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Numerical Stress Concentration Analysis of a Driven Gear Tooth Root With Two Fillet Radius

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Determining an optimal gear form relative to stress concentration is one of the main problems of gear design. Gear carrying capacity largely depends on main gear profile configuration parameters. The tooth root form and fillet radius have a great influence on gear tooth root strength which is primarily discussed in this paper. Special attention is given to the analysis of the impact of the gear tooth fillet radius at the critical cross section on stress values and distribution. The stress intensity factor and the gear working life depend directly on tooth root stress. It is known that a first initial crack appears on a gear tooth most affected by root stress concentration. Hence, this research was focused on finding an optimal fillet tooth root radius to minimize the tooth root stress intensity. However, in order to get a lower tooth root stress concentration value, one more fillet radius is involved as a "disencumber notch" [3]. In this paper a special attention will be dedicated to stress concentrations in a gear tooth root with two, instead of only one fillet radius ("two level approach" in a root). This paper provides the results achieved by the application of numerical methods - the finite element method (FEM) and the real working conditions simulation. The results are analyzed in order to form an effective numerical model for tooth root geometrical discontinuity phenomena at static loading. Although tooth root critical stress conditions were analysed for both driving and driven gears of one real transmitter, this paper presents the stress results only for the driven gear for both fillet radii ρ_{F1} and ρ_{F2}.

Key words: **tooth, stress state, stress concentration, pattern optimisation, numerical analysis, finite element method.**

Introduction

WHEN spur gears are concerned, the greatest stress concentration appears in tooth roots. The form and dimensions of a tooth root fillet radius have a great influence on stress concentration, crack initiation and even fatigue failure appearance. The most important errors may be flaws in materials such as: holes and disconnections, non-metal alloys, thermal handling errors, etc. Tooth root failures are mostly caused by these flaws. Elastic-plastic deformations, in a combination with some other unfavourable conditions, may cause crack initiation and catastrophic failures. If mashed gears are made of the same material, driving gear tooth root failures are often consequence of material fatigue.

The author of this paper investigated the influence of only one tooth root fillet radius on critical section stresses and the stress concentration factor in preliminary work [1, 2]. It is concluded that the tooth root stresses decrease as the fillet radius increases. It means that, with higher fillet radius values, the gear safety factor and its tooth root capacity become higher. Tooth root stresses are most important for crack initiation and failure appearance.

Characteristics of the analyzed gears

All analytical and numerical research is carried out on a real structure with a real condition simulation. The considered gears are a part of the structural elements of a large power planetary tra nsmitter (big excavator with a power of 2x550 kW) with the following characteristics:

- module m = 24mm,
- profile movement coefficient x_1 = 0 and x_2 = 0.326,
- face width b = 350mm,
- rotation moment T = 2528.8 kNm,
- number of rotations n = 4.1596min^{-1},
- number of teeth z_1 =20 and z_2 = 96,
- gear material is carburized steel 17CrNiMo6 (in accordance with DIN) with Young's modulus E=2.1x105MPa and Poisons's ratio ν=0.3.

All stress concentration analyses are carried out on tooth roots with only one fillet radius. However, in order to get lower tooth root stress concentration values, one more fillet radius as a "disencumber notch" is involved [3]. A special attention will be dedicated to stress concentration in gear tooth roots with two, instead of only one fillet radius ("two level approach" in a root).

The dimensions that define the tooth root geometry are presented in Fig.1.

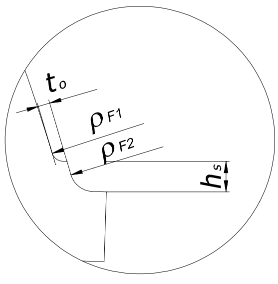

Figure 1. Dimensions of the gear tooth root fillet radius

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The numerical analysis is performed to determine the Von Mises stresses and the normal stresses for a driven gear of a planetary transmitter designed with two tooth root fillet radii ρ_{F1} and ρ_{F2} of the same or different values. The finite element method (FEM) for numerical analysis is used [6, 7], i.e. the finite element package FEMAP v.9.3 [4].

On the basis of the gear data, the 3D finite element model is made. One driven gear tooth is modeled by 11440 elements and 13000 nodes in order to have the same number of nodes and elements as the same analyzed gears with only one tooth root fillet radius. The mesh is refined in the tooth fillet region in order to show, as well as possible, the stress condition in that section. Only a 50mm wide layer of the tooth is analyzed, in accordance with a supposition that load is equally distributed along the instantaneous tooth side contact line. The driven gear tooth is loaded with the normal force F_{bn} which is acting with its whole value at the outer point of the single tooth contact (B) and with its half value at the inner point of the double contact (A).

The results of tooth root stresses for the driven gears and for both fillet radii ρ_{F1} and ρ_{F2} are shown in Table 1. The stress values in Table 1 present the tooth root critical section stress values. The stress on the tensile side of the gear tooth critical section causes crack initiation and failure appearance, and for that reason it is only presented in this Table.

