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Kinetic Impulses - Impacts On Rotating Rigid Rotor Bearings

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This paper is dedicated to the memory of Academician Valentin V. Rumyantsev (1921-2007) and contains a short review of his results in the area of stability of permanent rotations of a heavy rigid body, and the stability of rotation of a heavy rigid body with one fixed axis as well as the stability of gyroscopes.

Using the vector of the rotor's mass inertia moment, with respect to the rotation axis through the spherical bearing and the corresponding vector of the rotation rigid rotor mass deviational moment, the kinetic impulses- impacts to the rotor bearings are expressed. It is clearly visible that these two components are of opposite directions and that they constitute a deviational couple of kinetic impulses. The paper gives the graphical presentations of the rotator vector intensity as well as its angular velocity in a function of the angular coordinate

Key words: **vectorial analysis, vector calculus, impact loading, kinetic impulse, moment of inertia, rotor.**

Introduction

The year 2007 was dedicated to the **300th Anniversary** of the birth of Leonard Euler (1707-1783), *one of the leading scientists in the area of mathematics and mechanics of the 18th century.* The Euler equations for a rigid body rotation about a fixed point are very important, and represent applicable results for future new developments in dynamics. Three differential equations of Euler representing the rigid body motions about a fixed point are one of the many important contributions to mechanics by Leonard Euler (see Fig. 1.a*).

a) b**)** c**)**

Figure 1. a***** Leonard Euler (1707-1783)**;** b***** Aleksandr Mikhailovich Lyapunov (1857 – 1918); c***** Academician Valentin V. Rumyantsev (1921-2007)

In contrast to Newton's geometry-related procedure in the Principia, Euler formulated mechanical laws preferentially in terms of differential calculus. Euler claimed that "those laws of motion which a body observes when left to itself in continuing either rest or motion pertain properly to infinitely small bodies".

A scientific consideration of Leonard Euler and his variational principles of mechanics is the main content of [1] by Rumyantsev.

The stability of permanent rotations of a heavy rigid body and the stability of rotation of a heavy rigid body with one fixed point in the case of Kovalevskaya were investigated and presented in [2, 3] by Rumyantsev (see Fig.1. c*).

Lyapunov's theory of stability opened a new page in the history of global science. In particular, a contribution which should be mentioned here is that he worked as the editor of two volumes of ***Euler's collected works***. At that time he took part in the publication of Euler's selected works and was the editor of the 18th and 19th part of this miscellany (see Fig.1. b*).

To understand the place and the role of Rumyantsev in the development and augmentation of Lyapunov's scientific heritage, it is necessary to recall the development of the

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theory of stability in the 20th century. After Chetaev's death in 1959 Rumyantsev headed the Moscow Chetaev School. For almost half a century this School had incontestable authority not only in the USSR but also far abroad. The author met Rumyantsev during his participation in the Yugoslav Congress of Theoretical and Applied Mechanics at 1978 in Portorozh, and many times in Serbia during his visits to the Mathematical Institute SANU and also in the Shanghai during participation at the International Congress on Nonlinear Mechanics organized by academician Wei-Zang Chien, Rector of Shangai University. Rumyantsev was a member of the Serbian Academy of Sciences and Arts. Also, Rumyantsev was a supervisor of the Serbian yang researcher Vujicic's specialization in the area of Analytical Mechanics during his stay in Moscow. This collaboration was continued through exchange visits between these scientists during participation in scientific meetings.

The series of References [4-13] by Rumyantsev are devoted to the stability of steady motions of a gyroscope as well as to the stability of the rotational motions of a solid body with a liquid cavity. In [5] stability and control, with respect to the part of the coordinates of the phase vector of dynamical systems, are presented.

In [14] the non-linear dynamic response of a flexible rotor supported by ball bearings is studied by Sinou. The excitation is due to an unbalance inertia force. The non-linear unbalance responses and the associated orbits of the bearing rotor were investigated. The main intent of the paper [15] by Saruhan was to formulate, demonstrate, and validate a practical means of implementing an evolutionary optimization technique into a rotor-bearing system. The optimum design of a flexible rotor supported on three-lobe bearings was studied for optimal performance considering system stability along with other design criteria such as fluid film thickness, power loss, film temperature, and film pressure. This was achieved by using a genetic algorithm and the method of feasible directions. The results of Zou, Hua and Chen presented in reference [16] are in relation to a rigid coupling and a flexible connection made up of elastic coupling units, and are widely applied to a rotor-bearing system with a multi-branched shafting system. This paper proposes a modal synthesis method for the lateral vibration analysis of such a kind of rotor-bearing system. Cole, Keogh, Sahinkaya and Burrows in [17] considered a control system design for a rotor–magnetic bearing system that integrates a number of fault-tolerant control methods. The experimental results obtained from a flexible rotor system are used to demonstrate the effectiveness of the control implementations.

The objective of reference [18], by Bouzidane and Thoma, was to study the dynamic behavior of a rotor supported by a new hydrostatic journal bearing, and fed with a negative electro-rheological fluid. The bearing consists of four hydrostatic bearing flat pads, fed by capillary restrictors. The discussion of the results includes some thoughts on future trends. Reference [19] by Cavalca, Cavalcante and Okabe presents a methodology for analyzing the influence of the foundation, or supporting structure, on rotor-bearing systems. The mathematical procedure applies a modal approach using the modal parameters of generalised mass, damping ratio and natural frequencies. The Finite Elements Method is used to model the rotor. The linear model of the foundation is obtained by FEM and a modal approach is applied to reduce the number of degrees of freedom of the foundation model. The modal parameters of the foundation are estimated using frequency response functions and their respective Fourier Transforms, obtained experimentally.

In Reference [20], by Ganesan, information on the stability of vibratory motions becomes essential for ensuring safer designs for rotor-bearing systems, and obtained main criteria of operational safety were presented. In particular, the influence of shaft and bearing parameters on the stability characteristics has to be quantified for design and diagnostic purposes. Such an analytical investigation is the objective of the present paper. The expressions for amplitude and phase modulation functions that we derived quantify the effects of slowly varying the rotational speed on the motion characteristics. Based on the modulation functions, the stability regions in the parameter space are determined. The effects of bearing and shaft asymmetries on the stability of the rotor are illustrated.

A continuous model approach for cross-coupled bending vibrations of a rotor-bearing system with a transverse breathing crack was study by Chasalevris and Papadopoulos. Results of this model study was presented in their reference [21]. A local rubimpact fault diagnosis of rotor systems based on EMD was the research interest of Cheng, Yu, Tang and Yang, and the research results were presented in [22]. Numerical and experimental studies of a rotor–bearing–seal system were by Cheng, Meng and J Jing in [23].

From time to time it is useful to pay attention again to classical models of dynamics of mechanical systems and evaluate possibilities for new approaches to these classical results by using other than those methods usually used in the literature.

An interest in the study of vector and tensor methods with applications in ***Dynamics,*** especially in ***Kinetics*** of rigid and solid body rotational motions and deformation displacements as a new qualitative approach to the optimization of the time for universe teaching process, grew exponentially over the last few years. The short time for fundamental knowledge transfer during one term (semester) courses, with a high level of apparent teaching results, in the requirement for the optimization of the time, are focused to the new basic high level scientific ideas (logic and philosophical) which are easy to understand for most of students for engineering applications**.**

Also, we can conclude that the impact of different possibilities to establish the phenomenological analogy of different physical model dynamics expressed by vectors connected to the pole and the axis and the influence of such possibilities to applications allows professors, researchers and scientists to obtain lager views within their specialization fields.

This is the reason for introducing mass moment vectors to the rotor dynamics, and for expressions of the impact dynamics of a rotating heavy body.

A series of author's various published research results in the area of vector methods, with applications in Classical Mechanics is presented in the monograph [24, 25], and in a series of papers, and published scientific congresses communications [26-49]. The definitions of mass moment vectors coupled to the pole and the axis introduced the foundation for this vector method. The principal vector is $\vec{\mathfrak{J}}_0^{(\vec{n})}$ of the body mass inertia moment at the point $A = O$ for the axis oriented by the unit vector $\vec{n}$: $\vec{\mathfrak{J}}_A^{(\vec{n})} \overset{def}{=} \iiint\limits_V [\vec{\rho},[\vec{n},\vec{\rho}]]\,dm$, and there is a corresponding $\vec{\mathbf{D}}_A^{(\vec{n})}$

vector of the rotational rigid body mass deviational moment of the rotation axis through the spherical bearing A and for the rotation axis.

Also, there are two ***vector rotators***, pure kinematics vectors depending on angular velocity and angular acceleration of the rotational body. The vector rotators correspond to the rotation axis; one in the deviational plane through the axis, the other orthogonal to the deviational plane and both with intensity $\mathbf{R} = \sqrt{\dot{\omega}^2 + \omega^4}$. In the listed papers, many applications of the discovered vector method are presented by using mass moment vectors for expressing kinetic parameters of heavy rotors dynamics [48, 24- 26, 30-32, 47, 55, 56 and 57] as well as of coupled multi step rotors dynamics [37, 38, 41, 43, 44, 50] and for gyro rotors dynamics [38].

