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History and Present of Gyroscope Models and Vector Rotators

Ljiljana Veljović[1)]

The objective of this paper is to give a short historical overview of models and constructions of gyroscopes and present models as well as their component motions. All mechanical gyroscopes are based on coupled rotations around more axes with one point of intersection.

Therefore, this paper contains two related parts. The first part presents the kinetic parameters of the rigid body rotations around two axes without intersection when the rigid body is skew and eccentrically positioned to the spin axis.

In addition, the paper presents some of the original results in the area of nonlinear dynamics of gyro rotors applicable in the explanation of some nonlinear phenomena in gyroscope dynamics, obtained by Hedrih and Veljovic. Hedrih's mass moment vectors and vector rotators are used to present some characteristic members of vector expressions in the linear momentum and the angular momentum and their derivatives for the gyro rotor rotation around two axes without intersection. The paper also gives a graphical presentation of the phase portrait and the kinematical vector rotator for different normal distances between the axes and the skew position of the axes. Imprecision and errors in the function of gyroscopes are caused by eccentricity and unbalance of gyro rotors as well by the distance between the rotation axes. The given graphs are thus important for the explanation and necessary corrections of the gyroscope functioning.

In the second part, the paper presents an overview of the history and the present of gyroscope models and their construction realizations used to manage and stabilize the movement of ships, aircraft, vehicles and torpedoes. With the development of micro and nano technology, a new class of gyroscopes emerges.

Key words: **gyroscope, gyro stabilizer, gyrocompass, gyroscope rotor, gyro-stabilized platform, inertial platform, rotator, momentum, angular momentum, vector analysis.**

Introduction

THE properties of gyroscopes can be found in heavenly bodies in motion, artillery projectiles in motion, turbine rotors, different mobile installations on ships, aircraft propeller rotating, etc. The modern technique of gyroscopes is an essential element of powerful gyroscopic devices and accessories used for the automatic control of the movement of aircraft, missiles, ships, torpedoes, etc. They are used in navigation to stabilize the movement of ships in a seaway, to change their direction or the direction of angular and translatory velocity projectiles, and for many other special purposes. There are many devices applied in the military, and their design is based on the principles of gyroscopes [2, 4, 13, 14, 15 and 17]. Technical applications of gyros today are so manifold and diverse that there is a need to get out of the general theory of gyroscopes and to allocate a separate discipline called "applied theory of gyroscopes."

A gyroscope is a part of many scientific and transportation-related instruments including compasses, mechanisms that steer torpedoes toward their targets, equipment that keeps large ships such as aircraft carriers from rolling on the waves, automatic pilots on airplanes and ships as well as systems that guide missiles and spacecraft relative to the Earth (i.e., inertial guidance systems).

A well-known spinning top is a simple toy with an unusual property. When it rotates at sufficiently high angular velocity about its axis of symmetry it keeps in the state of stationary rotation around this axis. This feature has attracted scientists around the world and as a result of years of research many devices and instruments are created, from simple to very complex structures, which operate on the principle of a spinning top that plays an important role in stabilizing the movement. The characteristic of the gyroscope to keep the direction was used in many fields of mechanical engineering, mining, aviation, navigation, military industry and celestial mechanics. Gyroscopes are very important parts of instruments for aircraft, rockets, missiles, transport vehicles and many weapons. This gives them a significant role and needs to be under the strict control of the design and inner functioning because in case of damage it could lead to catastrophic consequences. A gyroscope (gyro, top) is a homogeneous, axis-symmetric rotating body that rotates at high angular velocity about its axis of symmetry. Today, it is one of the most important inertial sensors measuring angular velocities and small angular disturbances or angular displacement around the reference axis.

Gyroscopes for measuring angular velocity are called rate gyroscopes, and when they measure small angular disturbances they are called rate integrating gyroscopes. In English literature, the word gyro is used. The name *gyroscope* comes from the Greek words γυρο (turn) and σκοπεω (observed) and is related to the experiments carried out by Jean Bernard Leon Foucault in 1852. The principle of the gyroscope is based on the principle of a pseudo-regular precession. In physics this principle is known as gyroscopic inertia or rigidity of space. The device consists of a disk or a wheel the axis of which is free to rotate and take up any position in space. When the rotation rate of the gyroscope about its axis is much higher than the velocity of rotation of the gyroscope axis, this gyroscope keeps an unchanged direction of the axis of its rotation when the

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1) University of Kragujevac, Faculty of Mechanical Engineering, Sestre Janjić 6, 34000 Kragujevac, SERBIA

external forces are absent. When the gyroscope operates under an external torque, there is a tendency for the gyroscope axis to move in the direction of the torque action. The gyroscope opposes to the action of an external coupling with the coupling of the same intensity and direction and of opposite sense (gyroscopic torque, deviational torque). The coupling is proportional to its own, high angular velocity of the gyroscope. When the device is mounted in a Cardan ring, its position remains almost fixed, regardless of any motion of the platform it is mounted on.

Modern navigational aircraft equipment would be impossible without gyroscope devices. There are many civilian and military applications. Here are just devices for automatic control of different robots, excavators, drilling or other working machines. The application of the gyroscope technology is very wide. Today, gyrocompasses fully replace magnetic compasses in aircraft, ship and automobile industry. The tank drive stabilization and guidance as well as the guidance of submarines, ships and various projectiles along a specified path are some examples of the military usage requiring gyrocompasses. In ballistics, gyroscopes are basic measuring elements in the missile control. They show the missile flight deviation from the desired flight conditions.

Analysis of the coupled rotation properties

A gyroscope in general performs rotation about a fixed point so that it can be described by the vector equations of rigid body dynamics: changing the linear $\vec{K}$ and the angular $\vec{L}_0$ momentum for a fixed point around which the body performs the rotation, for example.

$$\frac{d\vec{K}}{dt} = \sum \vec{F}_i \,, \ \frac{d\vec{L}_0}{dt} = \vec{M}_0 \qquad (1)$$

where $\sum \vec{F}_i$ are resultant active and reactive forces and $\vec{M}_0$ is the vector sum moment of the active and reactive forces for the point O.

