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Application of Numerical Methods in Solving a Phenomenon of the Theory of Thin Plates

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The term thin plate implied an elastic body with a cylindrical or prismatic shape of small thickness in relation to other two dimensions. The basic dependences between geometrical and physical properties come mostly to setting up of relations between stress and strain conditions, which have been described by differential equations, both simple and partial ones. The methods used for solving established equations, with respect to outlined and initial conditions, may be classified into analytical and numerical methods. In case of complex and big construction systems subjected to arbitrary loads, including complex boundary conditions, solving differential equations by analytical methods is almost impossible. Then the solution is the application of numerical methods. One of the basic numerical methods is the Finite Element Method (FEM). In this paper, besides the analytical method, is also used for consideration of this phenomenon in a flat isotropic field, notably in thin plates with different boundary conditions and loading. In the end, more comments and further directions of investigations are given. This method reduces the problem to solving the system of paired algebraic equations, thus making it easier to solve.

Key words: **plate, thin plate, stress conditions, stress concentration, numerical methods, finite element method.**

Introduction

MANY problems in the field of deformable bodies are described by parameters of the condition of the systems which depend on coordinates, time, temperature, etc. Such systems are given in mathematical models by partial differential equations, for example, relation between stress condition and strain condition and external load in mechanics of continuum and others. For solving these equations, there are two approaches: analytical [1], [2],..., and numerical[5].

Analytical methods are of an earlier date and imply determination of mathematical functions which define solution in a closed form.

The basic characteristic of numerical methods is that the fundamental equations which describe the problem, including the boundary conditions, are solved in an approximate numerical way. In this paper, the finite element method is used for the analysis of stress-strain conditions in thin plates. The finite element method (FEM) implies discretization in a physical model and the final result is obtained by solving a system of algebraic equations. The FEM uses various forms of variation methods applied to a discrete model.

Display of basic equations in the finite element method

Unlike other numerical methods based on mathematical discretization of the equations of boundary problems, the finite element method (FEM) is based on the physical discretization of the continuum observed in the parts of finite dimensions and of simple shapes called finite elements (FE).

For modeling the continuum in the plane condition of stress and strain, where plates belong to, 2D finite elements are used. In the plane condition of strain the material is deformed in an identical way as in all planes parallel to one plane, for example, the *xy* plane. Internal distribution of displacement in the finite element itself is defined by interpolation functions.

The displacement vector $\{s\}$ in the plane (field) of the finite element has two components (u, v) which are continuous functions of the coordinates of the points, [2-8], so that it can be written

$$\{s\} = \mathbf{s} = \begin{Bmatrix} u \\ v \end{Bmatrix} = [h]\,\mathbf{q}_K = [h]\{q\}_{(K)} \tag{1}$$

The relation between the coordinates in the field of the element and the nodal coordinate is given as

$$x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} x \\ y \end{Bmatrix} = HX \tag{2}$$

where H is the interpolation matrix.

In tensor notation, Cauchy's kinematic equations can briefly be written

$$\{\varepsilon\} = [d]\{s\} \quad \text{or} \quad \varepsilon = ds \tag{3}$$

where d is the differential operator and s is the displacement vector.

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The equilibrium conditions between the internal and external forces on the part of the contour where the contour conditions are given by surface forces are expressed in Cauchy's equations

$$[d_s]^T\{\sigma\}=\{p_n\} \text{ or } d_s^T\sigma=p_n \tag{4}$$

where $[d_s]^T$ is the transposed matrix of the matrix $[d_s]$ the elements of which are cosines of the angles which the normal line *n* forms with the axes *x*, *y* at the points of the contour area.

The general form of constitutive equations, i.e. the tion between the components of stress and components of strain for elastic material which represents a generalization of the known *Hooke's* low, can be shown as

$$\{\sigma\}=[D]\{\varepsilon\} \text{ or } \sigma=D\varepsilon \tag{5}$$

If we ignore temperature stresses, we obtain the basic equation of the finite element which gives the relation between the nodal displacements and the l forces for the finite element. The matrix $\left[K^e\right]$ is the stiffness matrix of the finite element in the form

$$\{F\}=\left[K^e\right]\{S\} \tag{6}$$

The construction equation is obtained by joining the *e* equations (6) into a group equation in the form of

$$\{F\}=[K]\{S\} \tag{7}$$

where $\{F\}$ is the matrix of the columns the elements of which are components of the generalized nodal forces and $\{S\}$ is the matrix of the columns the elements of which are the components of the nodal displacement vectors.

