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An Analysis of Crack Propagation and a Plasticity-Induced Closure Effect

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In the present paper, a few computational models for the crack growth analysis are improved. The improvements consist of including the effect of the plasticity-induced crack closure, i.e. the effective stress intensity factor is computed through the finite element method in order to account the effect of plasticity-induced crack closure on fatigue crack growth. Thus, corrective factors for the effect of the plasticity-induced crack closure are determined here.
The improved models are noticed to provide a fairly good correlation with available experimental data. Furthermore, the calculated results show that the plasticity-induced crack closure has a significant effect on fatigue crack growth and flawed structure life.

Key words: **material fatigue, crack, fatigue crack, crack growth, technological improvements, plastification, calculation method, finite element method.**

Introduction

IN engineering practice, especially when dealing with fatigue, it is important to have suitable computational models for the crack growth analysis. The use of these models enables us to evaluate whether the conditions for safe design and maintenance of structural components are fulfilled. All computational models must be formulated so that they include and analyze all phenomena which appear during a cyclic loading process. One of these phenomena that should be analyzed in crack growth investigations is the plasticity-induced crack closure effect.

Theoretical analyses of the plasticity-induced crack closure carried out through analytical and/or numerical techniques can provide a vital insight into the effect of each of the different variables involved. In order to take into account the crack closure effect in fatigue crack growth evaluations, the effective stress intensity factor is applied instead of the stress intensity factor. As it is well-known, the effective stress intensity factor is defined as the value of the stress intensity factor reduced by means of the so-called opening stress intensity factor, K_{op} (value of the stress intensity factor when the crack is first fully opened).

Generally speaking, the value of K_{op} is dependent on the stress ratio R. Thus, in all crack closure models for crack growth evaluations, the effective stress intensity factor range can be written in a unified form:

$$\Delta K_{eff} = f\left(R,...\right)\Delta K \,, \tag{1}$$

where $f(R,...) \le 1.0$ is the a function dependent upon R - ratio, material, geometry, etc.

Many investigations have focused their attention on the calculation of the effective stress intensity factor range ΔK_{eff} and/or the opening stress intensity factor K_{op}, after Elber [1] introduced the crack closure concept. The crack closure effect can be analyzed by both analytical and numerical approaches [2-4]. Analytical methods appear quite useful but they generally require a number of crucial assumptions as well as simplistic versions of material models. As a consequence, numerical approaches are more often used today, since they produce very good results when compared with experimental data. Furthermore, these methods could be even used for correcting the existing analytical models. One of such numerical approaches is the elastic-plastic Finite Element Method (FEM), which can be used to study the plasticity-induced crack closure by simulating fatigue crack growth as a discrete number of incremental crack extensions. For the finite element simulations of the plasticity-induced crack closure, there are many variables that can be considered, such as: finite element type, mesh size, crack opening level determination, crack opening level stabilization, and a constitutive model [5].

One of the earliest finite element models related to the plasticity-induced crack closure under plane–stress condition was proposed by Ohji, Ogura and Ohkubo [6]. Their results indicated that the strain amplitude in the vicinity of the crack tip scaled with the effective stress intensity factor range. In a parallel study, Newman [7] also performed a two-dimensional finite element analysis for plain-stress condition using the incremental theory of plasticity. Later, Nakagaki and Atluri [8] proposed the finite element method based on special crack-tip elements and translation of the near-tip mesh, and examined the case of mixed mode and axial loading. Moreover, some researchers analyzed the crack closure effect in CT specimen [9, 10]. Sehitoglu and Sun [11, 12] formulated a

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new criterion to determine the stress state with crack opening and crack closure. Moreover, Wei and James [13] showed the difficulty of data convergence, between numerical and experimental results as well as between experimental results themselves.

The present paper investigates the effects of the plasticity-induced crack closure on fatigue crack growth and structure life estimation. The crack closure level is computed from a finite element simulation in order to improve some of the available crack closure models. As a result of such a finite element analysis, adequate corrective factors for different crack closure models are obtained here. Additionally, fatigue life is estimated, and numerical results obtained with or without corrections are compared.