In paper [31] the motion of a heavy body around a stationary axis, with turbulent damping, is investigated and kinetic pressures on bearings are expressed by mass moment vectors for the pole in the stationary bearing and for the axis of the body rotation. The motion equations of a variable mass object rotating around a fixed axis are expressed by a mass moment vector for the pole, and the axis and presented in [39]. Analogies between models of stress state, strain state and state of the body mass inertia moments are considered and presented in a series of references [54-57]. Some considerations of the derivatives of mass moment vectors for a dimensional coordinate system N, as well as in a real body system, are presented in [24, 25, 26, 27]. A trigger for coupled singularities, on an example of coupled rotors with deviational material particles, is presented in [40, 41, 42]. Nonlinear phenomena in rotor dynamics were investigated in the series of references [40-47]. These nonlinear phenomena include phase portraits and the homoclinic orbits visualization of nonlinear dynamics of multiple step reductor/multipliers; nonlinear dynamics of planetary reductors with turbulent damping; *the* nonlinear dynamics of a heavy material particle along a circle which rotates and optimal control; and homoclinic orbit layering in the coupled rotor nonlinear dynamics and the chaotic clock models.

Basic introduction to the theory of impulse-impact dynamics

Assumptions in the theory of impulse-impact dynamics

As a beginning, we consider some basic definitions, notations and assumptions in the theory of impulse-impact between two bodies. We take into account that:

1. time τ is the time of the body contact during the impulse-impact, and this time is very short;
2. impulse-impact forces $\vec{F}^{ud}$ are changeable and of very large intensity, with order as $\frac{1}{\tau}$, and a very short period time of action (excitation) in the time over τ .
3. the rate of impulse (linear momentum) change and angular momentum (kinetic moment) change of the bodies in the impulse-impact duration are finite;
4. the impulses of ’’sample forces’’ (no impact forces) are very small , and it is possible to neglect these impulses.
5. during the impulse-impact duration of duration τ the rigid bodies do not change positions in space.

In 1668 the Royal Society in London invited scientists to submit their work in the area of dynamics of impacting bodies for a scientific competition. The competition was entered with two manuscripts of the solution of the problem of dynamics of impacting bodies by ***Wallis*** (1616-1703), *Mechanica sive de mote*-1688 and ***Huygens*** – *De motu corporum ex percusione.* By using their work and his own inventions, Isaak Newton generalized their ideas and founded the new theory of the impacts, introducing into rigid body dynamics the hypothesis of impulse-impact period of body deformation as defined by the coefficient of restitution of bodies during the period of their impact. Also, it is necessary to point out the fact that in the works of Galileo Galilei we can find the first results in impact dynamics. Nowadays, there are many applications in the engineering systems theory for impact dynamics, especially those applied to the construction of vibro-impact machines.

Impact forces. Instanteneous impulse (linear momentum). Impulse-Impact.

The differential of the impulse of a motion $d\vec{p}(t)$ of the material particle is

$$d\vec{p}(t) = \vec{F}(t)\,dt = d\vec{K}_F(t) \tag{1}$$

where $\vec{K}(t)$ is the impulse of force. After integration, we obtain the following expression for the impulse of the force:

$$\vec{K}_F(t) = \vec{p}(t) - \vec{p}(0) = \int_0^t \vec{F}(t)\,dt = m\vec{v}(t) - m\vec{v}(0) = m\vec{v} - m\vec{v}_0 \tag{2}$$

The rate of change of the linear momentum of motion change $\Delta\vec{p}(t)$ for the duration of a finite time interval Δt is equal to the impulse of the force $\vec{K}_F(t)$, when the force $\vec{F}(t)$ is applied to the material particle along its motion at the path and over time.

The dimension of intensity of the impulse of force $\vec{K}_F(t)$, when the force $\vec{F}(t)$ is applied to the material particle is:

$$\dim\left|\vec{K}_F(t)\right| = MLT^{-1} \tag{3}$$

where M is a mass dimension with a unit $[kg]$, L is a length dimension with a unit $[m]$ and T is the dimension of time with a unit $[\sec]$. We can conclude that the unit of the impulse of the force is $\left[\mathrm{kg\,m\,sec}^{-1}\right]$ or $[\mathrm{N\,sec}]$.

We can conclude that the impulse of the force $\vec{K}_F(t)$ is a vector integral and that, in the general case, it is not in the same direction as the force $\vec{F}(t)$.

When the intensity of the force $\vec{F}(t)$ is finite, then the impulse of the force $\vec{K}_F(t)$ for force action in a short time interval, when that time interval tends to equal zero, $\Delta t = t - t_0 = \tau \to 0$, tends to be zero, too.

For the case that the rate of velocity change $\Delta\vec{v} = \vec{v}(t) - \vec{v}(t_0) = \vec{v} - \vec{v}_0$ is finite in a very short time interval $\Delta t = t - t_0 = \tau \to 0$, then the impulse $\vec{K}_F(t)$ of the force $\vec{F}(t)$ must be of finite intensity, and the intensity of the force $\vec{F}(t)$ must be infinite in this short time interval of action.

Forces $\vec{F}_{ud}(t)$ with previously defined properties, whose actions appear in the short time interval, and with infinite intensity and the finite impulse $\vec{K}_{Fud}(t)$, are named ***instantaneous impact forces with finite impulse.***

Now, we can define impact as an action on the body. *Action* as a result of the applied ***instantaneous impact forces*** $\vec{F}_{ud}$ ***with the finite impulse*** $\vec{K}_{Fud}(t)$ during a short time interval $\Delta t = t - t_0 = \tau \to 0$ is defined as the ***impact.***

For the ***impulse*** $\vec{K}_F(t)$ of the ***instantaneous impact forces*** $\vec{F}_{ud}$, we can write the following expression

$$\begin{aligned} \vec{K}_{Fud}(t_0+\tau) &= \vec{p}(t_0+\tau) - \vec{p}(t_0) = \\ &= m\vec{v}(t_0+\tau) - m\vec{v}(t_0) = \int_{t_0}^{t_0+\tau} \vec{F}_{ud}\,dt \end{aligned} \tag{4}$$

where $\vec{v}(t_0+\tau)$ and $\vec{v}(t_0)$ are the velocities before and after impact.

Mass moment vectors for the pole and the axis – a new vector view of classical mechanics

Vectors of the body mass moments for the pole and the axis

The monograph [24] and monograph paper [25] contain definitions of three kinetic vectors fixed to a certain point and an axis passing through the given space body point as a reference pole.

The definitions of these vectors for the pole and the axis are:

1. Vector $\vec{\boldsymbol{M}}_0^{(\vec{n})}$ of the body mass for the axis, oriented by the unit vector $\vec{n}$, through the point – pole O, in the form:

$$\vec{\boldsymbol{M}}_0^{(\vec{n})} \overset{def}{=} \iiint_V \vec{n}\,dm = \mathrm{M}\vec{n}\ , \quad dm = \sigma dV\ ; \tag{5}$$

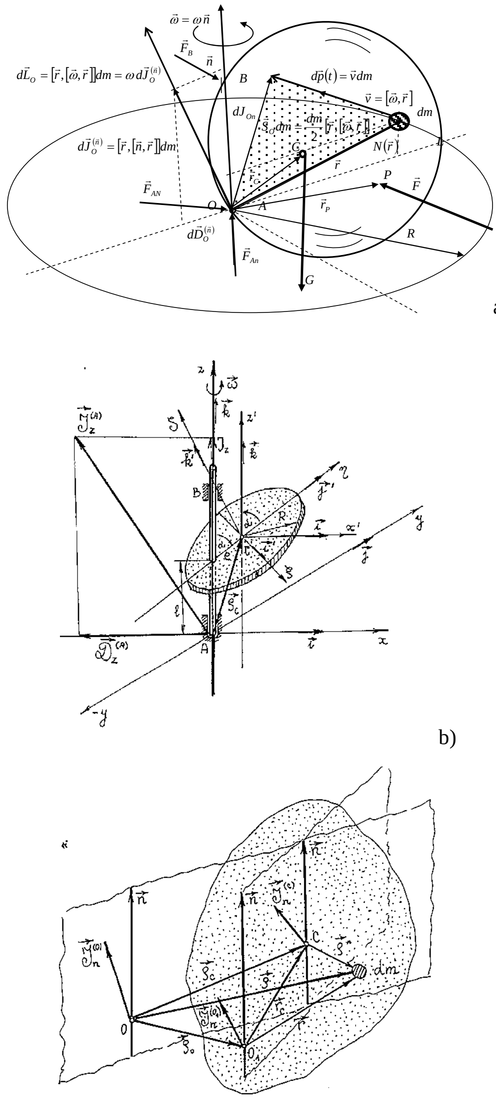
2. Vector $\vec{\mathbf{S}}_0^{(\vec{n})}$ of the body mass static (linear) moment for the axis, oriented by the unit vector $\vec{n}$, through the point – pole O, in the form:

$$\vec{\mathbf{S}}_0^{(\vec{n})} \overset{def}{=} \iiint_V [\vec{n},\vec{\rho}]\,dm\ , \quad dm = \sigma dV\ ; \tag{6}$$

