

Computational Modal Analysis of the LASTA Aircraft

Jelena Dimitrijević¹⁾
Predrag Kovačević¹⁾

This paper presents the results of the Computational Modal Analysis of a complete finite element model of the LASTA aircraft. The results of calculation, normal frequencies and normal mode shapes, shown here, will be used for the further aeroelastic analysis to confirm that this aircraft is free from flutter in the range of the designed speed. A short theoretical review of the Computational Modal Analysis and Lanczos Method is presented in this paper.

Key words: aircraft, propeller aircraft, trainer aircraft, eigen oscillations, modal analysis, numerical analysis, Finite Element Method.

Introduction

DURING designing, manufacturing and examination of an aircraft prototype it is required to confirm that the aircraft is free from flutter in the range of the designed speed. The flutter analysis and the flight test are used to confirm it.

This paper presents the results of the Computational Modal Analysis, i.e. the computation of the natural frequencies and the natural mode shapes of the LASTA aircraft by using the Finite Element Method. First, a finite element model of the LASTA aircraft is done. Then, the natural frequencies and the natural mode shapes are estimated using the Computational Modal Analysis. All this is done using the I-DEAS software. The results of the Computational Modal Analysis will be used for the Flutter Analysis, i.e. the calculation of unsteady generalized aerodynamic forces and critical flutter speeds of the LASTA aircraft.

Finite Element Method and Dynamic Analysis

Finite element modeling is used in the design cycle of new mechanical parts or systems because it offers a number of advantages [1], [2]. Using the Finite Element Method (FEM), a mathematical model of the structure can be built much more before the first prototype structure is even built. In addition, it can be determined how the mass or stiffness changes of the structure affect its static and dynamic characteristics and this is its great advantage.

All structures, regardless of their simplicity, have an infinite number of degrees of freedom (DOF). With generalization of the matrix methods and with the advantages of modern computers, engineers started using the Finite Element Method in the static and dynamic analysis of large structures. One of the main objectives of selecting an FE model is to reduce the infinite degree of freedom of systems with complex geometry and material properties, to a model with a limited DOF, but which will

capture the significant physical behavior of the system. The Finite Element Method is one of the most powerful popular mathematics modeling techniques. In brief, the basis of the FEM is the representation of a body or a structure by an assemblage of subdivisions called finite elements. These elements are interconnected at joints, which are called nodes or nodal points. Simple functions are chosen to approximate the distribution or variation of the actual displacements over each finite element. Such assumed functions are called displacement functions. The final solutions yield to displacements at nodes. The attraction of the FEM is that it is commercially available in the form of a user-oriented computer program.

In practice, the finite element model of any structure approximates a real structure using finite elements. If the behavior of a structure is too complex, the recipe is to divide it into substructures that are more manageable [3]. If these substructures are still too complex, the subdivision process is continued. The first step in the Finite Element Method is to subdivide a structure, or an assembly, into many smaller elements such as plates, beams, shafts, etc., or substructures. The accuracy of the model depends on a degree of the users' skill in choosing the discretization. The most important is to choose an appropriate element type and size for the parts or substructures, correctly connect all parts and choose appropriate boundary conditions. Then the overall equations of motion of the structure are made from the equations describing the motions of each of individual parts, or substructures, plus all the boundary conditions at the connection points between parts or substructures.

The Finite Element Method is typically used for the following analyses:

- Static analysis (static strength to observe the high stress and strain areas under static loads),
- Dynamic analysis (modal analysis, dynamic responses to external dynamic loads, dynamic strength and fatigue).

One of the most important fields of using the Finite Element Method is in the dynamic analysis [4]. By using

¹⁾ Military Technical Institute (VTI), Ratka Resanovića 1, 11132 Belgrade, SERBIA

the finite element model, it is possible to simulate the response of the real world to an external stimulus. External forces might be of short duration and high amplitude, i.e. impulse forces, which cause immediate structure crash. Furthermore, they may be of long duration and cyclic in nature and could cause fatigue damage of the structure during a long period. In that way, it is possible to predict structure damage, its crash or fatigue life even before the first prototype is constructed.

In the static analysis, the element size is smaller and a total number of elements is larger. In the dynamic analysis, the element size is bigger because the calculation method is more complex and then the total number of elements is smaller. Time-varying characteristics make the dynamic analysis more complicated and more realistic than the static analysis [5]. Two basic aspects of the dynamic analysis, which differentiate it from the static analysis, are:

- Dynamic loads are applied as a function of time.
- Time-varying external dynamic load induces a time-varying response (displacements, velocities, accelerations, forces, and stresses).

The primary task for the dynamic analyst is to determine the type of analysis to be performed. The choice of the appropriate mathematical approach depends on the nature of the dynamic analysis in many cases. The extent of the information required from a dynamic analysis also dictates the necessary solution approach and steps.

Two basic classifications of the dynamic analysis are:

- free vibrations and
- forced vibrations.

The free vibration analysis is used to determine the basic dynamic characteristics of the system when the external forces are zero, i.e., there is no applied load. Free vibrations can be damped and undamped. If damping is neglected, than it is an undamped free vibration.

Natural Frequency and Natural Mode Shape Computational Modal Analysis

The determination of natural (normal) frequencies and natural (normal) mode shapes of the structure with damping neglected is the first step in the dynamic analysis. According to natural frequencies and natural mode shapes that characterize the basic dynamic behavior of the structure, we have the indication of how the structure will respond to dynamic loading. The natural frequencies of a structure are the frequencies at which the structure naturally tends to vibrate if subjected to a disturbance. An appropriate deformed shape of the structure at a specific natural frequency of vibration is its normal mode shape, or the normal mode of vibration. Natural frequencies and natural mode shapes of any structure are estimated owing to the Computational Modal Analysis using finite element model of that structure [6]. They depend on the structural properties and boundary conditions.

