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Load Spectrum Determination for Computing and Laboratory Testing

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This paper presents a procedure for the load spectrum estimation based on the recorded working load. For a large number of cycles, the working load can be statistically analyzed and both a type and the parameters of the statistical distribution estimated. On the basis of such a suitable selection of load spectra and their step transformation, it is possible to introduce a systematic approach into this procedure in order to simplify the fatigue testing and the calculation of fatigue damage accumulation and finally the estimation of fatigue life. The load spectra for the wing-fuselage connection are presented.

Key words: **load, working load, load calculation, material fatigue, protection against load, statistical analysis.**

Introduction

THE loading of machine parts in real service or operational conditions, in general case, is a random time variable, with variable cycle parameters, Fig.1a). It is different from the traditional load testing in laboratory conditions where the parameters are mainly constant. Such random time variable load during real work of a machine under operational, i.e. service conditions is simply called working load. The elementary unit of each working load is one cycle with its main parameters: amplitude, mean value, frequency and form, i.e. cycle type. Besides, these parameters in each cycle of working load are different, i.e. they are also time dependent variables. The form of a loading cycle and its frequency have secondary influence on a fatigue process but the mean value and particularly the amplitude must be taken into consideration. Because of that, the complete working load process during machine operation is treated only through these two parameters, both as random occurrences. There are all conditions for a statistical approach analysis of working load in a way where the loading amplitude and the mean values in each cycle are recorded and then the probability distribution versus the intensity of parameter values can be estimated by the use of any counting method. Furthermore, this approach is used for the determination of loading or stress spectra which are the basis for laboratory simulations of testing under working conditions or for the estimation of fatigue life of machine parts and components [1, 3]. There are several different methods for representing a complex load history, such as: Peak counting, Peak between mean crossing counting, Level crossing counting, Range counting, Range-mean counting, Range pair cycle counting, and the Rain flow cycle counting method. All these methods give the results with no significant difference [4, 7, 9, 10].

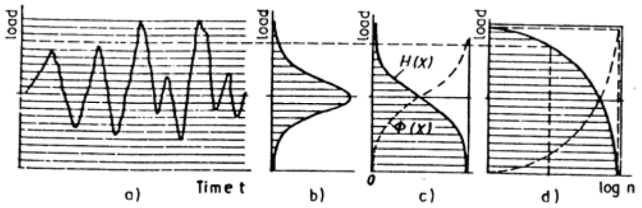

Figure 1. Working load (a), histogram of distribution (b), cumulative probability (c), continuously loading spectrum (d)

Working load recording and its statistical treatment

Let us designate the working load as X (force F, moment M or torque T). The working load is a time dependent variable of its parameters such as: amplitude, mean value, frequency and form, i.e. type of a cycle. The service load is a random process that should be recorded on the same type of machine (s) under selected representative service conditions. For a long work period it describes the typical representative working load with a large enough number of cyclic loading with a variety of different values of amplitude, mean value, frequency and type of cycle. The continuous fatigue process under such working load should be decomposed by using two primary parameters: the amplitude and the mean value. In other words, the variable random loading decomposition gives a set of large number of different events which are expressed through elementary loading cycles with different intensity of these two parameters and their relative frequencies during the whole working time [1, 3, 4, 8, 9, 11]. For a fatigue process, the amplitude and the mean value are of the primary influence. The random loading during the whole working time is

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decomposed into a big enough number of these two variable parameters: the intensity of amplitude and the mean value. However, what is the most significant for a fatigue process is the intensity of amplitude which can vary in a wide range.