Crack growth analysis based on crack closure concept

Developing an appropriate computational procedure for the crack growth analysis is one of the key issues for the assessment of the reliability of components and structures. In this Section, some of the computational models based on the crack closure phenomenon are analyzed.

As it is well-known, Elber [1] introduced the concepts of the crack closure and the effective stress intensity factor range ΔK_{eff} as the dominant driving force for fatigue crack growth. Elber considers that, as a crack propagates, the crack closure occurs as a result of plastically deformed material left in the path taken by the crack. This material is often referred to as the plastic wake. The plastic wake enables the crack to close before minimum load is reached, and Elber concluded that the stress intensity factor at the crack tip does not change while the crack is closed, even when the applied load is changed.

Elber argued that a load cycle is effective in driving the fatigue growth of a crack only if the crack is fully open. Additionally, he found that the crack opening stress level depends on the stress ratio R, and defined the effective stress intensity factor as a function of R and the stress intensity factor range ΔK, i.e.:

$$K_{eff} = (0.5 + 0.4R)\Delta K \ . \tag{2}$$

An analogous relationship, based on experimental tests, was proposed by Schjive [2]:

$$K_{eff} = \left(0.55 + 0.33R + 0.12R^2\right)\Delta K \ . \tag{3}$$

In the present paper, the crack closure model based on the crack-tip stress is also analyzed. Such a model was presented by Sehitoglu and Sun [11, 12], who proposed to determine the crack opening or closure in terms of the stress state in the crack tip vicinity, instead of the separation of nodes. Sehitoglu and Sun [11, 12] suggested that the crack opening point had to be taken where the whole crack plane was under tensile stress, $K_{tensile}$. Namely, they determined the point where the crack tip stress becomes tensile, $K_{tiptensile}$ or K_{tt} (TT model). They found that the tip tensile stress intensity factor K_{tt} can represent the effective stress intensity factor, i.e.:

$$\Delta K_{eff} = \Delta K_{tt} = K_{\max} - \frac{K_{ttop} + K_{ttcl}}{2} = \left(1 - 0.5\left(\frac{K_{ttop}}{K_{\max}} + \frac{K_{ttcl}}{K_{\max}}\right)\right) K_{\max} \tag{4}$$

where:

$$\frac{K_{ttop}}{K_{\max}} = 0.426 + 0.6058\,R - 0.7177\,R^2 \tag{5}$$

$$\frac{K_{ttcl}}{K_{\max}} = 0.1146 - 0.0699\,R + 0.4279\,R^2 \tag{6}$$

Equations (4), (5) and (6) show that the tip tensile stress intensity factor K_{tt} can be expressed as a function of the opening tip tensile stress intensity factor K_{ttop} and the closure tip tensile stress intensity factor K_{ttcl} as well as the maximum stress intensity factor K_{max}.

For the crack growth analysis, the basic relationship is that between the crack growth rate and the stress intensity factor. In order to include the plasticity-induced crack closure effect, Elber [1] proposed the following equation:

$$\frac{da}{dN} = C\left(\Delta K_{eff}\right)^m , \tag{7}$$

where C and m are parameters experimentally obtained.

Furthermore, it is often useful to estimate the component life for the purposes of the design and failure analysis. The number of loading cycles up to failure can be computed by integrating the equation related to the crack growth rate, with assumption that the initial and final flaw sizes are a_0 and a_f, respectively:

$$N = \frac{1}{C}\int_{a_0}^{a_f} \frac{da}{\left(\Delta K_{eff}\right)^m} \tag{8}$$

Since such an integration (Eq.8) is complex, numerical simulations have to be performed to compute fatigue life of structures up to failure.

Finite element analysis and plasticity induced crack closure

In the field of fracture mechanics, the stress intensity factor calculation is very important [14, 15]. The stress intensity factor is a parameter dependent on geometry (crack length, width of specimen, etc) and external loading (level of loading). As mentioned above, if a crack closure phenomenon is included in the crack growth analysis, the stress intensity factor should be substituted with the effective stress intensity factor. In this way, the stress ratio becomes another important parameter to be considered in the crack growth analysis. Crack growth models are usually referred to as empirical approaches but, in the case of the plasticity-induced crack closure analysis, numerical approaches are often employed besides the empirical ones.