Let us consider a heavy rigid body rotation around two axes without intersection and with a normal distance defined by the vector $\vec{r}_0 = \overrightarrow{O_1O_2}$. A heavy rigid body is eccentrically and skewly positioned on the spin axis (none of three main inertia axes of the body is parallel to the spin axis, as presented in Fig.1.). For this case, we denote with $\vec{\omega}_1 = \omega_1 \vec{n}_1$ and $\vec{\omega}_2 = \omega_2 \vec{n}_2$ two angular velocities around two axes oriented by the unit vectors $\vec{n}_1$ and $\vec{n}_2$. Using the mass moment vectors - the linear mass moment vectors $\vec{S}^{(O_2)}_{\vec{n}_1}$ and $\vec{S}^{(O_2)}_{\vec{n}_2}$, as well as the mass inertia moment vectors $\vec{J}^{(O_2)}_{\vec{n}_1}$, $\vec{J}^{(O_2)}_{\vec{n}_2}$, $\vec{J}^{(O_2)*C}_{\vec{n}_1}$, $\vec{J}^{(O_1)*}_{\vec{n}_1}$, $\vec{J}^{(O_1)*}_{\vec{n}_2}$ and $\vec{J}^{(O_2)}_{\vec{n}_1}$ for the pole and for the axis, the expressions for the linear momentum and the angular momentum can be expressed in he following vector forms (see Refs.[5] and [6]):

$$\vec{K} = [\vec{\omega}_1, \vec{r}_0] M + \omega_1 \vec{S}^{(O_2)}_{\vec{n}_1} + \omega_2 \vec{S}^{(O_2)}_{\vec{n}_2} \qquad (2)$$

for the linear momentum and

$$\begin{aligned} \vec{L}_{0_1} = {} & \omega_1 [\vec{r}_0, [\vec{n}_1, \vec{r}_0]] M + \omega_1 [\vec{\rho}_C, [\vec{n}_1, \vec{r}_0]] M \\ & + \omega_1 \left[\vec{r}_0, \vec{S}^{(O_2)}_{\vec{n}_1}\right] \omega_2 \left[\vec{r}_0, \vec{S}^{(O_2)}_{\vec{n}_2}\right] + \omega_1 \vec{J}^{(O_2)}_{\vec{n}_1} + \omega_2 \vec{J}^{(O_2)}_{\vec{n}_2} \end{aligned} \qquad (3)$$

for the angular momentum. In the previous vector expressions, $\vec{\rho}_C$ is the mass center vector position of the gyro rotor with respect to the pole O_2 on the spin axis.

It is not difficult to obtain the first derivative of the angular momentum. Then, using the first derivatives of the linear momentum (2) and the angular momentum (3) and applying the principle of dynamic equilibrium of the considered system dynamics, taking into account kinetic pressures on the rotor shaft bearings and active as well as other reactive forces, we obtain two vector equations based on vector equation (1). From these dynamic equilibrium vector conditions, it is easy to obtain vector expressions for kinetic parameters of the considered system dynamics. The obtained equations are possible to be used for a general case when a considered system has two degrees of freedom.

In the vector expressions for the derivatives for the linear momentum and for the angular momentum with respect to time, between other members, the following characteristic members appear: $\vec{R}_{01} \left| \vec{S}^{(O_2)}_{\vec{n}_1} \right|$ and $\vec{R}_{02} \left| \vec{S}^{(O_2)}_{\vec{n}_2} \right|$ in the expression for the linear momentum; $\vec{R}_{011} \left| \vec{D}^{(O_2)}_{\vec{n}_1} \right|$ and $\vec{R}_{022} \left| \vec{D}^{(O_2)}_{\vec{n}_2} \right|$ in the expression for the angular momentum. These characteristic members present a product between the pure kinematic vectors $\vec{R}_{01}$, $\vec{R}_{02}$, $\vec{R}_{011}$, $\vec{R}_{022}$ [for details see references by Hedrih [6]] and the absolute value of the deviation part of the linear mass moment vector $\left| \vec{S}^{(O_2)}_{\vec{n}_1} \right|$ and $\left| \vec{S}^{(O_2)}_{\vec{n}_2} \right|$ as well as the deviation part of the mass inertial moment vector $\left| \vec{D}^{(O_2)}_{\vec{n}_1} \right|$ and $\left| \vec{D}^{(O_2)}_{\vec{n}_2} \right|$ for the pole and the corresponding axis parallel to the component and the axes oriented by the direction of the component of angular velocities. The intensities of the kinematic vectors are: $R_{01} = R_{011} = \sqrt{\dot{\omega}_1^2 + \omega_1^4}$ for the axis oriented by the unit vector $\vec{n}_1$ and $R_{02} = R_{02} = \sqrt{\dot{\omega}_2^2 + \omega_2^4}$ for the axis oriented by the unit vector $\vec{n}_2$. These vectors are orthogonal to the axes of rotation and rotate at angular velocities different from the angular velocity of the corresponding axis.

Here we are going to analyze some properties of the vector rotators using a simple example of a gyro rotor with one degree of freedom.

For example, we take into consideration an eccentric disc (eccentricity is e), with the mass m and the radius r, inclined to the axes of its spinning by the angle β (see Fig.1).

In the special case the support shaft is vertical and the gyro-rotor shaft is horizontal and that shaft axes are without intersection. The normal distance between the axes is ℓ. The angle of spinning around the moveable horizontal axis oriented by the unit vector $\vec{n}_1$ is ϕ_1 and the angular velocity is ω_1. The angle of rotation around the vertical shaft support axis oriented by the unit vector $\vec{n}_2$ is ϕ_2 and the angular velocity is ω_2. The angular velocity of the rotor is $\vec{\omega}_1 = \omega_1 \vec{n}_1 + \omega_2 \vec{n}_2 = \dot{\phi}_1 \vec{n}_1 + \dot{\phi}_2 \vec{n}_2$. The angles ϕ_1 and ϕ_2 are generalized coordinates in the case when we investigate a system with two degrees of freedom. In this case ϕ_1 is an independent generalized coordinate, and the coordinate ϕ_2 is the rheonomic coordinate with kinematic excitation, programmed by the forced support rotation by constant angular velocity. When the angular velocity of the shaft support axis is constant, we have $\phi_2 = \omega_2 t + \phi_{20}$, $\dot{\phi}_2 = \omega_2 = const$, $\dot{\omega}_2 = 0$ (see Refs. [11, 12]).