Computation of natural frequencies and natural mode shapes of a structure could be of great significance. They tell us at what frequencies the structure can be excited into resonant motion. In many cases, this information is sufficient for modifying the structural design in order to reduce noise and vibration. In addition, a dynamic interaction between a component and its supporting structure is very important because the component could cause structural damage or failure if its operating frequency is close to one of the natural frequencies of the structure. A transient response, frequency response, response spectrum

and modal frequency analysis can be based on the results of the natural frequency and natural mode shapes. Another important field of use is in the experiment when the natural frequencies and natural mode shapes can be used to guide the experiment, for example to help the researchers to indicate at which frequencies to vibrate the structure and to indicate the best location for the accelerometers. In addition, design changes can also be evaluated by using natural frequencies and natural mode shapes. Finally, the knowledge of the natural frequencies and natural mode shapes for a particular structure is important for all types of dynamic analyses.

The dynamic response verification process [7] of the project is shown in Fig.1. The prototype can be virtual (finite element model) or real (physical prototype). Analytical, computational results (natural frequencies and natural mode shapes) are obtained based on the virtual prototype using the Computational Modal Analysis and the experimental results based on the real prototype using Experimental Modal Analysis, Vibration Tests, or Flight Tests. The verification of the finite element model and the results of the Computational Modal Analysis can be done after the experiments when it is possible to compare the modal parameters (natural frequencies and natural mode shapes) obtained computationally and experimentally. The correction of the finite element model and then natural frequencies and natural mode shapes can be done according to the experimental results, Fig.1. A new, calibrated virtual prototype may be further used for some other kinds of dynamic phenomena or for the dynamic analysis and design modification.

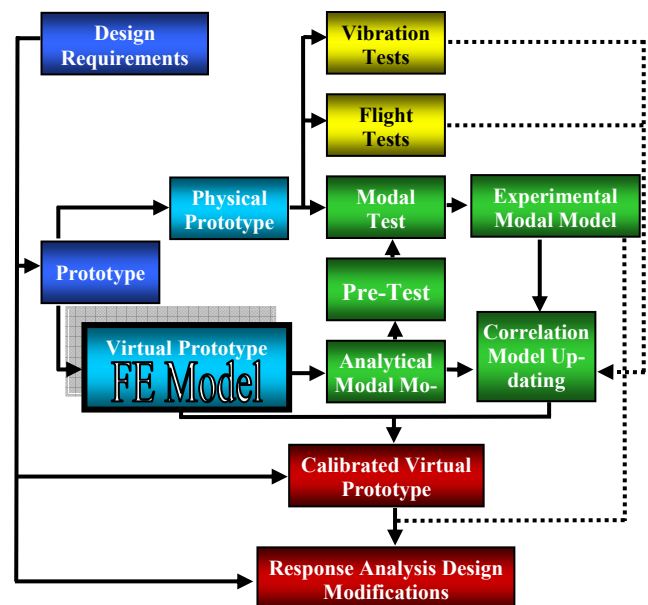


Figure 1. Dynamic response verification process

Theoretical review equation of motion for undamped free vibration

Natural frequencies and natural mode shapes are obtained as the results of the eigenvalue problem [8], i.e. equation of motion for undamped free vibrations.

The general equation of motion in the matrix form is:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P\} \quad (1)$$

where:

$[M]$	- mass matrix
$[C]$	- damping matrix
$[K]$	- stiffness matrix
$\{P\}$	- force vector
$\{u\}$	- displacement vector

Disregarding the dissipative forces $[C]\{\dot{u}\}$ and when the external (excitation) forces are zero, $\{P\}=0$, the equation of motion (1) reduces to equation of motion for undamped free vibrations and its matrix form is:

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \quad (2)$$

Dissipative forces are neglected because there is no possibility to estimate the damping matrix in this phase of the project. Damping effects can only be included approximately and that is way damping effects are neglected altogether in many analysis. Damping is obtained experimentally after the Experimental Modal Analysis when natural frequencies and natural mode shapes are obtained as well.

Using the Modal Decomposition Method and the condition that it is a harmonic vibration $\{\ddot{u}\} = -\omega^2 \{u\}$, eq.(2) reduces to the eigenvalue problem:

$$([K] - \omega^2 [M])\{\phi\} = 0 \quad (3)$$

where:

$\{\phi\}$	- eigenvector, or natural mode shape
ω	- circular natural frequency.

Eq. (3) is called eigenproblem and can be interpreted as a set of homogeneous algebraic equations. It represents base for the eigenvalue problem calculation. The first necessary step in the dynamic analysis of the structure is to solve the characteristic equation.

By pre-multiplying eigenequation (3) by the matrix $[M]^{-1}$, or by the matrix $[K]^{-1}$, it can be written as a standard eigenvalue problem:

$$[A]\{\phi\} = \lambda\{\phi\} \quad (4)$$

where:

$[A] = [M]^{-1}[K]$	- square dynamic matrix
$\lambda = \omega^2$	- eigenvalue

The matrix $[M]^{-1}$ must be non-singular. A consistent mass matrix always satisfies this condition. A stiffness matrix will satisfy this condition if the rigid body motion degrees of freedom are suspended. If the rigid body motion degrees of freedom exist, the Shift Frequency Method [9] is applied and it transforms eq. (3) to:

$$([K'] - (\omega^2 \pm S^2)[M])\{\phi\} = 0 \quad (5)$$

where:

$$[K'] = [K] \pm S^2 [M] \quad (6)$$

Eq. (3), eigenequation, has two possible solutions:

1. If $\det([K] - \omega^2 [M]) \neq 0$, the only possible solution is $\{\phi\}=0$. This is a trivial solution, which does not give any useful information from a physical point of view, except that there is no motion.