Because of the cyclical nature of both working and testing load, there are the same conditions for the occurrence of critical fatigue damage on machine parts due to the appearance of fatigue cracks or fatigue failure. Although there are considerable differences of amplitudes in the working load, the largest one is insufficient to cause a sudden and quick fracture of the part. It is also known that the fatigue of metals has a cumulative feature [5, 6, 7. 8, 9, 10, 11, 12]. This is the reason why the effect of rarely extreme high values of a loading amplitude can cause a relatively high fatigue damage degree in the material structure that is not even sufficient to cause a fatigue crack at once. For the analysis of a fatigue process it is of prime significance to estimate a damage degree of each single cycle, its level and sequence and then by using any of the convenient hypotheses of fatigue damage accumulation to establish the corresponding total damage during all loading cycles [1, 5, 6, 11, 13]. Thereafter, the corresponding whole number of all endured cycles needed to cause the critical fatigue damage (crack or fatigue fracture) represents a critical fatigue life that should be in analysis when testing or by calculation of fatigue damage accumulation. However, there are no reliable fatigue damage accumulation theories for such calculations but only various mostly very complex hypotheses the application of which for damage accumulation can be very problematic, especially by nonlinear and cycle sequence dependent hypotheses [2, 8, 9, 12]. Such estimated results are also as unreliable as a simpler Palmgren-Miner rule and both procedures need a testing verification.

Because of the foregoing reason, not only for computing but also for testing, there is a need for the determined service loading to be treated by its decomposition to each cycle level with its amplitude and mean values. The number of cycles obtained by this procedure can be considering as random, mutually independent events which represent a large set of data. Such a way is usually considered only for the distribution of the loading amplitude or the level, but the sequence appearance of each cycle is neglected [1, 3, 4, 7, 8, 9, 10, 11, 12].

Determination of working load and the methods for the step spectrum estimation

The recorded loading process during the whole working life is presented in Fig.1a). As mentioned above, due to the working load complexity, for a simple quantitative determination of fatigue processes, it is enough only to have an analysis taking its amplitude and eventually the corresponding mean value. These two main fatigue parameters define each cycle as an elementary unit event. The whole representative loading time history, in the whole interval of amplitude and mean value, should be decomposed applying a suitable counting method in a set of loading cycles with different amplitude, mean value and corresponding number of cycles (or half cycles). For this purpose, each of these cycle counting methods shows certain different deficiencies [4, 7, 8, 9, 10, 11]. Independently of various different counting methods, their statistical treatment is unique and common.

Let the whole interval of working load X with its total extreme working load levels, from the minimum X_{min} to the maximum value $X_{max}=X_1$, be decomposed into n_s various cycles or $2n_s$ half-cycles; the each n-th cycle or half cycle in its natural - on line working load cycle sequence is characterized with particular load mean value $X_{m,n}$, load amplitude $X_{a,n}$, load extreme value – upper $X_{\max,n} = X_{m,n} + X_a$ and lower $X_{\min,n} = X_{m,n} - X_a$. The interval of total extreme values of working load is divided into the mutual equal z subintervals, the each of them being characterized with the same width ΔX which is

$$\Delta X = \frac{X_{\max} - X_{\min}}{z} \tag{1}$$

If we assume that the total upper value is coincidentally the upper value for the first subinterval, $X_{\max} = X_{k=1} \equiv X_1$ which corresponds to a chosen occurrence probability of the greatest loading, for example in most 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, then the upper loading value in each arbitrary lower k-th subinterval is

$$X_k = X_1 - \left(k-1\right)\Delta X \ , \ k = 1, 2, 3, ..., z \tag{2}$$

By applying a chosen counting method with contemporary acquisition techniques, the corresponding numbers of cycles to the first, second, and k-th subinterval are: $n_{k=1}$, $n_{k=2}$, and n_k, k=1, 2,…,z, ($k_0 = 0$). By such selection in a ranked loading sequence the natural ordering (time rate sequence) of each loading cycle is fully neglected, which in a way reproduces a different fatigue process as under original ordering in random loading. The common great number of all subintervals z is then classified into a smaller number of j larger groups, each of them forming a corresponding loading step. So the first, the second, and the i-th step consists of: $k_1, k_2, ..., k_i$ subintervals with a corresponding number of cycles:

$$n_1 = \sum_{k=1}^{k_1} n_k \ , \ n_2 = \sum_{k=k_1+1}^{k_2} n_k \ ,\ldots,\ n_i = \sum_{k=k_{i-1}+1}^{k_i} n_k \ ,\ldots,$$
$$n_j = \sum_{k=k_{j-1}+1}^{k_j} n_k \tag{3}$$

where the sum of the number of subintervals in each step is equal to the whole number of all subintervals z :

$$k_1 + k_2 + ... + k_i + ... + k_j = \sum_{i=1}^{j} k_i = \sum_{i=1}^{j} \sum_{k=k_{i-1}+1}^{k=k_i} k_k = z \tag{4}$$

and similar for the corresponding number of cycles

$$n_1 + n_2 + ... + n_i + ... + n_j = \sum_{i=1}^{j} n_i = \sum_{i=1}^{j} \sum_{k=k_{i-1}+1}^{k_i} n_k = n_s \tag{5}$$

By dividing eq.(5) with n_s, we get

$$\frac{n_1}{n_s} + \frac{n_2}{n_s} + ... + \frac{n_i}{n_s} + ... + \frac{n_j}{n_s} = \sum_{i=1}^{j} \frac{n_i}{n_s} = \sum_{i=1}^{j} p_i = 1,0 \tag{6}$$

where p_i represents the probability for occurring loading values only in the $i-$th step between X_{i-1} and X_i with k_i subintervals ΔX_i is then as the ratio of the favorable n_i to the total possible events n_s

$$p_i \equiv p(X_i)=\frac{n_i}{n_s}\le 1 \qquad (7)$$

In each step there are more different values of loading and one value should be estimated as representative loading in a given step. At first [1], it was an algebraic mean value of the step, for example in the i-th step, the value $X_i=\left(X_{i-1}+X_i\right)/2$, or the value which gives the same "area" in the rectangular step as all different subintervals areas, i. e.

$$X_i=\frac{1}{n_i}\sum_{k_{i-1}+1}^{k_i} X_{k_i}n_{k_i} \qquad (8)$$

Later was suggested that the representative step loading value should be estimated by means of the same fatigue damage under a constant level step as the sum of fatigue damage of all different subintervals [8, 9, 11, 13]. This value represents the equivalent value in the given step, according to [11, 13], which is

$$X_{i,eqv}=X_i\left(D_{k_i sub\,\mathrm{int}\,ervals}\right)^{1/m}=X_i\left[\sum_{k_{i-1}+1}^{k_i}\left(\frac{X_k}{X_i}\right)^m\frac{n_k}{n_i}\right]^{1/m}\le X_i \qquad (9)$$

where are: $D_{k_i sub\,\mathrm{int}\,ervals}$ the fatigue damage in all k_i subintervals loading in the i-th step according to the Palmgren-Miner rule, and m is the exponent of the Wöhlers S-N curve. In such way the representing loading value can be estimated in all steps of the spectrum.

For each step of this representing loading value, the corresponding discrete probabilities $p_1,p_2,p_3,...,p_i,...,p_j$, create a histogram of discrete probability versus the corresponding step loading values $X_1,X_2,X_3,...,X_i,...,X_j$, and the probability density function $p=f(X)$.

The corresponding cumulative probabilities for given loading steps, from 1 to q, $X_q\le X\le X_1$ are respectively the ratio of the favorable events of all q loading steps, which with the current index $i=1,2,...,q$ gives $n_q=\sum_{i=1}^{i=q}n_i$, and the total number of all possible events which is equal to a total number of cycles in the loading spectrum n_s,

$$P_q\equiv P\left(X_q\le X\le X_1\right)=\frac{\sum_{i=1}^{q}n_i}{n_s}=\sum_{i=1}^{q}\frac{n_i}{n_s}=\sum_{i=1}^{q}p_i \qquad (10)$$