3. Vector $\vec{\mathbf{J}}_0^{(\vec{n})}$ of he body mass inertia moment for the axis, oriented by the unit vector $\vec{n}$, through the point – pole O, in the form (see Fig.2.a* and b*):

$$\vec{\mathbf{J}}_0^{(\vec{n})} \overset{def}{=} \iiint_V [\vec{\rho},[\vec{n},\vec{\rho}]]\,dm \tag{7}$$

where $\vec{\rho}$ is the position vector of the elementary body mass dm with respect to the common pole O. For special cases, the details can be seen in [24-30]. In the previously cited references, the spherical and deviational parts of the inertia mass moment vector and the inertia tensor are analysed. Monograph [24] gives the knowledge about the change (rate) in time and the derivatives of the vectors of the body mass linear moment, the body mass inertia moment for the pole and a corresponding axis for different properties of the body, on the basis of the results of the author's works [30, 39].



a)

b)

c)

Figure 2. a* Graphical presentation of the kinetic reactions to the rotor (rigid body) shaft bearings on the elementary mass particle dm of the rotating rigid body
b* A disc with respect to the axis of the shaft in an eccentrically skewed position, and the graphical presentation of the vector of the disc mass inertia moment for the reference point and an oriented axis of the shaft and of the corresponding deviational plane.
c* A graphical presentation of the theorem of material body mass inertia moment vectors for the two parallel axes through two reference points.

The "*supports*" vectors of the body mass linear moments, as well as of the body mass inertia moments for the pole O, and the axis oriented by the unit vector $\vec{n}$ are introduced by definition and expression. Detailed information on this can be found in [25, 30]. The "*support*" vector $\vec{\mathbf{S}}_0^{(\vec{n})}$ of the body mass linear moment, and the "*support*" vector $\vec{\mathbf{R}}_0^{(\vec{n})}$ of the body mass inertia moment of the body point $N : \overrightarrow{ON} = \vec{\rho}$, for the pole at the point O, and for the axis oriented by the unit vector $\vec{n}$, are defined by the following expressions:

$$\vec{\mathbf{S}}_0^{(\vec{n})} \overset{def}{=} \frac{\partial \vec{\mathbf{S}}_0^{(\vec{n})}}{\partial m} = [\vec{n}, \vec{\rho}], \quad \vec{\mathbf{N}}_0^{(\vec{n})} \overset{def}{=} \frac{\partial \vec{\mathbf{J}}_0^{(\vec{n})}}{\partial m} = [\vec{\rho}, [\vec{n}, \vec{\rho}]] \qquad (8)$$

The derivatives of these defined support vectors of the mass moment vectors for the pole and the axis are determined for different body properties, with respect to time. Detailed information on this can be found in [25,30].

This expression (see Fig.2. c*)

$$\vec{\mathbf{J}}_0^{(\vec{n})} = \vec{\mathbf{J}}_{0_1}^{(\vec{n})} + \left[\vec{\rho}_0, \vec{\mathbf{S}}_{0_1}^{(\vec{n})}\right] + \left[\vec{M}_C^{(0_1)}, [\vec{n}, \vec{\rho}_0]\right] + [\vec{\rho}_0, [\vec{n}, \vec{\rho}_0]]M \quad (9)$$

is **the vector form of the theorem** for the relation of material body mass inertia moment vectors, $\vec{\mathbf{J}}_0^{(\vec{n})}$ and $\vec{\mathbf{J}}_{0_1}^{(\vec{n})}$, for two parallel axes through two corresponding points, pole O and pole O_1. We can see that all the members in the last expression have the same structure. These structures are: $[\vec{\rho}_0, [\vec{n}, \vec{r}_C]]M$, $[\vec{r}_C, [\vec{n}, \vec{\rho}_0]]M$ and $[\vec{\rho}_0, [\vec{n}, \vec{\rho}_0]]M$.

In the case when the pole O_1 is the centre C of the body mass, the vector $\vec{r}_C$ (the position vector of the mass centre with respect to the pole O_1) is equal to zero, whereas the vector $\vec{\rho}_O$ turns into $\vec{\rho}_C$ so that the last expression (9) can be written in the following form:

$$\vec{\mathbf{J}}_0^{(\vec{n})} = \vec{\mathbf{J}}_C^{(\vec{n})} + [\vec{\rho}_C, [\vec{n}, \vec{\rho}_C]]M \qquad (10)$$

This expression (10) represents the mathematical and vector form of the theorem of the rate change of the mass inertia moment vector for the pole and the axis, when the axis is translated from the pole at the mass centre C to the arbitrary point, pole O.

The *Huygens-Steiner theorems* (see Refs. [25] and [30]) for the body mass axial inertia moment, as well as for the mass deviational moments, emerged *from this theorem* (10) on the change of the vector $\vec{\mathbf{J}}_0^{(\vec{n})}$ of the body mass inertia moment at the point O for the axis oriented by the unit vector $\vec{n}$ passing trough the mass center C and when the axis is moved translate to the other point O.

Linear and angular momentum of rigid body rotation

The classical literature gives a very well known definition for the rigid body linear momentum (motion quantity) and angular momentum (motion quantity moment). We shall consider it by means of the body mass moment vectors. We are following the classical definition, so that we write the following expression for the linear momentum in the vector form:

$$\vec{\mathrm{K}} = \iiint_V \vec{v}_N \, dm = \iiint_V (\vec{v}_A + [\vec{\omega}, \vec{\rho}])\, dm = \mathrm{M}\, \vec{v}_A + \omega \vec{\mathrm{S}}_A^{(\vec{n})}. \quad (11)$$

The expression (11) for the linear momentum $\vec{\mathrm{K}}$ of a rigid body, whose points have translational velocity $\vec{v}_A$ of the reference point A, and relative velocity $[\vec{\omega}, \vec{\rho}]$ due to the rotation around the axis oriented by the vector $\vec{\omega} = \omega\vec{n}$ through the point A, has two parts: 1. a translational component equal to the product of the translation velocity $\vec{v}_A$ of the reference point A and the body mass, this being the linear momentum due to the translatory motion with the velocity of the reference point A; and the rotational component equal to the product of the magnitude ω of the angular velocity $\vec{\omega} = \omega\vec{n}$ and the vector $\vec{\mathbf{S}}_{\vec{n}}^{(A)}$ of ***the body mass linear moment*** at the reference point A, and for the axis oriented by the unit vector $\vec{n}$.

The second kinetic vector connected to the reference point which plays an important part in the rigid body dynamics is the rigid body angular momentum for the given pole, $\vec{\mathbf{L}}_0$. Following the classic definition according to [50], the rigid body angular momentum, in the vector form, is calculated by means of the following expression:

$$\vec{\mathbf{L}}_0 = \iiint_V [\vec{r}, \vec{v}_N]\, dm = \iiint_V [\vec{r}_A + \vec{\rho}, \vec{v}_A + [\vec{\omega}, \vec{\rho}]]\, dm \quad .(12)$$

The angular momentum for the point A, $\vec{\mathbf{L}}_A$, is connected not only to the pole A but to the axis oriented by the momentary angular velocity vector $\vec{\omega}$, depending on the chosen reference point A to which we connect the vectors $\vec{\mathbf{S}}_A^{(\vec{n})}$ and $\vec{\mathbf{J}}_A^{(\vec{n})}$ of the rigid body mass linear and inertia moments by connecting the body mass to the translatory velocity $\vec{v}_A$ of the reference point A. Therefore, we write the following:

$$\vec{\mathbf{L}}_A = \left[\vec{\mathbf{S}}_{\vec{n}}^{(A)}, \vec{v}_A\right] + \vec{\omega}\left(\vec{n}, \vec{\mathbf{J}}_A^{(\vec{n})}\right) - \omega \vec{\mathbf{D}}_A^{(\vec{n})}. \qquad (13)$$

Fig.2 a* shows the rigid body with the fixed rotation axis through the fixed point A around which it rotates with the angular velocity $\vec{\omega}$ which changes in time so that the angular acceleration $\dot{\vec{\omega}}$ appears. The kinetic energy is expressed as $2\mathrm{E}_k = \omega\left(\vec{\omega}, \vec{\mathrm{J}}_A^{(\vec{n})}\right) = \omega^2\, \mathrm{J}_A^{(\vec{n})}$. For this case of a rigid body rotation around a fixed axis, the linear momentum and angular momentum are given n the following forms:

$$\vec{\mathbf{K}} = [\vec{\omega}, \vec{\rho}_C]M = \omega \vec{\mathbf{S}}_A^{(\vec{n})}, \qquad (14)$$

$$\vec{\mathbf{L}}_A = \vec{\omega}\left(\vec{n}, \vec{\mathbf{J}}_A^{(\vec{n})}\right) + \omega\left[\vec{n}\left[\vec{\mathbf{J}}_A^{(\vec{n})}, \vec{n}\right]\right] = \vec{\omega}\left(\vec{n}, \vec{\mathbf{J}}_A^{(\vec{n})}\right) + \omega \vec{\mathbf{D}}_A^{(\vec{n})} \quad .(15)$$