2. If $\det([K] - \omega^2 [M]) = 0$, the solution is non-trivial $\{\phi\} \neq 0$.

The eigenvalue problem is then reduced to the following equation:

$$\det([K] - \omega^2 [M]) = 0 \quad (7)$$

and for $\omega^2 = \lambda$ is:

$$\det([K] - \lambda [M]) = 0 \quad (8)$$

The determinant is zero only at the set of discrete eigenvalues λ_i , or ω_i^2 . The corresponding i -th eigenvector is $\{\phi\}_i$. So the eq. (3) can be rewritten as:

$$([K] - \omega_i^2 [M])\{\phi\}_i = 0 \quad i=1,2,3\dots \quad (9)$$

The number of eigenvalues and eigenvectors is equal to the number of degrees of freedom. In structural dynamics, each eigenvalue corresponds to the natural frequency and each eigenvector to the natural mode shape. Together they define one free vibration mode of the structure. The i -th eigenvalue λ_i is related to the i -th natural frequency as follows:

$$f_i = \frac{\omega_i}{2\pi} \quad (10)$$

where:

f_i	- i -th natural frequency
-------	-----------------------------

$$\omega_i = \sqrt{\lambda_i}$$

The mode shape of a linear elastic structure that is in free or forced vibration, at any given time, is a linear combination of all of its natural mode shapes, or mathematically:

$$\{u\} = \sum_i \{\phi\}_i \xi_i \quad (11)$$

where:

$\{u\}$	- vector of physical displacements
$\{\phi\}_i$	- i -th natural mode shape
ξ_i	- i -th modal displacement

There is a property of orthogonality between eigenvectors. It is very easy to present the evidence for these properties. From eq. (9) with respect to the equation $\omega_i^2 = \lambda_i$, for one vector $\{\phi\}_i$ is obtained:

$$[K]\{\phi\}_i = \lambda_i [M]\{\phi\}_i \quad (12)$$

and after pre-multiplying by another transposed vector, $\{\phi\}_j^T$ the previous equation becomes:

$$\{\phi\}_j^T [K]\{\phi\}_i = \lambda_i \{\phi\}_j^T [M]\{\phi\}_i \quad (13)$$

In the same way, starting from the vector $\{\phi\}_j$ and pre-multiplying by vector $\{\phi\}_i^T$ equation (12) becomes:

$$\{\phi\}_i^T [K]\{\phi\}_j = \lambda_j \{\phi\}_i^T [M]\{\phi\}_j \quad (14)$$

If $[K]$ and $[M]$ are real symmetric matrixes, it can be written:

$$\{\phi\}_i^T [M] \{\phi\}_j = \{\phi\}_j^T [M] \{\phi\}_i \quad \{\phi\}_i^T [K] \{\phi\}_j = \{\phi\}_j^T [K] \{\phi\}_i$$

Therefore, after subtracting eq. (14) from eq. (13) it is obtained:

$$0 = (\lambda_i - \lambda_j) \{\phi\}_i^T [M] \{\phi\}_j$$

And then if $(\lambda_i - \lambda_j) \neq 0$, i.e. if $i \neq j$:

$$\{\phi\}_i^T [M] \{\phi\}_j = 0 \text{ and } \{\phi\}_i^T [K] \{\phi\}_j = 0 \quad (15)$$

Eq. (15) define the orthogonal properties of the vectors with respect to the system mass or stiffness matrixes.

If $i = j$, then these two vectors are not orthogonal and then eq. (14) is equal to some non zero scalar constant:

$$\{\phi\}_j^T [M] \{\phi\}_j = m_j, \quad m_j - j\text{-th generalized mass} \quad (16)$$

and

$$\{\phi\}_j^T [K] \{\phi\}_j = k_j,$$

$$k_j = \omega^2 m_j - j\text{-th generalized stiffness} \quad (17)$$

In addition, from eq. (16) and eq. (17), Rayleigh's equation is obtained:

$$\omega_j^2 = \frac{\{\phi\}_j^T [K] \{\phi\}_j}{\{\phi\}_j^T [M] \{\phi\}_j} \quad (18)$$

Eq. (15) are known as the orthogonal property of normal modes, which means that each normal mode shape is unique and different from any other. In addition, one normal mode shape cannot be obtained through a linear combination of any other normal mode shapes.

For practical examination natural mode shapes can be scaled. If one of the elements of the eigenvectors is assigned a certain value, the rest of the $(n-1)$ elements are also fixed because the ratio between any two elements is constant. Thus, process of adjusting the elements of the natural mode shapes to make their amplitude unique is called normalization. The resulting scaled natural modes are called orthonormal modes. The mass (inertial) normalization is the default method of eigenvector normalization and scales each eigenvector, natural mode shape, to a unit value of the generalized mass:

$$\{\phi\}_i^T [M] \{\phi\}_j = 1 \quad (19)$$

When the structure is free, without support, then, besides elastic modes, it has rigid-body modes. The natural frequency of the rigid-body mode is equal to zero.

Methods for natural frequency and natural mode shape estimation

There are large numbers of numerical methods of the real eigenvalue extraction for its different type and size. While most of the methods can be applied to all problems, the choice is often based on the efficiency of the solution process.

The methods of eigenvalue extraction belong to one of the following groups [10]:

- Methods with direct eigenvalue extraction finding zero determinants.
- Iterative methods (Inverse Iteration Method, Vector Iteration Method, Subspace Iteration Method) - In an iterative method, a certain number of roots are extracted at a time by iterative procedures applied to the original dynamic matrix.
- Transformation methods (Jacobi Method, Ritz Method, **Lanczos Method**, Givens Method, QL Hausholder Method, QZ and QR Method) - In the transformation method, the matrix of coefficients is first transformed into a special form (diagonal or tridiagonal) from which the eigenvalues may easily be extracted.
- Reduction method (Guyan Method) - In the reduction method, the structure is divided into substructures and after the eigenvalue extraction, they are synthesis again.

The characteristics of the Sturm series, the procedure for reorthogonalization and deflection, can be used to improve methods. Some of these methods, as presented in Fig.2, are better for small degree of freedom systems (QL Hausholder Method, QZ and QR Method, Guyan Method and Jacobi Method), while the others (Subspace Iteration Method, Givens and Lanczos Method) are superior for large structures.

