UDK: 504.055:571.9, 534.1.01
COSATI: 12-01, 20-11

Energy Analysis of Vibro-Impact System Dynamics Based on a Heavy Mass Particle Free Oscillations Along Curvilinear Rough Trajectories

Srđan Jović[1)]
Vladimir Raičević[1)]

This work is based on the motion analysis of vibro-impact systems moving freely along non-ideal lines-rough curvilinear paths in a vertical plane in the shapes of parabola, cycloid and circle. The non-ideal character of the relation is due to the Coulomb's type friction force with the coefficient $\mu = tg\alpha_0$. The oscillator is composed by one heavy mass particle (the observed systems have one degree of freedom of motion) whose free motion was limited by one or two elongation fixed limiters. The analytical-numerical results for certain kinetic parameters of the observed vibro-impact systems are a basis for the visualization of the motion analysis and energy analysis, which are the subject of this analytical research. In this paper the methodology of the energy transfer investigation among the elements of the observed vibro-impact system is presented.

Key words: **vibro-impact system, vibro-impact dynamics, impact effect, free oscillations, process dynamics, dynamic analysis, rough surface, singular point, kinetic energy, potential energy.**

Introduction

VIBRO-IMPACT processes in the dynamics of the systems and properties and specifications of non-linear phenomena with discontinuous conditions were investigated by many researchers all around the world. Based on the previous knowledge about the theory of vibro-impact systems and on the original works by: František Peterka [41-43], Katica (Stevanović) Hedrih [9-17], Alz Nayfeh et al [36, 37], Dimentberg M.F [7], Foole S., Bishop S. [8], Luo G.W., Xie J.H. [30], Hinrichs N., Oestreich M., Popp K. [28], Nordmark A.B. [38], Pavlovskaia E., Wiercigroch M. [39, 40], and others, it can be concluded that there is a great interest today for the investigation of energy transfers within complex systems and non-linear modes. That is the reason for the importance of the energy analysis of the dynamics of vibro-impact processes in vibro-impact systems with one or more degrees of freedom as well as non-ideal relations.

The problems of dynamics of vibro-impact systems represent a separate area of the applied theory of oscillations. The theory of vibro-impact systems is specially important for engineering practice for a wide application of vibro-impact actions used for realization of technology processes. The collisions occuring in the procedure of kinematic couples oscillation motions cause increased dynamic impact loadings, decreasing durability and liability of the system, as well as alternating dissipative system features. The studies about vibo-impact systems and vibro-impact actions are essential because of some very harmful impacts to the gaps and wearing in kinetic systems. The investigation of such vibro-impact effects is important for achieving expected motion regimes and system stability, i.e. for the regulation of the system motion.

The introducing theory for this paper was taken from the books of D. Rašković [45], where the motion of mechanic systems in ideal conditions and without limitations was analyzed, as well as the motion of a curvilinear oscillator in the presence of sliding Coulomb's friction, then from papers [14, 15] by Katica (Stevanović) Hedrih referring to the movement of heavy mass particles along rough curvilinear paths. In order to perform the analysis of the dynamics of a vibro-impact system with curvilinear paths and non-ideal relations, an explanation of free oscillations of heavy mass particles along curvilinear paths and non-ideal links must be done first.

Free mass particle oscillations along curvilinear paths and non-ideal constraints

A vibro-impact system represents a dynamic system with the oscillating motion in the periods with impacts occurences. In order to study the motion and analyze the energy of a corresponding vibro-impact system, the non-impact motion i.e. the motion between the impacts must be analyzed first. A non-impact motion is described by differentail (double) motion equations and by double phase trajectories equations, free oscillations of heavy mass particles along curvilinear rough lines and vertical planes and with non-ideal links, as well as the particular examples of motions along the rough parabolic line, rough cycloid line and rough circle line, based on the results of Professor dr Katica (Stevanović) Hedrih [13-14].

1) Faculty of Technical Sciences, Knjaza Miloša 7, 38220 Kosovska Mitrovica, SERBIA

Free motion of a heavy mass particle along a rough curbilinear line

Let us study free motions (oscillations) of a heavy mass particle M, mass m, along a rough curvilinear line with a sliding Coulomb's friction force and the coefficient type μ (Fig.1).

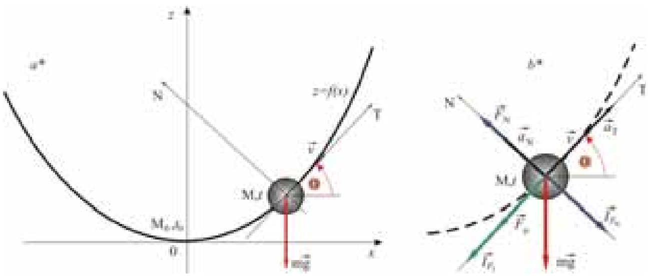


Figure 1. Free motion of a heavy mass particle along a curvilinear path: a* derived position of a heavy mass particle; b* force plan

The curvilinear line equation situated in the vertical plane Oxz is written in a form $z = f(x)$. By using the principle of dynamic equilibrium, with acting forces presented in Fig.1 b*, the vector's equation of motion of heavy mass particles along curvilinear paths can be written in a form :

$$\left(-m\ddot{s}\vec{T}\right)+\left(-m\frac{v^2}{R_k}\vec{N}\right)+mg\left(-\sin\alpha\vec{T}-\cos\alpha\vec{N}\right)+F_N\vec{N}-\mu\left|\vec{F}_N\right|\frac{\vec{v}}{|\vec{v}|}=0$$

After scalar multiplication of this vector equation with the unit vectors $\vec{T}$ and $\vec{N}$ and by completing the obtained scalar vectors equations, one differential (double) equation of motion of heavy mass particles is obtained as a function of the curvilinear (arc) coordinate s, $\left(ds = dx\sqrt{1+z'^2}\right)$, in a form:

$$\ddot{s} + g\sin\alpha \pm \mu\left(\frac{v^2}{R_k} + g\cos\alpha\right) = 0 \qquad (1)$$

By solving the differential (double) equation of motion (1) we get a (double) phase trajectory equation [14]:

$$\dot{x}^2(x) = e^{-\int \frac{2}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2} \pm \mu \frac{z''}{\sqrt{1+z'^2}}\right]dx} \cdot \left[-2g\int \frac{1}{\left(1+z'^2\right)}\left(z' \pm \mu\right) e^{\int \frac{2}{\sqrt{1+z'^2}}\left[\frac{d}{dx}\sqrt{1+z'^2} \pm \mu \frac{z''}{\sqrt{1+z'^2}}\right]dx} dx + C\right] \qquad (2)$$

Eq. (1) and (2) are the basis for the formation of diffrential (double) motion equations and (double) phase trajectory equations for any shape of curvilinear lines. In this paper, rough parabolic, cycloid and circle lines are considered on the source paper [14].

Free motion of heavy mass particles along parabolic rough lines is presented in Fig.2

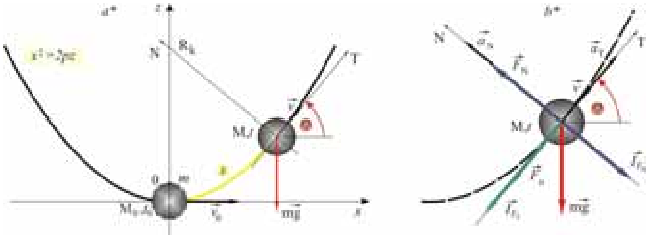


Figure 2. a* Initial and derived position of a heavy mass particle; b* force plan

Based on the theory of the motion of heavy mass particles along rough curvilinear lines in general, the analysis of this motion represents a special case.

The general equation of the parabola has a form $x^2 = 2pz$, where $2p[m]$ - is the parabola parameter equal to the quadratic distance of the focus from the top of the parabola. The observed system has one degree of freedom of motion. For a generalized coordinate, we take the parameter φ (the angle between the tangent direction and the direction paralel to the axis Ox). Based on equations (1) and (2) and the general parabola equation, using many mathematic operations, the differential (double) equation of motion and the phase trajectory of the heavy mass particle equation along the parabolic rough line (as a function of generalized coordinate φ) were obtained.

$$\ddot{\phi} + (3tg\phi \pm \mu)\dot{\phi}^2 + \frac{g\cos^3\phi}{p}(\sin\phi \pm \mu\cos\phi) = 0, \begin{cases} za\ v>0 \\ za\ v<0 \end{cases}, \qquad (3)$$

$$\dot{\phi}^2 = \cos^6\phi\left(-\frac{g}{p\cos^2\phi} + Ce^{\mp 2\mu\phi}\right) \begin{cases} za\ v>0 \\ za\ v<0 \end{cases} \qquad (4)$$

Where C - integration constant depending of initial motion conditions (valid also in equations for cycloid and circle lines). The integral constant has some alternation depending on a period of motion of a heavy mass particle along a parabolic rough line limited by points on the phase trajectory where the velocity is equal to zero. That alteration is related to the alternation of the direction of the heavy mass particle motion, i.e. the alternation of velocity direction of a heavy mass particle causing the alternation of the friction force.

Free motion of a heavy mass particle along a cycloid rough line is presented in Fig.3

Based on the theory derived for the motion of heavy mass particles along the curvilinear rough lines in general, the analysis of this motion represents a special case.