When the number of cycles is large, such histogram can be transformed into a continuous line as shown in Fig.1b). The cumulative probability for a load value X in the domain between the minimum value $X_{\min}$ and the value $X\ge X_{\min}$ is defined by the cumulative distribution function as

$$\phi(X_{\min}\le X)=\int_{X_{\min}}^{X}f(X)dX=\phi\left(X\right)-\phi\left(X_{\min}\right) \qquad (11)$$

The cumulative probability for the total range of all possible loading values from $X_{\min}$ to $X_{\max}\equiv X_1$ is equal to 1.0, so that the cumulative probability for the loading range from X to $X_{\max}\equiv X_1$ is

$$\phi(X\le X_1)=\int_{X}^{X_1}f(X)dX=\phi(X_1)-\phi(X) \qquad (12)$$

The cumulative probability $H\left(X_1\ge X\right)$ for the remaining loading range can be defined in a similar way, so then

$$H\left(X_1\ge X\right)=1-\phi\left(X_{\min}\le X\right) \qquad (13)$$

is valid for the total loading interval $X_{\min}\le X\le X_{\max}\equiv X_1$.

In that way, the cumulative distribution function $H\left(X\right)=1-\phi\left(X\right)$ enables the integration beginning with the maximum load value which is the most important in the whole spectrum (Fig.1c). If the cumulative probability is expressed by the total - instantaneous running number of the cycle n throughout the chosen total number of cycles for the given spectrum n_s, and if it is represented usually as the log-axes versus the loading or the stress value as an absolute X or a relative presentation $\left(X/X_1\right)$, in the linear or the log axis, we obtain a continuously falling (decreasing) sequence (ordering) in a form of cumulative distribution function (Fig.1d). This diagram in an appropriate form represents the given spectrum in the same coordinate system as the corresponding Wöhlers- or $S-N$ curve.

In this way the following can be estimated for each step of the loading spectrum: the loading X_i, the corresponding number of cycles n_i and finally the total greatest level of the loading interval in the counting method as the greatest step level which is at the same time the spectrum level.

The loading spectra are sometimes presented in a relative loading or corresponding stress level as the ordinate as

$$x_i=\frac{X_i}{X_1}\le 1 \qquad (14)$$

and the relative number of cycles as the relative abscissa

$$f_i=\frac{n_i}{n_s}\le 1 \qquad (15)$$

Such relative stress spectrum with relative stress as the ordinate, and the running number of cycles as the abscissa, shows in any way the so-called fulfillment degree of the given relative loading spectrum to one full one step spectrum with a constant amplitude and a mean value the fulfillment degree of which is 1.0. According to the linear Palmgren-Miner rule, [11, 13], for a known exponent value m of theWöhlers S-N curve, the fatigue damage degree for the given loading or stress spectrum is

$$D_s=\sum_{i=1}^{j}\left(\frac{X_i}{X_1}\right)^m\cdot\frac{n_i}{n_s}=\sum_{i=1}^{j}x_i^m\cdot f_i\le D_{one\ step\ spectrum}=1{,}0 \qquad (16)$$

Based on the loading spectrum, a so-called block loading is formed which in any way represents the natural ordering (time rate sequence) of the loading levels in the original working load.

Depending on working loads, step loads spectra can be represented by several typical forms [5]. Fig.2 shows the general spectra with variable amplitudes and variable mean values, but the constant amplitude within each steps. Based on these general spectra, especially in aviation industry, there are some several test methods in a form of standardized block loading spectra such as TWIST, FALSTAFF, and others for which the testing results are known [1, 2, 3, 4, 8, 9, 10, 14].