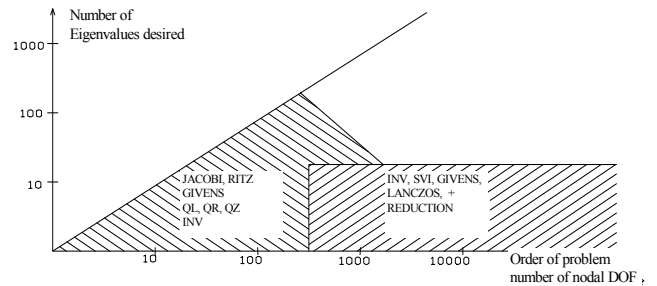


Figure 2. Selection of the method for eigensolution

When the number of degrees of freedom increases, then the necessary computer memory and time for extraction also increase. The Iterative Methods and the Lanczos Method are the most efficient for eigenvalue extraction for the structures with a few hundred thousand degrees of freedom.

Lanczos Method

The Lanczos Method overcomes the limitations and combines the best features of the transformation and iterative methods [11]. It does not miss roots (transformation methods), and it only find the roots requested by the user (iterative methods). The Lanczos Method is the preferred method for most medium and large sized problems, since it has a performance advantage over other methods. In addition, the advantage is its capability of parallel processing.

In 1950, Lanczos published his procedure for efficient determination of m first or last in series eigenvalues and appropriate eagevectors of $n \times n$ symmetric matrix. However, not until 1975, Lanczos Method was included in software NASTRAN and in many other known FEM software [12]. Lanczos Method is included in the static and dynamic analysis software that is developed in theVTI.

The symmetric matrix $[A]$ is obtained (eq. (4)) using certain transformation of the matrixes $[M]$ and $[K]$ so this procedure can be applied to natural frequency extraction.

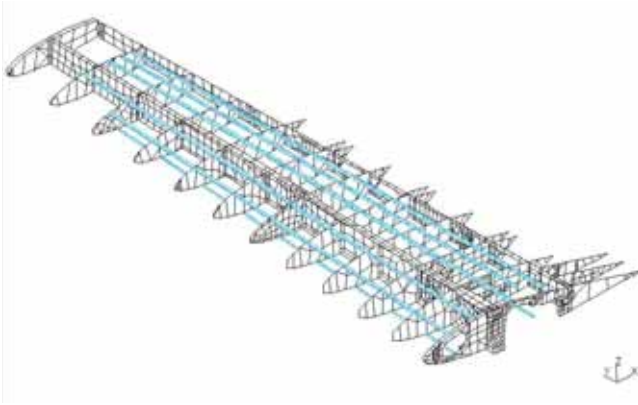


Figure 4. Wing rib and the stringer FE model

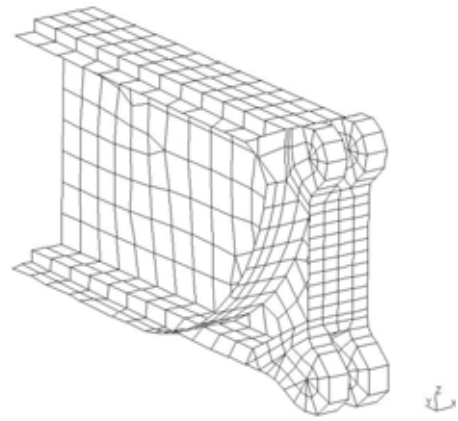


Figure 8. Fitting FE model

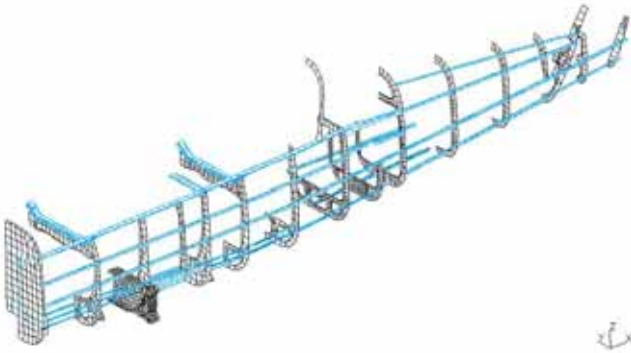


Figure 5. Fuselage frame and the stringer FE model

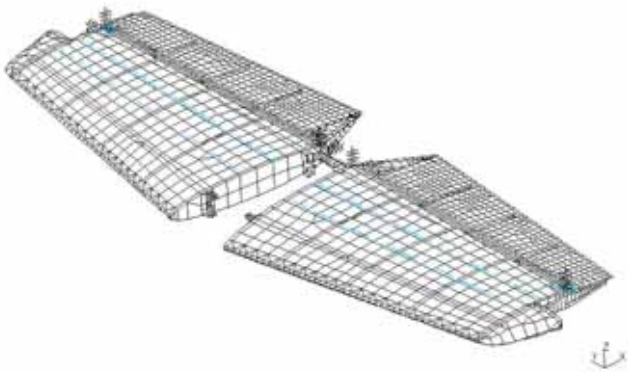


Figure 6. Horizontal stabilizer and the elevator FE model

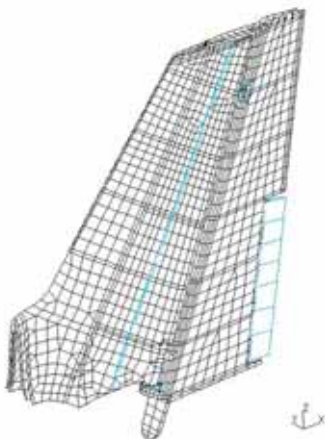


Figure 7. Fin and the rudder FE model

The main assembly of the aircraft finite element model is made by join subassemblies [15]. The subassemblies are connected in points where the tie-down fittings are positioned. These connections are modeled as beams and strings. In the end, the masses of pilots, fuel in reservoir, oil, equipment and flight controls are added to the aircraft FE model. The complete FE model of the LASTA aircraft has 42.129 finite elements, 37.367 nodes and 220.536 degrees of freedom and it is shown in Fig.9. The boundary conditions are free-free and the Shift Frequency Method is applied.