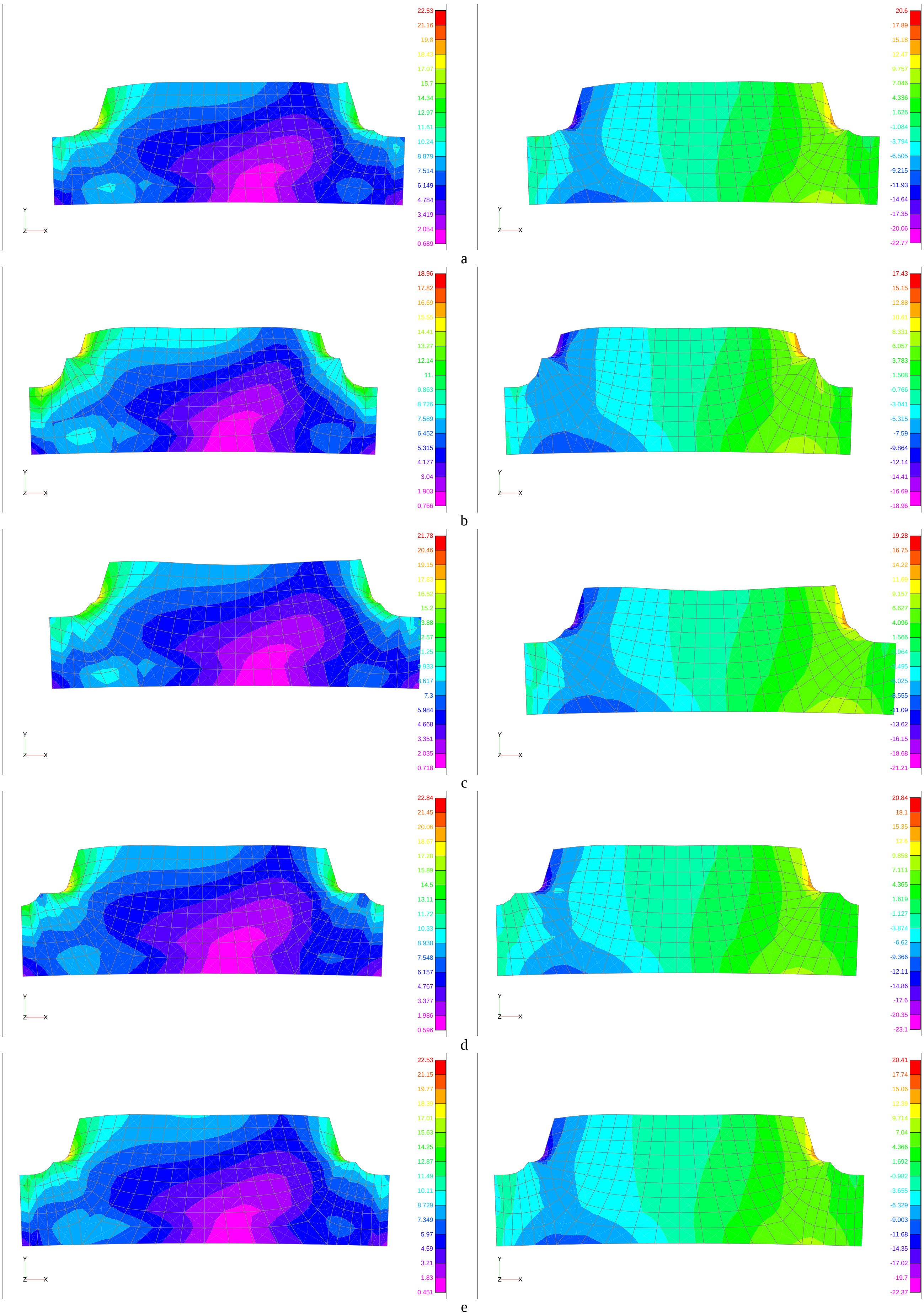
The tooth root critical cross section, in accordance with the ISO recommendation, is determined by the tooth fillet radius tangent positioned at an angle of 30° to the gear symmetry line, and its dimensions are the critical cross section width s_{Fn} and the face width b.

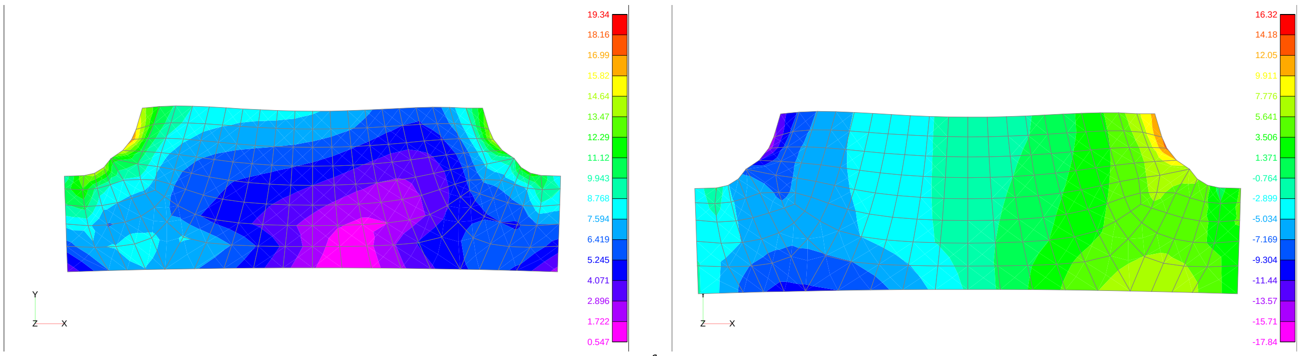
At the beginning, the first six examples were investigated [5], and then, depending on the stress results, the dimensions of the tooth root (Fig.1) were modified in order to get lower stress values than the first ones. Some of the investigated cases were successful and some not. It is presented in Table 1 (m is the gear module).

Table1. Equivalent and normal stress values for the tensile side of the driven gear for different tooth root shapes with two fillet radii ρ_{F1} and ρ_{F2}