A number of researchers have simulated the plasticity-induced crack closure using finite element analyses. Such studies are based on the following algorithm. An elastic-plastic model is built with a suitably refined mesh, and remote tractions are applied to simulate cyclic loading. The crack tip node is released during each cycle, advancing the crack one element length and allowing a plastic wake to form. Crack closure is evaluated by monitoring the contact between crack faces. This process is repeated until the crack opening stress values stabilize.

The present authors have simulated the plasticity-induced crack closure in the same way, by performing a finite element analysis. For the geometry shown in Fig.1

(CT specimen), the stress intensity factors as well as the corrective factors to include the effect of the plasticity-induced crack closure have been computed through such a numerical analysis.

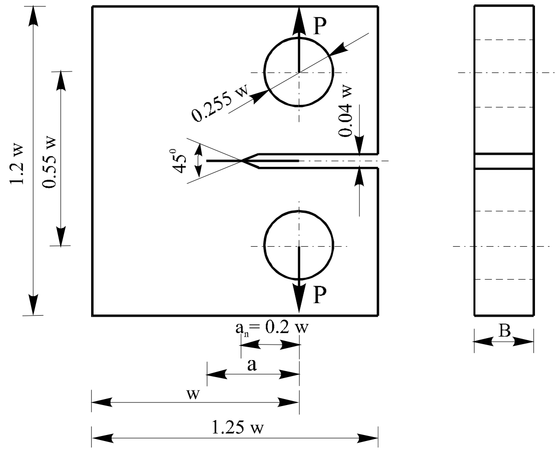


Figure 1. Geometry of CT specimen.

First of all, some finite element meshes for a few different values of external force ranging from 3000 N to 15000 N have been defined. In more detail, five finite element meshes have been modelled for five different values of force. One of these meshes is that plotted in Fig.2.

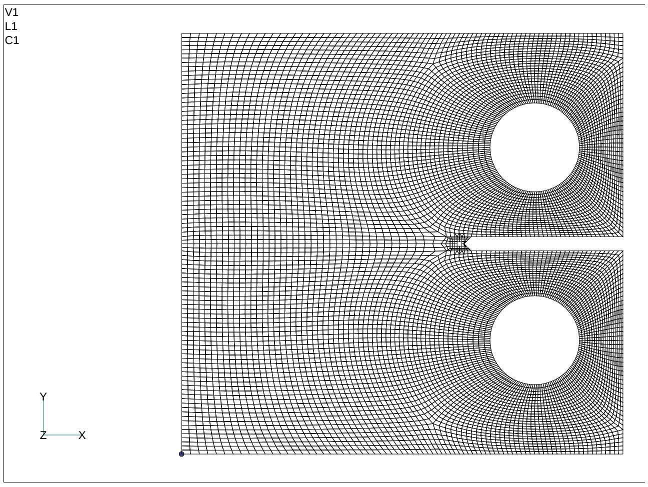


Figure 2. Finite element mesh for the investigated CT specimen (w = 0.075 m, a_0 = 0.016 m , B = 0.010 m).

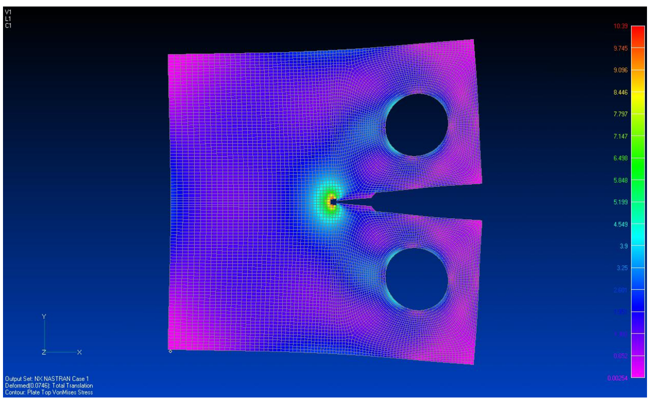


Figure 3. Finite element analysis for the CT specimen examined (w = 0.075 m, a_0 = 0.016 m , B = 0.010 m).