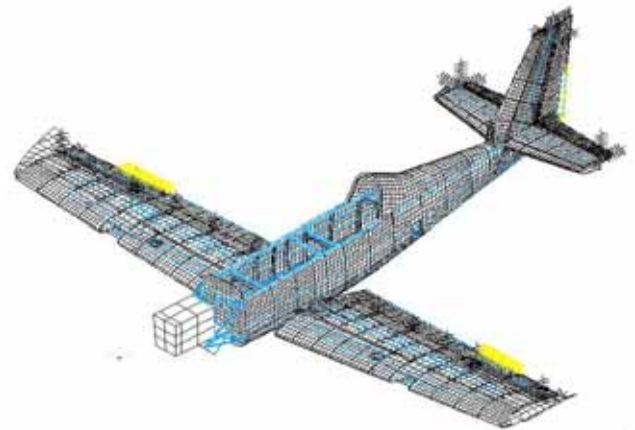


Figure 9. LASTA aircraft FE model

Computational Modal Analysis

The Computational Modal Analysis or the calculation of the natural frequencies and the natural mode shapes are done for the symmetric and anti-symmetric case according to the recommendation from [16]. The difference between these two cases is in the aileron string stiffness. The aileron string stiffness is $k=200\text{kN/m}$ for the symmetric case and $k=20\text{kN/m}$ for the anti-symmetric case.

The frequencies of the elevator and rudder rotation are fitted according to the command control circuits stiffness obtained experimentally.

After the modal analysis, symmetrical natural mode shapes are chosen from the symmetric case calculation and anti-symmetric natural mode shapes from the anti-symmetric case calculation. All local mode shapes are neglected. All natural mode shapes are inertial normalized.

Natural Mode Shapes

The frequencies and mode shapes description of the finite element model of the LASTA aircraft are presented in Table 1. All natural mode shapes are shown from Fig.10 to Fig.26.

Table 1. Natural mode shapes

No.	f	Mode shape description	Figure
	(Hz)		
1	4.913	Aileron anti-symmetric rotation	Fig.10
2	7.316	Rudder rotation	Fig.11
3	8.140	Elevator rotation	Fig.12
4	11.397	First wing bending	Fig.13
5	11.745	Fuselage torsion	Fig.14
6	14.596	Aileron symmetric rotation	Fig.15
7	15.428	Horizontal stabilizer anti-symmetric bending	Fig.16
8	17.529	Fuselage symmetric bending in the vertical plane	Fig.17
9	20.688	Anti-symmetric coplanar wing and fuselage bending	Fig.18
10	22.711	Second wing bending	Fig.19
11	23.094	Symmetric coplanar wing	Fig.20
12	23.831	Horizontal stabilizer symmetric bending	Fig.21
13	24.367	Flap anti-symmetric rotation	Fig.22
14	24.759	Flap symmetric rotation	Fig.23
15	26.335	Fuselage anti-symmetric torsion	Fig.24
16	29.380	Fin bending	Fig.25
17	32.384	Wing symmetric torsion	Fig.26