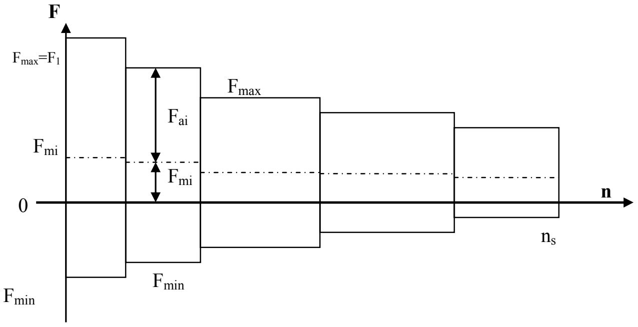

Figure 2. General spectra with variable amplitudes and variable mean values, but with a constant amplitude within each step

Numerical example

In order to illustrate the procedure for the estimation of loading spectra, one numerical example is included here. One similar loading spectrum is estimated by a wing-fuselage joint of an airplane[15], Fig.3a. The loading spectrum represents 50 flight hours and has 6 steps. The loading value and the number of cycles in each step is given in the Figure. For these cases the working load shows its natural level sequence from the minimum to the maximum levels and vice versa. In this way, a block loading spectrum is then formed as seen in Fig.3b) and with data shown in Table 1. Thus estimated block loading spectrum is directly used for fatigue testing of one part of this joint, Fig.4.

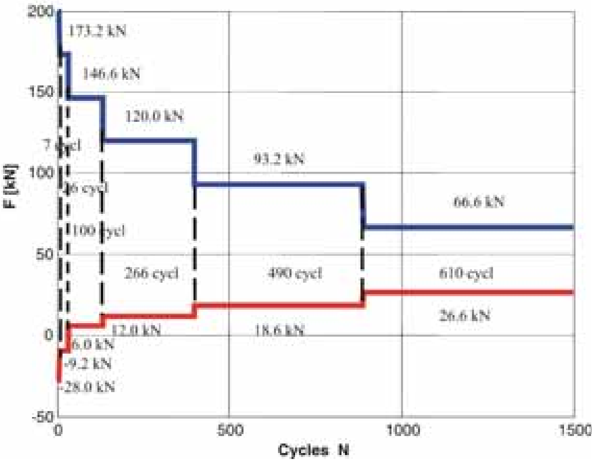

Figure 3a). Loading spectra of a wing-fuselage joint

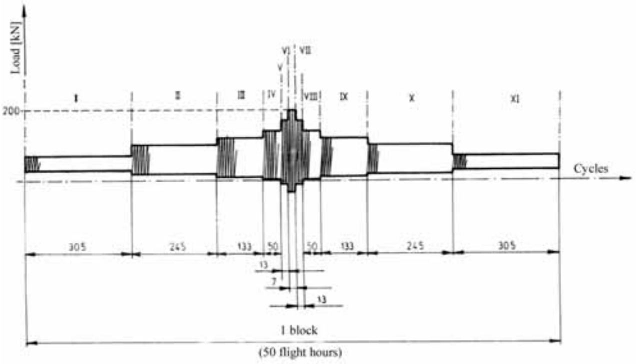

Figure 3b). Testing blocks - Loading spectra of a wing-fuselage joint

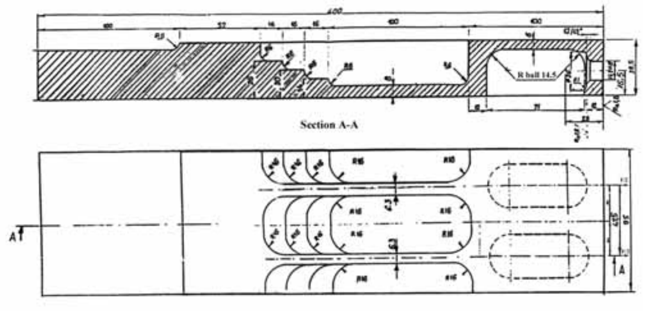

Figure 4. Testing part of a wing –fuselage joint

Table 1. Load spectra and corresponding stresses in the joint critical region

Load level	n_i	F_{min}[kN]	F_{max}[kN]	σ_{min}[MPa]	σ_{max}[MPa]
I	305	26.6	66.6	58.8	147.2
II	245	18.6	93.2	41.0	205.5
III	133	12.0	120.0	28.0	279.8
IV	50	6.0	146.6	13.3	313.8
V	13	-9.2	173.2	20.3	336.5
VI	7	-28.0	200.0	61.9	347.0
VII	13	-9.2	173.2	20.3	336.5
VIII	50	6.0	146.6	13.3	313.8
IX	133	12.0	120.0	28.0	279.8
X	245	18.6	93.2	41.0	205.5
XI	305	26.6	66.6	58.8	147.2