Tooth root fillet radius form	Driven gear				Tooth root fillet radius form	Driven gear			
	Equivalent stresses [MPa]		Normal stresses [MPa]			Equivalent stresses [MPa]		Normal stresses [MPa]	
	A	B	A	B		A	B	A	B
a	Gear tooth root fillet radius dimensions				**f2**	Gear tooth root fillet radius dimensions			
	t_o	hs	ρ_{F1}	ρ_{F2}		t_o	h_s	ρ_{F1}	ρ_{F2}
	0,1m	0,05m	0,1m	0,2m		0,1m	0,125m	0,3m	0,2m
	2,4mm	1,2mm	2,4mm	4,8mm		2,4mm	3mm	7,2mm	4,8mm
ρ_{F1}	145,4169	194,0668	140,6825	184,7749	ρ_{F1}	130,9370	165,6977	117,9687	144,7214
ρ_{F2}	78,1971	119,9051	35,3467	48,8787	ρ_{F2}	87,6935	128,8913	53,0514	70,1433
b	Gear tooth root fillet radius dimensions				**f3**	Gear tooth root fillet radius dimensions			
	t_o	h_s	ρ_{F1}	ρ_{F2}		t_o	h_s	ρ_{F1}	ρ_{F2}
	0,1m	0,25m	0,1m	0,2m		0,1m	0,125m	0,1m	0,2m
	2,4mm	6mm	2,4mm	4,8mm		2,4mm	3mm	2,4mm	4,8mm
ρ_{F1}	137,1894	167,4429	131,6179	157,7621	ρ_{F1}	141,7121	184,3699	136,0559	173,6164
ρ_{F2}	84,0820	118,8338	60,0325	78,3314	ρ_{F2}	74,6192	109,3108	45,7623	59,9301
c	Gear tooth root fillet radius dimensions				**f4**	Gear tooth root fillet radius dimensions			
	t_o	h_s	ρ_{F1}	ρ_{F2}		t_o	h_s	ρ_{F1}	ρ_{F2}
	0,05m	0,1m	0,1m	0,2m		0,1m	0,125m	0,2m	0,2m
	1,2mm	2,4mm	2,4mm	4,8mm		2,4mm	3mm	4,8mm	4,8mm
ρ_{F1}	144,1836	137,3641	135,2524	173,0252	ρ_{F1}	138,9922	177,9510	124,8129	154,5481
ρ_{F2}	101,6971	120,8788	66,6635	92,1991	ρ_{F2}	79,8214	118,7840	47,2910	63,8579
d	Gear tooth root fillet radius dimensions				**b2**	Gear tooth root fillet radius dimensions			
	t_o	h_s	ρ_{F1}	ρ_{F2}		t_o	h_s	ρ_{F1}	ρ_{F2}
	0,2m	0,1m	0,1m	0,2m		0,1m	0,3m	0,1m	0,2m
	4,8mm	2,4mm	2,4mm	4,8mm		2,4mm	7,2mm	2,4mm	4,8mm
ρ_{F1}	149,5880	196,5911	144,7330	186,8748	ρ_{F1}	142,3826	169,4541	132,7316	154,5374
ρ_{F2}	148,1100	17,0655	25,9666	148,1100	ρ_{F2}	106,8901	153,9703	70,9650	94,6647
e	Gear tooth root fillet radius dimensions				**b3**	Gear tooth root fillet radius dimensions			
	t_o	h_s	ρ_{F1}	ρ_{F2}		t_o	h_s	ρ_{F1}	ρ_{F2}
	0,1m	0,1m	0,1m	0,2m		0,1m	0,25m	0,1m	0,3m
	2,4mm	2,4mm	2,4mm	4,8mm		2,4mm	6mm	2,4mm	7,2mm
ρ_{F1}	146,8134	193,4468	140,0568	181,2204	ρ_{F1}	129,5774	163,5687	93,4367	110,7312
ρ_{F2}	73,3257	112,8184	35,8602	49,4948	ρ_{F2}	84,0020	121,2024	53,3515	70,7222
f	Gear tooth root fillet radius dimensions				**b4**	Gear tooth root fillet radius dimensions			
	t_o	h_s	ρ_{F1}	ρ_{F2}		t_o	h_s	ρ_{F1}	ρ_{F2}
	0,1m	0,1m	0,3m	0,2m		0,1m	0,25m	0,2m	0,2m
	2,4mm	2,4mm	7,2mm	4,8mm		2,4mm	6mm	4,8mm	4,8mm
ρ_{F1}	132,0178	169,6420	118,7619	148,2602	ρ_{F1}	134,5689	160,9604	122,6621	142,9562
ρ_{F2}	92,2675	139,2327	49,8163	68,2389	ρ_{F2}	106,4536	153,4819	73,8484	99,1061
a2	Gear tooth root fillet radius dimensions				**b5**	Gear tooth root fillet radius dimensions			
	t_o	h_s	ρ_{F1}	ρ_{F2}		t_o	h_s	ρ_{F1}	ρ_{F2}
	0,125m	0,05m	0,1m	0,2m		0,1m	0,25m	0,3m	0,2m
	3mm	1,2mm	2,4mm	4,8mm		2,4mm	6mm	7,2mm	4,8mm
ρ_{F1}	155,8758	210,1066	147,0918	194,8825	ρ_{F1}	129,6407	150,4770	116,3271	131,0013
ρ_{F2}	62,5591	99,2857	22,2166	31,4905	ρ_{F2}	111,0235	158,5784	78,4686	104,6536

The tooth root Von Mises stress and the normal stress for the inner mesh contact point B (only these stresses are presented because they are the highest and most dangerous for crack initiation) for the driven gear are shown in Fig.2.





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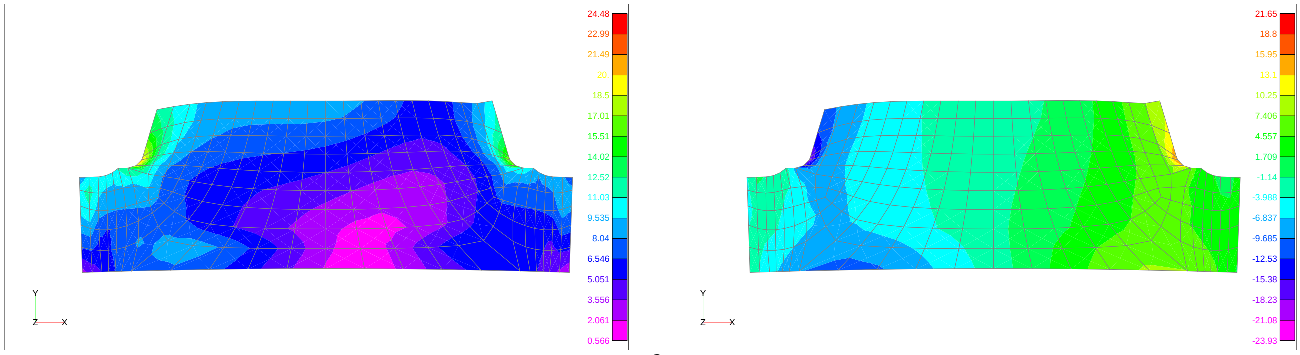