Thereafter, such meshes have been used for calculations of stress distribution and for the analysis of the plasticity-induced crack closure effect. Fig.3 shows the stress distribution in a CT specimen made of Al Alloy 2024 T351, for the crack length a = 0.02625 m, the maximum external force P_{max}= 3300 N and the stress ratio R = 0.1. Note that the symbol a denotes the crack length measured from the line of application of the external load, B represents the thickness of the CT specimen, and w is the distance between the applied force P and the left edge of the specimen (Fig.1).

Owing to the finite element analysis related to different values of external force, the geometry of the CT specimen as well as type of material and the corrective factors for different crack closure models have been determined (Table 1).

Table 1. Calculated corrective factors for crack closure models using FEM

Equation	Corrective factor
$\Delta K_{eff} = (0.5+0.4R)\Delta K$	0.925
$\Delta K_{eff} = (0.55+0.33R+0.12R^2)\Delta K$	0.927
$\Delta K_{eff} = \left(1-0.5\left(\frac{K_{ttop}}{K_{\max}}+\frac{K_{ttcl}}{K_{\max}}\right)\right)K_{\max}$	0.9574

Note that the equations for the effective stress intensity factors depend on the stress intensity factor (Table 1). Therefore, analytical expressions for the stress intensity factor are needed. The stress intensity factor equation (for the considered CT specimen, Fig.1) used in the present paper is given by:

$$\Delta K = \frac{\Delta P}{B\sqrt{w}}\left(\frac{2+\frac{a}{w}}{\left(1-\frac{a}{w}\right)^{3/2}}0.886+4.64\left(\frac{a}{w}\right)-13.32\left(\frac{a}{w}\right)^2 + 14.72\left(\frac{a}{w}\right)^3-5.6\left(\frac{a}{w}\right)^4\right) \quad (9)$$

Numerical results

Now the above three different crack closure models are analyzed and improved using the finite element method. In the following examples, numerical crack growth calculations are performed on a CT specimen subjected to axial loading with constant amplitude in order to verify and compare these three models.

Stress intensity factor calculation

In this example, the effective stress intensity factor calculations for different clack closure models are carried out. The CT specimen analyzed is made of 2024 T351 Al Alloy with E = 74000 MPa, C = 1.51 10^{-10} and m = 4. The configuration of the CT specimen is shown in Fig.1. The geometry parameters are as follows: w = 0.075 m; B =0.010 m, a_0 = 0.016 m. The CT specimen is subjected to cyclic axial loading with a constant amplitude, P_{max} = 3300 N and the stress ratio R = 0.1.

For the known specimen geometry and material parameters, the computed values of the effective stress intensity factors (using equations (2), (3), (4), (5), (6) and (9) with/without corrective factors (Table 1)) are shown in Fig.4.

In Fig.4, it can be seen that the calculated effective stress intensity factors are significantly lower than the stress intensity factors, and the results are different for the different crack closure models examined.

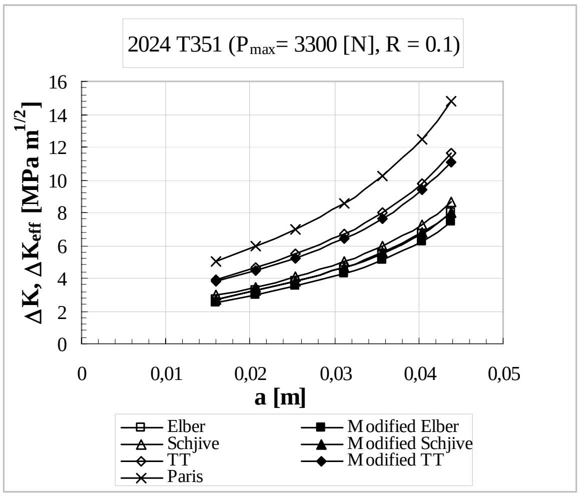


Figure 4. Crack length a against the effective stress intensity factor range ΔK_{eff}.

