UDK: 623.465.3
COSATI: 19-07, 16-04, 17-07

Guidance of Ground to Ground Rockets Using Flight Path Steering Method

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The ballistic flight of ground-to-ground rockets is sensitive to disturbances (total impulse variation, wind, thrust misalignment, etc.). As the range of ground-to-ground rockets increases, the accuracy of rockets free flight decreases. Requirements for the increase of the range and minimization of the impact point dispersions can be solved by adding the guidance and control system to rockets. Based on the differences between the real flight parameters and the calculated parameters for the nominal trajectory, the flight path angle correction algorithm is given in the paper. The efficiency of the proposed algorithm is verified by the numerical simulation of the rocket flight with the guidance and control system for a hypothetical rocket. The results of the simulation are obtained with the assumption that the SDINS instruments measure accelerations and angular rates of rockets without errors and noises.

Key words: **multiple launch rocket system, guided rocket, rocket guidance, ground to ground rocket, impact point dispersion, flight path guidance.**

Notation and symbols

$\mathbf{r}$ – Vector of the rocket position
$\mathbf{v}$ – Rocket velocity
$\mathbf{\Gamma}$ – Flight path angle
$\mathbf{r}_T$ – Vector of the target position
x_T, y_T, h_T – Coordinates of the target
x_i, y_i, h_i, V_i – Nominal trajectory coordinates and velocity at the sampling intervals
x_i', y_i', h_i', V_i' – Coordinates of the rocket and velocity due to disturbances
$\hat{x}_i, \hat{y}_i, \hat{h}_i, \hat{V}_i$ – Coordinates of the rocket and velocity estimated by SDINS
$x(\hat{x}_i), y(\hat{x}_i)$ $h(\hat{x}_i), V(\hat{x}_i)$ – Nominal trajectory coordinates and velocity at the estimated down range position
γ_i, χ_i – Nominal trajectory flight path angles at the sampling intervals
γ_c, χ_c – Correlated (demanded) flight path angles
$\gamma(\hat{x}_i)$, $\chi(\hat{x}_i)$ – Nominal trajectory flight path angles at the estimated down range position
$\frac{\partial \mathbf{r}}{\partial \mathbf{r}_i}$ – Partial derivatives of the rocket range relative to the rocket position
$\frac{\partial \mathbf{r}}{\partial \mathbf{v}_i}$ – Partial derivatives of the rocket range relative to the rocket velocity
$\frac{\partial \mathbf{r}}{\partial \mathbf{\Gamma}_c}$ – Partial derivatives of the rocket range relative to the rocket flight path angles
$\frac{\partial \mathbf{r}}{\partial t_i}$ – Partial derivatives of the rocket range relative to the time
$\Delta\mathbf{r}_i$ – Difference between the rocket actual position and the position on the nominal trajectory
$\Delta\mathbf{v}_i$ – Difference between the rocket actual velocity and the nominal velocity
$\Delta\mathbf{\Gamma}_c$ – Difference between the rocket required flight path angle and the nominal flight path angles
Δt_i – Error in time measurements
$\frac{\partial \mathbf{\Gamma}_c}{\partial \mathbf{r}_i}$ – Partial derivatives of the rocket flight path angles relative to the rocket position
$k_\gamma(\hat{x}_i)$, $\tau(\hat{x}_i)$ – Gain and time constant of the flight path control loop at the estimated down range position
a_{zd}, a_{yd} – Demanded normal acceleration
a_z, a_y – Realized normal acceleration
$\frac{\Delta q}{\Delta \eta}(s)$ – Rocket pitch rate transfer function
$\frac{\Delta a_z}{\Delta \eta}(s)$ – Rocket lateral acceleration transfer function
$\frac{\Delta a_z'}{\Delta \eta}(s)$ – Transfer function for the acceleration (a_z'), measured by an accelerometer
K_q – Pitch rate gain
U_k – Axial component of the kinematic velocity
ω_n – Natural frequency of the rocket transfer function
ς_n – Damping factor of the rocket transfer function

Introduction

BASIC requirements for the ground-to-ground rockets are the range increase and the accuracy increase. As the range of the ground-to-ground rockets increases, the accuracy of the rockets free flight decreases. Even the increase of the range from 32 km to 50 km of the Multiple Launch Rocket System (MLRS) doubles the circular error probable (CEP) [1]. This increase of the dispersion of the rockets free flight requires the increase of the number of the

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rockets necessary to hit the target.