Since the velocity $\vec{v}$ and the acceleration $\vec{a}$ of the body elementary mass at the point N are: $\vec{v} = [\vec{\omega}, \vec{\rho}]$ and $\vec{a} = [\dot{\vec{\omega}}, \vec{\rho}] + [\vec{\omega}, [\vec{\omega}, \vec{\rho}]]$ (see [24] and [25]), then the main vector $\vec{\mathrm{F}}_{\mathrm{rj}}$ of the inertia force of the overall rigid body rotating around the axis with the angular velocity $\vec{\omega}$ is:

$$\vec{\mathrm{F}}_{\mathrm{rj}} = -\iiint_V \vec{a}\, dm = -\dot{\omega} \vec{\mathbf{S}}_A^{(\vec{n})} - \omega\left[\vec{\omega}, \vec{\mathbf{S}}_A^{(\vec{n})}\right]. \qquad (16)$$

For the main reduction moment of the inertia forces of the overall rigid body rotating around the axis, and for the point A, we calculate the following:

$$\vec{\mathbf{M}}_{\mathrm{Aj}} = \iiint_V \left[\vec{\rho}, d\, \vec{\mathrm{F}}_{\mathrm{rj}}\right] = -\dot{\omega} \vec{\mathbf{J}}_A^{(\vec{n})} - \omega\left[\vec{\omega}, \vec{\mathbf{J}}_A^{(\vec{n})}\right], \quad (17)$$

The vector equations of the body rotation around the fixed axis thought a point A can be obtained by using theorems of changing linear and angular momentum with respect to time. By differentiating expression (14) for the linear momentum and expression (15) for angular momentum with respect to time, we obtain:

$$1.\ \frac{d\vec{\mathbf{K}}}{dt} = \dot{\omega}\vec{\mathbf{S}}_{\vec{n}}^{(A)} + \omega\left[\vec{\omega}, \vec{\mathbf{S}}_{\vec{n}}^{(A)}\right] = -\vec{F}_{rj} = \vec{F}_{r},$$

$$\frac{d\vec{\mathbf{K}}}{dt} = \left|\vec{\mathbf{S}}_{\vec{n}}^{(A)}\right|\left(\dot{\omega}\vec{u}_1 + \omega^2\vec{v}_1\right) = \vec{\mathbf{R}}_1\left|\vec{\mathbf{S}}_{\vec{n}}^{(A)}\right| = \mathbf{R}\left|\vec{\mathbf{S}}_{\vec{n}}^{(A)}\right|\vec{r}_1, \quad (18)$$

where

$$\vec{\mathbf{R}}_1 = \mathbf{R}\,\vec{r}_1, \quad \mathbf{R} = \sqrt{\dot{\omega}^2 + \omega^4}. \quad (19)$$

The vector rotator $\vec{\mathbf{R}}_1 = \mathbf{R}\,\vec{r}_1$ for the rigid body rotation axis and the pole A, is orthogonal to the rotation axis with one component orthogonal to the deviational plane through the body rotation axis and other in the deviational plane (for detail see [24]).

Equation (18) for the changing linear momentum with respect to time is equal to the main resultant vector of the active and reactive forces and shows that the change of the linear momentum of motion is the vector normal to the rotation axis and has two components: one due to the angular velocity change, which is normal to the rotation axis and the deviational plane, and contains the body mass centre and the rotation axis, and the other component which depends on the angular velocity squared, which is normal to the rotation axis and lies in the plane formed by the rotation axis and the rigid body mass centre undergoing rotation.

$$2.\ \frac{d\vec{\mathbf{L}}_A}{dt} = \dot{\omega}\vec{\mathbf{J}}_A^{(\vec{n})} + \omega\left[\vec{\omega}, \vec{\mathbf{J}}_A^{(\vec{n})}\right] = -\vec{\mathbf{M}}_{Aj} = \vec{\mathbf{M}}_A,$$

$$\frac{d\vec{\mathbf{L}}_A}{dt} = \dot{\vec{\omega}}J_A^{(\vec{n})} + \dot{\omega}\vec{\mathbf{D}}_A^{(\vec{n})} + \omega\left[\vec{\omega}, \vec{\mathbf{D}}_A^{(\vec{n})}\right] = \dot{\vec{\omega}}J_A^{(\vec{n})} + \left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|\vec{\mathbf{R}}, \quad (20)$$

$$\vec{\mathbf{R}} = \mathbf{R}\vec{r}, \quad \mathbf{R} = \sqrt{\dot{\omega}^2 + \omega^4}. \quad (21)$$

The vector rotator is $\vec{\mathbf{R}} = \mathbf{R}\vec{r}$, for the rigid body rotation axis and the pole A. This is rotating and also increasing by the angular velocity and by the angular acceleration, and, at the same, it causes the inertia force deviational moment to increase. The vector rotator $\vec{\mathbf{R}} = \mathbf{R}\vec{r}$ and the vector rotator $\vec{\mathbf{R}} = \mathbf{R}\vec{r}_1$ are orthogonal to one another and rotate with the same velocity

$$\dot{\gamma} = \omega + \dot{\vartheta} = \omega\frac{\omega^4 + \ddot{\omega}\omega - \dot{\omega}^2}{\omega^4 + \dot{\omega}^2} \quad (22)$$

This expression (20) immediately shows that:

a) the first component depending on the angular acceleration $\dot{\vec{\omega}}$ is co-linear with the rotation axis;

b) the second component, which also depends on the angular acceleration intensity $\dot{\omega}$, is normal to the rotation axis, and is in the direction of the deviational part of the vector $\vec{\mathbf{J}}_{\vec{n}}^{(A)}$ of the rigid body mass inertia moment at the pole in the spherical (stationary, fixed) bearing A. For the rotation axis it is proportional to the magnitude of the angular acceleration $\dot{\vec{\omega}}$ and the vector $\vec{\mathbf{D}}_{\vec{n}}^{(A)}$ of the rotation rigid body mass deviational moment of the rotation axis at the spherical bearing A.

c) the third component is proportional to the square of the angular velocity ω^2 and to the magnitude of the vector $\vec{\mathbf{D}}_{\vec{n}}^{(A)}$ of the rotation rigid body mass deviation moment of the rotation axis at the spherical bearing A and for the rotation axis. It is like a vector normal to the rotation axis and the vector $\vec{\mathbf{D}}_{\vec{n}}^{(A)}$ of the deviation mass load to the rotation axis, which means it is normal to the deviation plane.

For the case of a rigid body rotation around a fixed axis, under the action of active forces $\vec{F}_k$, $k = 1, 2, 3, ..., N$, with the vector positions $\vec{\rho}_k$, $k = 1, 2, 3, ..., N$ at the points of application on the rigid body, we can write the following two-vector equations of the rigid body's dynamic equilibrium:

$$\frac{d\vec{\mathbf{K}}}{dt} = \left|\vec{\mathbf{S}}_A^{(\vec{n})}\right|\left(\dot{\omega}\vec{u}_1 + \omega^2\vec{v}_1\right) = \vec{\mathbf{R}}_1\left|\vec{\mathbf{S}}_A^{(\vec{n})}\right| = \mathbf{R}\left|\vec{\mathbf{S}}_A^{(\vec{n})}\right|\vec{r}_1 = = \sum_{k=1}^{k=N}\vec{F}_k + \vec{F}_A + \vec{F}_B, \quad (23)$$

$$\frac{d\vec{\mathbf{L}}_A}{dt} = \dot{\vec{\omega}}J_A^{(\vec{n})} + \dot{\omega}\vec{\mathbf{D}}_A^{(\vec{n})} + \omega\left[\vec{\omega}, \vec{\mathbf{D}}_A^{(\vec{n})}\right] = \dot{\vec{\omega}}J_A^{(\vec{n})} + \left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|\vec{\mathbf{R}} = = \sum_{k=1}^{k=N}\left[\vec{\rho}_k, \vec{F}_k\right] + \left[\vec{\rho}_B, \vec{F}_B\right]. \quad (24)$$

where $\vec{F}_{AN}$ and $\vec{F}_B$ are the reaction of the rotating rigid body bearings and $\vec{r}_B$ vector position bearing B with relation to the hinged bearing A.