Crack growth rate and life estimation

This example deals with crack growth rate and life calculation. The CT specimen and material used in this example are the same as in the previous one (see example 1). External cyclic loading is axial with a constant amplitude, P_{max}= 3300 N, the stress ratio R= 0.1 and R= 0.3. Since geometry, material and type of loading are known, the crack growth rate can be determined by Eq.(7) together with different equations (2), (3) or (4) with (5) and (6) for the effective stress range and the corrective factors (Table 1) as well as Eq.(9) for the stress intensity factor.

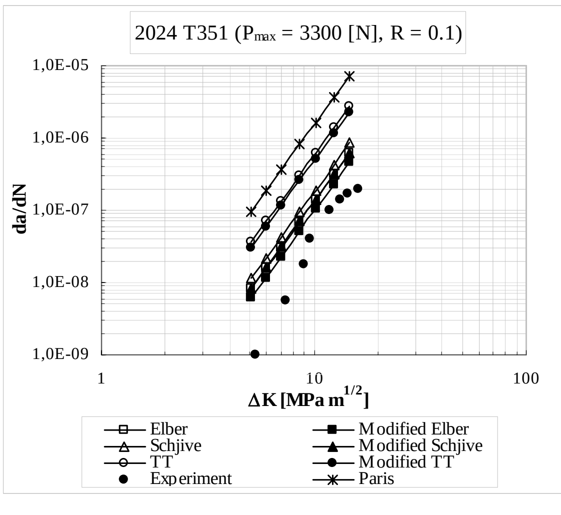
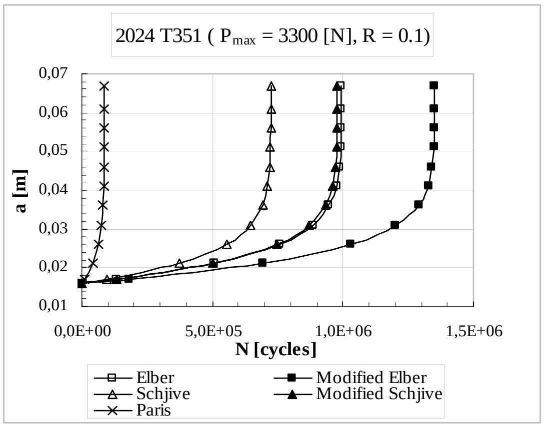


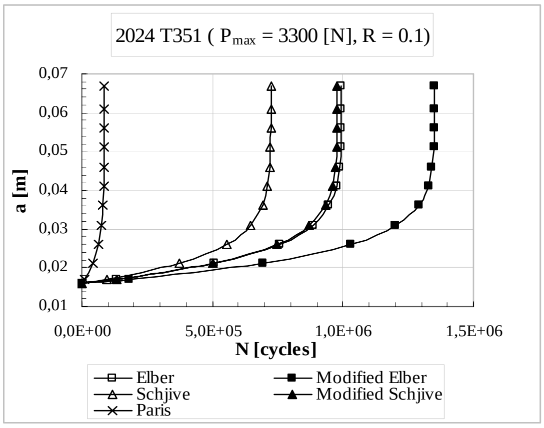
Figure 5. Crack growth rate versus the stress intensity factor.

Fig.5 presents the comparison of the considered crack closure models as well as the Paris' model ($da/dN=C\Delta K^m$) with experimental data [16,17]. It can be observed that the evaluations of crack growth rates using different crack closure models have good accuracy when compared with experimental results. Additionally, the Modified Elber's and Modified Schjive's models including the crack closure effect provide better results for fatigue crack growth rates, especially the Modified Elber's model. Moreover, the TT and Modified TT models seem to be very conservative but less than the Paris' model, when compared to experimental data.

Another important aspect to tackle is the structure life estimation. Based on known characteristics of material, geometry and loading, the number of loading cycles up to failure can be evaluated by using Eq.(8) and different equations (2), (3), (4) with (5) and (6) for the effective stress intensity factor (Elber, Schjive, TT and their modifications) as well as Eq.(9) for the stress intensity factor.



(a)



(b)

Figure 6. Crack growth analysis of CT specimen using different crack closure models ((a) R = 0.1, (b) R = 0.3).

Fig.6 presents the number of loading cycles up to failure, obtained from the above three crack closure models with and without improvements. Note that (see Fig.6 and Table 2) modifications/improvements that include the effects of the plasticity-induced crack closure increase the evaluated number of loading cycles up to failure.