Some of disturbances, such as thrust misalignments and asymmetric aerodynamic effects, are eliminated by the rocket spinning. However, winds, total impulse variations, launcher interactions and atmospheric density variations still have a significant influence on the impact accuracy.

In order to minimize the influence of disturbances on the impact accuracy, an extensive study is oriented to the development of a cost effective guidance and control packages which can be added to the ground-to-ground rockets.

The cost effectiveness of the Guided Multiple Launch Rocket System (GMLRS) relative to the Multiple Launch Rocket System (MLRS) is analyzed in [1]. The dscription of the GMLRS system with the description of the type of the SDINS (strapdown inertial navigation system), autopilot and guidance and control system are given in the same paper. The results of four experiments with different types of the guidance and trajectory correction are presented. The guidance of these rockets is based on the Flight Path Steering (FPS) and Instantaneous Impact Point (IIP).

The concept of the correlated velocity is used for the guidance of the long range rockets in the part of the trajectory where influence of the atmosphere can be neglected. This concept requires control of the velocity vector (direction and magnitude of the velocity) based on two boundary problems [2].

One of the published GMLRS guidance laws is based on the idea that the control system eliminates the differences between measured and prescribed values of the velocity vector angles and of the accelerations in the lateral (y and z) directions of the rocket [3]. The prescribed values are obtained from the calculation of the nominal trajectory for a required range. In order to alleviate influence of total impulse deviation for the rockets with no thrust-termination mechanism, the correction of the prescribed flight path angle ($\gamma^*(t)$) is necessary by a function $\Delta\gamma(t)$ which is defined empirically by trial and error. The proposed guidance law is not efficient because it is necessary to know in advance disturbances in order to determine the correction of the flight path angle in function of time ($\Delta\gamma(t)$).

The application of the correlated velocity concept on the ground-to-ground missile which trajectory is entirely in dense layers of the atmosphere requires calculation of the reference trajectory and differential coefficients in the control points [4]. This concept requires cut-off of the missile engine at the point where all conditions, necessary to hit the target by a ballistic flight, are fulfilled.

Based on the known nominal trajectory for a required range of the ground to ground rockets, the mathematical model of the flight path angles correction for compensation of the disturbances is the subject of this paper. The verification of the propoused mathematical model is done by numerical simulation. The numerical simulation of the rocket flight is based on the six degree of freedom mathematical model. A strap-down system is used for the actual trajectory parameters determination with an assumption that there are no errors in the angular rate and the acceleration measurements. The rocket is guided by the flight path steering method.

Guidance Law for Ground to Ground Rockets with Thrust no Cut Off

The nominal trajectory for the required range of the ground to ground rockets is a ballistic trajectory in the standard atmosphere without disturbances and with nominal thrust. This trajectory is obtained by the numerical simulation of the ballistic flight by applying the six degree of freedom mathematical model of the rocket flight.

The deviation of the actual trajectory relative to the nominal trajectory is given in Fig.1. The position of the target on the ground is defined by the coordinates x_T and y_T .

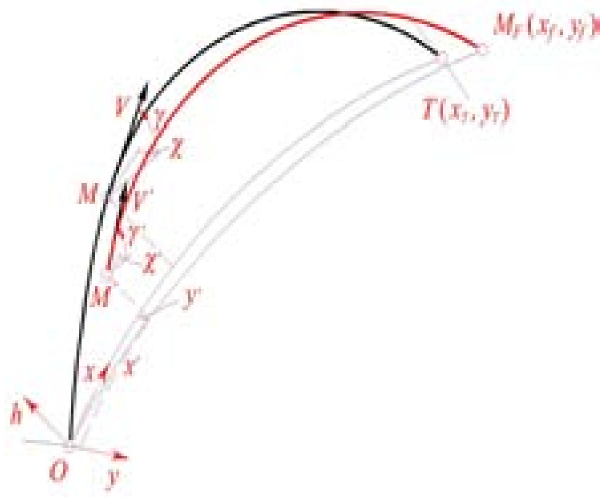

Figure 1. Trajectory dispersion

In order to simplify the derivation of the mathematical model for the flight path angle correction, the following matrix notation will be used in the paper.