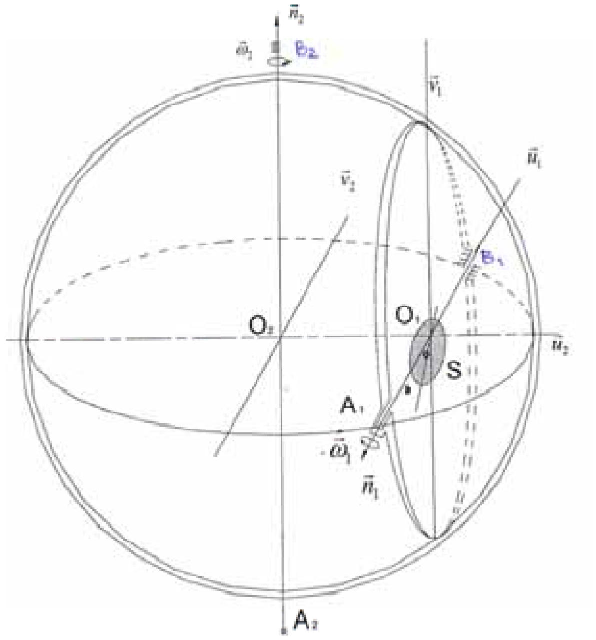


Figure1. Model of a gyro rotor with two-component rotation around the orthogonal axes without intersections

For that example, the differential equation of motion of the gyro rotor spinning reviewed model with two orthogonal axes of rotation without intersection (see Fig.1) is in the form:

$$\ddot{\phi}_1 + \Omega^2(\lambda - \cos\phi_1)\sin\phi_1 + \Omega^2\psi\cos\phi_1 = 0 \tag{4}$$

Where

$$\Omega^2 = \omega_2^2\frac{\left(\varepsilon\sin^2\beta - 1\right)}{\left(\varepsilon\sin^2\beta + 1\right)}$$

$$\varepsilon = 1 + 4\left(\frac{e}{r}\right)^2 \tag{5}$$

$$\lambda = \frac{g(\varepsilon - 1)\sin\beta}{e\omega_2^2\left(\varepsilon\sin^2\beta - 1\right)}$$

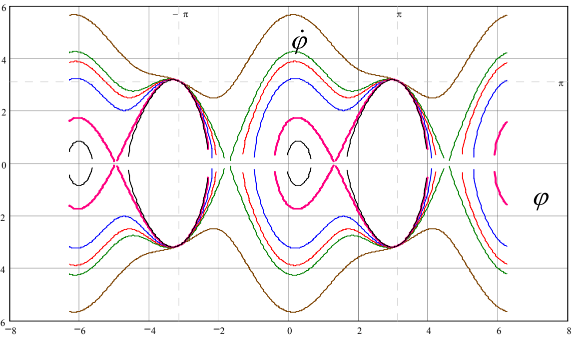
$$\psi = \frac{2me\ell\sin\beta}{J_u - J_v}$$

where: e - eccentricity of the skewly positioned disc, r-radius of the disc, β - angle of inclination, ℓ - normal distance between two rotating axes

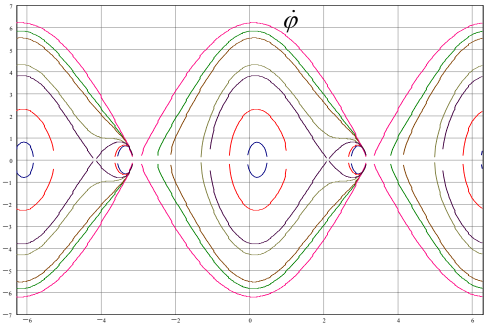
This equation is nonlinear and it was considered in references [6 - 12].

By using the differential equation and the Math Cad program and corresponding equations for phase trajectories, a numerical analysis is carried out. The numerical results are used for the following presentation of the phase trajectory transformation as a result of changing the normal distance between the axes and also the angle of the skew position of the spin axis. Fig.2. shows the transformation of the phase trajectory of the heavy gyro-rotor with the rotating axis without intersection for different values of the disk inclination angle β to the spin axis and for two different initial conditions: **(a*)** $\phi_0 = \pi[rad]$; $\dot{\phi}_0 = \pi[rad/\sec]$ and **(b*)** $\phi_0 = \pi[rad]$, $\dot{\phi}_0 = 0$. Fig.3 presents the transformation of a phase trajectory presentation of the heavy gyro-rotor with the rotating axis without intersection for different values of the normal distance between the axes and for the corresponding initial condition [11, 12].

a* $\phi_0 = \pi[rad]$; $\dot{\phi}_0 = \pi[rad/\sec]$ **b*** $\phi_0 = \pi[rad]$; $\dot{\phi}_0 = 0$



a* $\phi_0 = \pi[rad]$; $\dot{\phi}_0 = \pi[rad/\sec]$



b* $\phi_0 = \pi[rad]$; $\dot{\phi}_0 = 0$

Figure 2. Transformation of the phase trajectory of the heavy gyro-rotor with the rotating axis without intersection for different values of the disk inclination angle β to the spin axis and for two different initial conditions: **(a*)** $\phi_0 = \pi[rad]$; $\dot{\phi}_0 = \pi[rad/\sec]$ and **(b*)** $\phi_0 = \pi[rad]$; $\dot{\phi}_0 = 0$

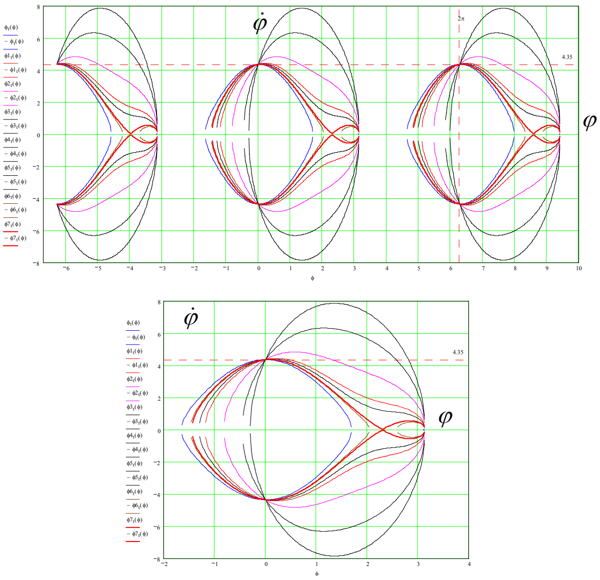
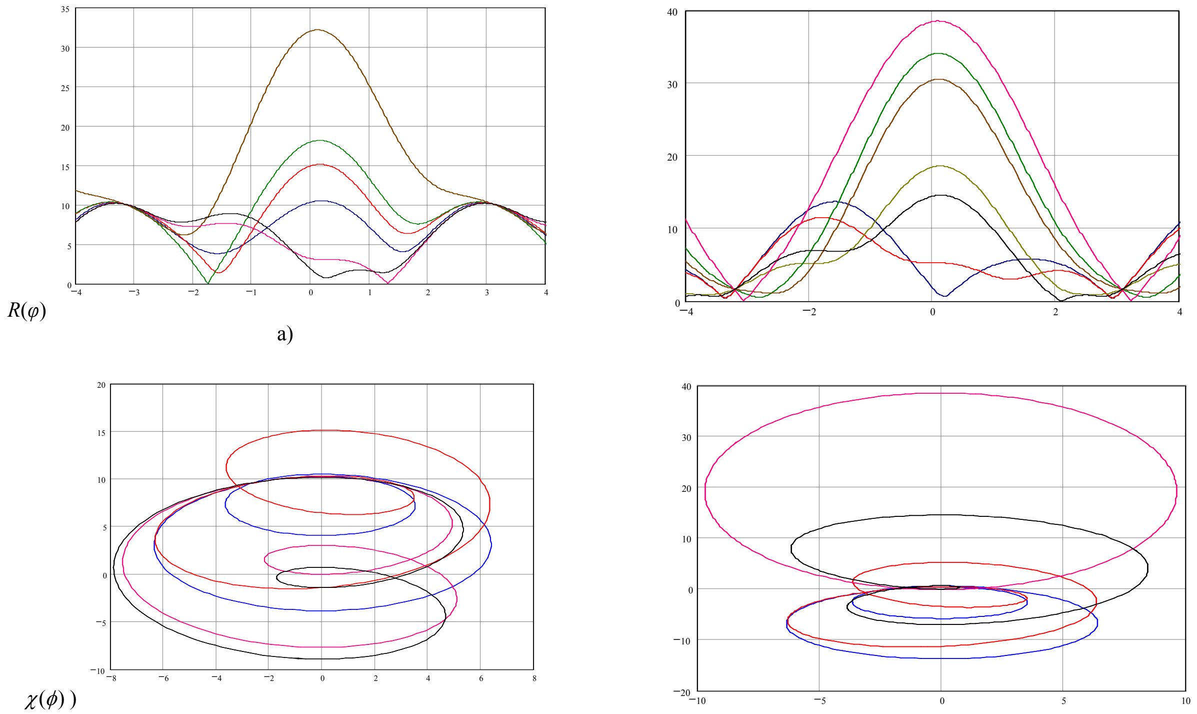