Table 2. Evaluated number of loading cycles up to failure (a_0= 0.016 m, R = 0.3)

	Number of loading cycles up to failure (for final crack length a_f = 0.066 [m])	Δ [%]
Elber	$1.56\ 10^6$	
Modified Elber	$2.12\ 10^6$	26.415
Schjive	$1.22\ 10^6$	
Modified Schjive	$1.64\ 10^6$	25.610
TT	$6.28\ 10^5$	
Modified TT	$7.48\ 10^5$	16.043

By taking into account crack closure effects, models are improved (in the range from 16 to 26%). Additionally, as determined in the previous example, the Modified Elber's model is that in the best agreement with the experimental results (Fig.5): in other words, a much lower number of loading cycles up to failure is obtained by employing improvements and the TT model. Namely, the number of loading cycles computed using the Modified Elber's model is about 70% higher when compared to that by the TT model and about 65% when compared to that by the Modified TT model.

Evaluation of the number of loading blocks up to failure

This example deals with the calculation of the number of loading blocks up to failure. One block of loading (i.e. load spectrum) is shown in Fig.7. The load spectrum consists of three levels (P_{max} = 4000 N, P_{max}=10000 N, P_{max}= 8000 N) with the stress ratio R = 0.3. Such a load spectrum is applied on a CT specimen (with the same properties as that in example 2). To evaluate the number of loading blocks, Elber's and Modified Elber's models as well as TT and Modified TT models are applied.

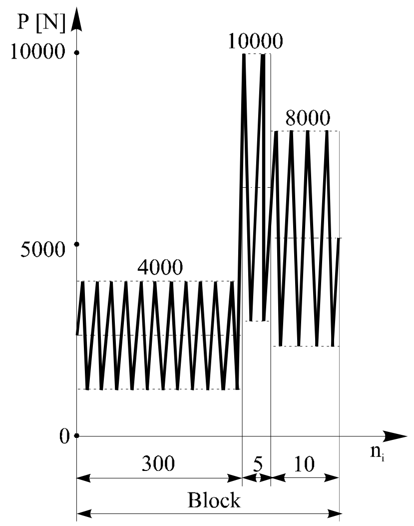


Figure 7. Load spectrum (R = 0.3).

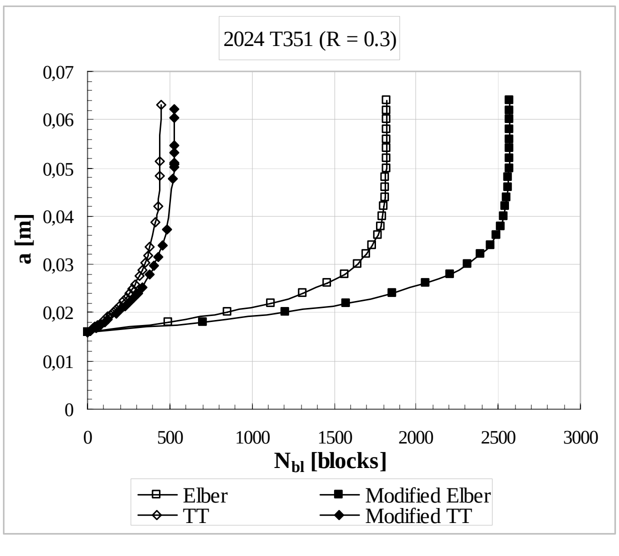


Figure 8. Fatigue crack growth analysis (a_0 = 0.016 m).

Fig.8 shows the comparison, related to the number of loading blocks up to failure between the Elber and TT crack closure models and between the Modified Elber and Modified TT models.

For the load spectrum shown in Fig.7, the computed values of the number of blocks up to failure using the Elber and TT models are listed in Table 3.

From Table 3, it can be deduced that the crack closure effect can significantly increase the number of loading blocks up to failure (in the range from 16 to 29%). Moreover, the introduced improvements for the effective stress intensity factors have more impact on the number of loading blocks up to failure than on the number of loading cycles up to failure (Table 2 and Table 3).