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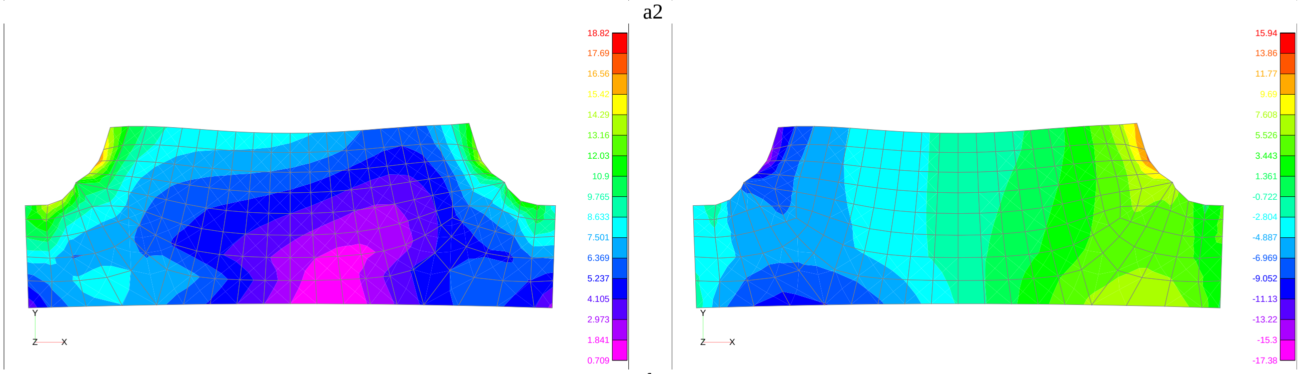

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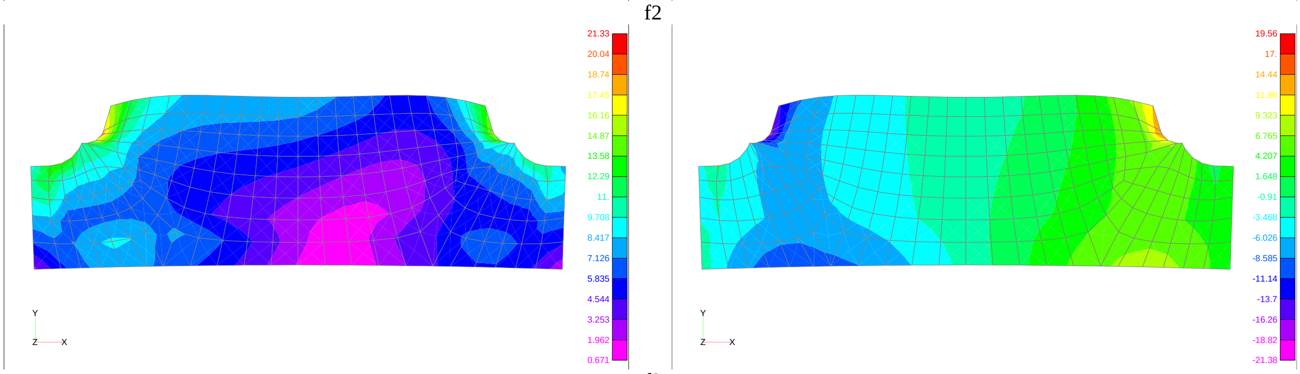

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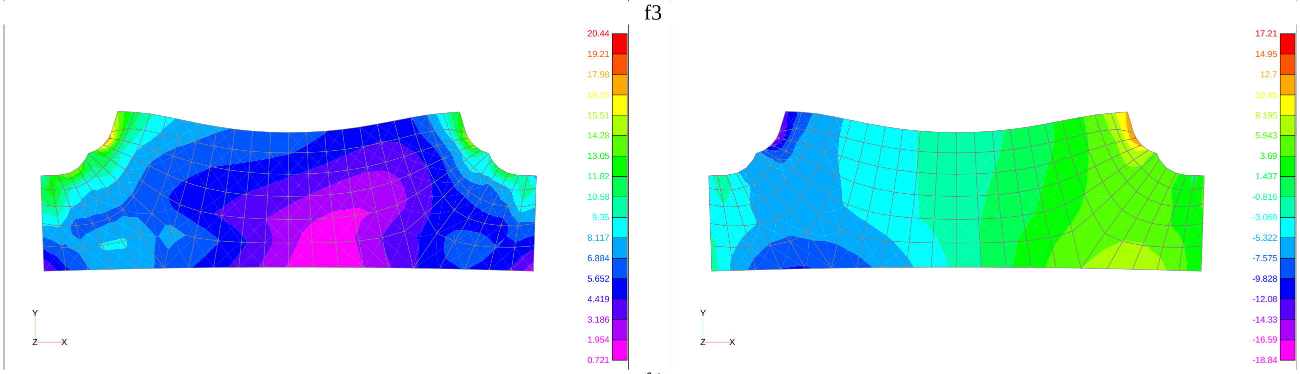