Figure 3. Transformation of the phase trajectory presentation of the heavy gyro-rotor with the rotating axis without intersection for different values of the normal distance between the axes and for the corresponding initial condition.



a)

b)

Figure 4. Vector rotator of the heavy gyro rotor: a) the intensity portrait; b) the hodograph; c) the angular velocity for different values of the angle β and for different initial conditions

Fig.4a) shows the dependence on the vector rotator intensity in the function of the elongation and for different values of the initial parameters h of the energy. The rotator is different from zero so the dynamic pressures on the bearings are different from zero as well. The lowest values of the rotator correspond to the position of the unstable static equilibrium position, while the highest values of the rotator correspond to the position of the stable static equilibrium position.

Fig.4b) shows the rotator trajectories. The shapes of trajectories depend on the parameters of the system.

In practice, it is possible to use the approximation of the derived expressions so that the mathematical description does not change the physicality of the problem.

The approximate theories of the gyroscope dynamics very often analyze problems occurring in practice. Thus, the angular momentum can be expressed by an approximate value, the product of moment of inertia and angular velocity of spinning, which is much higher than the angular velocity of precession.

$$\vec{L}_O \approx \omega_1 \vec{J}^{(O)}_{\vec{n}_1},\ \omega_2 << \omega_1 \qquad (6)$$

This vectorial expression of the angular momentum for the fixed point is approximate and it is acceptable only when the current angular velocity of spinning coincides with the gyro rotors main body axis of inertia, i.e. when the angular velocity of the transmission precession of the movement is zero.

The change of momentum can be attributed to the top of the velocity vector that is proportional to the applied force. Due to the applied force, the vector momentum peak moves in a plane perpendicular to the force plane. Due to the tendency of the gyroscope own axis to deviate, a couple appears leading the axis to overlap with the spinning axis, which is known as the gyroscopic effect. This effect occurs in turbines, vehicles, aircraft, compasses and even children's toys.

Gyrocompass

The review of gyroscopes can start with historical models.

A gyrocompass (see Figs.5, 6, 7) is an instrument that appears at the beginning of the twentieth century in order to be used in polar expeditions for identifying the meridians. When a gyroscope is used as a signal indicator, then the axis around which it spins must always be horizontal and must show the same north-south direction on the Earth. Due to the curvature of the Earth and the friction, the gyroscope axis deviates from the horizontal plane. In order to preserve a constant direction, a free gyroscope is controlled to perform the correction of its deviations in height and side so that its axis returns to the main north-south position. This correction is permanent or occasional so that short-term deviations (during rotation, for example) are not significant. It is a consequence of the effects of gyroscopic pressures caused by the existence of electro-magnetic or electric motors that are integral parts of the device. Compasses are used primarily on aircraft and on smaller fast-moving ships.

German inventor H. Anschutz-Kaempfe constructed a gyrocompass that was applied in an underwater missile. The Sperry gyrocompass, Fig.5, appeared in 1911, was a gyro rotor with a three-phase asynchronous motor with 10000 rpm, placed in a vacuum enclosure, with two connected containers in which mercury (ballistic element) served as a vibration damper and also limited the possibility of rotor movement.

A gyrocompass produced in the form of Anschütz gyrorotor, Fig.6, has two three-phase induction motors. It weighs 2.2 kg and has rotating angular velocities of opposite senses of more than 20,000 rpm. The sensitive element floats freely in the liquid that is a mixture of distilled water, glycerin and acid.

Figure 5. Sperry gyrocompass

Figure 6. Anschütz gyrocompass

Figure 7. Typical gyrocompass

Gyro repeaters are tools that work in conjunction with gyrocompasses. They are often used when measuring the azimuth in order to determine the position or the control. A gyro magnetic compass is a combination of a gyroscope and a magnetic compass. A gyroscope with three degrees of freedom indicates the direction of the continent. To make this possible, the rotation axis must be permanently horizontal. Friction in the shaft bearings and the curvature of the Earth make the axis constantly rise. The axis is returned to the horizontal north-south position by forces caused by an electromagnet or electric motor. The direction is corrected by a magnetic compass. Corrections are made regularly or occasionally by short-term impulses.

Gyro stabilization of the projectile motion – Torpedo

A torpedo (see Figs.8, 9) is an underwater missile used for shooting at ships at long distances. A missile is ejected from the torpedo towards the target. Having flown some distance in the air, the missile dives into the water. From that point it is necessary for the torpedo axis to be constantly directed towards the goal. Therefore, the stern-mounted torpedo stabilizers are coupled with the gyroscopic device. Since the gyroscope axis keeps its direction in space, the gyroscope frame retains the direction and remains focused on the target when the torpedo deviates from the given direction. The frame is connected to a pneumatic device that regulates compressed air intake into the cylinder. The air in the cylinder puts pressure on the piston and make it move thus activating vertical stabilizers that return the torpedo to a specified path. The resistance of air affects the movement of the projectile. Drag force is proportional to the speed of the missile and lies in the plane of motion in the direction of the tangent line at the center of the missile trajectory and in the opposite sense of velocity. This force creates a torque that tends to rotate the missile around the horizontal axis. On the other hand, while in the gun barrel, the projectile acquires high angular velocity, becoming a gyroscope, and deviates from the firing plane. The angular momentum of the projectile falls in the direction of the axis of symmetry of the projectile, which does not coincide with the direction of the tangent of the projectile center trajectory.