Table 3. Comparison of number of loading blocks up to failure (a_0= 0.016 m, R = 0.3)

	Number of blocks up to failure (for final crack length a_f = 0.063 [m])	Δ [%]
Elber	1.819 10^3	
Modified Elber	2.568 10^3	29.167
TT	0.445 10^3	
Modified TT	0.527 10^3	15.556

Conclusion

A crack growth analysis is numerically carried out by taking into account the plasticity-induced crack closure effect. Three different crack closure models (Elber, Schjive and TT) are improved through corrective factors for the effective stress intensity factors. In addition, improved/modified methods are compared with standard models.

The following conclusions can be drawn:

1. Introduction of the crack closure effect in the crack growth analysis enables more realistic and accurate life estimation up to failure (actually, the considered crack closure models **(**Elber, Schjive, TT and their modifications**)** are much less conservative than the conventional Paris' model);
2. As far as the considered crack closure models are concerned, the best agreement with experimental data is obtained when using the Modified Elber's model and the worst is obtained when using the TT model;
3. The finite element method is a powerful and useful tool for the analysis of the plasticity-induced crack closure;
4. The effect of the plasticity-induced crack closure (analyzed with the introduced corrective factors) is greater when evaluating the number of loading blocks up to failure than when estimating the number of loading cycles up to failure;
5. By introducing the plasticity-induced crack closure effect in the crack growth analysis, the quality of crack growth estimations for structural components can be improved.

References

[1] ELBER,W.: *Fatigue crack closure under cyclic tension*. Engng Fract Mech, 1970, Vol.2, pp.37-44.

[2] SCHJIVE,J.: *Some formulas for the crack opening stress level*. Engng Fract Mech, 1981, Vol.14, pp.461-465.

[3] HUANG,X.P., CUI,W.C., LENG,J.X.: *A model of fatigue crack growth under various load spectra*. In: MESO International Conference, Montreal, Canada, August 1-4, 2005, pp.303-311.

[4] BOLJANOVIĆ,S., MAKSIMOVIĆ,S.: *Fatigue life analysis of cracked structural components using crack closure effects*. Proceedings of the 2nd South – East European Conference on Computational Mechanics (SEECCM 2009), Rhodes, Greece, June 22-24, 2009, (SE276).

[5] SKINNER,J.D., DONIEWICZ,S.R.: *Simulation of plasticity-induced fatigue crack closure in part-through cracked geometries using finite element analysis*. Engng Fract Mech, 2002, Vol.69, No.1, pp.1-11.

[6] OHJI,K., OGURA,K., OHKUBO,Y.: *Cyclic analysis of a propagating crack and its correction with fatigue crack growth*. Engng Fract Mech, 1975, Vol.7, pp.457-464.

[7] NEWMAN,J.C.: *A crack opening stress equation for fatigue crack growth*. Int J Fract, 1984, Vol.24, pp.R131-135.

[8] NAKAGAKI,M., ALTURI,S.N.: Fatigue crack closure and delay effects under mode I spectrum loading: an efficient elastic-plastic analysis procedure. Fatigue Fract Engng Mater Struct, 1979, Vol.1, pp.421-429.

[9] NAKAMURA,H., KOBAYASKHI,H., YAMASE,S., NAKAZAWA,H.: *Finite element analysis of fatigue crack closure in compact specimen.* Mechanical Behaviour of Materials (ICM4), Fourth Int Conf Mechanical Behaviour of Materials, Stockholm, Sweden, 1983, Vol.2, pp.817-823.

[10] BLOM,A.F., HOLM,D.K.: *An experimental and numerical study of crack closure.* Engng Fract Mech, 1985, Vol.22, pp.997-1011.

[11] SEHITOGLU,H, SUN,W.: *Modeling of plane strain fatigue crack closure*. ASME J Engng Mater Technol: 15, 113, 1991, pp.31-41.

[12] SUN,W., SEHITOGLU,H.: *Residual stress fields during fatigue crack growth*. Fatigue Fract Engng Mater Struct, 1992, Vol.15, pp.115-128.

[13] WEI,L.W., JAMES,M.N.: *A study of fatigue crack closure in polycarbonate CT specimen.* Engng Fract Mech, 2000, Vol.66, pp.223-242.