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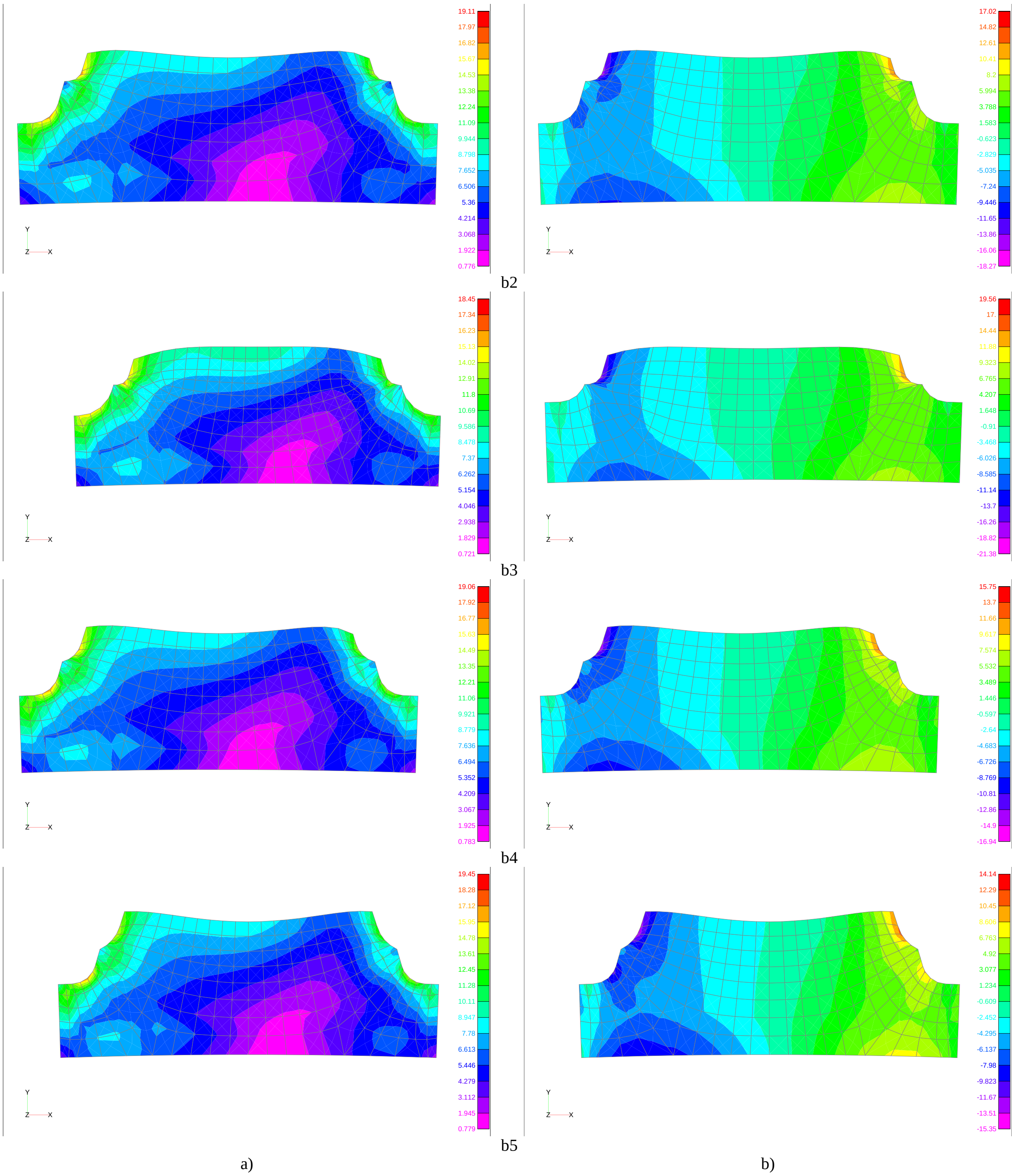

Figure 2. a) Von Mises and b) normal tooth root stresses for the load in the inner mesh contact point B

The highest stress occurs during the contact of two gears in the outer point of the single tooth contact and then the tooth root critical section stress is at the highest; therefore, this paper will consider only the stress caused by load in that contact point.

Spur gears with only one tooth root fillet radius have the lowest Von Mises and normal stress values for the fillet radius value of ρ_F=10.943mm. However, the graphic line of tooth root stress values for a fillet radius higher than ρ_F=10.031mm have disagreement of linear dependence and because of that this paper will consider only stress values lower than this one [1, 2].

The lowest stress values for the driven gear are σ_e=142.4273MPa and σ_y=119.4627MPa (σ_e - Von Mises stress, σ_y - normal stress) [1, 2]. In order to obtain lower critical section stress values, the gear tooth roots with two fillet radii are analyzed. It is supposed that two fillet radii ("two level approach" in a root) act as a "disencumber notch" for stress and the tooth root stress concentration will be lower. Nevertheless, the analyses show that it is not always correct and it is possible to get higher tooth root stress with two fillet radii rather than only with one for the same determined gear.

Fig.1 presents a tooth root with the dimensions of two fillet radii where ρ_{F1} is the upper radius and ρ_{F2} is the lower radius. The critical section position is determined in the same way as for the tooth root with only one fillet radius. It is always possible to determine the critical section position for the upper radius ρ_{F1}, but for the lower radius ρ_{F2}, the critical section is near the top of the fillet radius and it is often impossible to be determined below the angle of 30°.

The Von Mises stress values, for gears with only one fillet radius, are always higher than normal stress values [1, 2]. It is the same for the driven gear with two tooth root fillet radii – the Von Mises stress values are always higher than normal stress values for both fillet radii. For the majority of the analyzed samples, the stress in the lower fillet radius is much lower than in the upper radius, i.e. it is lower than the stress on the tooth root with only one fillet radius.

For the first analyzed sample ***a*** the critical stress values are higher than stress values for the same gears but with only one fillet radius. For the driven gear, the Von Mises stress is 14% higher and the normal stress is 31% higher.

For the sample ***b***, for the driven gear on the upper fillet radius, the equivalent stress value is 1% lower, but the normal stress value is 13% higher. For the driven gear on the upper fillet radius the equivalent stress value is lower than 1%, but the normal stress value is 13% higher.