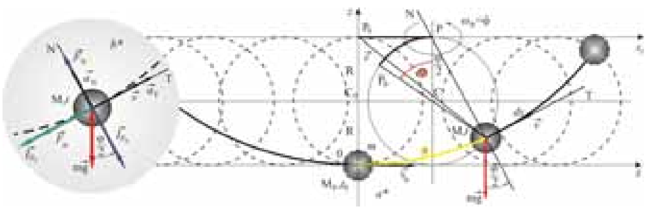


Figure 3. a* Initial and derived position of a heavy mass particle; b* force plan

The observed system has one degree of freedom of motion. For a generalized coordinate we adopt the parameter ϕ (the angle between the direction $M\overline{P}_0$ and the vertical line) which defines the position of the heavy mass particle M positioned on the circle of the radius R moving equally along the axis Ox. The heavy mass particle in this case describes the line representing a geometrically clear cycloid path. Based on equations (1) and (2) together with the parameters of the cycloide equation $x = R(\phi + \sin\phi)$ and $z = R(1-\cos\phi)$ and using the chain of mathematical operations, we obtain a differential (double) equation of motion a and (double) equation of the phase trajectory of a heavy mass particle along a cycloid rough line (as a function of the generalized coordinate ϕ).

$$\ddot{\phi}-\dot{\phi}^2\left(\frac{1}{2}tg\frac{\phi}{2}\mp\mu\right)+\left(tg\frac{\phi}{2}\pm\mu\right)\frac{g}{2R}=0, \quad \begin{cases} za\ v=2R\cos\frac{\phi}{2}\dot{\phi}>0 \\ za\ v=2R\cos\frac{\phi}{2}\dot{\phi}<0 \end{cases} \tag{5}$$

$$\dot{\phi}^2=-\frac{\left(\frac{g}{2R}\right)}{1+4\mu^2}\frac{1}{\cos^2\frac{\phi}{2}}\left[(\pm3\mu)\sin\phi-\left(1-2\mu^2\right)\cos\phi+\frac{1+4\mu^2}{2}+Ce^{\mp2\mu\phi}\right]$$

$$\begin{cases} za\ v=2R\cos\frac{\phi}{2}\dot{\phi}>0 \\ za\ v=2R\cos\frac{\phi}{2}\dot{\phi}<0 \end{cases}$$

where C - is the integration constant.

Free motion of a heavy mass particle along a circle rough line is presented in Fig.4

The angle φ represents the generalized coordinate of the observed non-conservative mechanical system with one degree of freedom of motion.

By using the coordinate system of the references with the axes in the direction of perpendicular and tangent, and based on the procedure conducted for the motion of heavy mass particles along rough curvilinear paths (1), the differential (double) equation of motion of a heavy mass particle along the circle rough line can be written in a form of:

$$\ddot{\phi}\pm\dot{\phi}^2 tg\alpha_0+\frac{g}{R\cos\alpha_0}\sin\left(\phi\pm\alpha_0\right)=0 \quad \begin{cases} za\ \dot{\phi}>0 \\ za\ \dot{\phi}<0 \end{cases}, \tag{7}$$

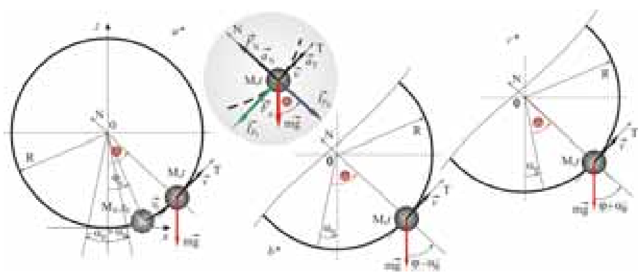


Figure 4. a*The initial and derived position of a heavy mass particle, force plan; b* and c* Presentation of the 'relative' equilibrium positions with the alternation properties $\pm\alpha_0$

By solving equation (7) we get a (double) phase trajectory equation of a heavy mass particle moving along a circle rough line

$$\dot{\phi}(\phi)^2=\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\cdot\left[\cos\left(\phi\pm\alpha_0\right)-2tg\alpha_0\sin\left(\phi\pm\alpha_0\right)\right]+Ce^{\mp2\phi tg\alpha_0} \tag{8}$$

$$\begin{cases} za\ \dot{\phi}>0 \\ za\ \dot{\phi}<0 \end{cases}$$

where C - is the integrating constant.

Vibro-impact system based on a heavy mass particle free oscillations along curvilinear paths and non-ideal constraints

The dynamics of vibro-impact systems based on oscillators with free motion along non-ideal links-rough curvilinear lines, in a shape of parabola and a circle was analyzed by the application of the analytical method of 'adjustment' and the phase plane method. One or two heavy mass particles–pellets moving freely along the rough curvilinear route with a sliding Coulomb's type friction force were also used as a part of the oscillator. The system becomes a vibro-impact system when one or two elongation limiters for each of them are positioned and taken as mobile and stable limiters.

Vibro-impact system based on heavy mass particle free oscillations along a parabolic rough line

A heavy mass particle moving along a parabolic rough line in the vertical plane, with a sliding Coulomb's type friction force coefficient $\mu=tg\alpha_0$, with one elongation limiter on the right and one elongation limiter on the left side (Fig.5).

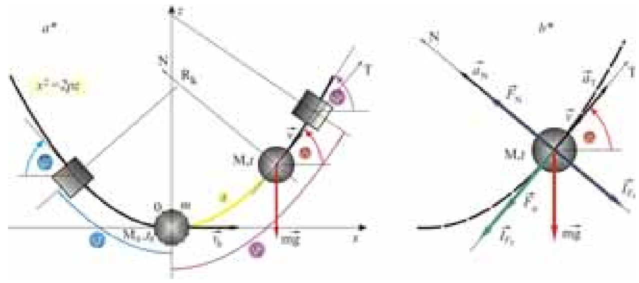


Figure 5. The system with two stabile elongation limiters based on the oscillator with one pellet: a* initial and derived position of the pellet; b* force plan

The positions of the limiters are determined by the arc coordinates $s_{ul,1}=s_1\left(\phi_1\right)$ and $s_{ul,2}=s_2\left(\phi_2\right)$ and are measured from the equilibrium position of the heavy mass particle. The arc (curvilinear) coordinates are given as a function of the angle φ.

For the complete description of the dynamics of heavy mass particles, the differential (double) equation of motion (3) is coupled to:

a* initial conditions

$$s_{(0)}\left(\phi_{(0)}\right)=s_0\left(\phi_0\right)$$

and

$$v_{(0)}\left(\phi_{(0)},\dot{\phi}_{(0)}\right)=\dot{s}_{(0)}\left(\phi_{(0)},\dot{\phi}_{(0)}\right)=v_0\left(\phi_0,\dot{\phi}_0\right);$$

b* angular elongation limitation conditions, and impact conditions

$$s_{ul,i}=s_i\left(\phi_i\right),\ s_{ul,(i+1)}=s_{(i+1)}\left(\phi_{(i+1)}\right),$$

$$\dot{s}_{odl,i}\left(\dot{\phi}_{odl,i}\right)=-k\dot{s}_{ul,i}\left(\dot{\phi}_{ul,i}\right),$$

$$\dot{s}_{odl,(i+1)}\left(\dot{\phi}_{odl,(i+1)}\right)=-ks_{ul,(i+1)}\left(\dot{\phi}_{ul,(i+1)}\right),\ i=1,2,3,...,n,$$

where: k - impact coefficient in the range between $k=0$, for an ideal plastic impact, and $k=1$, for an ideal elastic impact; n - number of impacts until the arrestment of a heavy mass particle on the parabolic rough line, or until the interval where a heavy mass particle continues to move without impact to the limiter.

Free motion of heavy mass particles along parabolic rough lines is divided into the corresponding intervals and subintervals of motion:

First: from the initial moment of motion to the impact into the right elongation limiter;

Second: from the right elongation limiter to the impact to the left elongation limiter, until the direction alternation (motion intervals limited by the friction force direction alternation).

The motion analysis is conducted by using the phase trajectory equation (4) with the corresponding argument in dependance of the motion interval.

Grafic visualization of the phase portrait of heavy mass perticles in the observed vibro-impact system

Based on real values of kinetic and geometry parameters of the system,

$$\phi_1 = \frac{\pi}{4}[rad], \phi_2 = -\frac{\pi}{6}[rad], \ \phi_0 = 0, \ \dot{\phi}_0 = 7\left[\frac{rad}{s}\right],$$

$$p = 1[m], \alpha_0 = 0{,}05, \ g = 9{,}81\left[\frac{m}{s^2}\right] \text{ and } m = 0{,}2[kg],$$

the phase portrait of a heavy mass particle moving along the parabolic rough line is showed (Fig.6).