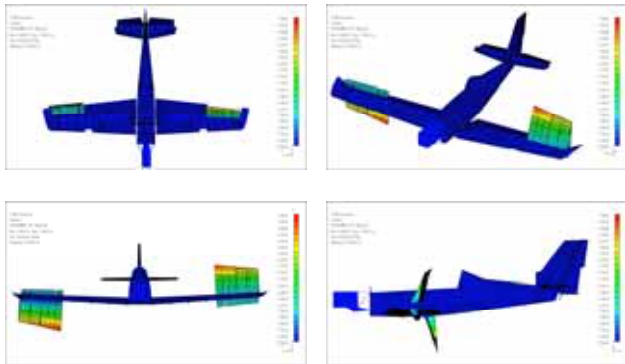


Figure 10. Mode 1 - Aileron anti-symmetric rotation

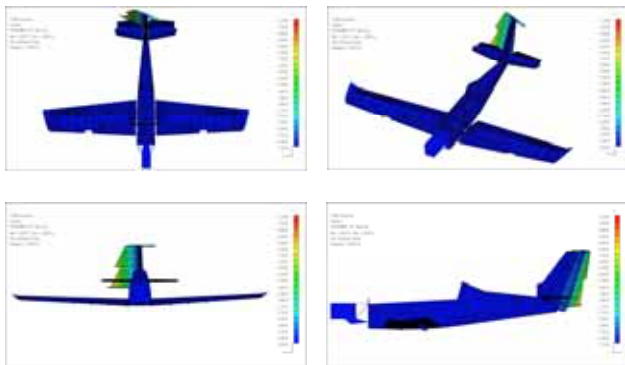


Figure 11. Mode 2 - Rudder rotation

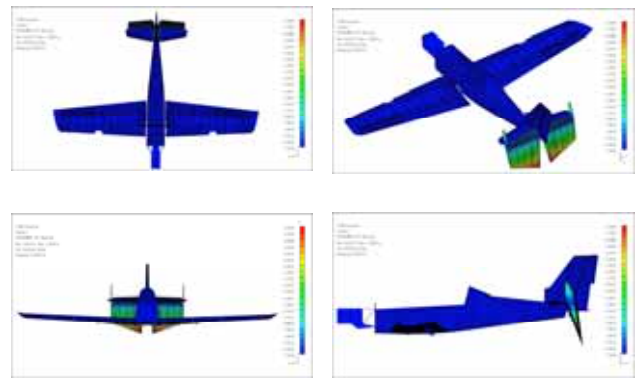


Figure 12. Mode 3 - Elevator rotation

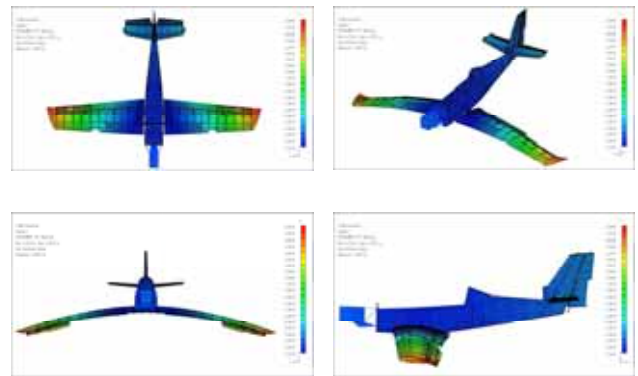


Figure 13. Mode 4 - First wing bending

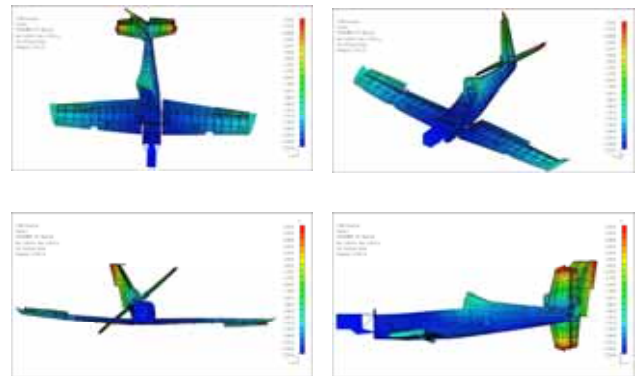


Figure 14. Mode 5 - Fuselage torsion

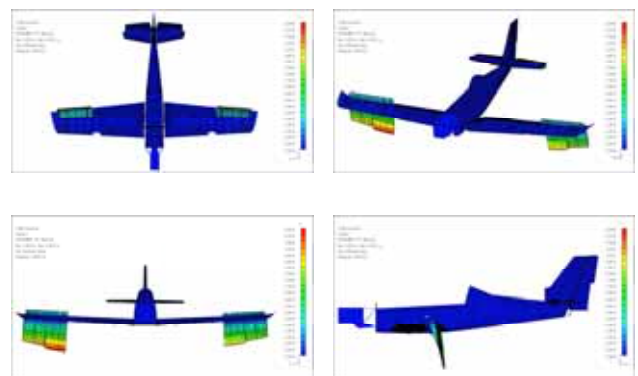


Figure 15. Mode 6 - Aileron symmetric rotation

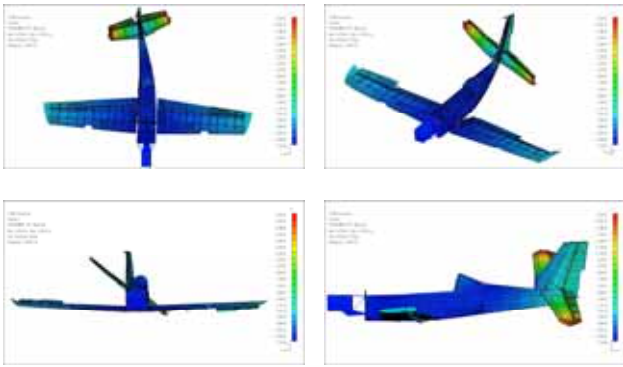


Figure 16. Mode 7 - Horizontal stabilizer anti-symmetric bending

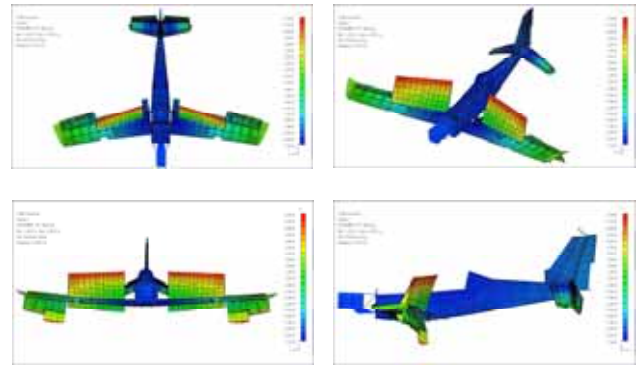


Figure 20. Mode 11 - Symmetric coplanar wing

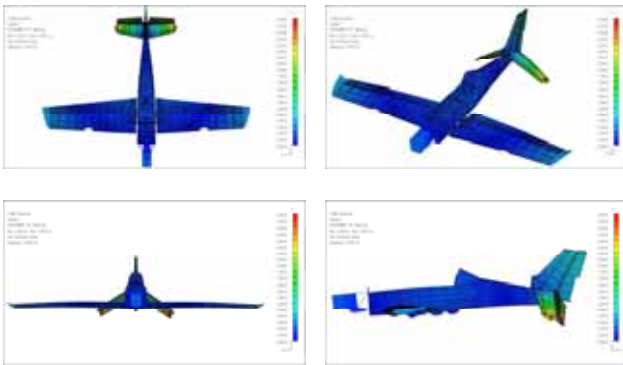


Figure 17. Mode 8 - Fuselage symmetric bending in the vertical plane