For the driven gear, for the case ***c***, for the driven gear, both of stress values are lower on the lower radius, but on the upper radius, the Von Mises stress and the normal stress values are 12% and 23% higher, respectively.

For the sample ***d***, the stress values are higher for the upper tooth root fillet radius: the normal stress values and the eqivalent stress values are 32% and 17% higher, respectively. On the lower tooth root fillet radius the stress values are lower than the stress values in the compared gears.

The sample ***e*** shows that normal and Von Mises stress values are higher than the compared stress values on the upper fillet radius. For the driven gear, the equivalent stress is 15% higher on the upper fillet radius and 24% on lower fillet radius. The normal stress for this gear is 29% higher on the upper fillet radius and 26% lower on the lower fillet radius.

Considering the case ***f***, it can be seen that the Von Mises and normal stress are less significant on the lower fillet radius, but they are higher on the upper fillet radius. For the driven gear, on the upper tooth root fillet radius, the equivalent stress values are 1% higher and the normal stress values are 6%, while on the lower fillet radius the equivalent stress values and the normal stress values are 18% and 52% lower, respectively.

The last mentioned cases are first analyzed. The cases ***b*** (lower stress for both gears) and ***f*** (approximately the same stress values for both gears) have the best results. Moreover, although stress values in the sample ***a*** are significantly higher than the compared ones, tooth root dimensions (sample ***a*2**) are varied in order to get lower tooth root stress. It was just partially successful, because the stress values for the case ***a*2** are lower than for the case ***a***, but further more significantly higher than the stress values in the compared gear with only one tooth root fillet radius. Because of that, further examinations are done only for the cases with lower or approximately the same stress values as the compared ones (***b*** and ***f***).

In order to that, the sample ***f*2** has a higher dimension h_s, while other dimensions are the same as the dimensions of the sample ***f***. It reflected well on the tooth root stress because it became lower for both gears and it was possible to make a mesh between driving and driven gears of the same tooth root dimensions. The equivalent stress is 1% lower, but the normal stress is 9% higher.

For the sample ***f*3**, all tooth root dimensions except the upper fillet radius ρ_{F1} are left the same as in the sample ***f*2** (ρ_{F1} is now 0.3 *m* instead of 0.1 *m*, *m* being a gear module). It increases overall stress on the upper fillet radius comparing to the samples ***f*2** and ***f***, even on the compared gear tooth with one tooth root fillet radius. The overall stress for the driven gear on the lower fillet radius is lower than the compared one. The equivalent stress and the normal stress are 10% and 24% higher, respectively.

The results of the sample ***f*2** were good and that was the reason to analyze one more variant (***f*4**) of its dimensions. In the ***f*4** sample the fillet radii ρ_{F1} and ρ_{F2} are equal, but the rest of dimensions is left the same as in the sample ***f*3**. However, stress in that case is higher than in the case ***f*2**. For the driven gear, the equivalent stress and the normal stress are 6% and 10% higher, respectively. These results are for the upper fillet radius, but for the lower fillet radius the stress values for both gears are lower than the compared ones.

Considering all variants of the ***f*** samples, it can be concluded that both Von Mises and normal stress become lower as the upper fillet radius ρ_{F1} decreases, so the ***f*2** sample is the most available, especially for the driving gear [4]. If we keep the rest of dimensions unchanged, it is possible to make a meshing of driving and driven gear of the same tooth root dimensions. The ***b*** sample of the tooth root geometry has good results of stress values and that is the reason for further investigation of this sample and its geometry. The sample ***b*2** has the different dimension h_s (from 0.25 *m* to 0.3 *m*) while other dimensions are left unchanged. In this case, for the driven gear on the upper fillet radius the equivalent stress values are higher for 2% and the normal stress values for 11%. On the lower fillet radius the equivalent stress values are lower for 2% and the normal stress values are lower for 44%.

The case ***b*3,** compared to the sample ***b*,** has a different lower radius ρ_{F2} (changed from 0.2*m* to 0.3*m*). The driven gear has lower stress on the lower fillet radius: The equivalent stress is 14% lower. The driven gear tooth on the upper fillet radius has lower stress values; the equivalent stress for 2% and the normal stress for 22%. It means that ***b*3** tooth root form, in accordance with stress condition, is quite available for the driven gear, because this gear has lower stress values on both fillet radii, the upper as well as the lower one.

The sample ***b*4** tooth root form has an equal fillet radius $\rho_{F1} = \rho_{F2} = 0.2m$ (other dimensions are the same as in the ***b*** sample). Namely, the driven gear has insignificantly higher normal stress on the upper fillet radius while other stresses are lower than the compared ones. On the upper fillet radius, the normal stress values are higher for 3%; and the equivalent stress values are lower for 3% while at the same time on the lower fillet radius of the same gear the normal stress values are lower for 29% and the equivalent ones for 8%.

The last ***b*** sample variants are more available, in accordance with stress condition, for the driven gear and that is the same for the sample ***b*5** [4]. When compared to the ***b*** sample, the ***b*5** sample has a different upper fillet radius which is now ρ_{F1}=0.3 *m*. The driven tooth gear has lower stress values even on the upper and on the lower fillet radius. The equivalent stress values are 10% lower on the upper fillet radius and 1% on the lower fillet radius. The normal stress values are 7% lower on the upper fillet radius and 36% on the lower fillet radius.

These analyses show that stress is reduced as the h_s dimension increases, but it has to take care of its value to allow meshing of gears. It is not necessary that tooth root dimensions are the same for both the driving and the driven gear in order to allow the gear meshing. Investigations of gears with two tooth root fillet radii show that the increase of the upper fillet radius ρ_{F1} results in the reduction of stress. So, in order to reduce stress and achieve an appropriate tooth root state condition, these two dimensions have to be modified, which every constructor aims for..