These load spectra or block load spectra can be also used for a numerical analysis in fatigue life prediction, stress-strain behavior in a notch root, fatigue crack initiation and propagation phases and assessment of fulfillment and a fatigue damage degree as well as in an experimental testing procedure. On the basis of the known results for some loading spectra it is possible to estimate the corresponding data for other loading spectra with higher reliability in various cases of machine works.

Conclusions

Real working load is of probabilistic nature and can be represented by a statistical variable. Working load is, in a general case, a random time variable magnitude which must be statistically treated to obtain some appropriate procedures which are more suitable for various engineering purposes. It is thus necessary to record the on-line working load, and, by using a chosen favorable cycle counting method, to obtain a statistical set of loading parameters. By applying the statistical methods, it is possible to form a function of probability and a corresponding loading spectrum with necessary parameters of its distribution, such as amplitude, mean value and number of cycles for each step level. Above all, this enables computing fatigue damage, estimating fatigue life and predicting fatigue crack initiation and propagation phases based on the known experimental fatigue data. Especially for the verification of computing data, the starting bases are a load spectrum and a block load spectrum with its natural time rate sequence, i.e. its ordering of a loading level. In this way, for the established loading spectra for various cases of machine working loads, it is possible to estimate the characteristics of machine parts with higher reliability.

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Određivanje spektra opterećenja za proračune i laboratorijaska ispitivanja

U ovom radu je izložen postupak određivanja spektra opterećenja na bazi zapisa radnog opterećenja. Za veliki broj ciklusa, radno opterećenje se može statistički analizirati i ordediti raspodela i parametri raspodele. Bazirano na pogodnom odabiru spektra opterećenja i transformaciji u određene intervale broja cilusa, moguće je uvesti određenu sistematičnost u postupak, u cilju pojednostavljenja i prilagođenja za ispitivanje zamora, za proračune akumulacije zamornog oštećenja i procene veka. Prestavljen je spektar veze krilo-trup.

Ključne reči: **opterećenje, radno opterećenje, proračun opterećenja, zamor materijala, zaštita od opterećenja, statistička analiza.**

Определение спектра нагрузки для расчётов и исследований в лабораториях

В настоящей работе представлен поступок определения спектра нагрузки на основе заметок рабочей нагрузки. Для большого количества циклов, рабочую нагрузку возможно статистически анализировать и определить оценку распределения и параметры распределения. Обосновано на подходящем отборе спектра нагрузки и превращении в определённые интервалы количества циклов, возможно ввести определённую систематичность в поступок, с целью упрощения и приспособления для исследований усталости, для расчётов накопления усталостных повреждений и оценки срока службы. Здесь представлен сректр связи крыло-фюзеляж.

Kly~evwe slova: **нагрузка, рабочая нагрузка, расчёт нагрузки, усталость материала, защита от нагрузки, статистический анализ.**

Détermination du spectre de la charge pour les computations et les essais de laboratoires

On a présenté le procédé pour la détermination du spectre de charge basé sur l’enregistrement de la charge de travail dans le cadre de ce papier. Pour un grand nombre de cycles la charge de travail peut être analysée de point de vue statistique et l’on peut déterminer la distribution et les paramètres de cette distribution. Se basant sur le choix adéquat du spectre de la charge et la transformation en intervalles du nombre des cycles, il est possible d’introduire la systématique dans ce procédé afin de simplifier et d’adapter les recherches sur la fatigue pour le calcul de l’accumulation de défaillance de fatigue et l’estimation de la durée de vie. On a présenté le spectre de la connexion entre l’aile et le fuselage.

Mots clés: **charge, charge pratique, calcul de la charge, fatigue de matériel, protection contre la charge, analyse statistique**