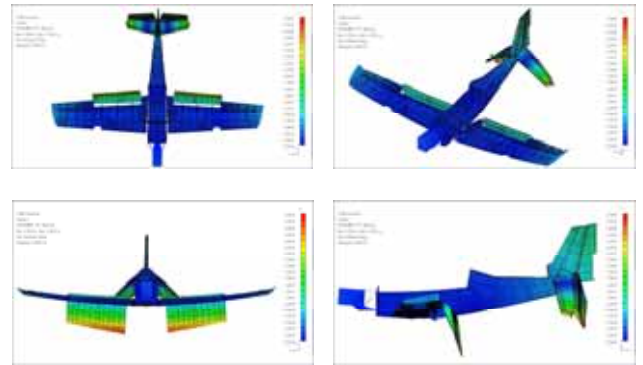


Figure 21. Mode 12 - Horizontal stabilizer symmetric bending

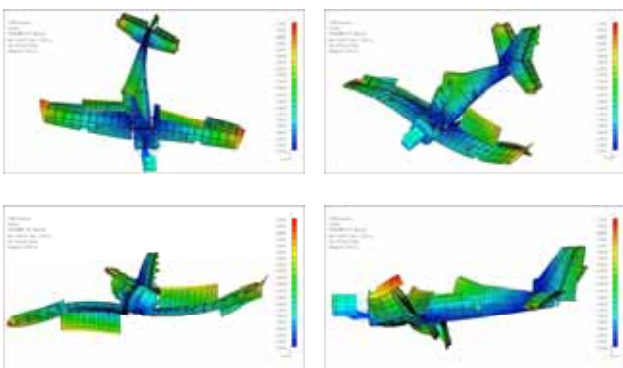


Figure 18. Mode 9 - Anti-symmetric coplanar wing and fuselage bending

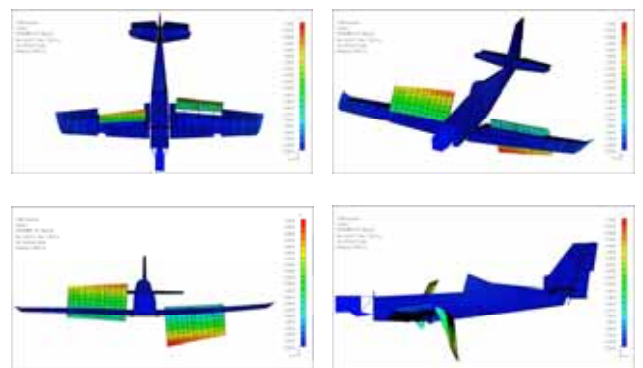


Figure 22. Mode 13 - Flap anti-symmetric rotation

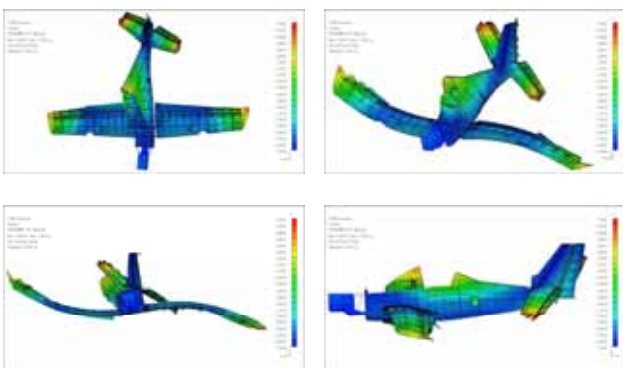


Figure 19. Mode 10 - Second wing bending

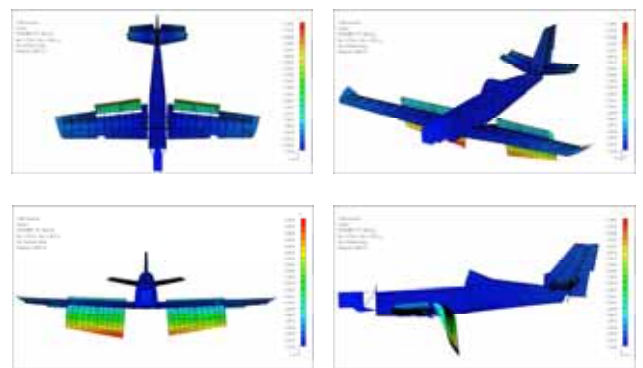


Figure 23. Mode 14 - Flap symmetric rotation

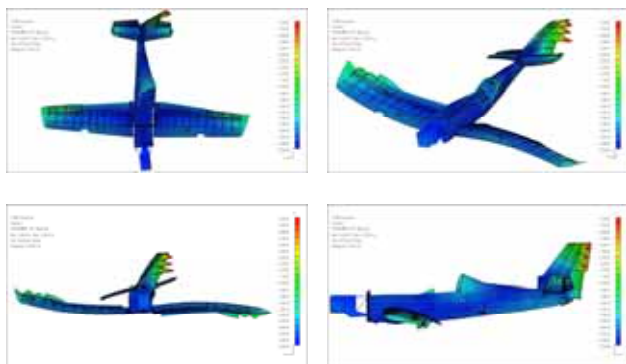


Figure 24. Mode 15 - Fuselage anti-symmetric torsion

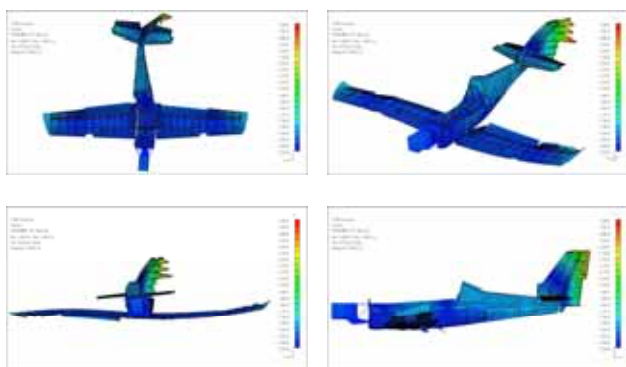


Figure 25. Mode 16 - Fin bending

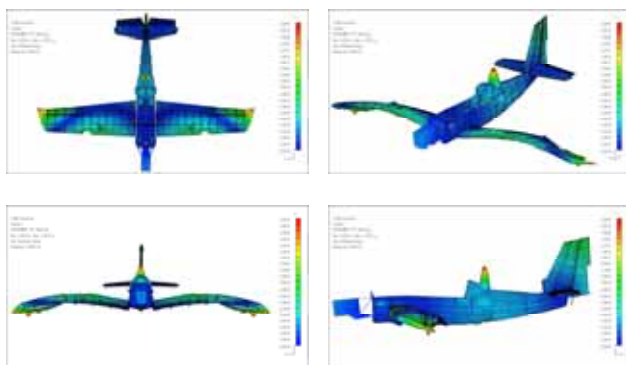


Figure 26. Mode 17 - Wing symmetric torsion

Conclusion

The results of the Computational Modal Analysis of the LASTA aircraft are presented in this paper.