Figure 8. Torpedo

During firing, the projectile acquires high angular velocity in the gun barrel. The projectile motion is a type of a gyroscopic motion. It leaves the target plane with some deviation of the programmed path motion. The angular momentum as a vector is in the direction of the projectile axis of symmetry that is not in the tangent direction to the programmed projectile trajectory. The deviational momentum of the projectile motion is in the opposite direction to the dumping forces induced by the air to the projectile motion. These dumping forces induced by air are the forces of projectile motion stabilization of rotation around the transversal axis but a precession motion of the axis of projectile symmetry occurs around the tangent of the projectile center trajectory. Precession is the same direction as the rotation of the projectile [3].

Figure 9. Gyroscope for torpedo stabilization, produced in Germany

The gyroscope for this torpedo stabilization (Fig. 9) is modeled on the principle of the coupled rotation around the corresponding number of axes with intersection in one point. In the analysis of the kinetic parameters of its motion it is possible to use the vector equations presented in Part 2 when the distance between the axes is equal to zero.

Gyro stabilization motion of the ship

The ship can oscillate about the longitudinal axis (rolling), about the transverse axis (swing, pitching) and the vertical axis (turn, yawing). The gyroscopic effects can occur in all the working parts of the vessel and the steam turbine rotor with high angular velocity (speed turbines) which causes additional pressure on the turbine bearings.

Propellers of large ships are brought into motion by turbines placed along the length of the ship and spin with n rpm in the clockwise direction. The angular momentum vector of the turbine motion is in the Oz axis direction and the angular velocity of rotation is in the direction of their axis of rotation. When the vessel is pitching in length (about the axis Oy1), then the law of rotation can be, $\phi = \phi_0 \sin(2\pi t / T)$, where T is the period of rocking of the boat and the maximum deviation. Thus defined, the law of rotation correspondents to the angular velocity, which is $\dot{\phi} = (2\pi\phi_0 / T)\cos(2\pi t / T)$. It is orthogonal to the axis of rotation of their axis of rotation. Due to the rocking of the ship there is a gyroscopic torque that is focused on the negative axis Oz, which causes a high gyroscopic pressure on the turbine bearings.

The appearance of gyroscopic moments is used for the stabilization of mobile structures. Namely, a movable bearing can be moved, due to the action of gyroscopic forces, in the direction along which the vessel axis moves by the shortest route to the Oy_1 axis.

The effects of wind, waves, and water currents can cause additional movement of the ship, rolling, climbing and turning the ship off the given course. On the ship there are devices whose purpose is to maintain the ship at a given speed. These devices are known as autopilots. Autopilots can be used when changing the course to a new direction (changing course). The basic elements of autopilot systems are shown in Fig.10.

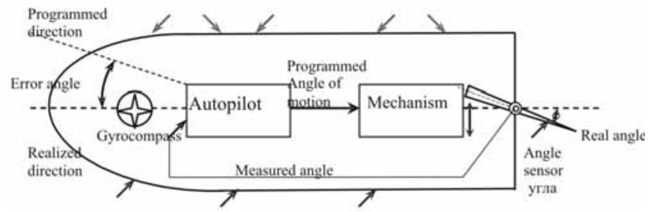


Figure 10. The system for the automatic control of ship motion [4]

The actual direction of the ship is measured by a gyrocompass (or a magnetic compass on small vessels) and compared with the desired direction of the autopilot entries, “skipper "of the ship. The autopilot account requires a steering angle and sends a control signal to the steering mechanism. By monitoring the steering angle and its comparison with the desired angle a control loop can be formed. The rudder provides the steering torque on the ship hull to point to the actual direction of the vessel in accordance with the desired rate, while the winds, waves or currents provide moments that can help or hinder such actions.

Gyro stabilization motion of the aircraft

Aircraft propellers rotate at high angular velocity values about the longitudinal axis of the aircraft. When the plane turns, for example to the left, the pilot has to act to rudders for directions to provide a torque that is perpendicular to the longitudinal axis of the aircraft. The aircraft axis turn creates a couple which a deviation couple is opposed to, causing the front part of fuselage to lift and the rear part to drop. During the right turn, the aircraft tend to dive. These motions are compensated for by rudders. Fig.12 presents an aircraft on a curvilinear trajectory.

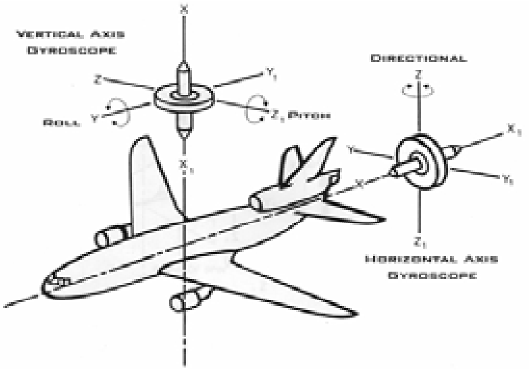


Figure 11. Aircraft with gyro stabilization

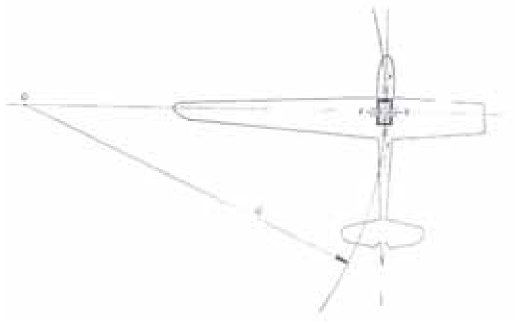


Figure 12. Aircraft on a curvilinear trajectory

Inertial navigation platform

Information on the position, speed and altitude of aircraft or vehicles is very important. When these data are known, the autopilot can keep the plane on a given course or a guided missile system can launch missiles into a particular orbit. Such data are provided by inertial navigation systems that comprise the inertial measurement unit and the control mechanism. Inertial navigation systems (see Fig.13) measure the position of aircraft controlled by appropriate mechanisms. These systems are primarily developed for navigation and missiles and are known as inertial platforms (see Ref. [16]).