These two-vector equations (23)-(24) are the kinetic equations of dynamic equilibrium for the rotation of the body around a stationary axis under the action of an active force system $\vec{F}_k$ (for more information see [24] and [25]). For the examples of the applications of the previously introduced mass moments vectors and the linear and angular momentum expressed by these vectors, see the following [26-45, 48-49].

The orthogonal part of the derivative of the rotating rigid body angular momentum, with respect to time, for a certain pole in the spherical bearing A on the rotation axis, to the rotation axis is:

$$\frac{d\vec{\mathbf{L}}_A^d}{dt} = \dot{\omega}\vec{\mathbf{D}}_A^{(\vec{n})} + \omega\left[\vec{\omega}, \vec{\mathbf{D}}_A^{(\vec{n})}\right] = \left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|\vec{\mathbf{R}}. \quad (25)$$

We can write the following relations:

$$\frac{\left|\vec{F}_{rj}\right|}{\left|\vec{\mathbf{M}}_A^{(dev)}\right|} = \frac{\left|\frac{d\vec{\mathbf{K}}}{dt}\right|}{\left|\frac{d\vec{\mathbf{L}}_A^d}{dt}\right|} = \frac{\left|\vec{\mathbf{S}}_A^{(\vec{n})}\right|}{\left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|} = const. \quad (26)$$

Kinetic pressures on shaft bearings excited by the body rotation

For the kinetic reactions $\vec{F}_{AN(kin)}$ and $\vec{F}_{B(kin)}$, of the rotor bearings (or pressures - $\vec{F}_{AN(kin)}$ and - $\vec{F}_{B(kin)}$ on the rotor bearings) it is possible to write the following simple expressions (for details see [24], [51-53]):

$$\vec{F}_{AN(kin)} = \left|\vec{\mathbf{S}}_0^{(\vec{n})}\right| \vec{\mathbf{R}}_1 - \frac{1}{r_B}\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right| \vec{\mathbf{R}}_2 \qquad (27)$$

$$\vec{F}_{B(kon)} = \frac{1}{r_B}\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right| \vec{\mathbf{R}}_2 \qquad (28)$$

where $\vec{\mathbf{R}}_2 = \left[\vec{n}, \vec{\mathbf{R}}\right]$.

From the previous expression for kinetic reactions, $\vec{F}_{AN(kin)}$ and $\vec{F}_{B(kin)}$, of the rotor bearings (or pressures - $\vec{F}_{AN(kin)}$ and - $\vec{F}_{B(kin)}$ on the rotor bearings) in the rotor bearings, it is clearly visible that there are two parallel and opposite direction components $\pm\frac{1}{r_B}\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right|\vec{\mathbf{R}}_2 = \left|\vec{\mathbf{D}}_0^{(\vec{n})}\right|\left[\vec{n}, \vec{\mathbf{R}}\right]$ of the distance r_B which constitute **a deviational couple of kinetic reaction** $\vec{F}_{AN(kin)}$ and $\vec{F}_{B(kin)}$ of the rotor bearings (or pressures - $\vec{F}_{AN(kin)}$ and - $\vec{F}_{B(kin)}$ on the rotor bearings) and with the intensity $\left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|\left|\vec{\mathbf{R}}\right|$, but in the vector form $\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right|\vec{\mathbf{R}}$.

Kinematics vector rotators

From expressions (27) and (28), we can conclude that the kinetic reactions, $\vec{F}_{AN(kin)}$ and $\vec{F}_{B(kin)}$, of the rotor bearings (or pressures - $\vec{F}_{AN(kin)}$ and - $\vec{F}_{B(kin)}$ on the rotor bearings) and their action to the rotor shaft bearings are proportional to the intensity of the vector rotator. Also, we can see that kinetic reactions are larger for larger values of angular velocities and angular accelerations of the shaft rotation. It is possible to conclude that greater than kinetic reactions (or pressures) on the rotor shaft bearings which have low angular velocities appear for the case of the larger deviation mass distribution properties of the rotor.

Then, it is easy to analyze the intensity of the vector rotator using expression (21). Also using expression (22), we can analyze the angular velocity and the angular acceleration of the vector rotator, and also angular velocity and angular acceleration of the direction of the kinetic reaction of the shaft bearing (or pressures on the shaft bearings) for different angular velocities of the rotor.

For that reason, and for the first example, see Fig.3 for the rigid body rotation about a fixed axis with a constant angular acceleration $\dot{\omega} = \dot{\omega}_0 = const$, and the angular velocity $\omega = \dot{\omega}_0 t + \omega_0$, and with the angle of rotation $\phi = \frac{\dot{\omega}_0}{2}t^2 + \omega_0 t$, the expression for the relative angular velocity of the Rotator's rotation about the axis of the rotor shaft is in the form:

$$\dot{\vartheta} = -\frac{2\dot{\omega}_0^2\left(\dot{\omega}_0 t + \omega_0\right)}{\left(\dot{\omega}_0 t + \omega_0\right)^4 + \dot{\omega}_0^2}. \qquad (29)$$

The final form of the expression for the relative angular acceleration of the vector rotator for the case of the rigid body rotation with a constant angular acceleration is given by:

$$\ddot{\vartheta} = \frac{\left(6\dot{\omega}_0^3\omega^4 - 2\dot{\omega}_0^5\right)}{\left(\omega^4 + \dot{\omega}_0^2\right)^2} = \frac{2\dot{\omega}_0^3\left[3\left(\dot{\omega}_0 t + \omega_0\right)^4 - 2\dot{\omega}_0^2\right]}{\left[\left(\dot{\omega}_0 t + \omega_0\right)^4 + \dot{\omega}_0^2\right]^2} \qquad (30)$$

In Fig.3. a graphical representation of the angular velocity $\omega = -\dot{\omega}_0 t + \omega_0$ of a rigid rotor rotation for the case where the angular acceleration of the rigid rotor is constant, is presented, as well as the corresponding graphs of the relative angle $\vartheta(t)$, the angular velocity $\dot{\vartheta}(t)$, and the angular acceleration $\ddot{\vartheta}(t)$ of the rotator vector $\vec{\mathbf{R}}$ rotation about the rotor shaft axis, and the corresponding intensity $\mathbf{R}$ of this vector rotator.

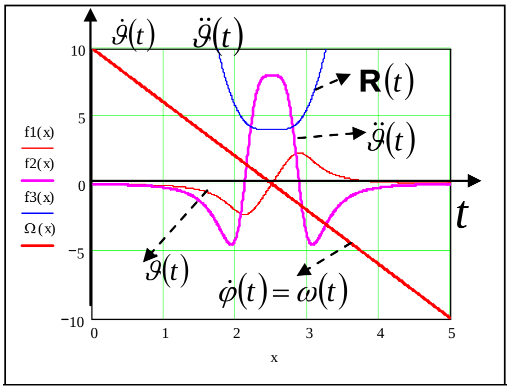


Figure 3. Graphical representation of the angular velocity $\omega = -\dot{\omega}_0 t + \omega_0$ of a rigid rotor, for the case that the angular acceleration of the rigid rotor is constant, and the corresponding graphs of the relative angle $\vartheta(t)$, angular velocity $\dot{\vartheta}(t)$, and angular acceleration $\ddot{\vartheta}(t)$ of the rotator vector $\vec{\mathbf{R}}$ rotation about the rotor shaft axis, and the corresponding intensity $\mathbf{R}$ of this vector rotator.

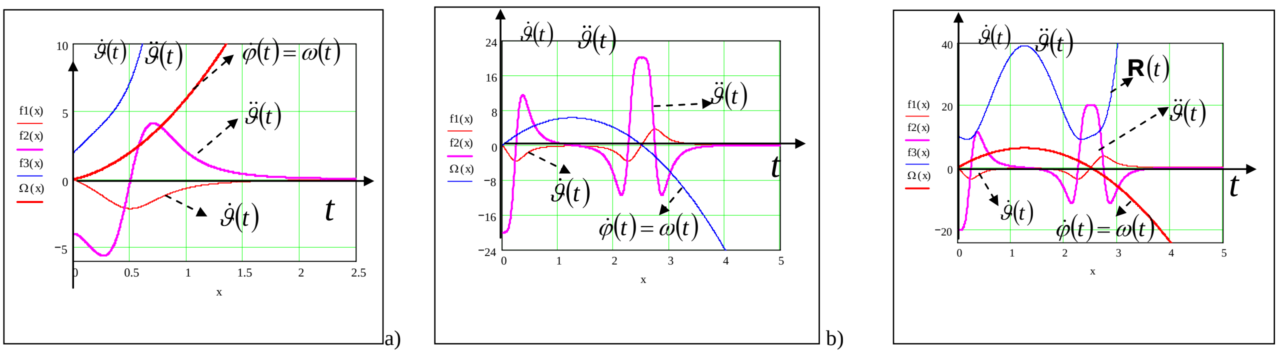


Figure 4. Graphical representation of the angular velocity $\omega = \frac{\ddot{\omega}_0}{2}t^2 + \dot{\omega}_0 t + \omega_0$ of the rigid rotor, for the case where the angular acceleration second kind (jerk) of the rigid rotor rotation is constant, and corresponding graphs of the relative angle $\vartheta(t)$, angular velocity $\dot{\vartheta}(t)$ and angular acceleration $\ddot{\vartheta}(t)$ of the rotator vector $\vec{\mathbf{R}}$ rotation about the rotor shaft axis and the corresponding intensity $\mathbf{R}$ of this vector rotator. a* , b* and c* for different initial conditions.