All aircraft parts are modeled by finite elements and they are joined into the aircraft finite element model that has 220.536 degrees of freedom. The natural frequencies and the natural mode shapes of seventeen modes are obtained by the Computational Modal Analysis and they are presented in Table 1. All natural mode shapes are shown from Fig.9 to Fig.16.

The results of the Computational Modal Analysis for the

LASTA aircraft will be used for the calculation of unsteady generalized aerodynamic forces and critical flutter speeds. The natural frequencies and the natural mode shapes demonstrate where to expect modes on the Ground Vibration Test (GVT) – Experimental Modal Analysis (EMA).

A real value of the natural frequencies, the natural mode shapes and the damping will be obtained after the Ground Vibration Test. These results will be then used for the final calculation of the unsteady generalized aerodynamic forces and critical flutter speeds. In addition, according to these results of frequencies and damping, the aircraft finite element model will be verified and fit for further use in the analysis of the aeroelastic phenomena (aeroelastic twisting, reverse, critical flutter speed calculation, aeroelastic response to gust, etc.) and aircraft dynamic response (under armament action, landing, ground roll and taxiing).

Literature

- [1] BATHE, K.J., WILSON, E.L.: *Numerical Methods in Finite Element Analysis*, Prentice-hall, inc., Englewood Cliffs, New Jersey, 1976.
- [2] KIENTZY, D.W., RICHARDSON, M.H.: *Combined Testing and Analysis on a PC using MSC/pal and STAR*, Structural Measurement Systems, Inc., San Jose, California MacNeal-Schwendler World Users' Conference March 23, 1988.
- [3] FELIPPA, C.A.: *Introduction to Finite Element Methods*, Department of Aerospace Engineering Sciences, September 2006.
- [4] WILSON, E.L.: *Numerical Methods in Offshore Engineering*, John Wiley & Sons, 1987.
- [5] NX Nastran, *Basic Dynamics Analysis User's Guide*
- [6] RICHARDSON, M.H.: *Measurement and Analysis of the Dynamics of Mechanical Structures*, Hewlett-Packard Conference for Automotive and Related Industries, Detroit, MI, October 1978.
- [7] PEETERS, B., CLIMENT, H., DE DIEGO, R., ... : *Modern Solutions for Ground Vibration Testing of Large Aircraft*, IMAC 26, Orlando, USA, February 2008.
- [8] WILKINSON, J.H.: *The Algebraic Eigenvalue Problem*, Clarendon press, Oxford, 1965.
- [9] OJALVO, I.U.: *Proper Use of Lanczos Vectors for Large Eigenvalue Problems*, Computers & Structures, 1985, Vol.20, No.1-3.
- [10] KOVAČEVIĆ, P.: *Analiza dinamike strukture letelica metodom konačnih elemenata*, Vojnotehnički institut VJ, 1995.
- [11] WEINGARTEN, V.I., RAMANATHAN, R.K., CHEN, C.N.: *Lanczos Eigenvalue Algorithm for Large Structures on a Minicomputer*, Computers & Structures, 1983, Vol.16, No.1-4.
- [12] KOVAČEVIĆ, P.: *Razmatranje uticaja statičkih aeroelastičnih pojava i sopstvenih oblika oscilovanja na minimizaciju mase strukture uzgonskih površi*, Univerzitet u Beogradu, Mašinski fakultet, Beograd, 1989.
- [13] HUGHES, T.J.R.: *The Finite Element Method, Static and Dynamic Finite Element Analysis*, Prentice Hall, 1987.
- [14] DIMITRIJEVIĆ, J., KOVAČEVIĆ, P.: *Računska modalna analiza aviona LASTA*, Vojnotehnički institut VS, Beograd, 2009.
- [15] CRAIG, R.R., BAMPTON, M.C.C.: *Coupling of Substructures for Dynamic Analysis*, AIAA Journal, Vol. 6, No. 7, July 1968.
- [16] FEDERAL AVIATION ADMINISTRATION: *Advisory Circular, Means of Compliance With Section 23.629-1B, Flutter*, 2004.

Received: 03.03.2010.

Računska modalna analiza aviona LASTA

U ovom radu su prikazani rezultati računске modalne analize kompletnog modela konačnih elemenata aviona LASTA. Rezultati proračuna, sopstvene frekvencije i sopstveni oblici oscilovanja, koji su ovde prikazani, biće korišćeni za dalju analizu aeroelastičnih pojava radi provere njegove bezbednosti od pojave flatera u okviru anvelope upotrebnih brzina leta.

Ključne reči: avion, elisni avion, školski avion, sopstvene oscilacije, modalna analiza, numerička analiza, metoda konačnih elemenata.

Расчётный модальный анализ самолёта "Ласточка"

В настоящей работе показаны результаты расчётного модального анализа целостной модели конечных элементов самолёта "Ласточка". Результаты расчёта, собственные частоты и собственные формы колебаний, здесь показанные, будут использованы для дальнейшего анализа аэроупругих явлений с целью проверки её надёжности из-за возникновения флаттера в границах диапазона употребительных скоростей полёта.

Ключевые слова: самолёт, винтовой самолёт, учебный самолёт, собственные колебания, модальный анализ, цифровой анализ, метод конечных элементов.

Analyse modale de computation de l'avion LASTA

L'analyse de computation modale pour le modèle complet des éléments finis de l'avion LASTA est exposée dans ce travail. Les résultats du calcul, de la fréquence naturelle et des formes de l'oscillation naturelle, décrits ici, seront utilisés pour les analyses futures des phénomènes aériens élastiques pour confirmer que cet avion n'est pas soumis au battement dans le cadre de la vitesse prévue du vol.

Mots clés: avion, avion à hélice, avion école, oscillations naturelles, analyse modale, analyse numérique, méthode des éléments finis