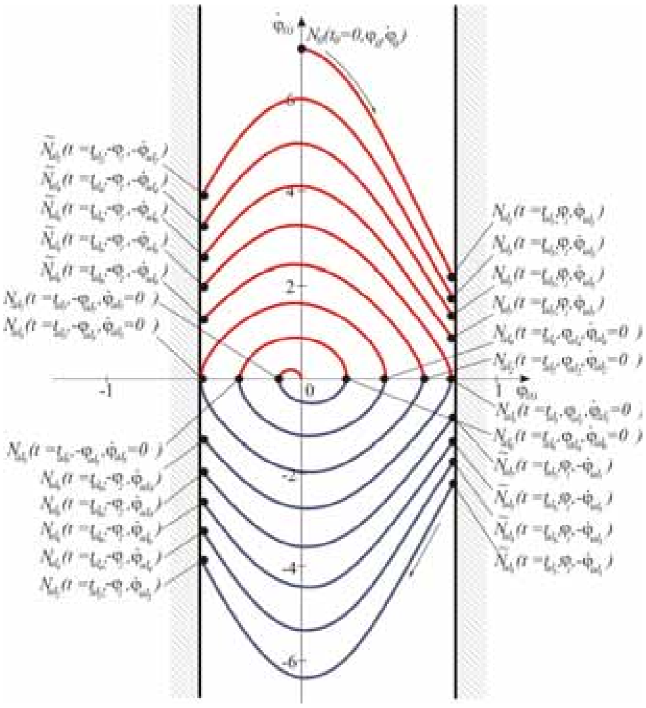


Figure 6. Phase portrait ofa heavy mass particle moving along the parabolic rough line with the sliding Coulomb's type friction coefficient $\mu = tg\alpha_0$ with the limited elongations in the plane $\left(\phi, \dot{\phi}\right)$

Graphic visualization of the energy analysis of the observed vibro-impact system

By using the analytical expressions for the peripendicular pressure force $F_{N,i}$, the power originated from the sliding *Coulomb's* type friction force $P_{\mu,i}$ to the heavy mass particle on the parabolic rough line, the kinetic energy $E_{k,i}$, the potential energy $E_{p,i}$ and the total mechanical energy E_i, $(i = 1, 2, .., n.)$,

$$F_{N,i} = mg\cos\phi + mp\cos^3\phi\left(-\frac{g}{p\cos^2\phi} + C_i e^{\mp 2\mu\phi}\right),$$

$$P_{\mu,i} = -\mu F_{N,i}\dot{s} = -\mu F_{N,i}\frac{p}{\cos^3\varphi}\dot{\varphi} =$$

$$= -\mu mp\left(g\cos\varphi + p\cos^3\varphi\left(-\frac{g}{p\cos^2\varphi} + C_i e^{\mp 2\mu\varphi}\right)\right)\sqrt{-\frac{g}{p\cos^2\varphi} + C_i e^{\mp 2\mu\varphi}},$$

$$Ek_i(\phi) = \frac{1}{2}mv_i^2 = \frac{1}{2}m\frac{p^2}{\cos^6\phi}\dot{\phi}_i^2 = \frac{1}{2}mp^2\left(-\frac{g}{p\cos^2\phi} + C_i e^{\mp 2\mu\phi}\right),$$

$$Ep_i(\phi) = \frac{1}{2}\frac{mgp}{\cos^2\phi} \text{ and}$$

$$E_i(\phi) = Ek_i(\phi) + Ep_i(\phi) =$$

$$= \frac{1}{2}mp^2\left(-\frac{g}{p\cos^2\phi} + C_i e^{\mp 2\mu\phi}\right) + \frac{1}{2}\frac{mgp}{\cos^2\phi},$$

For every separate branch of the phase portrait, there is a graphic of alternation of F_N, P_μ, E_k, E_p and E from the the initial moment of motion until the moment when heavy mass particles return to the equilibrium position (Fig.7-11).

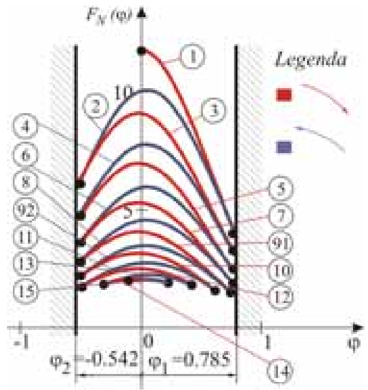


Figure 7. Curve of pressure force alternation as a function of the angle ϕ

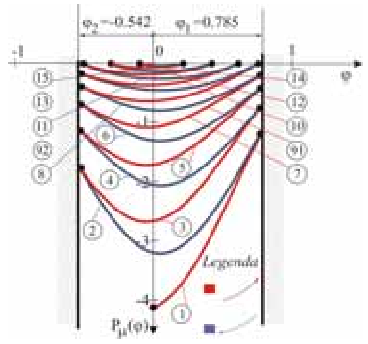


Figure 8. Curve of power alternation P_μ as a function of the angle ϕ

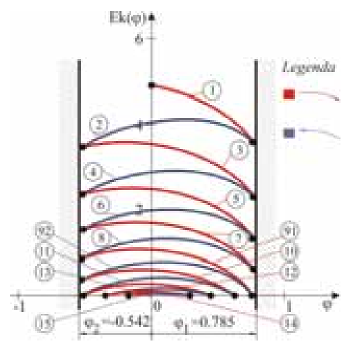


Figure 9. Graphic presentation of the kinetic energy alternation in the plane (Ek, ϕ)

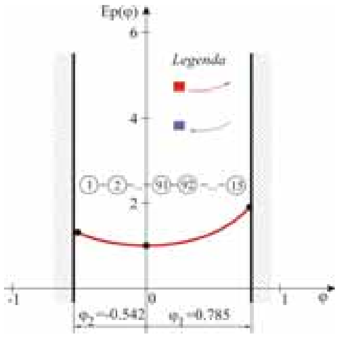


Figure 10. Graphic presentation of the potential energy alternation in the plane (Ep,ϕ)

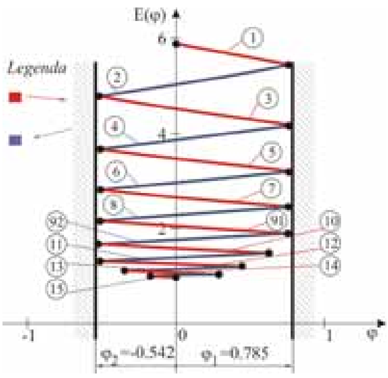


Figure 11. Graphic presentation of the total mechanical energy alternation in the plane (E,ϕ)

Vibro-impact system based on heavy mass particle free oscillations along the cycloid rough line

A heavy mass particle is moving along the cycloid rough line in the vertical plane with the sliding Coulomb's type friction coefficient $\mu = tg\alpha_0$, with the elongation limiters on the left and on the right side (Fig.12). The limiter positions are determined by the arc coordinates $s_{ul,1} = s_1(\varphi_1)$ and $s_{ul,2} = s_2(\varphi_2)$ and measured from the equilibrium position of the heavy mass particle. The arc (curvilinear) coordinates are given as a function of the angle φ.

For the complete description of the dynamics of the heavy mass particle, there are conditions matched to the differential (double) motion equation:

*a** initial conditions

$$s_{(0)}(\varphi_{(0)}) = s_0(\varphi_0) \text{ and}$$

$$v_{(0)}(\varphi_{(0)},\dot{\varphi}_{(0)}) = \dot{s}_{(0)}(\varphi_{(0)},\dot{\varphi}_{(0)}) = v_0(\varphi_0,\dot{\varphi}_0);$$

*b** angular elongation limitation conditions, and collision conditions

$$s_{ul,i} = s_i(\varphi_i)\ ,\ s_{ul,(i+1)} = s_{(i+1)}(\varphi_{(i+1)}),$$

$$\dot{s}_{odl,i}(\dot{\varphi}_{odl,i}) = -k\dot{s}_{ul,i}(\dot{\varphi}_{ul,i}),$$

$$\dot{s}_{odl,(i+1)}(\dot{\varphi}_{odl,(i+1)}) = -ks_{ul,(i+1)}(\dot{\varphi}_{ul,(i+1)}),\quad i = 1,2,3,...,n,$$

where: *k*- is the impact coefficient within the range from $k = 0$, for the ideal plastic impact, to $k = 1$, for the ideal elastic impact; *n*- number of impacts until the heavy mass particle stopping on the parabolic rough line or to the interval where the heavy mass particle continues to move without impact to the limiter.

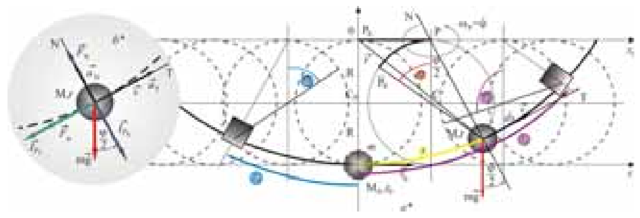


Figure 12. System with two stable elongation limiters, based on the oscillator with one pellet: a* initial and derived position of the pellet; b* force plan

A free motion of the heavy mass particle along the cycloid rough line is divided into corresponding motion intervals and sub intervals:

First: from the initial moment of motion to the impact to the right elongation limiter;

Second: from the right elongation limiter to the impact to the left elongation limiter, etc., until the direction alternation (motion intervals limited by friction force direction alternation).

The motion analysis is conducted by using the phase trajectory equation (6) with the corresponding argument in dependance of the motion interval.

Grafic visualization of the phase portrait of the heavy mass perticle in the observed vibro-impact system

Based on real values of the kinetic and geometry parameters of the system,

$$\varphi_1 = \frac{\pi}{4}[rad], \varphi_2 = -\frac{\pi}{6}[rad], \varphi_0 = 0, \dot{\varphi}_0 = 8\left[\frac{rad}{s}\right],$$

$$R = 0{,}05[m], \alpha_0 = 0{,}05, g = 9{,}81\left[\frac{m}{s^2}\right], m = 0{,}2[kg].$$

The phase portrait of the heavy mass particle moving along the cycloid rough line is showed (Fig.13).