Examples of the solutions

Sngle-axis tensioned plates with a single opening of a variable shape are chosen as examples which demonstrate the finite elemnt method. In all examples the intesity of the tensioning surface forces is p=1 N/m^2. Such an analysis is intended to show wich shape of the opening is more favourable in terms of the magnitude of stress and strain. Taking into account the occurrence of stress concentration at the points of opening, i.e. geometrical discontinuity, then such an analysis is especially important in technical practise.

The first example shows a single-axis tensioned plate weakened by a circular opening of *d*=100 mm and an angle between the lines of aplication of forces and *x* - axis α=90^o. Figures 1 - 3 illustrate the details of the analysis conducted.

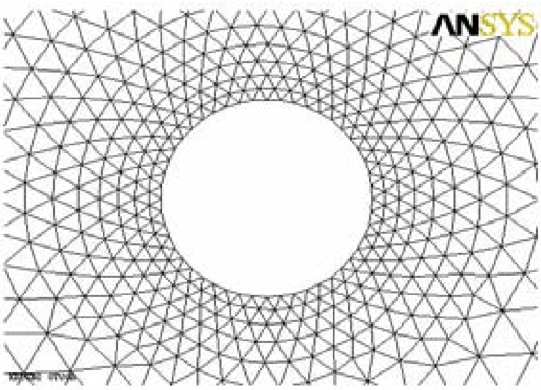

Figure 1. The mesh of FE around the aperture

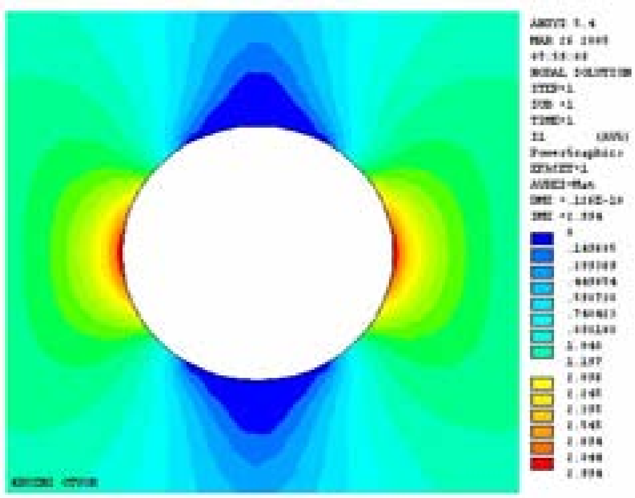

Figure 2. Lines of equal $\sigma_{\max}$ stress

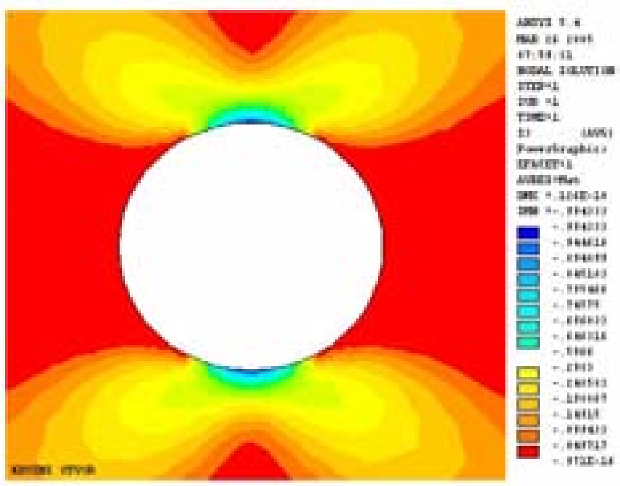

Figure 3. Lines of equal $\sigma_{\min}$ stress

The second example applies to a single-axis tensioned plate weakened by an ellyptic opening with relation of semi-axes $k=b/s=2/3$, where the semi-axis $a=100$ mm in the direction of the *x*-axis and $b=66.67$ mm in the direction of the y-axis. The angle between the lines of application of forces and the *x*-axis is $\alpha=90^\circ$. Figures 4 - 6 show the results for stress values.