For the second example, see Fig.4, we take into consideration that the rigid body rotation about a fixed axis with a constant angular acceleration second order (jerk), $\ddot{\omega} = \ddot{\omega}_0 = const$ and the angular acceleration is $\dot{\omega} = \ddot{\omega}_0 t + \dot{\omega}_0$, the angular velocity is $\omega = \frac{\ddot{\omega}_0}{2}t^2 + \dot{\omega}_0 t + \omega_0$ and the angle of rotor rotation is $\phi = \frac{\dddot{\omega}_0}{6}t^3 + \frac{\dot{\omega}_0}{2}t^2 + \omega_0 t$.

Fig.4. gives a graphical representation of the angular velocity $\omega = \frac{\ddot{\omega}_0}{2}t^2 + \dot{\omega}_0 t + \omega_0$ of the rigid rotor rotation for the case when the angular acceleration second kind (jerk) of the rigid rotor rotation is constant as well as the corresponding graphs of the relative angle $\vartheta(t)$, the angular velocity $\dot{\vartheta}(t)$, the angular acceleration $\ddot{\vartheta}(t)$ of the rotator vector $\vec{\mathbf{R}}$ rotation about the rotor shaft axis, and the corresponding intensity $\mathbf{R}$ of this vector rotator a*, b* and c* for different initial conditions.

Kinetic impulses - impacts on shaft bearings excited by impact forces due to the rigid body rotation

Now, let us consider a rigid body with two bearings, one stationary hinged at the point A at the pole O ($A \equiv O$), and the other one cylindrical at the point B, with the possibility to rotate about the axis through the points A and B, oriented by unit vector $\vec{n}$. It is loaded by the ***instantaneous impulse-impact forces*** $\vec{F}_{ud}$ ***with the finite impulse*** $\vec{K}_{Fud}(t)$ during the short time interval $\Delta t = t - t_0 = \tau \to 0$, and also by the active forces $\vec{F}$ and $\vec{G}$, with finite intensity and with zero impulse. Then we have the case when the rigid body is under an impact (see Fig.5).

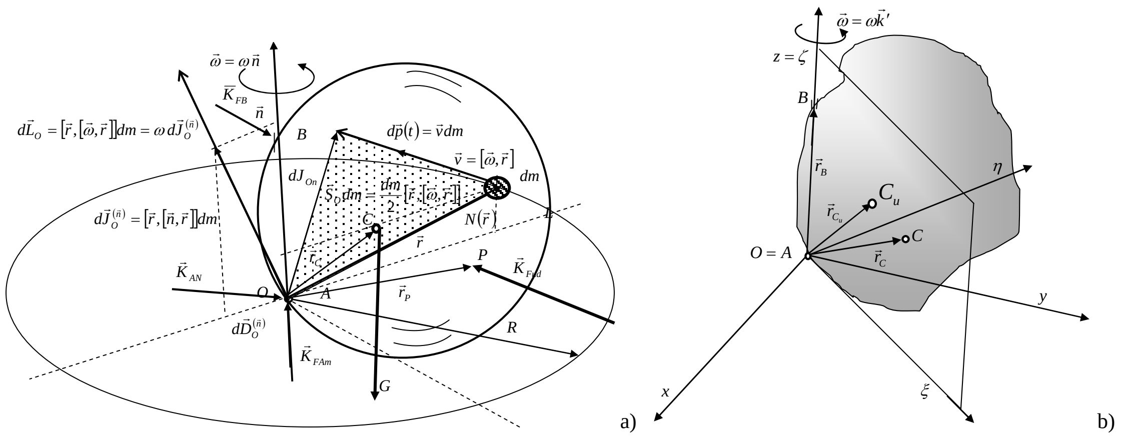


Figure 5. a***** Graphical presentation of the impact reactions to the rotor (rigid body) shaft bearings on the elementary mass particle dm of the rotating rigid body b***** Center of impact of the rigid body rotating around the fixed axis

Taking into account that the previous rigid body is in a dynamical state with the angular velocity ω and the angular acceleration $\dot{\omega}$ (type rotation about a fixed axis oriented by the unit vector $\vec{n}$), then using the considerations of the previous paragraph, on the basis of the expressions for the derivatives of the linear momentum (23) and angular momentum (24), we can write two vector equations in differential forms (for the elementary change – differential rate of the impulse of motion in time as well as for the elementary change - differential rate of the moment of impulse of the motion) with respect to time:

$$d\vec{p}(t) = \left|\vec{\mathbf{S}}_A^{(\vec{n})}\right| \vec{\mathbf{R}}_1 dt = \vec{F}_{AN}dt + \vec{F}_{An}dt + \vec{F}_B dt + \\ +\vec{F}_{ud}dt + \vec{G}dt + \vec{F}dt \tag{31}$$

$$d\vec{L}_A = \dot{\omega}\left(\vec{\mathbf{J}}_A^{(\vec{n})}, \vec{n}\right)\vec{n}dt + \left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|\vec{\mathbf{R}}dt = \\ = \left[\vec{r}_P, \vec{F}_{ud}\right]dt + \left[\vec{r}_B, \vec{F}_B\right]dt + \left[\vec{\rho}_C, \vec{G}\right]dt + \left[\vec{\rho}_N, \vec{F}\right]dt \tag{32}$$

After the integration of the previous vector equations in a differential form for the short time interval $\Delta t = t - t_0 = \tau \to 0$, the duration of the impact under the ***instantaneous impulses-impact forces*** $\vec{F}_{ud}$ ***with the finite impulse*** $\vec{K}_{Fud}(t)$, we obtain the following two vector expressions:

$$\Delta\vec{p}(t) = \vec{p}(t) - \vec{p}(t_0) + \\ = \left|\vec{\mathbf{S}}_A^{(\vec{n})}\right| \int_{t_0}^{t} \vec{\mathbf{R}}_1 dt = \int_{t_0}^{t} \vec{F}_{AN}dt + \int_{t_0}^{t} \vec{F}_{An}dt + \int_{t_0}^{t} \vec{F}_B dt + \\ + \int_{t_0}^{t} \vec{F}_{ud}dt + \int_{t_0}^{t} \vec{G}dt + \int_{t_0}^{t} \vec{F}dt \tag{33}$$

$$\Delta\vec{L}_A = \vec{L}_A(t) - \vec{L}_A(t_0) = \\ = \int_{t_0}^{t} \dot{\omega}\left(\vec{\mathbf{J}}_A^{(\vec{n})}, \vec{n}\right)\vec{n}dt + \left|\vec{\mathbf{D}}_A^{(\vec{n})}\right| \int_{t_0}^{t} \vec{\mathbf{R}}dt = \int_{t_0}^{t} \left[\vec{r}_P, \vec{F}_{ud}\right]dt + \\ + \int_{t_0}^{t} \left[\vec{r}_B, \vec{F}_B\right]dt + \int_{t_0}^{t} \left[\vec{\rho}_C, \vec{G}\right]dt \int_{t_0}^{t} \left[\vec{\rho}_N, \vec{F}\right]dt \tag{34}$$

and after taking into account the active forces $\vec{F}$ and $\vec{G}$, with finite intensity and zero impulse, as well as assumptions of the theory of impact dynamics listed from paragraph II.1*, we can write the following two vector equations (of the impulses of impact dynamics state equilibrium) of the rotating rigid body in the following form:

$$\Delta\vec{p}(t)=\vec{p}(t)-\vec{p}(t_0)=\left|\vec{\mathbf{S}}_A^{(\vec{n})}\right|\int_{t_0}^{t}\vec{\mathbf{R}}_1 dt= \\ =\vec{K}_{FAN}+\vec{K}_{FAn}+\vec{K}_{FB}+\vec{K}_{Fud} \tag{35}$$

$$\Delta\vec{L}_A=\vec{L}_A(t)-\vec{L}_A(t_0)=(\omega-\omega_0)\left(\vec{\mathbf{J}}_A^{(\vec{n})},\vec{n}\right)\vec{n}+ \\ +\left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|\int_{t_0}^{t}\vec{\mathbf{R}}dt\approx\left[\vec{r}_P,\vec{K}_{Fud}\right]+\left[\vec{r}_B,\vec{K}_{FB}\right] \tag{36}$$

wher $\vec{\mathbf{R}}=\dot{\omega}\vec{u}+\omega^2\vec{w}=\mathbf{R}\vec{\mathbf{R}}_0$ as well aso $\vec{\mathbf{R}}_1=\dot{\omega}\vec{u}_1+\omega^2\vec{w}_1=\mathbf{R}\vec{\mathbf{R}}_{01}$, and also:

$$\int_{t_0}^{t}\vec{\mathbf{R}}dt=\int_{t_0}^{t}\left[\dot{\omega}\vec{u}+\omega^2\vec{w}\right]dt$$

and

$$\int_{t_0}^{t}\vec{\mathbf{R}}_1 dt=\int_{t_0}^{t}\left[\dot{\omega}\vec{u}_1+\omega^2\vec{w}_1\right]dt \tag{37}$$