It is possible to present the graphs of all above mentioned samples by presenting the Von Mises (Fig.3) stress and the normal stress (Fig.4.) as a function of the tooth root fillet radii ρ_{F1} and ρ_{F2}.

It has to be pointed out that, due to tooth root dimensions, it is not possible to realize every gear meshes. It concerns especially the cases where the dimension h_s has higher values (all ***b*** tooth root samples), since although it is more appropriate case from the aspect of stress (for these samples stress values are reduced), it is not possible to realize the mesh of the same dimension gears because the gears would intermesh with each other. However, even these samples are important for final conclusions in this investigation. In some cases, lower h_s values reduce stress and it can be concluded that with a good tooth root dimension selection - although it is not necessary for driving and driven gears to have the same form and dimensions - stress reduction can be significant.

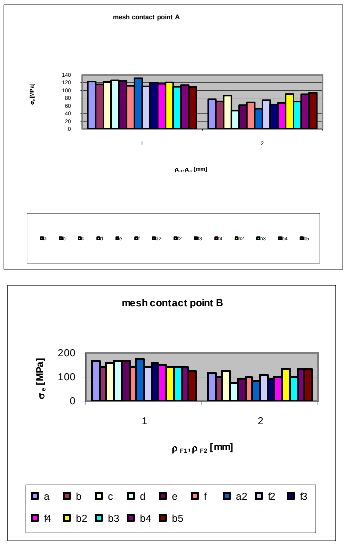

Figure 3. Equivalent stress values in the driven gear tooth root critical section for different fillet radius ρ_{F1} and ρ_{F2} values

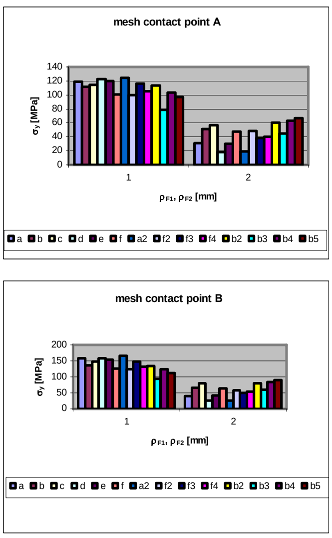

Figure 4. Normal stress values in the driven gear tooth root critical section for different fillet radius ρ_{F1} and ρ_{F2} values

This investigation confirmed that it is possible to get critical section stress reduction with a correct gear tooth root selection. It means that stress reduction produces better tooth root load capacity and prolongs a gear service life, which was the object of this paper.

Conclusion

The topic of this paper is forming an efficient and reliable numerical model for the determination of tooth root phenomena caused by geometrical discontinuity under static and cyclic loads. Critical tooth root stress concentration is caused, in the first place, by an inner contact point load of a single mesh and then by an outer contact point of double mesh. These loads are the main cause of crack initiation and they are only investigated in this paper. The most important one is tooth root stress caused by load in the inner contact point of the mesh since it is the highest and the most dangerous for failure appearance.

Reduction of stress concentration acts directly on gear service life elongation because it deflects the danger of crack initiation and increases the safety factor SF at that place. Although the stress on the pressured side is higher than on the tensile one, the stress concentration on the tensile side is more important for initial crack appearance.

It is supposed that two fillet radii (“two level approach” in a root) act as a “disencumber notch” for stress and that the tooth root stress concentration will be lower. Nevertheless, the analyses show that is not always correct and that it is possible to get higher tooth root stress with two fillet radii than only with one radius for the same determined gear.

The Von Mises stress values, for gears with only one fillet radius, are always higher than the normal stress values [1, 2]. It is the same for the driven gear with two tooth root fillet radii. The Von Mises stress values are always higher than those of the normal stress for the both of fillet radii. For the most of analyzed samples, stress values on the lower fillet radius are much lower than on the upper radius, which means that they are lower than stress values on the tooth root with only one fillet radius.

It is concluded that stress is reduced as the h_s dimension increases, but it has to take care of its value to allow gears mesh. It is not necessary that tooth root dimensions are the same for both the driving and the driven gear in order to allow the gear mesh. Investigations of gears with two tooth root fillet radii show that the increase of the upper fillet radius ρ_{F1} results in the reduction of stress. So, in order to reduce stress and achieve an appropriate tooth root state condition, these two dimensions have to be modified, which every constructor aims for.

It is important to point out that, due to tooth root dimensions, it is not possible to realize every gears mesh. It concerns especially the cases where the dimension h_s has higher values (all ***b*** tooth root samples), since although it is more appropriate case from the aspect of stress (for these samples stress values are reduced), it is not possible to realize the two same dimensions gear mesh because the gears would intermesh with each other. However, even these samples are important for final conclusions in this investigation. It can be concluded that with a good tooth root dimension selection, even with h_s values, it is possible to reduce stress. In the case of the “two level approach“ in a root it is not necessary for driving and driven gears to have the same form and dimensions and stress reduction can be significant if the dimension selection is appropriate.

This investigation confirmed that it is possible to obtain critical section stress reduction with a correct gear tooth root selection. It means that stress reduction results in better tooth root load capacity and prolongs gear service life, and that was the object of this paper.