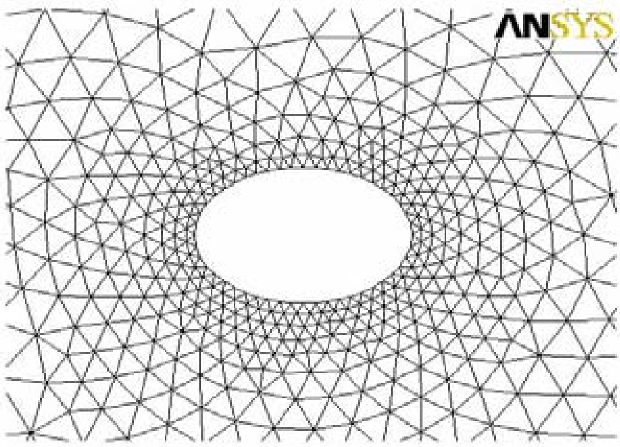

Figure 4. The mesh of FE around the aperture

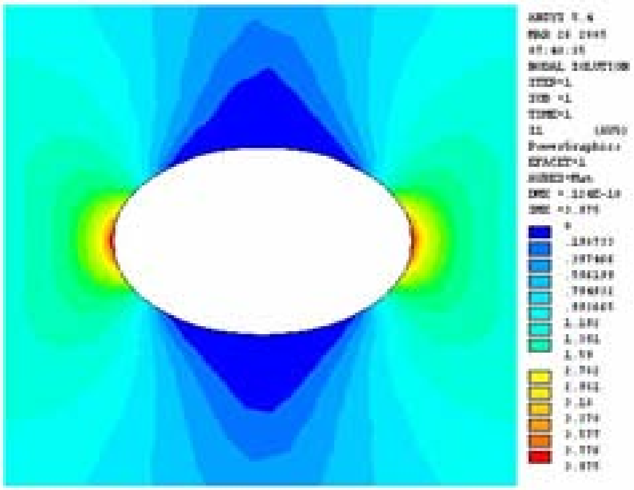

Figure 5. Lines of equal $\sigma_{\max}$ stress

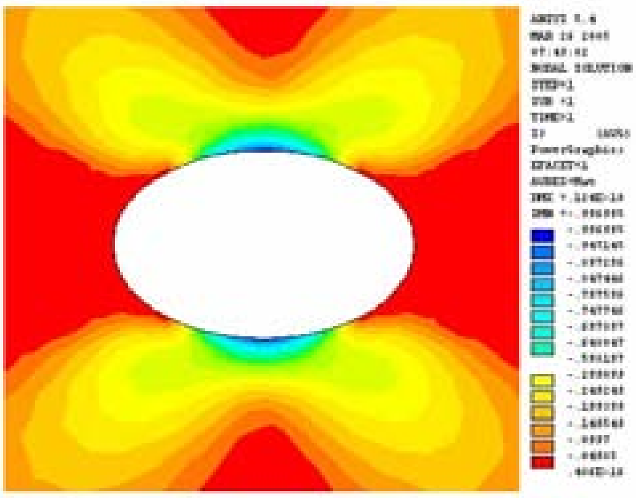

Figure 6. Lines of equal $\sigma_{\min}$ stress

The third example shows a single-axis tensioned plate weakened by a square opening of the side $a = 100$ mm, with the radius of the angle rounding $r_{\theta=45^\circ} = 0.0245a = 2.45$ mm. Figures 7 - 9 illustrate the mesh and stress values around the opening.