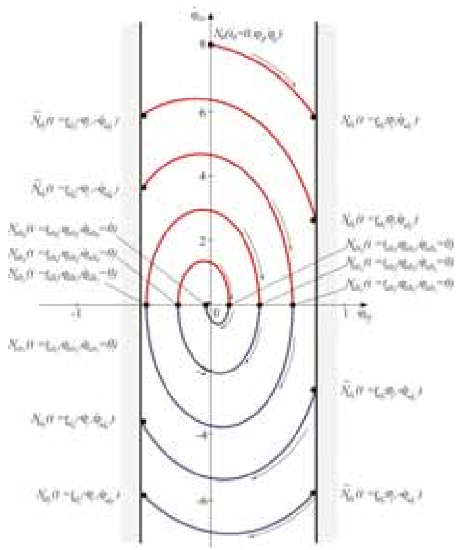


Figure 13. Phase portrait of the heavy mass particle moving along the cycloid rough line with the sliding Coulomb's type friction coefficient $\mu = tg\alpha_0$ with the limited elongations in the plane $(\phi,\dot{\phi})$

Graphic visualization of the energy analysis of the observed vibro-impact system

By using the analytical expressions for the peripendicular pressure force $F_{N,i}$, the power originated from the sliding *Coulomb's* type friction force $P_{\mu,i}$ of the heavy mass particle on the parabolic rough line, the kinetic energy $E_{k,i}$, the potential energy $E_{p,i}$ and the total mechanical energy E_i, $(i=1,2,..,n.)$,

$$E_i,\ (i=1,2,..,n.),$$

$$F_{N,i}=mg\cos\frac{\phi}{2}+m2R\cos\frac{\phi}{2}\cdot\left(-\frac{\left(\frac{g}{2R}\right)}{1+4\mu^2}\frac{1}{\cos^2\frac{\phi}{2}}\left[(\pm 3\mu)\sin\phi-\left(1-2\mu^2\right)\cos\phi+\frac{1+4\mu^2}{2}+C_ie^{\mp 2\mu\phi}\right]\right),$$

$$P_{\mu,i}=-\mu F_{N,i}2R\cos\frac{\varphi}{2}\dot{\varphi}=-\mu F_{N,i}2R\cos\frac{\varphi}{2}\cdot\sqrt{-\frac{\left(\frac{g}{2R}\right)}{1+4\mu^2}\frac{1}{\cos^2\frac{\varphi}{2}}\left[(\pm 3\mu)\sin\varphi-\left(1-2\mu^2\right)\cos\varphi+\frac{1+4\mu^2}{2}+C_ie^{\mp 2\mu\varphi}\right]},$$

$$Ek_i(\phi)=2mR^2\cos^2\frac{\phi}{2}\dot{\phi}_i^2(\phi)=2mR^2\cos^2\frac{\phi}{2}\cdot\left(-\frac{\left(\frac{g}{2R}\right)}{1+4\mu^2}\frac{1}{\cos^2\frac{\phi}{2}}\left[(\pm 3\mu)\sin\phi-\left(1-2\mu^2\right)\cos\phi+\frac{1+4\mu^2}{2}+C_ie^{\mp 2\mu\phi}\right]\right),$$

$$Ep_i(\phi)=mgR(1-\cos\phi)\ \text{and}$$

$$E_i(\phi)=Ek_i(\phi)+Ep_i(\phi)=2mR^2\cos^2\frac{\phi}{2}\cdot\left(-\frac{\left(\frac{g}{2R}\right)}{1+4\mu^2}\frac{1}{\cos^2\frac{\phi}{2}}\left[(\pm 3\mu)\sin\phi-\left(1-2\mu^2\right)\cos\phi+\frac{1+4\mu^2}{2}+C_ie^{\mp 2\mu\phi}\right]\right)+ +mgR(1-\cos\phi).$$

For every separate branch of the phase portrait, there is a graphic presentation of the alternation of F_N, P_μ, E_k, E_p and E from the initial moment of motion until the moment when the heavy mass particle returns into the equilibrium position (Figs.14-18).

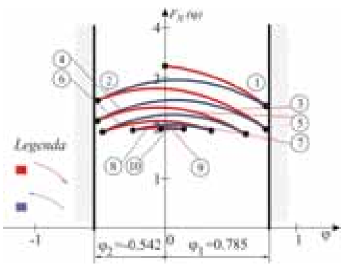


Figure14. Curve of pressure force alternation as a function of the angle ϕ

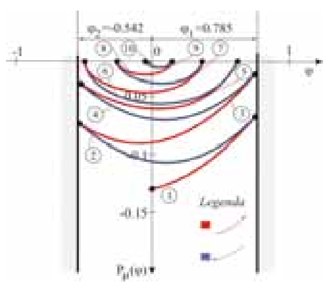


Figure 15. Curve of the power alternation P_μ as a function of the angle

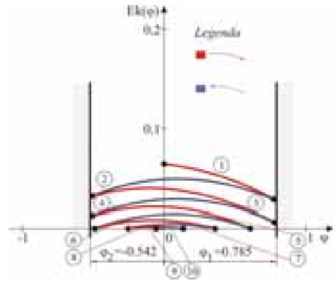


Figure 16. Graphic presentation of the kinetic energy alternation in the plane (Ek,ϕ)

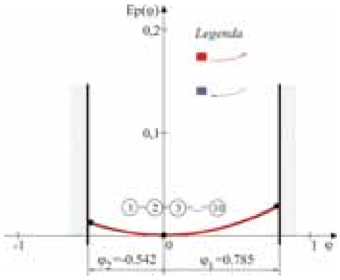


Figure 17. Graphic presentation of the potential energy alternation in the plane (Ep,ϕ)

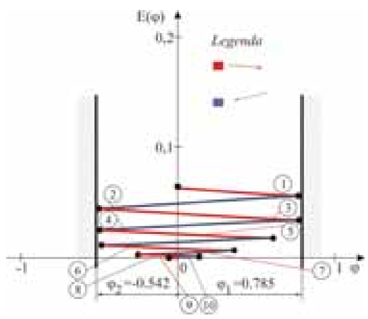


Figure18. Graphic presentation of the total mechanical energy alternation in the plane (E,ϕ)

Vibro-impact system based on heavy mass particle free oscillations along the circle rough line

A heavy mass particle is moving along the circle rough line in the vertical plane withthe sliding Coulomb's type friction coefficient $\mu=tg\alpha_0$, with one elongation limiter on the right side (Fig.19). The limiter position is determined by the angle δ measured from the equilibrium position of the heavy mass particle i.e. from the vertical line driven through the center of the circle.

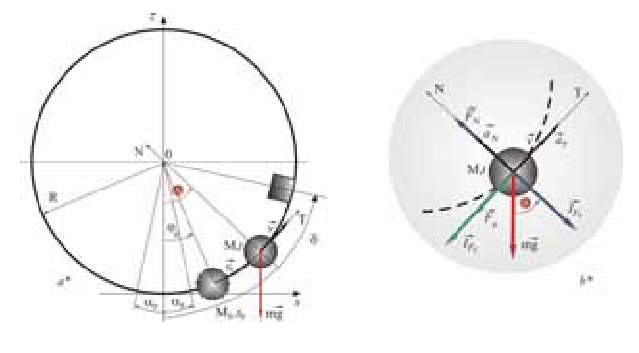


Figure 19. System with one stable elongation limiter, based on the oscillator with one pellet: a* initial and derived position of the pellet; b* force plan

For the complete description of the dynamics of heavy mass particles there are conditions matched to the differential (double) motion equation:

*a** initial conditions $\varphi_{(0)} = \varphi_0$ and $\dot{\varphi}_{(0)} = \dot{\varphi}_0$;

*b** angular elongation limitation conditions, and collision conditions

$$\varphi_{ul_i} = \delta \ , \ \dot{\varphi}_{odl_i} = -k\dot{\varphi}_{ul_i} \ , \ i = 1,2,3,...,n \ ,$$

where: *k*- is the impact coefficient within the range from $k = 0$, for the ideal plastic impact, to $k = 1$, for the ideal elastic impact; *n*- the number of impacts until the heavy mass particle stopping on the circle rough line or to the interval where the heavy mass particle continues to move without impact to the limiter.

Free motion of heavy mass particles along circle rough lines is divided into corresponding motion intervals and sub intervals limited by the direction of the friction force alternation.

The motion analysis is conducted by using the phase trajectory equation (8) with the corresponding argument in dependance of the motion interval.

Grafic visualization of the phase portrait of the heavy mass perticle in the observed vibro-impact system

Based on real values of the kinetic and geometry parameters of the system,

$$\delta = \frac{\pi}{4}[rad],\ \phi_0 = \frac{\pi}{12}[rad],\ \dot{\phi}_0 = 3,8\left[\frac{rad}{s}\right], R = 0,5[m],$$
$$\alpha_0 = 3, g = 9,81\left[\frac{m}{s^2}\right], m = 0,2[kg].$$

The phase portrait of the heavy mass particle moving along the circle rough line is showed (Fig.20.).

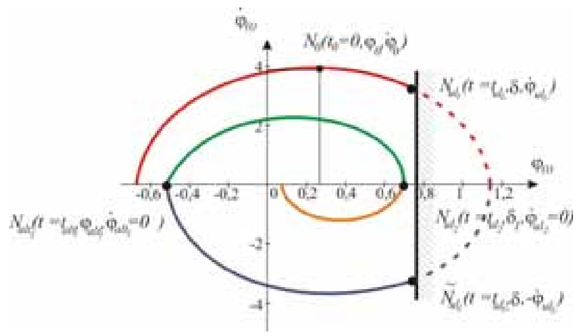


Figure 20. Phase portrait of the heavy mass particle moving along the circle rough line with the sliding Coulomb's type friction coefficient $\mu = tg\alpha_0$ with the limited elongations in the plane $(\phi, \dot{\phi})$

For the selected initial conditions and friction coefficient 3 (high degree of resistance), in the observed case there were one impact and one oscillation until the moment of arrestment.