First, applying scalar multiplication of both previous equations (35) and (36) by the unit vector $\vec{n}$, and second, the applying vector multiplication with respect to the right hand side of both the previous equations (35) and (36) by unit vector $\vec{n}$, the four equations are obtained. Obtained system of the four obtained equations contain two in the scalar form and two in the vector form. The obtained equations are in the function along the unknown reactive impulses $\vec{K}_{FAN}$, $\vec{K}_{FAn}$ and $\vec{K}_{FB}$ of the impact forces to the rotate body bearings. We can obtain the following expressions of reactive impulses-impact forces on the rotating body bearings $\vec{K}_{FAN}$ and $\vec{K}_{FAn}$ as well as $\vec{K}_{FB}$ in the following forms:

$$\vec{K}_{FAN}=\left|\vec{\mathbf{S}}_A^{(\vec{n})}\right|\int_{t_0}^{t_0+\tau}\vec{\mathbf{R}}_1 dt-\left[\vec{n},\left[\vec{K}_{Fud},\vec{n}\right]\right]- \\ -\frac{1}{r_B}\left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|\int_{t_0}^{t_0+\tau}\vec{\mathbf{R}}_2 dt+\frac{1}{r_B}\left[\left[\vec{r}_P,\vec{K}_{Fud}\right],\vec{n}\right] \tag{38}$$

$$\vec{K}_{FB}=\frac{1}{r_B}\left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|\int_{t_0}^{t_0+\tau}\vec{\mathbf{R}}_2 dt-\frac{1}{r_B}\left[\left[\vec{r}_P,\vec{K}_{Fud}\right],\vec{n}\right] \tag{39}$$

$$\vec{K}_{FAn}=-\left(\vec{n},\vec{K}_{Fud}\right)\vec{n} \tag{40}$$

for the time duration of the interval $\Delta t=t-t_0=\tau\to 0$ and the difference equation of the impulse rotation of the rigid body in the impact state in the form:

$$(\omega-\omega_0)\left(\vec{\mathbf{J}}_0^{(\vec{n})},\vec{n}\right)=\left(\vec{n},\left[\vec{r}_P,\vec{K}_{Fud}\right]\right) \tag{41}$$

From the expressions for the reactive impulses of the impulse-impact forces to the rotating body bearings $\vec{K}_{FAN}$ and $\vec{K}_{FB}$, we can separate the two parts depending on the kinetic parameters of the rigid body dynamics in the following forms:

$$\vec{K}_{FAN\,DEV}=-\frac{1}{r_B}\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right|\int_{t_0}^{t}\vec{\mathbf{R}}_2 dt \tag{42}$$

$$\vec{K}_{FB\,DEV}=\frac{1}{r_B}\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right|\int_{t_0}^{t}\vec{\mathbf{R}}_2 dt=-\vec{K}_{FAN\,DEV} \tag{43}$$

They are parallel in the direction $\int_{t_0}^{t}\vec{\mathbf{R}}_2 dt$, of the same intensity and in the opposite direction. These two deviational components of the impulse of the impact forces on the rotating body bearings generate a deviational couple with the intensity $\mathbf{M}^{dev}_{\vec{K}(\vec{F}_A,\vec{F}_B)}=\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right|\left|\int_{t_0}^{t}\vec{\mathbf{R}}dt\right|$. This couple depends only on the rigid body mass distribution around the axis of rotation, and the angular velocity and the angular acceleration during the time interval $\Delta t=t-t_0=\tau\to 0$, in accordance with difference equation (38).

This is the deviational couple impact on the rotating body bearings, and in engineering practice with a very dangerous effect, which can destroy the shaft bearings.

Also, from the expressions for reactive impulses – reactive impact forces of the rotating body bearings $\vec{K}_{FAN}$ and $\vec{K}_{F_B}$,(or kinetic impacts - $\vec{K}_{FAN}$ and - $\vec{K}_{FAn}$ on the rotating body bearings), we can separate two parts depending on the kinetic parameters of the rigid body dynamics and on the impact force in the following form:

$$\vec{K}_{FB\,DEV}=\frac{1}{r_B}\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right|\int_{t_0}^{t}\vec{\mathbf{R}}_2 dt-\frac{1}{r_B}\left[\left[\vec{r}_P,\vec{K}_{Fud}\right],\vec{n}\right]= \\ =-\vec{K}_{FAN\,DEV} \tag{44}$$

There are two parallel components, of the same intensity and in the opposite direction. These two deviational components of the reactive impulses of impact forces on the rotate body bearings also build a deviational couple with the intensity

$$\mathbf{M}^{dev}_{\vec{K}(\vec{F}_A,\vec{F}_B),1}=\left|\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right|\int_{t_0}^{t}\vec{\mathbf{R}}_2 dt-\left[\left[\vec{r}_P,\vec{K}_{Fud}\right],\vec{n}\right]\right| \tag{45}$$

Centre of impulse-impact of the rigid body rotating around a fixed axis

Let's consider the problem of finding the position of the point C_u of the application of the ***instantaneous impulse-impact forces*** $\vec{F}_{ud}$ ***with the finite impulse*** $\vec{K}_{Fud}(t)$ under which an impact on the rigid body rotation, reactive impact impulses on the bearing $\vec{K}_{FAN}$ and $\vec{K}_{FAn}$ as well as $\vec{K}_{FB}$ are all equal to zero. This point C_u is the centre of the impulse-impact of the rotating rigid body around a fixed axis. Let us denote the vector position of the centre of impact C_u, with $\vec{r}_{C_u}=\xi_{C_u}\vec{u}+\eta_{C_u}\vec{w}+\zeta_{C_u}\vec{n}$, where ξ_{C_u} ,

η_{C_u} and ζ_{C_u} are their coordinates in a moving coordinate system with the basic unit vectors $\vec{u}$, $\vec{w}$ and $\vec{n}$.

$$\vec{K}_{FAN} + \vec{K}_{FB} = \left|\vec{\mathbf{S}}_A^{(\vec{n})}\right| \int_{t_0}^{t_0+\tau} \vec{\mathbf{R}}_1 dt - \left[\vec{n}, \left[\vec{K}_{Fud}, \vec{n}\right]\right] = 0 \quad (46)$$

$$\vec{K}_{FB} = \frac{1}{r_B}\left|\vec{\mathbf{D}}_A^{(\vec{n})}\right| \int_{t_0}^{t_0+\tau} \vec{\mathbf{R}}_2 dt - \frac{1}{r_B}\left[\left[\vec{r}_{C_u}, \vec{K}_{Fud}\right], \vec{n}\right] = 0 \quad (47)$$

$$\vec{K}_{FAn} = -\left(\vec{n}, \vec{K}_{Fud}\right)\vec{n} = 0 \quad (48)$$

$$(\omega - \omega_0)\left(\vec{\mathbf{J}}_0^{(\vec{n})}, \vec{n}\right) = \left(\vec{n}, \left[\vec{r}_{C_u}, \vec{K}_{Fud}\right]\right) \quad (49)$$

From the previous condition or equation (46), we can conclude that the reactive ***impulse*** $\vec{K}_{Fud}(t)$ of ***instantaneous impact forces*** $\vec{F}_{ud}$ must be orthogonal to the rotation axis, and must lie in the plane orthogonal to the rotation axis, $\vec{K}_{F_{ud}} \perp \vec{n}$. Then we obtain:

$$\vec{K}_{Fud}(t) = K_{Fud,\xi}(t)\vec{u} + K_{Fud,\eta}(t)w\vec{w} \quad (50)$$

and

$$K_{Fud,\zeta}(t) = 0 \quad (50^*)$$

From two first equations (47) and (48) it is possible to eliminate $\vec{K}_{Fud}(t)$ by taking into account that $\vec{K}_{F_{ud}} \perp \vec{n}$, and also equation (49), and to obtain an expression for the $\vec{r}_{C_u}$-vector position of the centre of impact. From equation (49), we obtain:

$$\omega(t_0+\tau) = \omega_0(t_0) + \frac{\left(\vec{n}, \left[\vec{r}_{C_u}, \vec{K}_{Fud}\right]\right)}{\left(\vec{\mathbf{J}}_0^{(\vec{n})}, \vec{n}\right)} \quad (51)$$

Also we can take into account that:

$$\int_{t_0}^{t_0+\tau} \omega^2 dt = 0,$$

$$\int_{t_0}^{t_0+\tau} \dot{\omega} dt = \omega - \omega_0 = \frac{\left(\vec{n}, \left[\vec{r}_{C_u}, \vec{K}_{Fud}\right]\right)}{\left(\vec{\mathbf{J}}_0^{(\vec{n})}, \vec{n}\right)} \neq 0 \quad (52)$$

As we know that $\vec{u} = \vec{i}\cos\phi + \vec{j}\sin\phi$ and $\vec{v} = -\vec{i}\sin\phi + \vec{j}\cos\phi$ for the general case it is necessary to find the following integrals: $\int_{t_0}^{t_0+\tau} \vec{\mathbf{R}}_2 dt$ and $\int_{t_0}^{t_0+\tau} \vec{\mathbf{R}}_1 dt$, but then by obtaining the vector position $\vec{r}_{C_u} = \xi_{C_u}\vec{u} + \eta_{C_u}\vec{w} + \zeta_{C_u}\vec{n}$ of the centre of impact it is possible to eliminate these integrals.

