностных трещин. Эти результаты были использованы для определения аналитического выражения для коэффициента интенсивности напряжений, каковы они необходимы чтобы проанализировать распространение трещин и оценки остаточного ресурса. При использовании производных аналитических выражений для коэффициентов интенсивности напряжений на основе метода конечных элементов был проведён анализ расширения трещин.

Kly~evwe slova: **механика разрушения, анализ напряжений, прочность на разрыв, стержень, растрескивание, рост трещины, метод конечных элементов.**

Le facteur de l'intensité de tension chez la barre circulaire à la fissure elliptique superficielle et l'estimation du reste de durée de vie

Les recherches sur le comportement des éléments structuraux aux fissures superficielles sont présentées dans ce papier. L'attention est portée sur la barre circulaire ayant une fissure elliptique superficielle sous la tension d'extension. Le but de ce travail est l'analyse de l'influence de la forme et de la taille de fissure superficielle quant aux paramètres de la mécanique de fracture. Les facteurs de l'intensité de la tension (FIT) sont déterminés par la méthode des éléments finis (MEF). A cet effet, l'élément structural en forme de la barre circulaire sous la tension d'extension, est considéré dans ce travail. Les facteurs d'intensité de tension chez l'élément cité sont calculés via l'analyse à trois dimensions de la méthode MEF aux éléments singuliers finis avec 20 nœuds. On a donné le tableau des résultats pour les facteurs d'intensité de la tension déterminés par utilisation de MEF. Ces facteurs ont été calculés pour les différentes dimensions des fissures superficielles. Ces résultats ont été utilisés pour déterminer la formule analytique du facteur d'intensité qui est nécessaire pour analyser la propagation de la fissure et pour l'estimation du reste de la durée de vie. On a réalisé l'analyse de la fissure au moyen de la formule de FIT basée sur MEF.

Mots clés: **mécanique de fracture, analyse de tension , extension, barre, fissure, propagation de la fissure, méthode de éléments finis.**