The conditions needed for the heavy mass particle to have several impacts onto the angular elongation limiter in the observed vibro-impact system are:

$$\phi_0 < \delta \ \text{ and } \ \sqrt{2\frac{g}{R}(1-\cos\delta)} < \dot{\phi}_0 < \sqrt{4\frac{g}{R} - 2\frac{g}{R}(1-\cos\delta)} \ .$$

It can be concluded that for a lower friction coefficient and a larger initial velocity, there is a large number of impacts and oscillations before the arrestment of the heavy mass particle moving along the rough circle line.

In order to get a better graphic visualization of the motion analysis and the energy analysis of the observed vibro-impact system, the values for the sliding friction coefficient will be changed (instead of $\alpha_0 = 3$ there is $\alpha_0 = 0,05$) and also for the initial velocity of the heavy mass particle (instead of $\dot{\phi}_0 = 3,8\,[\mathrm{rad/s}]$ there is $\dot{\phi}_0 = 7\,[\mathrm{rad/s}]$). The rest of kinetic and geometry parameters remained the same.

The phase portrait of the heavy mass particle moving along the circle rough line in this case is given in Fig. 21.

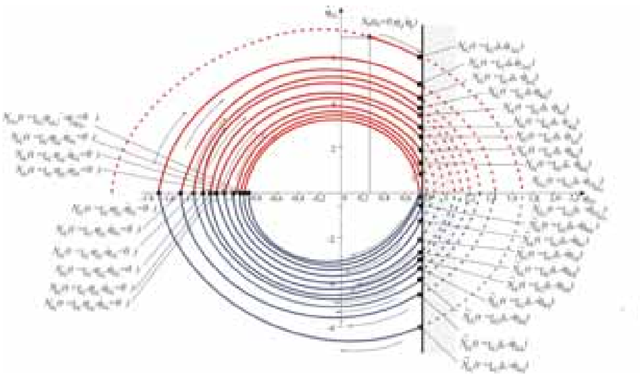


Figure 21. Phase portrait of the heavy mass particle moving along the circle rough line with the sliding Coulomb's type friction coefficient $\mu = 0,05$ with the limited elongations in the plane $(\phi, \dot{\phi})$

Graphic visualization of the energy analysis of the observed vibro-impact system

By using the analytical expressions for the peripendicular pressure force $F_{N,i}$, the power originated from the sliding *Coulomb's* type friction force $P_{\mu,i}$ of the heavy mass particle on the parabolic rough line, the kinetic energy $E_{k,i}$, the potential energy $E_{p,i}$ and the total mechanical energy E_i,

$$E_i\,,\ (i = 1,2,..,n.),$$

$$F_{N,i} = mg\cos\varphi + mR\cdot$$
$$\cdot\left(\frac{2g}{(1+4tg^2\alpha_0)R\cos\alpha_0}\left[\cos(\varphi\pm\alpha_0) - 2tg\alpha_0\sin(\varphi\pm\alpha_0)\right] + C_i e^{\mp 2\varphi tg\alpha_0}\right),$$

$$P_{\mu,i} = -\mu F_{N,i} R\dot{\varphi} = -\mu F_{N,i} R\cdot$$
$$\cdot\left\{-\sqrt{\frac{2g}{(1+4tg^2\alpha_0)R\cos\alpha_0}\left[\cos(\varphi\pm\alpha_0) - 2tg\alpha_0\sin(\varphi\pm\alpha_0)\right] + C_i e^{\mp 2\varphi tg\alpha_0}}\right\},$$

$$Ek_i(\varphi) = \frac{1}{2}mR^2\dot{\varphi}_i^2(\varphi) = \frac{1}{2}mR^2\cdot$$
$$\cdot\left(\frac{2g}{(1+4tg^2\alpha_0)R\cos\alpha_0}\left[\cos(\varphi\pm\alpha_0) - 2tg\alpha_0\sin(\varphi\pm\alpha_0)\right] + C_i e^{\mp 2\varphi tg\alpha_0}\right),$$

$$Ep_i(\varphi) = mgR(1-\cos\varphi) \ \text{ and}$$

$$E_i(\phi) = Ek_i(\phi) + Ep_i(\phi) = \frac{1}{2}mR^2\cdot$$
$$\cdot\left(\frac{2g}{(1+4tg^2\alpha_0)R\cos\alpha_0}\left[\cos(\phi\pm\alpha_0) - 2tg\alpha_0\sin(\phi\pm\alpha_0)\right] + C_i e^{\mp 2\phi tg\alpha_0}\right) +$$
$$+ mgR(1-\cos\phi).$$

For every separate branch of the phase portrait, there is a graphic presentation of the alternation of F_N , P_μ , E_k , E_p and E from the initial moment of motion until the moment when the heavy mass particle returns into the equilibrium position (Figs.22-26).

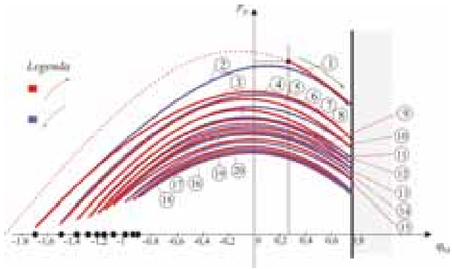


Figure22. Curve of the pressure force alternation as a function of the angle ϕ

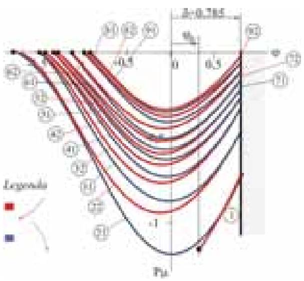


Figure 23. Curve of the power alternation P_μ as a function of the angle ϕ

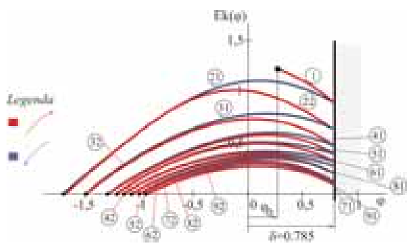


Figure 24. Graphic presentation of the kinetic energy alternation in the plane (Ek,ϕ)

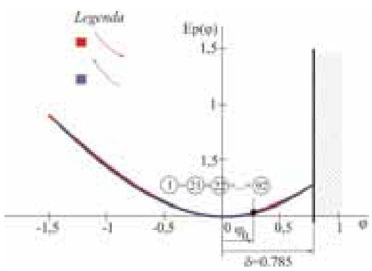


Figure 25. Graphic presentation of the potential energy alternation in the plane (Ep,ϕ)

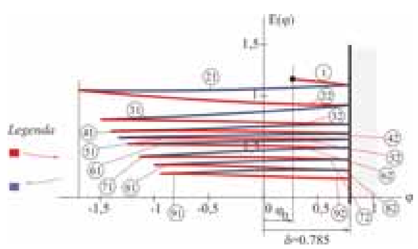


Figure 26. Graphic presentation of the total mechanical energy alternation in the plane (E,ϕ)

The application of vibro-impact systems to the construction of vibro-machines

A vibro-impact motion is necessary for technology processes in many devices [31-35]. The next paragraph presents the schematic presentations of vibro-impact machines (Figs.27-41) for conducting the presented motion analysis and the energy analysis of the observed vibro-impact systems with the appropriate graphic visualization by the corresponding dynamic models [1-6,29]. It should be mentioned that this paper represents the sequel of the paper in reference [18], but with a difference that the paper referred to the straight line oscillator which can be also included as a special case of a heavy mass particle motion along rough curvilinear routes (the oscilation motion is enabled by an elastic spring force instead of gravity).

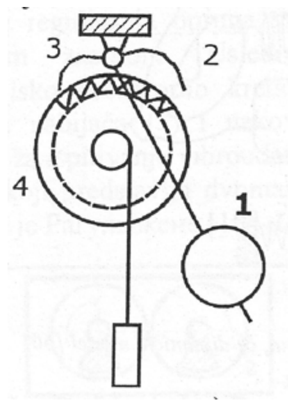


Figure 27. Clock mechanism

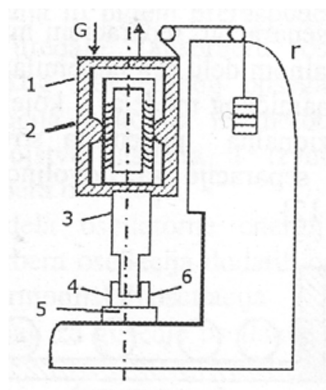


Figure 28. Machine for ultra sound cutting

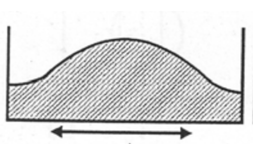


Figure 29. Sieve

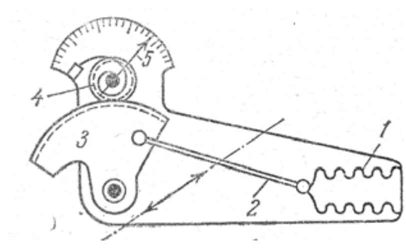
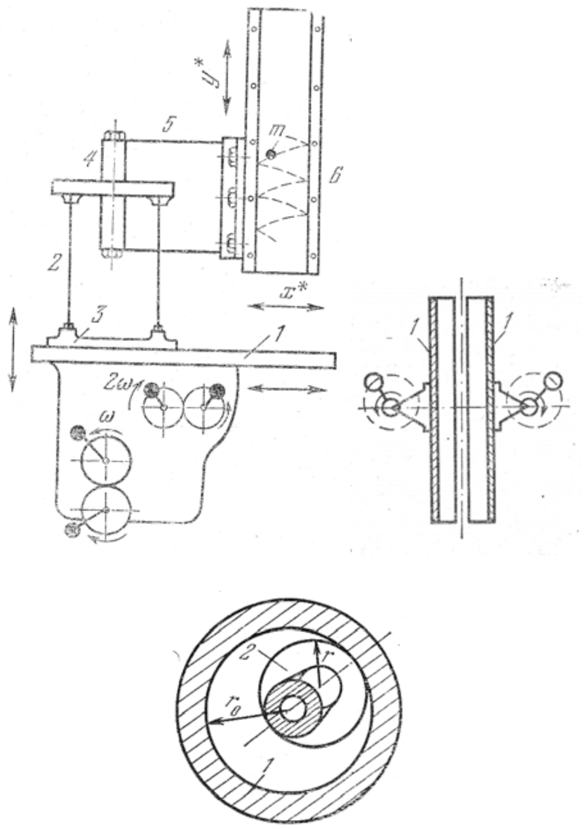