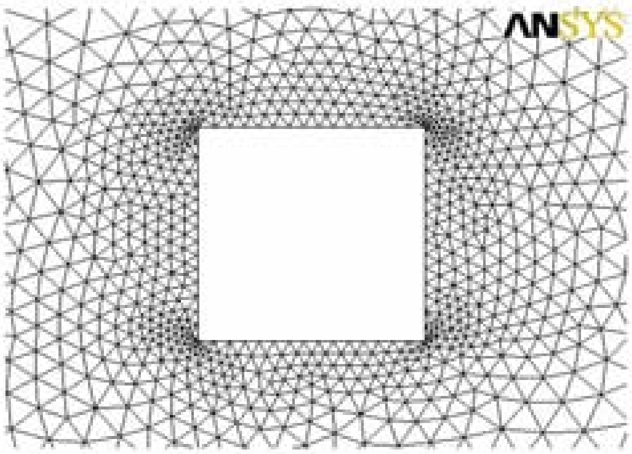

Figure 7. The mesh of FE around the aperture

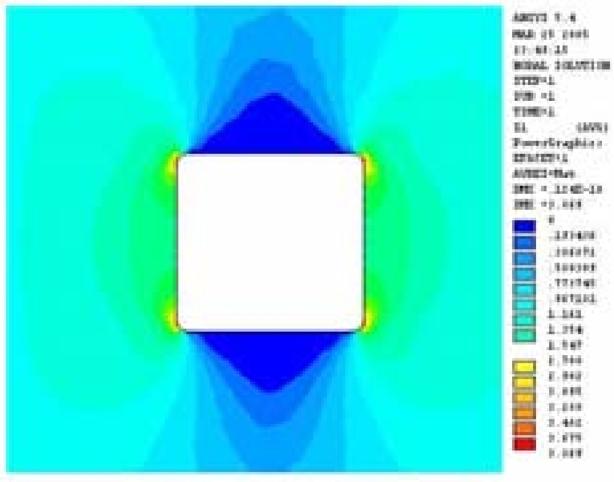

Figure 8. Lines of equal $\sigma_{\max}$ stress

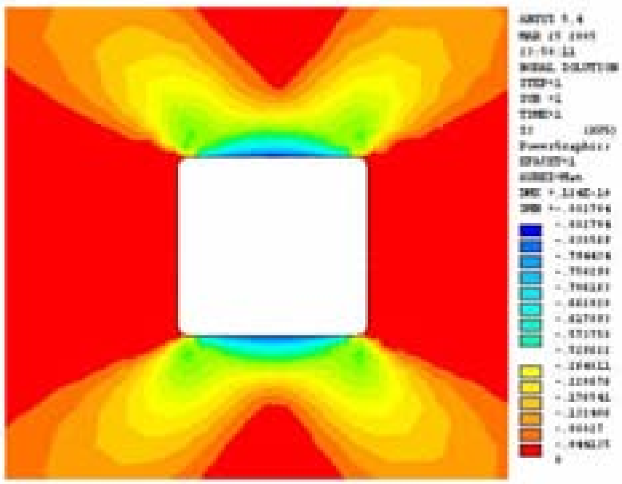

Figure 9. Lines of equal $\sigma_{\min}$ stress

The Fourth example applies to a single-axis tensioned plate weakened by a rectangular opening with relation of the sides $a/b = 5$, where the sides are $a = 100$ mm, $b = 20$ mm, the radius of the angle rounding is $r = 0.06a = 6$ mm and the angle between the lines of application of forces and the axis is $\alpha = 90^\circ$.

Figures 10 - 12 illustrate the mesh and stress values around the opening.