[14] CARPINTERI,A.: Shape change of surface cracks in round bars under cyclic axial loading. Int J Fatigue, 1993, Vol.15, pp.21-26.

[15] CARPINTERI,A., BRIGHENTI,R., VANTADORI,S.: *Notched shells with surface cracks under complex loading*. Int J Mech Sciences, 2006, Vol.48, pp.638-649.

[16] RANGANATHAN,N.: *Certain aspects of* variable *amplitude fatigue*. IFC-8-Fatigue 2002, Stockholm, 3–7 June 2002, pp.613 - 623.

[17] MAKSIMOVIĆ,K.: *Fatigue Crack Growth Analysis of Damaged Structural Components Under Mode-I and Mixed Modes*, Scientific Technical Review, ISSN 1820-0206, 2009, Vol.59,No.1, pp.35-40.

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Analiza širenja prskotine i uticaj plastifikacije na zatvaranje prskotine za vreme širenja

U prezentovanom radu su izvršena poboljšanja nekoliko proračunskih modela za analizu širenja prskotine. Poboljšanja su ostvarana zahvaljujući uključivanju uticaja plastifikacije koji prouzrokuje efekat zatvaranja oko vrha prskotine. Ustvari, metoda konačnih elemenata je korišćena da bi se analizirao efekat zatvaranja oko vrha prskotine, a samim tim odredio efektivni factor intenziteta napona. Prema tome, u radu su određeni korektivni faktori kojima se uključuje efekat zatvaranja oko vrha prsline.

U radu su upoređene proračunate vrednosti određene primenom poboljšanih modela sa raspoloživim eksperimentalnim rezultatima i dobijeno je dobro slaganje rezultata. Osim toga, vrednosti dobijene putem proračuna ukazuju da efekat zatvaranja oko vrha prsline ima značajan uticaj na širenje prsline, kao i procenu preostalog veka pri zamoru kod elemenata struktura sa inicijalnim oštećenjima.

Ključne reči: **zamor materijala, prskotina, zamorna prskotina, rast prskotine, tehnološka poboljšanja, plastifikacija, metoda proračuna, metoda konačnih elemenata.**

Анализ расширения трещины и влияние пластификации на замыкание трещины во время распространения

В настоящей работе улучшено несколько расчётных моделей для анализа расширения трещины. Улучшения содержаны во вводе эффекта пластификации вокруг вершины трещины, которые приводят к замыканию трещины, а это реализовано применением метода конечных элементов при вводе эффективного фактора интенсивности напряжения. Значит, в работе определены факторы коррекции, при помощи которых учитываются эффекты пластификации. Улучшеные расчётные модели сравнены с располагающими экспериментальными результатами и при этом получена хорошая согласованность. При этом результаты расчётных анализов указывают, что эффект пластификации вокруг вершины трещины значительно влияет на усталостное расширение трещины, а в том числе и на оценку остаточного усталостного срока службы у элементов структуры с инициальными трещинами.

Ключевые слова: **Усталость материала, трещина, усталостная трещина, рост трещины, технологические улучшения, пластификация, метод расчёта, метод конечных элементов.**

Analyse de la propagation de la fissure et l'influence de la plastification sur la fermeture de la fissure au cours de la propagation

Dans ce papier on a présenté quelques modèles améliorés de computation pour l'analyse de la propagation de la fissure. Les améliorations consistent en introduction des effets de plastification qui produisent la fermeture de la fissure et cela a été réalisé par la méthode des éléments finis en introduisant le facteur effectif de l'intensité de la charge. Les facteurs correctifs qui prennent en considération les effets de la plastification ont également été déterminés dans ce travail. Les modèles améliorés de computation ont été comparés avec les résultats expérimentaux disponibles et on a obtenu bon accord. En outre les résultats de computation démontrent que l'effet de la plastification a l'influence signifiante sur la propagation de fatigue chez la fissure ainsi que sur l'évaluation de la durée de future fatigue pour les éléments structuraux aux fissures initiales.

Mots clés: **fatigue des matériaux, fissure, fissure de fatigue, croissance de la fissure, améliorations technologiques, plastification, méthode de computation, méthode des éléments finis.**