Inertial navigation systems have angular and linear accelerometers. Angular accelerometers measure the rotation of the spacecraft in the space around all three axes: pitch (nose up or down), yaw (nose left and right) and roll (in the direction or opposite clockwise direction). Linear accelerometers measure the vehicle moving in the direction of all three axes. Computers process the received data to determine the current position of the aircraft.

Some systems place the linear accelerometers on PTO gyro stabilized platforms, which allows the platform to rotate around all three axes of rotation. In order to neutralize the gyroscopic precession, two gyroscopes with axes intersected at the right angle are mounted (a gyroscopic couple: two gyroscopes of the same inertial properties and of angular velocities of the same intensity). Two gyroscope pairs that support and strengthen the platform remain solid even when the props turn in any direction.

This is the basis of inertial navigation systems. The platform turning results in the action of the appropriate sensors on the axis support. If there are three accelerometers on the platform, it is possible to determine where the vehicle is going and how to change the direction of its motion.

An Apollo spacecraft had a gyro stabilized platform with three axes with a particular attention paid to moving mechanical parts that are very sensitive to wear and can stop motion which may cause a delay of propellers. To avoid this, a fluid bed chamber or a float mounting gyrostabilizer platform can be used. Such systems have high accuracy.

Fluid bearings are very slippery because oil or inert gas is let through holes for spherical platforms to rotate freely.

Solar cells or transformers are used to move the platforms.

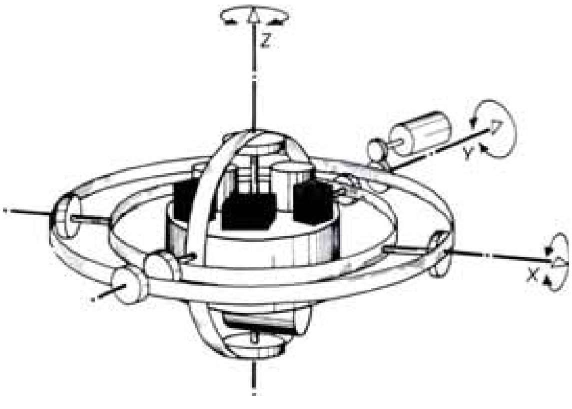


Figure 13. Inertial navigational platform

Gyro stabilized platforms are applied in long-range missiles because of the need that some devices are in a fixed position relative to the absolute coordinate system. A scheme of the Titan missile gyro stabilized platform is given in Fig.14 [3]. Gyroscope (1) and motor (2) stabilize the platform in relation to the rolling axis (x - axis), gyroscope (3) and motor (12) stabilize the platform in relation to the pitch axis (z- axis) and gyroscope (4) and motor (2) stabilize the platform in relation to the turning axis (y - axis). When the platform (11) is affected by a disturbance aiming at deviating the platform from the given direction, the precession occurs. The signal goes from the precession transducer angle through the amplifier to the stabilization motor. The motor torque is transferred to the platform that is contrary to the disturbance torque and thus the platform retains its position.

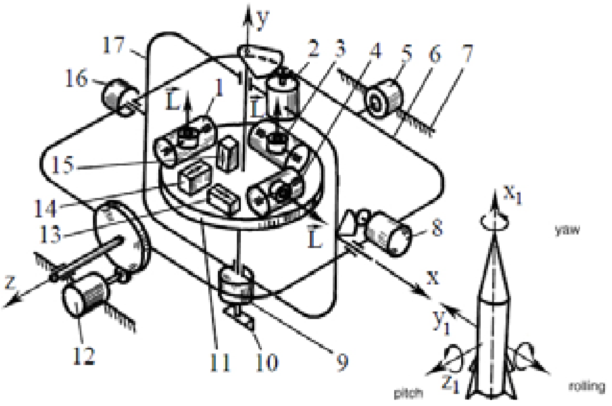
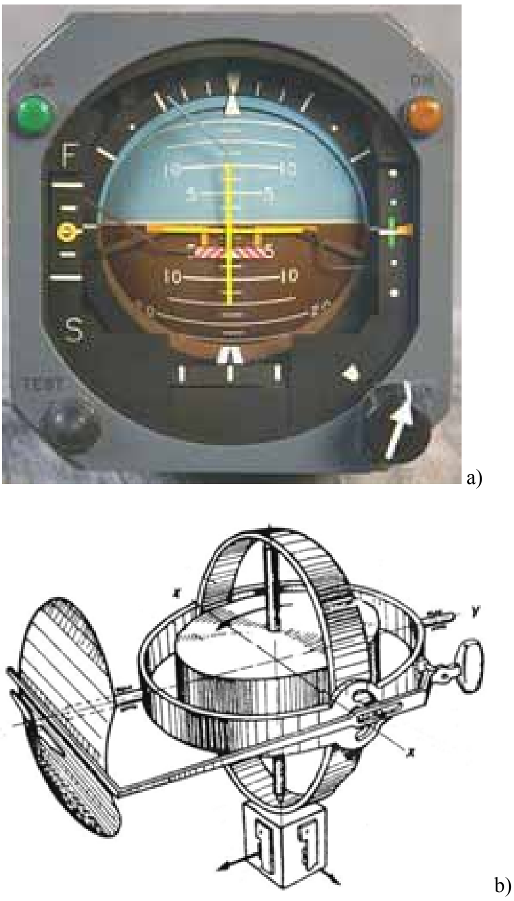


Figure 14. Gyro stabilized platform [3]

For each gyroscope of this gyro stabilized platform (Fig.14) it is possible, using the principle of coupled rotations around a corresponding number of axes with the intersection in one point, to analyse the kinetic parameters of their motion presented by vector equations in Part 2 when the distance between the axes is equal to zero.

Artificial horizon

When weather conditions are such that flight pilots lose the horizon, visual flight conditions suitable for replacing the real horizon should be created. There is an instrument that shows the aircraft position in relation to the natural horizon, no matter whether the aircraft is tilted around its longitudinal or transverse axis. The main element of this instrument is a gyroscope with three degrees of freedom (three axes of rotation). The axis of rotation of the gyroscope tends to keep the direction perpendicular to the surface of the Earth. On the front part of the instrument, an aircraft is represented by a stylized silhouette in front of the moving line of the artificial horizon. Thus the pilot may have information on the transverse and longitudinal position of the aircraft. In order to prevent tilting its gyroscope axis, special rectifier devices generate an appropriate moment. In the aircraft designed to perform the evolution, artificial horizons (see Fig.15) have additional devices for gyroscope blocking. This is necessary because at higher pitch some parts of the gyroscope hit into special limiters, which leads to their damage.



a)

b)

Figure 15. a) Artificial horizon, b) Physical model of the artificial horizon

Turn indicators

Turn indicators are gyroscopic instruments that show the changing of aircraft direction in a straight line. The basic operating element is a gyroscope with two degrees of freedom. Fig.16 shows the gyroscope (1) rotating at high angular velocity around its main axis x-x, while its frame (2) can rotate around the axis y-y perpendicularly to the main axis. Both axes lie in the horizontal plane, but the main one is normal to the longitudinal axis of the aircraft. In turn, the axis X-X forcibly change its position in space, its consequence being a force that tends to turn the frame around the axis Y-Y. The embedded springs (4) resist to the free movement of the frame so that the strain produces the torque about the axis Y-Y. At a particular frame tilt, the torques are equal and the hand (3), tightly connected to the frame, resumes its balanced position that determines the shift size. A larger shift corresponds to a higher torque and a greater spring deformation.