Figure 30. Measuring aparture



a* b* c*

Figure 31. Schematic presentation of the vibro-impact system with the curvilinear motion of heavy mass particles

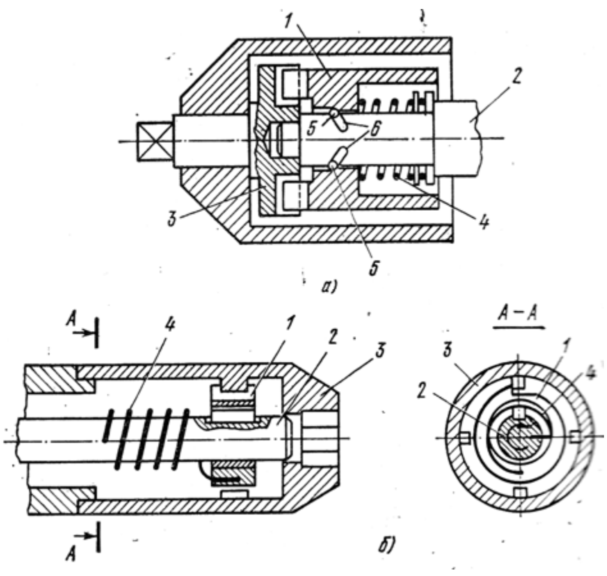


Figure 32. The example of the mechanism for the technical realization in constructions such as elevators, holding tools, etc.

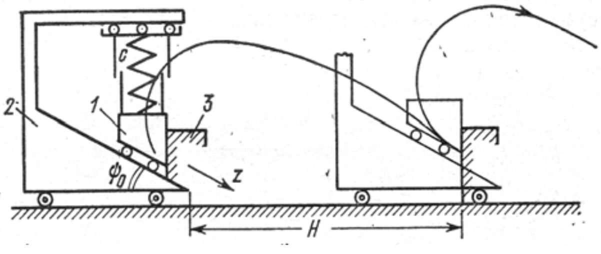


Figure 33. Plane analog mechanism

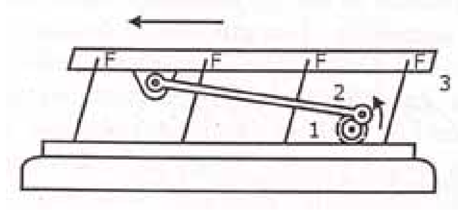


Figure 34. Vibro-transporter

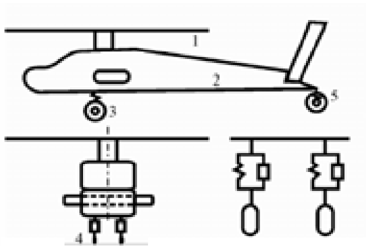


Figure 35. Helicopter

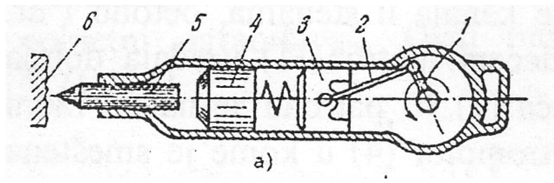


Figure 36. Manual vibro-impact hammer

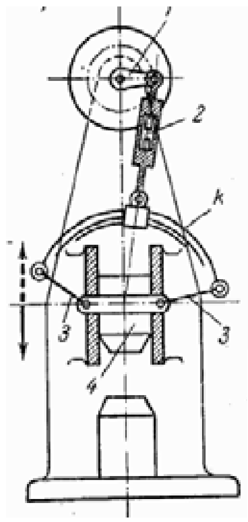


Figure 37. Vibro-impact hammer

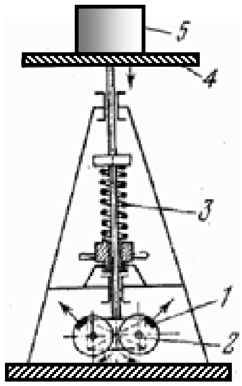


Figure 38. Vibro-impact platform

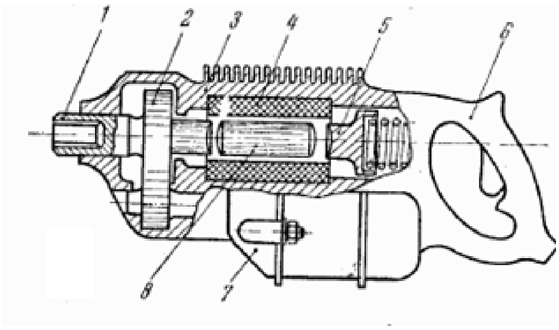


Figure 39. Manual stroke-rotary hammer

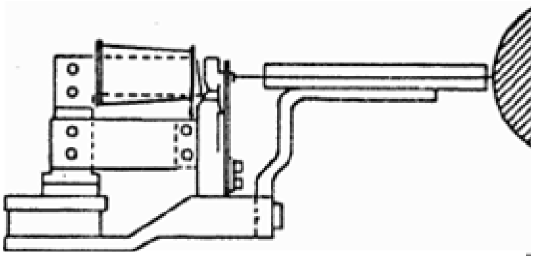


Figure 40. Printer

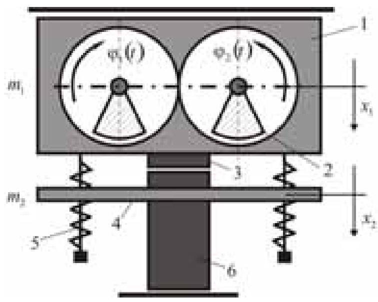


Figure 41. Vibro-rammer

Concluding remarks

Non-linearity of the observed vibro-impact systems originate from the discontinuity of the angular velocity of heavy mass particles moving along rough curvilinear routes. Discontinuities of the angular velocity occur at the moment of impact of a heavy mass particle into the angular elongation limiters set on the right and on the left side, at the moment of the direction alternation of the motion of heavy mass particles (when the alternation occurs) causing the angular velocity and friction force alternation. This non-linearity is described mathematically for heavy mass particles by an ordinary differential equation, more precisely by the second member, representing the square angular velocity of the generalized coordinate $\dot{\phi}^2$. This corresponds to the case known in literature as a case of „turbulent“ damping.

It should be pointed out that, in the observed vibro-impact systems, coupled singularities [13, 16] are triggered, i.e. phenomena of bifurcation of equilibrium positions, because of the influence of the sliding Coulomb's type friction force and the angular velocity direction alternation.

For the considered vibro-impact systems there is a common conclusion that the non-impact motion represents free supressed oscillations of the dynamic system with one degree of freedom. Rough curvilinear lines are a mutually detaining bond.

Free oscillations of heavy mass particles along rough curvilinear lines, representing vibro-impact systems, are divided into corresponding motion intervals and sub-intervals. Each interval or sub-interval is matched to a differential motion equation from the group of regular homogenous non-linear differential equations.

Differential (double) motion equations for corresspponding interval and sub-intervals are coupled with initial motion conditions, impact conditions into elongation limiters and direction alternation conditions, which cause alternations of the friction force direction.

By solving analytically differential (doble) motion equations, the analytical expressions for the phase trajectories in the plane $(\varphi, \dot{\varphi})$, are made, which is necessary for the energy analysis, together with the equation of curves of mechanical dependance for the energy analysis of the dynamics of vibro-impact systems.

The authors presented a good quality graphic visualization of the curves of alternations of the components of mechanical energies of vibro-impact system dynamics and motion analysis of a representative point of the system kinetic state during the kinetic (dynamics), by applying the analytical expressions and the software package MathCad and user's package CorelDraw.

By the phase portrait analysis and the graphic of the kinetic energy E_K, the potential energy E_P, the total mechanical energy E, the pressure force F_N and the power originated of the sliding Coulomb's type friction force alternations, it can be concluded for all examples of free motion of heavy mass perticles along rough curvilinear lines, with one degree of freedom:

The perpendicular pressure force on a rough parabola, cycloid and circle line does not change its value.

* In the moment of impact of the heavy mass particle onto the elongation limiter (any position), when the impact is ideally eleastic, the motion velocity intensity is not changed.

* In the case of the heavy mass particle direction alternation, the velocity is equal to zero.

In the case of mutually retaining bonds in the point of alternation, the pressure force is of the local minimum value and the corresponding friction force alternates its direction.

The friction force direction is altered: in the point where the angular velocity of heavy mass particles is equal to zero and at the point of impact of heavy mass particles onto the elongation limiter.

Power alternation, due to the sliding friction Coulomb's type force, follows the graphic of the friction force alternation, but the power is always with a negative argument and in subsequent representative points has lower values (decreasing from a higher level to a lower level). In the heavy mass particle motion along rough curvilinear lines with elongation limiters, assuming that the impact is ideally elastic, from the initial moment to the moment when the particle returns into the equilibrium position, the maximum value of power of the sliding Coulomb's type force decreases constantly, no matter how many degrees of freedom there are in the observed system.