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Numerička analiza koncentracije napona u korenu zupca gonjenog zupčanika sa dva podnožna zaobljenja

Nalaženje optimalnog oblika zubaca, odnosno parametara ozubljenja, sa aspekta koncentracije napona je jedan od ključnih problema kod zupčastih prenosnika. Nosivost zupca u velikoj meri zavisi od glavnih parametara koji određuju oblik profila zupca. Kad je reč o nosivosti podnožja zupca zupčanika, o čemu će se najviše govoriti u ovom radu, vrlo veliki uticaj ima oblik podnožja zupca, odnosno radijus prelaznog zaobljenja u korenu zupcu. Posebna pažnja usmerena je na radijus podnožnog zaobljenja korena zupca u kritičnom preseku. To direktno utiče na faktor koncentracije napona i sam radni vek zupčanika. Poznato je da prva prslina u podnožju zupca zupčanika nastaje zbog koncentracije napona na tom mestu. Upravo zato, predmet daljeg istraživanja ovog rada je usmeren na iznalaženje optimalnog oblika prelaznog zaobljenja u podnožju zupca zupčanika sa ciljem da se dobije minimalna vrednost koncentracije napona. Međutim, u cilju smanjenja koncentracije napona u korenu zupca uvodi se još jedan radijus kao „zarez rasterećenja“. U ovom radu će se posebna pažnja posvetiti izučavanju koncentracije napona u podnožju zubaca, gde je prelazno zaobljenje izvedeno (umesto sa jednim radijusom) sa dva radijusa različitih vrednosti („two level approach“ in root). U cilju brzog i efikasnog rada u narednom ispitivanju će se primeniti metoda konačnih elemenata (MKE) pri simulaciji stvarnih radnih uslova.

Predmet ovog rada je formiranje efikasnog i pouzdanog numeričkog modela za saznavanje fenomena koji nastaju usled geometrijskih diskontinuiteta u podnožju zupca zupčanika a pri statičkim opterećenjima. Iako su ispitivani kritični naponi zubaca oba zupčanika u sprezi [5], u ovom radu će biti prikazani samo rezultati za gonjeni zupčanik za oba podnožna radijusa ρ_{F1} i ρ_{F2}.

Ključne reči: **zupčanik, naponsko stanje, koncentracija napona, optimizacija oblika, numerička analiza, metoda konačnih elemenata.**

Цифровой анализ концентрации напряжения в корне зубца верхней зубчатой передачи со двумя предельными валиками

Обнаружение оптимальной формы зубцов, т. е. параметров нарезания, со стороны концентрации напряжений представляет одну из главных проблем у зубчатых передаточных механизмов. Несущая способность зубчатого колеса в значительной степени зависит от основных параметров, определяющих форму профиля зубцов. Когда речь идёт о несущей способности ножки зубчатого колеса, о чём больше всего и говорит настоящая работа, большое влияние оказывает форма ножки зубчатого колеса, т.е. радиус переходного закругления в корне зубца. А особое внимание направлено на радиус закругления в ножке корня зубчатого колеса в критическом сечении. Это непосредственно влияет на фактор концентрации напряжений и на сам ресурс зубчатого колеса. Уже известно, что первая трещина в ножке зубца зубчатого колеса появляется из-за концентрации напряжений на этом месте. Именно из-за этого тема дальнейших исследований настоящей работы направлена к обнаруживанию оптимальной формы переходного закругления в ножке зубца зубчатого колеса, с целью получить минимальное значение концентрации напряжений. Но, с целью снижения концентрации напряжений в корне зубца вводится ещё один радиус в роли Чзапятой разгруженияЧ. В настоящей работе особое внимание будет направлено на исследование концентрации напряжений в корне зубцов, где сделано переходное закругление (вместо с одним радиусом) со два радиуса различных значений (Итыо левел аппроацхг ин роот).

Ради быстрой и эффективной работы в будущих исследованиях будет использован метод конечных

элементов (МКЭ) при имитационном моделировании действительных рабочих условий.
Предметом настоящей работы является формирование эффективного и надёжного цифрового метода с целью узнавания феноменов, которые возникают из-за геометрических дисконтинуитетов в ножке зубца зубчатого колеса, а при статическими нагрузками.
Хотя бы исследованы критические напряжения зубцов обоих zub~atwh kolës в запряжке [5], в nasto}|ej rabote будут показаны только результаты для зубчатой передачи для обоих радиусов в ножке зубчатого колеса ρ_{F1} и ρ_{F2} •

Ключевые слова: **зубчатая передача, состояние напряжённости, концентрация напряжения, оптимизация формы, цифровой анализ, метод конечных элементов.**

Analyse numérique de la concentration de tension dans la racine de la dent chez l'engrenage actionné à deux racines de filet

La découverte de la forme optimale de l'engrenage est l'un des problèmes principaux relatifs à la concentration de la tension. La portance de l'engrenage et l'état de tension dépendent en grande partie du profil de l'engrenage et des paramètres de sa configuration ce qui est le domaine principal pour l'analyse scientifique. La forme de la racine de dent et le filet de radius ont grande influence sur la force de la racine de dent chez l'engrenage et c'est le sujet principal de ce travail. L'attention particulière est portée sur l'analyse du radius chez la racine de filet dans l'intersection critique. Cela influe directement sur le facteur de la concentration de la tension et sur la durée du travail de l'engrenage. Il est connu que la fissure apparaît chez la dent d'engrenage affectée le plus par la concentration de la tension dans la racine. Pour cela l'objet de cette recherche est centré sur la découverte de la forme optimale pour le radius de la racine de filet dans le but de minimiser l'intensité de tension chez la racine de la dent. Cependant, afin de diminuer la concentration de tension on introduit un radius de plus comme „entaille de décharge". Dans ce papier, l'attention spéciale sera prệtée à la concentration de tension dans la racine de la dent avec deux racines de filet („two level approach" in root). Pour effectuer un travail rapide et efficace , la méthode des éléments finis (MEF) sera utilisée pour simuler les conditions réelles de travail. L'objet de ce travail est la création d'un modèle numérique sûr et efficace pour étudier les phénomènes qui se produisent à cause des discontinuités géométriques chez la racine de la dent au cours des charges statiques. Bien que les charges critiques étudiées chez les dents des deux engrenages soient en relation [s] ce papier présentera seulement les résultats pour l'engrenage actionné pour deux racines des filet ρ_{F1} et ρ_{F2}.

Mots clés: **engrenage, état de tension, concentration de tension, optimisation de la forme, analyse numérique, méthode des éléments finis.**