New trends in construction

Today, the gyroscope can be found in a number of devices: mobile phones, navigation systems and other portable devices. The Hubble Space Telescope (see Fig.17) is in the orbit around the Earth and monitors cosmic bodies and other phenomena in the universe. It circles around the Earth under an inclination of 28.5 degrees and the average visit is 96 minutes at a speed of 28,000 km / h. Hubble has gyroscopes that enable very precise measurements.

Modern gyroscopes have high performances. Not only do they feature high accuracy, but they are of small size and low cost.

Figure 17. The Hubble Space Telescope

Being a part of important instruments, a gyroscope is subjected to research and innovation. The gyroscope instruments work in a variety of conditions. Many research projects study them, especially the behavior of the rotor in some conditions such as magneto-hydrodynamic fields [2]. It is shown that under certain conditions the chaotic motion of the rotor can occur. Mechanical gyroscope parts are exposed to wear. Nowadays many studies pay attention to a new generation of gyroscopes and dynamical properties [1]. The first gyroscopes were gyroscopes of large dimensions. Now, it is possible to have very small gyroscopes, micro and nano ones.

In inertial systems and suspension systems in cars, mainly optical gyroscopes of high performances are built in but they are big and expensive. Gyroscopes functioning on the basis of Coriolis force are cheaper and of lower performance. Therefore, among these two categories of gyroscopes there is space for a new type of gyroscopes. It is designed with a magnetic levitating gyroscope rotor without a bearing that rotates very fast (see Fig.18). The high-speed rotation of the rotor has such a thin angular momentum to behave like a gyroscope. With its simple design, the device can be mass-produced based on microelectronics.

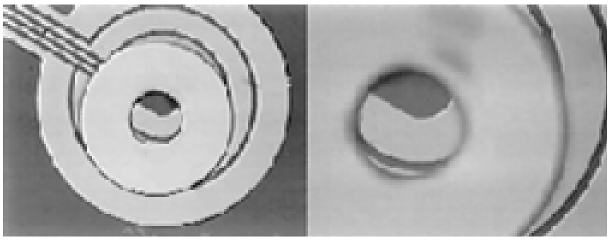


Figure 18. Disc of 400 μm in diameter during levitation

Micro gyroscopes and nano gyroscopes

At the Tel Aviv University (TAU) researchers have promoted a gyroscope device that can fit on the top of a pin. This will lead to revolutionary changes in smart phones and devices in medicine and lead to new solutions in software and hardware technology. Professor Koby Schener from this University carried out research with intention to scale down optical gyroscopes. Schener's gyroscope consists of thin semiconductor lasers with a diameter of several tens of micrometers, which rotate around three axes forming an angle of 90 degrees. The changes in the wavelength are measured depending on coherent light produced by the rotation. This information is then translated into GPS coordinates. Such a device can be used not only for positioning without a GPS signal (important for regions with no signals) but, very importantly, for medical applications, installation in pill cameras that do not need artificial radio-tracking. Schener has developed a laboratory device that is under investigation. It is believed that in a few years the device will find its purpose outside laboratories (Optic Express).

Conventional gyroscopes weigh around 1 - 2 kg and as such can be installed in airplanes. If necessary, they can be installed in small objects, products of modern technology such as telephones. The installation of such gyroscopes would be strictly limited.

Concluding Remarks

In the end, we conclude that our intention to present a short historical review of models and constructions of gyroscopes and present models, as well as their component motions, is realized. Since functioning of all mechanical gyroscopes is based on coupled rotations around more axes with one point of intersection, we present the basic vector equations of the dynamic equilibrium of the rigid body with coupled rotations around two axes without intersection as a generalized case.

Since imprecision and errors in the functioning of gyroscopes occur due to unbalancing, eccentricity and lack of intersection of the axes in one point, it is very useful to investigate nonlinear phenomena in the dynamics of the rigid body coupled rotations around two axes without intersection, as presented in Chapter 2 of this paper. The graphical presentation of phase trajectory transformation shows the changes of position of stationary gyro rotors depending on the normal distance between the axis and the skew angle between the spin axis and one central main inertial axis.

The first part contains applications of mass moment vectors and vector rotators introduced by K. Hedrih for expressing vector expressions for the linear momentum and the angular momentum and their derivatives for describing the coupled rotation of the rigid body around two axes of rotations that are without intersection. We conducted an analysis of characteristic meanings of some members in vector expressions for the linear momentum and the angular momentum and their connection with the deviation moment. The deviation moment is the main parameter in mechanical gyro stabilization. The deviational couple contains the vector rotator which shows that the direction of the deviational moment generally rotates with different angular velocity from the angular velocity of the gyro rotor. This difference is a result of errors in construction of gyrostabilizers.

The second part presents an overview of the history and the present of gyroscope models and their construction realization based on rigid body coupled rotations. Most of the listed gyrostabilizers are mechanical systems which rotate around a fixed point by coupled rotation around a number of axes with the intersection in one point.

With the development of micro and nano technology, a new class of gyroscopes appears. It points to the sources of gyroscopic moments that enable the stabilization and control of movement. In a recent class of gyro-rotors and gyrostabilizers, new kinds of motion appear.

With the development of micro and nano technology, there is a need for a new class of high precision gyroscopes of small dimensions. Current trends of gyro-technologies are being developed in two directions. According to one group of researchers, traditional gyroscopes should be smaller, lighter and more mobile. The integration of optics and mechanics was made in optical mechanic gyroscopes [1, 2]. Instead of rotating disks, new types of gyroscopes have a very thin plate (see ref. [15] by. Martynenko and all, (2008)) (solid state) and computer chips replace mechanical components. A machined hollow ball of piezo-electric materials such as quartz is an essential element of the Brandy Sniffer gyroscope. This gyroscope almost has no moving parts; it is very accurate but expensive.

New and cheaper high-performance gyroscopes will be far more significant in car, submarine and aircraft navigation. Scientists all over the world are already working in this field.