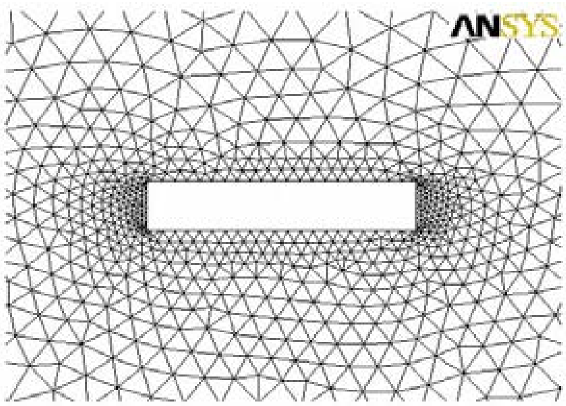

Figure 10. The mesh of FE around the aperture

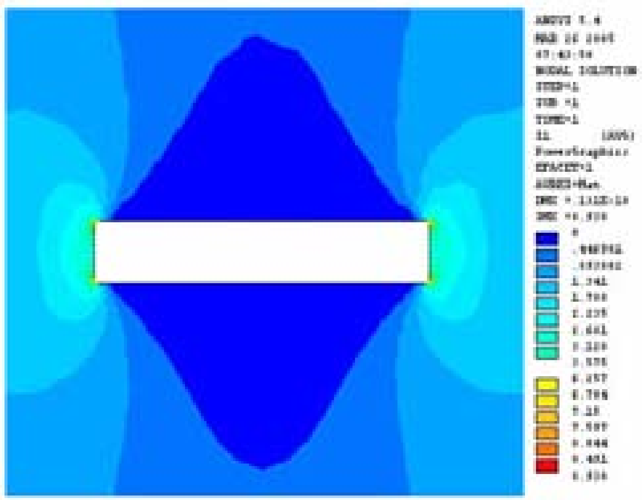

Figure 11. Lines of equal $\sigma_{\max}$ stress

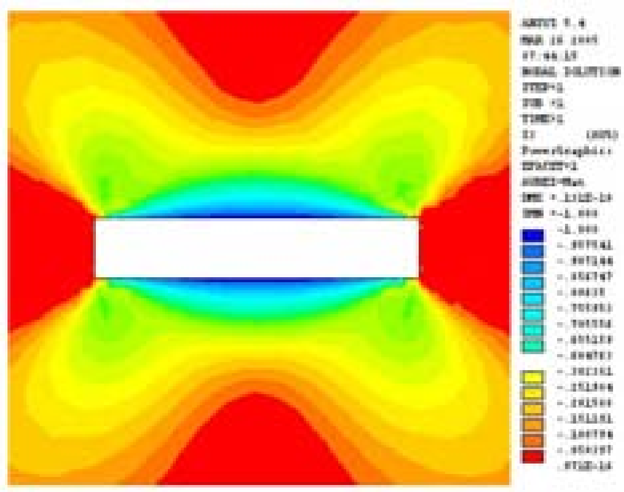

Figure 12. Lines of equal $\sigma_{\min}$ stress

By comparing the values obtained we can conclude that with all other conditions identical the most unfavourable case is when there are rectangular openings with the longer side in the direction of the *x*-axis.

In order to verify numerically the obtained results, in this chapter we made a comparison with the theoretically obtained results of stress distribution for each of the above given examples, Table 1.

Table 1. The effects of shape of hole on stress concentration

Applied method	Maximum stress values around holes [N/m^2]			
	Circular hole	Elliptical hole	Square hole	Rectangular hole
Analytical	$\sigma_{\max(anal)} = 3.00$	$\sigma_{\max(anal)} = 4.00$	$\sigma_{\max(anal)} = 3.860$	$\sigma_{\max(anal)} = 8.050$
FEM	$\sigma_{\max(MKE)} = 2.994$	$\sigma_{\max(MKE)} = 3.975$	$\sigma_{\max(MKE)} = 3.869$	$\sigma_{\max(MKE)} = 8.938$

Geometrical factor of stress concentration can be determined according to these values [5], [7].

Conclusion

Summarizing the results presented in this paper, we come to the conclusion that the application of the finite element method can be used very efficiently in solving the problems of stress-strain conditions of thin plates with different shapes of openings.

Either longitudinal or transversal, the openings are sources of stress concentration, i.e. the points which cause an increase of stress intensity. Investigations have shown that the intensity of these stresses depends not only on shape and dimensions of the openings but also on their position in regards to the lines of application of external load.

The values obtained have led to the conclusion that openings of a rectangular shape are most unfavourable as far as stress concentration is concerned.

The developed methodology and application of the finite element method make finding solutions for stresses, strain, etc. significantly easier, which gives great advantage over classical analytical methods.