Kinetic energy, depending explicitly on the angular velocity of heavy mass particles, permanently changes and its maximum value in the sequence motion intervals is decreased.

Potential energy depends on the elongation which is identical for all identical motion intervals, due to the fact that it depends on the heavy mass particle weight and the generalized coordinates, and that the impact does not influence the potential energy, since it results from the effect of the conservative forces on the system.

Total mechanical energy of the system constantly decreases, i.e. in each subsequent motion interval, the total mechanical energy of the system dynamics has a lower value (at the point of impact onto the elongation limiter and the point of the angular velocity alternation).

The fourth section of this paper presents a series of models of technology processes, with real engineering constructions. The selected models are characteristic, presented in scientific monographies of leading scientists and researchers from the field of vibro-impact dynamics. In every real model, the motion and energy analysis of the corresponding vibro-impact system can be done, by the application of the methodology presented in this paper which is the continuation of the author’s own research presented in reference [18].

Acknowledgement: Parts of this research were supported by the Ministry of Sciences and Technology of the Republic Serbia through the Mathematical Institute SANU Belgrade (Grant ON174001 Dynamics of hybrid systems with complex structures, Mechanics of materials) and the Faculty of Technical Sciences Kosovska Mitrovica, University of Priština with the seat in Kosovska Mitrovica.

Special thanks to the scientific and professional support in the analytical research we owe to Professor Katica (Stevanović) Hedrih, project leader and co-supervisor of the doctoral dissertation entitled "Energy analysis of the dynamics vibro-impact curvilinear paths and non-ideal links” by doctorand Mr. Srđan Jović [27].

References

[1] BABICKII,V.I., KOLOVSKII,M.Z.: *Vibrations of linear system with limiters, and excited by random excitation*, Mehanika tverdogo tela, ,1967, No.3.

[2] BABICKII,V.I.: *Theory of vibro-impact systems*, Moskva, "Nauka", 1978.

[3] BABICKII,V.I., KOLOVSKII,M.Z.: *Investigation of the vibro-impact systems by resonant regimes*, Mehanika tverdogo tela, 1976, No.4.

[4] BAPAT,C.N., POPPLEWELL,N.: *Several Similar Vibroimpact Systems*, Journal of Sound and Vibration, 1987, 113 (1), pp. 17-28.

[5] BAČLIĆ,B.S., ATANACKOVIĆ,T.M.: *Stability and creep of a fractional derivative order viscoelastic Rod*, Bulletin T, CXXI de L'Academie Serbe des Sciences st de Arts - 2000, Class des Sciences mathematiques et naturelles Sciences mathematiques, 2000, No.25, pp.115-131.

[6] BAPAT,C.N.: *Impact-Pair under Periodic Exitation*, Journal of Sound and Vibration, 1988, 120 (1), pp. 53-61.

[7] DIMENTBERG,M.: *Pseudolinear vibro-impact systems: Non-white random excitation*, Nonlinear Dynamics, Volume 9, Number 4/ April, 1996, pp. 327-332.

[8] FOOLE,S., BISHOP,S.: *Bifurcation in Impact Oscillators*, UTAM, London, 1998.

[9] HEDRIH (STEVANOVIĆ) K.: *Optimal control in nonlinear system with no ideal constraints*, Commun Nonlinear Sci Numer Simulat (2010), doi:10.1016/j.cnsns.2010.04.053

[10] HEDRIH (STEVANOVIĆ),K.: (2005), *Nonlinear Dynamics of a Heavy Material Particle Along Circle which Rotates and Optimal Control*, Chaotic Dynamics and Control of Systems and Processes in Mechanics (Eds: G. Rega, and F. Vestroni), p. 37-45. IUTAM Book, in Series Solid Mechanics and Its Applications, Edited by G.M.L. Gladwell, Springer,. 2005, XXVI, 504 p., ISBN: 1-4020-3267-6. DOI 10.1007/1-4020-3268-4_4. ISBN 978-1-4020-3267-7 (Print) 978-1-4020-3268-4 (Online)

[11] HEDRIH (STEVANOVIĆ),K.: (2008), *The optimal control in nonlinear mechanical systems with trigger of the coupled singularities*, pp.174-182, in the book: Advances in Mechanics: Dynamics and Control : Proceedings of the 14th International Workshop on Dynamics and Control / [ed. by F.L. Chernousko, G.V. Kostin, V.V. Saurin] : A.Yu. Ishlinsky Institute for Problems in Mechanics RAS. – Moscow : Nauka, 2008. –ISBN 978-5-02-036667-1

[12] HEDRIH (STEVANOVIĆ),K.: (Serbia), Homoclinic Orbits Layering in the Coupled Rotor Nonlinear Dynamics and Chaotic Clock Models, SM17 – Multibody Dynamics (M. Geraldin and F. Pfeiffer), p. Lxiii – CD - SM10624, Mechanics of the 21st Century (21st ICTAM, Warsaw 2004) - CD ROM INCLUDED, edited by Witold Gutkowski and Tomasz A. Kowalewski, IUTAM, Springer 2005, ISBN 1-4020-3456-3, Hardcover., p. 421+CD. ISBN-13 978-1-4020-3456-5 (HB), ISBN-10 1-4020-3559-4(e-book), ISBN-13-978-1-4020-3559-3 (e-book). www. springeronline3.com.

[13] HEDRIH (STEVANOVIĆ),K.: *A Trigger of Coupled Singularities*, MECCANICA, Vol.39, No. 3, 2004., pp. 295-314. International Journal of the Italian Association of Theoretical and Applied Mechanics, Meccanica, Publisher: Springer Science+Business Media B.V., Formerly Kluwer Academic Publishers B.V. ISSN: 0025-6455 (Paper) 1572-9648 (Online) DOI: 10.1023/B:MECC.0000022994.81090.5f, Issue: Volume 39, Number 3, Date: June 2004, Pages: 295 - 314

[14] HEDRIH (STEVANOVIĆ),K.: *Free and forced vibration of the heavy material particle along line with friction: Direct and inverse task of the theory of vibrorheology*, 7th EUROMECH Solid Mechanics Conference, J. Ambrósio et.al. (eds.), Lisbon, Portugal, September 7-11, 2009, CD –MS-24, Paper 348, pp. 1-20.

[15] HEDRIH (STEVANOVIĆ),K.: *Vibrations of a Heavy Mass Particle Moving along a Rough Line with Friction of Coulomb Type*, ©Freund Publishing House Ltd., International Journal of Nonlinear Sciences & Numerical Simulation 10(11): 1705-1712, 2009. Vol.11, No.3 March 2010, pp. 203-210.

[16] HEDRIH (STEVANOVIĆ),K.: *Discontinuity of kinetic parameter properties in nonlinear dynamics of mechanical systems,* Invited Keynote Lecture, Proceedings of the 9th Brazilian Conference on Dynamics, Control and Their Applications, DINCON , Serra Negra, 2010, pp. 8-40. (SP - ISSN 2178-3667).

[17] HEDRIH (STEVANOVIĆ),K.: (2009), *Phase plane method applied to the optimal control in nonlinear dynamical systems – Heavy material particle oscillations along rough circle line with friction*: Phase portraits and optimal control 10th conference on DYNAMICAL SYSTEMS THEORY AND APPLICATIONS, December 7-10, 2009. Łódź, Polan (submitted)

[18] HEDRIH (STEVANOVIĆ),K., JOVIĆ,S.: *Models of Technological Processes on the Basis of Vibro-impact Dynamics*, Scientific Technical Review, 2009, ISSN 1802-0206, Vol.LIX,No.2, pp.51-72.

[19] HEDRIH (STEVANOVIĆ),K., RAIČEVIĆ,V., JOVIĆ,S.: *Vibro-impact of a Heavy Mass Particle Moving along a Rough Circle with Two Impact Limiters*, ©Freund Publishing House Ltd., International Journal of Nonlinear Sciences & Numerical Simulation ISSN: 1565-1339, 2010, Vol.11, No.3, pp.211-224.

[20] HEDRIH (STEVANOVIĆ),K., RAIČEVIĆ,V., JOVIĆ,S.: *Vibroimpact System Dynamics: Heavy Material Particle Oscillations Along Rough Circle with Two Side Moving Impact Limits*, The Symposium DyVIS (Dynamics of Vibroimpact Systems) ICoVIS -2th International Conference on Vibroimpact Systems, 6-9 January 2010. School of Mechanical Engineering & Automation North-eastern University, Shenyang, Liaoning Province, P. R. China, pp.79-86.

[21] HEDRIH (STEVANOVIĆ),K., RAIČEVIĆ,V., JOVIĆ,S.: *Phase Trajectory Portrait of the Vibro-imact Forced Dynamics of Two Mass Particles along Rough Circle*, 3rd International Conference on Nonlinear Science and Complexity, Ankara, Turkey, Çankaya Üniversitesi, 28-31 Jul, 2010 (to appear).

[22] HEDRIH (STEVANOVIĆ),K., RAIČEVIĆ,V., JOVIĆ,S.: *The phase portrait of the vibro-imact dynamics of two mass particle motions along rough circle*, 3rd International conference “Nonlinear Dynamics – 2010”, Kharkov, Ukraine, 21-24 Septembar, 2010, pp.84-89.

[23] HEDRIH (STEVANOVIĆ),K., JOVIĆ,S.: *Vibroimpact system dynamics: Heavy material particle oscillations along rough circle with one side impact limit*, 10th Conference on Dynamical systems-theory and applications (DSTA) Lodz, December 7-10, 2009 Poland. Proceedings Volume 1, pp.213-220.