A U.S. team of the scientists from the Oak Ridge National Laboratory has constructed a highly sensitive silicon gyroscope. This team believes that this new generation of gyroscopes will be very important for navigation and geolocation where global positioning systems cannot be used.

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Istorija i sadašnjost modela žiroskopa i vektori rotatori

Cilj ovog rada je da se da kratak pregled modela i konstrukcija žiroskopa kao i njihovih komponentnih kretanja. Svi mehanički žiroskopi baziraju na spregnutim rotacijama oko nekoliko osa koje se seku u jednoj tački.

Upravo je to razlog što se ovaj rad sastoji od dva dela. U prvom delu dati su kinetički parametri rotacije krutog tela oko dve mimoilazne ose u slučaju kada je telo ekscentrično i koso postavljeno u odnosu na osu sopstvene rotacije. Takođe su dati i neki rezultati K.Hedrih i LJ. Veljović u oblasti nelinearne dinamike koji se mogu primeniti za objašnjenje pojedinih nelinearnih fenomena u dinamici žiroskopa. Na osnovu vektora momenata mase i vektora rotatora, uvedenih od strane K. Hedrih, dati su neki karakteristični članovi u izrazima za vektor količine kretanja i vektor momenta količine kretanja kao i u njihovim izvodima u slučaju kada se girorotor obrće oko dve mimoilazne ose. Takođe su date grafičke prezantacije faznih portreta i kinematičkog vektora rotatora za različite vrednosti rastojanja između mimoilaznih osa kao i ugla nagiba. Nepreciznosti i greške u radu žiroskopa nastaju zbog ekscentričnosti i debalansa girorotora u odnosu na osu sopstvene rotacije, kao i usled razmaka između obrtnih osa, pa su dati grafici važni za objašnjenje i donošenje korektivnih rešenja neophodnih za pravilan rad žiroskopa.

U drugom delu rada dat je istorijski pregled modela žiroskopa i njihovih konstrukcionih realizacija kao i prikaz savremenih modela, a koji se primenjuju za upravljanje i stabilizaciju kretanja brodova, letilica, vozila i torpeda. Sa razvojem mikro i nano tehnologije javile su se i nove vrsta žiroskopa.

Ključne reči: **žiroskop, žirostabilizator, žirokompas, rotor žiroskopa, žirostabilisana platforma, inercijalna platforma, rotator, količina kretanja, moment količine kretanja, vektorska analiza.**

История и настоящее модели гироскопа и векторы – вращающие устройства

Цель этой работы – дать короткий обзор моделей и конструкций гироскопов, а в том числе и движений их составных частей. Все механические гироскопы основывают на вращении пары сил вокруг нескольких осей, пересекающихся в одной точке.

Именно это является причиной, почему настоящая работа состоит из двух частей. В первой части приведены кинетические параметры вращения жёсткого тела вокруг двух непересекающихся осей в случае когда тело установлено эксцентрично и косо по отношению к осе собственного вращения. Здесь тоже приведены и некоторые результаты К. Хедрих и Л. Велёвич в области нелинейной динамики, применимые для пояснений отдельных нелинейных феноменов в динамике гироскопа. На основании вектора момента массы и вектора вращающего устройства, введеных со стороны К. Хедрих, здесь тоже приведены некоторые характерные элементы (члены) в выражениях для вектора количества движения и для вектора момента количества движения, а в том числе и в их выводах в случае когда гироскоп вращается вокруг двух непересекающихся осей. Тоже приведены графические представления фазовых изображений и кинетического вектора вращающего устройства для различных величин растояний между непересекающимися осями, а в том числе и углом наклона.

Неточности и ошибки в работе гироскопа возникают из-за эксцентричности и незабаллансированности гидроротора по отношению к осе собственного вращения, а в том числе и из-за промежутка между вращающимися осями, из-за чего приведённые графики очень важны для пояснений и принимания коррективных решений, необходимых для правильной и постоянной функции и работы гироскопа.

Во второй части настоящей работы показан исторический обзор моделей гироскопов и их структурных осуществлений, а в том числе показаны и современные модели, употребляющиеся для управления и стабилизации движения кораблей, летающих аппаратов, перевозочных средств и торпед. Наряду с развитием микро и нано технологий появились и новые виды гироскопов.

Kly~evwe slova: **гироскоп, гиростабилизатор, гирокомпас, ротор гироскопа, гироустановившаяся площадка, инерционная площадка, вращающее устройство, количество движения, момент количества движения, векторный анализ.**

Histoire et présent des modèles du gyroscopes et les vecteurs rotateurs

Le but de ce travail est de donner un court tableau des modèles et des constructions des gyroscopes ainsi que leurs mouvements composants. Tous les gyroscopes mécaniques sont fondés sur les rotations couplées autour de plusieurs axes qui ont un point d'intersection commun. C'est pourquoi ce travail se compose de deux parties. La première partie présente les paramètres cinétiques pour la rotation du corps rigide autour de deux axes sans intersection lorsque le corps excentrique est posé en biais par rapport à l'axe de sa propre rotation. On a donné aussi quelques résultats obtenus par K.Hedrih et Lj.Veljović dans le domaine de la dynamique non linéaire qui peuvent s'appliquer dans l'explication de certains problèmes dans la dynamique de gyroscope. Basés sur les vecteurs du moment de masse et des vecteurs de rotation, introduit par K.Hedrih, on a présenté des articles caractéristiques dans les expressions pour le vecteur de la quantité du mouvement ainsi que dans leurs dérivées pour le cas ou le gyroscope tourne autour de deux axes sans intersection. On a donné aussi les présentations graphiques des portraits de phase et le vecteur cinématique de rotateur pour les différentes valeurs des distances entre les axes sans intersection ainsi que l'angle d'inclinaison. Les non précisions et les erreurs dans le fonctionnement du gyroscope sont dus aux excentricités et déséquilibre des rotateurs du gyroscope par rapport à l'axe de rotation et à l'écart entre les axes tournant. Pour cela les graphiques donnés sont importants pour l'explication et l'élaboration des solutions correctives nécessaires pour le bon fonctionnement du gyroscope. Dans la seconde partie du travail on a donné le tableau historique des modèles de gyroscope et leurs réalisations constructives ainsi que le tableau des modèles actuels utilisés pour la commande et la stabilisation du mouvement des bateaux, avions, véhicules et torpilles. Avec le développement des macro et nano technologies les nouveaux types de gyroscopes ont apparu.

Mots clés: **gyroscope, stabilisateur gyroscopique, gyrocompas, rotor de gyroscope, plateforme gyroscopique stabilisée, plateforme inertielle, rotateur, quantité de mouvement, moment de la quantité de mouvement, analyse vectorielle.**