For the simplest expressions in the calculation, without loss of generality, we can take into account that the centre C of the rotating rigid body mass in the form of a thin rigid plate lies in a plane defined by $\vec{n}, \vec{u}$, and that is $\eta_C = 0$, and for the vector position $\vec{r}_C = \xi_C\vec{u} + \zeta_C\vec{n}$. Then, we obtain:

$$\int_{t_0}^{t_0+\tau} \omega^2 dt = \frac{K_{Fud,\xi}}{M\xi_C} = 0,$$

$$\int_{t_0}^{t_0+\tau} \dot{\omega} dt = \frac{K_{Fud,\eta}}{M\xi_C} = \omega - \omega_0 \neq 0$$

$$\left|\vec{\mathbf{S}}_0^{(\vec{n})}\right| = M\xi_C, \quad (53)$$

$$\left|\vec{\mathbf{D}}_0^{(\vec{n})}\right| = D_{0n\xi} = -J_{0n\xi}$$

$$K_{Fud,\xi} = 0, \; K_{Fid,\eta} \neq 0$$

$$K_{Fud,\zeta} = 0$$

and

$$(\omega - \omega_0) J_{0\zeta}^{(\vec{n})} = \left(\xi_{C_u} K_{Fud,\eta} - \eta_{C_u} K_{Fud,\xi}\right) \quad (54)$$

The coordinates of the vector position $\vec{r}_{C_u} = \xi_{C_u}\vec{u} + \eta_{C_u}\vec{w} + \zeta_{C_u}\vec{n}$ of the centre of body's impact are:

$$\eta_{C_u} = 0,$$

$$\xi_{C_u} = \frac{\left(\vec{\mathbf{J}}_0^{(\vec{n})}, \vec{n}\right)}{\left|\vec{\mathbf{S}}_A^{(\vec{n})}\right|} = \frac{J_{0\zeta}^{(\vec{n})}}{M\xi_C}, \quad (55)$$

$$\zeta_{C_u} = \frac{\left|\vec{\mathbf{D}}_A^{(\vec{n})}\right|}{\left|\vec{\mathbf{S}}_A^{(\vec{n})}\right|} = \frac{D_{0n\xi}}{M\xi_C} = -\frac{J_{0n\xi}}{M\xi_C}, \quad (56)$$

On the basis of the obtained expressions (55)-(56), we can conclude that the vector position $\vec{r}_{C_u} = \eta_{C_u}\vec{w} + \zeta_{C_u}\vec{n}$ of the centre of body's impact of the rotating rigid plate is in the same plane as the centre C of the rotating rigid body mass – plane plate, and that the ***instantaneous impulse-impact forces*** $\vec{F}_{ud}$ ***with the finite impulse*** $\vec{K}_{Fud}(t)$ are orthogonal to the rotating rigid plate and the axis of rotation.

Concluding remarks

The paper presents a new approach to the classical knowledge in dynamics for a rotating rigid body, as well as for the dynamics of impact on a rotating rigid body around a fixed axis by using mass moment vectors and vector rotators coupled for the pole and the axis thought this pole in a fixed-hinged shaft bearing. By using mass moment vectors, simpler and shorter expressions for the kinetic parameters of the rotating rigid body are obtained as well as kinetic pressures. Kinetic impacts on the rotor shaft bearings have much simpler forms than the scalar expressions published in numerous university books and monographs. The expressions of the deviational couple caused by pair of the kinetic pressures and the deviational impact couple of the kinetic impacts on the rotor shaft bearings are simply visible through "one short look".

This new view on classical mechanics of rotors can be a new approach in the university teaching process for the simplest transfer of classical knowledge of a rotating rigid body, and with very visible kinetic parameter properties, without additional explanation, or only with a short explanation, especially when speaking about the deviational phenomena of rotating body dynamics and coupled rotors.

This view on vectors can be connected with the view on vectors of the theory of elasticity (see [21l 43, 44, 45, 46-50]) by using an analogy between models of the stress state and strain state in the theory of elasticity and the vector model of the mass moment state around an arbitrary pole (and transfer knowledge from one area of science to other). The knowledge about the main stress and strain directions and main stresses and main strains (see [55], [56] and [58]) as well as about the corresponding expressions is possible on an analogy for applying expressions for the main inertia directions and main mass inertia moments. Also by using knowledge of the extreme values of shear stresses in planes with an angle of 45 degrees to the main stress direction in an analogy shows that the directions of the mass inertia moment asymmetry are with an angle of 45 degrees to the main mass inertia directions, and that the corresponding axial mass inertia moments are equal to the middle of the sum of the two corresponding main axial mass inertia moments. For the corresponding pair of these axis deviational components of the mass inertia moment for the pole, and for the axis in the direction of the mass inertia asymmetry, it is found to be equal to half of the difference between two corresponding main mass axial inertia moments.

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Kinetički impulsi – udari na ležišta rotora

Rad je posvećen sećanju na akademika Valentina V. Rumyantseva (1921-2007) i sadrži kratak prikaz njegovih rezultati o stabilnosti permanentnih rotacija teških tela, kao i o stabilnosti rotacija teških krutih tela sa fiksiranim osama, a takodje i o stabilnosti žiroskopa.
Koristeći vektor momenta inercije mase rotora za osu rotacije kroz sferno ležište i odgovarajući devijacioni vektor momenta mase rotora, izrazi za kinetičke impulse - udari na ležišta rotora su određeni. Iz dobijenih izraza je lako videti da su dve devijacione komponente kinetičkih impulsa na ležišta suprotnog smera i da obrazuju jedan devijacioni udarni spreg. Prikazana je serija grafika intenziteta vektora rotatora, kao i odgovarajućih ugaonih brzina, kojima se rotatori obrću, u funkciji ugaone koordinate.

Ključne reči: **vektorska analiza, vektorski račun, udarno opterećenje, udarni impuls, moment inercije,rotor.**

Кинетические импульсы - удары на подшипники ротора

Эта работа посвящена памяти академика Валентина В. Румянцева (1921-2007 гг) и содержит короткий обзор его результатов о устойчивости неизменных вращений твёрдых тел, а в том числе и о устойчивости вращений твёрдых жёстких тел с неподвижными осями, а также и о устойчивости гироскопов.
Пользуясь вектором момента инерции массы ротора в роли оси вращения через сферический подшипник и соответствующий вектор отклонения момента массы ротора, определены выражения для кинетических импульсов – удары на подшипники ротора. Из полученных выражений легко заметить, что пара составных частей отклонения кинетических импульсов удары на противоположные подшипники и что образуют одну ударную смычку отклонения. Здесь графически показана серия интенсивности вектора вращающего устройства, а в том числе и соответствующих угловых скоростей, которыми вращаются вращающие устройства в роли и функции угловых координат.

Ключевые слова: **векторный анализ, векторный счёт, ударная нагрузка, ударный импульс, момент инерции, ротор.**

Les impulses cinétiques – impacts sur le palier du rotor

Ce travail est dédié à la mémoire de l'académicien Valentin V.Rumyantsev (1921-2007) et comprend un court tableau des ses résultats dans le domaine de la stabilité des rotations permanentes des corps lourds, la stabilité des rotations des corps lourds rigides aux axes fixes et la stabilité des gyroscopes. En utilisant le vecteur du moment inertiel de la masse du rotor pour l'axe de la rotation à travers le palier sphérique et le vecteur de déviation correspondant du moment de la masse du rotor les impulses cinétiques – impacts sur les paliers de rotor sont déterminés. Il est facile de constater, à partir des expressions obtenues, que les deux composantes de déviations des impulses cinétiques sur les palier ont le sens opposé et qu'ils forment un couple impact de déviation. On a présenté une série de graphiques d'intensité du vecteur de rotor ainsi que les vitesses d'angle correspondantes dont les rotors tournent en fonction de la coordonnée de l'angle.

Mots clés: **analyse vectorielle, calcul vectoriel, charge d'impact, impact d'impulse, moment d'inertie, rotor.**