Further work can include anisotropy of material and temperature stresses, dynamic stress and the like.

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Primena numeričkih metoda pri rešavanju fenomena u teoriji tankih ploča

Pod tankom pločom u Teoriji elastičnosti podrazumeva se elastično telo cilindričnog ili prizmatičnog oblika male debljine u odnosu na druge dve dimenzije. Osnovne zavisnosti između geometrijskih i fizičkih veličina svode se uglavnom na postavljanje veza između stanja napona i stanja deformacije i spoljašnjeg opterećenja, što se opisuje diferencijalnim jednačinama, običnim ili parcijalnim. Metode koje se koriste za rešavanje postavljenih jednačina, uz zadovoljenje odgovarajućih konturnih i početnih uslova, mogu da se svrstaju u dve grupe: analitičke i numeričke. U slučaju složenih i velikih sistema konstrukcija izloženih dejstvu proizvoljnog opterećenja uključujući i složene granične uslove, rešavanje diferencijalnih jednačina analitičkim putem je veoma teško ili nemoguće. Tada se rešenje traži uz pomoć numeričkih metoda.

U ovom radu, za razmatranje fenomena u ravanskom izotropnom polju, odnosno kod tankih ploča pri različitim graničnim uslovima i opterećenjima, primenjuje se metoda konačnih elemenata (MKE).

Na kraju rada su dati komentari i pravci daljih istraživanja.

Ključne reči: **ploča, tanka ploča, naponsko stanja, raspodela napona, numeričke metode, metoda konačnih elemenata.**

Применение цифрового метода при разрешении феномена в теории тонких плит

Под тонкой плитой в Теории эластичности подразумевается упругое тело цилиндрической или призматической формы маленькой толщины по отношению к другим двумя размерам. Основные зависимости между геометрическими и физическими значениями в основном обосновываются на установлении связей между состоянием напряжённости и состоянием деформации и внешней нагрузки, что определяется (описывается) дифференциальными уравнениями, либо обыкновенными, либо частичными. Методы, использовавшие для решений установленых уравнений, со соблюдением соответствующих контурных и исходных условий, возможно классифировать в две группы: аналитические и цифровые. В случае конструкций сложных и масштабных систем, подвергших к действию произвольной нагрузки, включая и сложные предельные условия, решение дифференциальных уравнений аналитическим способом очень трудно или даже невозможно. Тогда решение надо искать при помощи цифровых методов.

В настоящей работе, для рассматривания феномена в плоском изотропном поле, т.е. у тонких плит при различных предельных условиях и нагрузках, применяется метод конечных элементов (МКЭ). В конце работы приведены комментарии и пути будущих исследований.

Kly~evwe slova: **плита, тонкая плита, состояние напряжённости, распределение напряжения, цифровые методы, метод конечных элементов.**

Application des méthodes numériques pour résoudre les phénomènes dans la théorie des plaques minces

Dans la théorie de l'élasticité sous le terme de plaque mince on entend un corps élastique, de forme cylindrique ou prismatique, de petite épaisseur par rapport aux deux autres dimensions. Les dépendances basiques entre les propriétés géométriques et celles physiques sont généralement en établissement des relations entre l'état de tension et l'état de déformation et la charge externe, ce qu'on décrit par les équations différentielles, simples ou partielles. Les méthodes utilisées pour résoudre les équations posées et avec accomplissement des conditions initiales et de contour, sont classées en deux groupes: analytiques et numériques. Au cas des grands systèmes complexes de constructions qui sont exposés à l'action de la charge arbitraire, y compris les conditions limites complexes, la résolution des équations différentielles par la voie analytique est très difficile ou impossible. La solution est alors recherchée à l'aide des méthodes numériques. Dans ce travail , pour l'étude des phénomènes dans le champ isotrope plan, c'est-à-dire chez les plaques minces dans les différentes conditions limites et charges, on utilise la méthodes des éléments finis (MKE).

Mots clés: **plaque, plaque mince, état de tension,distribution de la tension, méthodes numériques, méthode des éléments finis.**