[24] HEDRIH (STEVANOVIĆ),K., JOVIĆ,S.: *Energy of the vibroimpact system dynamics: Heavy material particle oscillations along rough circle with one side impact limit*, 6-th IFAC INTERNATIONAL WORKSHOP on Knowledge and technology transfer in/to developing countries: automation and infrastructure,

Hotel Metropol Resort, Ohrid Republic of Macedonia, September 26-29, 2009 (is uploaded to the DECOM-TT 2009 web site with ID number D65).

[25] HEDRIH (STEVANOVIĆ),K., JOVIĆ,S.: *Energy of the vibro-impact systems with Coulomb's type frictions*, ESMC Lisbon 2009 Minisymosium MS-24 Kinetics, Control and Vibrorheology KINCONVIB-2009, 7-11 Septembar 2009. pp. 43-47.

[26] JOVIĆ,S.: *Energijska analiza dinamike vibroudarnih sistema, Magistarski rad*, str. 239, odbranjen 06. Novembra 2009. Fakultet tehničkih nauka u Kosovskoj Mitrovici Univerziteta u Prištini, Srbija.

[27] JOVIĆ,S.: (2010). *Energijska analiza dinamike vibroudarnih sistema sa krivolinijskim putanjama i neidealnim vezama*, Doktorska disertacija, str. 335, bila na uvid javnosti, očekuje se odbrana do završetka kalendarske godine. Fakultet tehničkih nauka u Kosovskoj Mitrovici Univerziteta u Prištini, Srbija.

[28] HINRICHS,N., OESTREICH,M., POPP,K.: *Dynamics of oscillators with impact and friction*, Chaos, Solitons and Fractals, 1997, 8, 4, pp.535-558.

[29] KOBRINSKII,A.E., KOBRINSKII,A.A.: *Vibroudarni sistemi*, Moskva, Nauka, 1973.

[30] LUO,G.W., XIE,J.H.: *Hopf Bifurcation of a Two-Degree-of-Freedom Vibro-impact System*, Journal of Sound and Vibration, 1998, 213 (3), pp.391-408.

[31] Mitić,S.: *Стабилност детерминистичких и стохастичких процеса у виброударним системима*, 1994. Mašinski fakultet Niš, Srbija, Doktorska disertacija.

[32] MITIĆ,S., HEDRIH,K.: *Pregled savremenih saznanja o vibroudarnim sistemima*, I deo-Klasifikacija vibroudarnih sistema sa dinamičkim modelima, Tehnika, obl.Mašinstvo, 1991, Vol.15, No.11-12, pp.758-763.

[33] MITIĆ,S., HEDRIH,K.: *Pregled savremenih saznanja o vibroudarnim sistemima*, II deo-Pregled metoda istraživanja, Tehnika, obl. Mašinstvo, 1992, Vol.41, No.3-4, pp.231-235.

[34] MITIĆ,S., HEDRIH,K.: *Pregled savremenih saznanja o vibroudarnim sistemima, III deo-Fazne trajektorije regulisanih vibroudarnih procesa kod jednodimenzionih udarnih prigušivača*, 8 str., Tehnika, obl. Mašinstvo, 1994, Vol.43, No.1-2, M1-M5.

[35] MITIĆ,S., HEDRIH,K.: *Pregled savremenih saznanja o vibroudarnim sistemima*, IV deo-Fazne trajektorije vibroudarnih procesa u torzionom oscilatoru sa medjusobno kruto vezanim udarnim masama, 8 str., Tehnika, obl. Mašinstvo, 1994, Vol.43, No.7, M25-M29.

[36] NAYFEH,A.H.: *Transfer of energy from high-frequency to Low-frequency modes*, Book of Abstracts, The Second International Conference "Asymptotics in Mechanics" St. Petersburg Marine Technical University, Russia, 13-16 October, 1996, pp. 44.

[37] NAYFEH,A.H., BALACHANDRAN,B.: *Applied Nonlinear Dynamics*, Wiley Interscience, 1995 .

[38] NORDMARK,A.B.: *Nonperiodic Motion Caused by Grazing Incidence in an Impact Oscillator*, Journal of Sound and Vibration, 1991, 145(2), pp. 279-297.

[39] PAVLOVSKAIA,E., WIERCIGROCH,M.: *Periodic solution finder for an impact oscillator with a drift*, Journal of Sound and Vibration, 2003, 267, 4, pp.893-911

[40] PAVLOVSKAIA,E., WIERCIGROCH,M., WOO,K.C., RODGER,A.A.: *Modelling of ground moling dynamics by an impact oscillator with a frictional slider*, Meccanica, 2003, 38, 1, pp.85-97.

[41] PETERKA,F.: *Bifurcations and transition phenomena in an impact oscillator*, Chaos, Solitons and Fractals, 1996, 7, 10, pp.1635-1647.

[42] PETERKA,F.: *Laws of Impact Motion of Mechanical Systems with one Degree of Freedom: Part I - Theoretical Analysis of n- Multiple (1/n) - Impact Motions*, Acta Technica CSAV, 1974, No.4, pp.462-473.

[43] PETERKA,F.: *Laws of Impact Motion of Mechanical Systems with one Degree of Freedom: Part II - Resalts of Analogue Computer Modelling of the Motion*, Acta Technica CSAV, 1974, No 5, pp 569-580

[44] RAGULSKENE,V. L.: *Stereomehaničeskoi modeli udara*, Vibrotehnika, 1 (1), 1967.

[45] RAŠKOVIĆ,P.D.: *Teorija oscilacija*, Naučna knjiga, 1965, str. 503.

[46] STOKER,J. J.: (1950), *Nonlinear Vibrations*, Interscience Publish.

[47] TUNG,P.C., SHAW,S.W.: *A Method for the Improvement of Impact Printer Performance*, Journal of Vibration, Acoustics, Stress and Reliability in Design, oct. 1988, Vol. 110, pp.528-532.

Received: 17.10.2010.

Energijska analiza dinamike vibroudarnih sistema na bazi sopstvenih oscilacija teške materijalne tačke po krivolinijskim hrapavim putanjama

Rad se zasniva na analizi kretanja vibro-udarnih sistema na bazi oscilatora sa neidealnim vezama-hrapavim krivim linijama u vertikalnoj ravni, oblika: parabole, cikloide i kruga. Neidealnost veze potiče od trenja klizanja Coulumb-ovog tipa koeficijenta $\mu = tg\alpha_0$. Oscilator se sastoji od jedne teške materijalne tačke (posmatrani sistemi imaju jedan stepen slobode kretanja) čije je sopstveno kretanje ograničeno sa jednim ili dva nepokretna ograničivača elongacija. Analitičko-numerički rezultati za određene kinetičke parametre posmatranih vibro-udarnih sistema su osnova za vizuelizaciju analize kretanja i energijske analize što je i bio predmet ovog analitičkog istrazivanja. U ovom radu je prikazana metodologija izučavanja transfera energije iz kinetičke u potencijalnu u režimu vibroudara.

Ključne reči: **vibroudarni sistem, vibroudarna dinamika, udarno dejstvo, sopstvene oscilacije, dinamika procesa, dinamička analiza, hrapava površina, singularna tačka, kinetička energija, potencijalna energija.**

Энергический анализ динамики виброударных систем на основе собственных колебаний тяжёлой массированной точки по криволинейным неровным траекториям

Эта работа основывается на анализе движения виброударных систем на основе осциллятора с неидеальными связями – неровными кривыми линиями в вертикальной плоскости в форме: параболы, циклоиды и круга. Неидеальность связи происходит из трения скольжения типа коэффициента Колумба $\mu = tg\alpha_0$. Осциллятор состоит из одной тяжёлой массированой точки (у рассматриваемых систем одна степень свободы движения), чьё собственное движение ограничено одним или двумья неподвижными ограничителями удлинений.

Аналитико- цифровые результаты для определённых кинетических параметров рассматриваемых виброударных систем являются основой для визуализации анализа движения и энергического анализа, что и было содержанием - темой этого аналитического исследования. В настоящей работе показана методология выучивания переноса энергии из кинетической в потенциальную в режиме виброудара.

Ключевые слова: **виброударная система, виброударная динамика, ударное действие, собственные колебания, динамика процесса, динамический анализ, неровная поверхность, единственная точка, кинетическая энергия, потенциальная энергия.**

Analyse d'énergie de la dynamique des systèmes vibro - impact à la base des oscillations naturelles du point lourd matériel sur les trajectoires rêches curvilignes

Ce travail se base sur l'analyse du mouvement des systèmes vibro- impact à la base de l'oscillateur aux liens non idéals – des lignes rêches dans le plan vertical en formes des paraboles, cycloïdes et cercles. Non idéalité du lien est due à la friction du glissement du type Coulumb coefficient $\mu = tg\alpha_{\circ}$. L'oscillateur se compose d'un point matériel lourd (les systèmes observés possèdent un degré de la liberté du mouvement) dont le mouvement est limité par un ou deux limiteurs fixes d'élongation. Les résultats analytiques numériques pour les paramètres cinétiques déterminés des systèmes vibro impact observés présentent la base pour la visualisation d'analyse du mouvement et l'analyse d'énergie ce qui était l'objet de présente recherche analytique. Dans ce travail on a présenté la méthodologie pour l'étude du transfert de l'énergie cinétique à l'énergie potentielle dans le régime vibro-impact.

Mots clés: **système vibro-impact, dynamique vibro-impact, effet d'impact, oscillations naturelles, dynamique du processus, surface rêche, point singulier, énergie cinétique, énergie potentielle**