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ h \end{bmatrix}, \ \mathbf{V} = [V] \text{ and } \mathbf{\Gamma} = \begin{bmatrix} \gamma \\ \chi \end{bmatrix} \tag{1}$$

In the time instance $t = t_i$, the position of the rocket is defined by the point M_i (Fig. 1). The ballistic flight of the rockets from the point M_i to the target is defined by the velocity $\mathbf{V}_i$ and the flight path angles γ_i and χ_i . This ballistic flight provides rocket impact points at the position of the target. The equality of the rocket and target position at the impact point can be written in the vector form

$$\mathbf{r}_T - \mathbf{r}_F\left(\mathbf{r}_i, \mathbf{V}_i, \mathbf{\Gamma}_i, t_i\right) = 0 \tag{2}$$

where:

t_i - initial time for the balistic flight from the arbritary point M_i ,

$\mathbf{r}_T = \begin{bmatrix} x_T \\ y_T \end{bmatrix}$ - target position (for the surface target $h_T = 0$),

$\mathbf{r}_F = \begin{bmatrix} x_F \\ y_F \end{bmatrix}$ - impact point of the rocket at the end of the flight $t = t_f$,

$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ h_i \end{bmatrix}$ - rocket position on the nominal trajectory in the time instance $t = t_i$,

$\mathbf{\Gamma}_i = \begin{bmatrix} \gamma_i \\ \chi_i \end{bmatrix}$ - flight path angles in the vertical and horizontal plane at the time instance $t = t_i$,

If the flight path angles at the initial instance of the ballistic flight are equal to the required (correlated) ones, then the rocket position at the end of the flight ($t = t_{T_f}$) is equal to the target position

$$\gamma_i = \gamma_c\,, \ \chi_i = \chi_c \tag{3}$$

$$\mathbf{r}_T - \mathbf{r}\left(\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_i, t_i\right) = 0 \tag{4}$$

In the case of disturbances, the rocket is not at the point M_i on the nominal trajectory but at some point M_i' away from the nominal trajectory. The point M_i on the nominal trajectory corresponds to the time instance t_i. Since there are some errors in the measurement of the current time, the measured time t_i' corresponds to the point M_i'. The values of the flight path angles, at the time instance $t = t_i'$, can be determined from the condition that the ballistic flight of the rocket from this time instance can hit the target

$$\gamma_i^{'} = \gamma_c\,, \quad \chi_i^{'} = \chi_c \tag{5}$$

The correlated flight path angles at the point M_i can be defined in the matrix form $\mathbf{\Gamma_c} = \begin{bmatrix}\gamma_c\\ \chi_c\end{bmatrix}$.

The general condition that a rocket with the ballisitc flight from the point M_i' hits the target is defined by the equation

$$\mathbf{r}_T - \mathbf{r}_F\left(\mathbf{r}_i', \mathbf{v}_i', \mathbf{\Gamma}_c, t_i'\right) = 0 \tag{6}$$

The second term in equation (6) can be developed in the Taylor's series relative to the point M_i on the nominal trajectory

$$\mathbf{r}_T - \mathbf{r}_F\left(\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_i, t_0\right) - \\ -\left[\frac{\partial \mathbf{r}_F}{\partial \mathbf{r}_i}\Delta \mathbf{r}_i + \frac{\partial \mathbf{r}_F}{\partial \mathbf{v}_i}\Delta \mathbf{v}_i + \frac{\partial \mathbf{r}_F}{\partial \mathbf{\Gamma}_i}\Delta \mathbf{\Gamma}_c + \left(\frac{\partial \mathbf{r}_F}{\partial t_i}\right)\Delta t_i\right] = 0 \tag{7}$$

where the partial derivatives are given for the nominal trajectory parameters $\mathbf{r}\left(\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_c, t_i\right)$

$$\frac{\partial \mathbf{r}_F}{\partial \mathbf{r}_i} = \begin{bmatrix} \dfrac{\partial x_f}{\partial x_i} & \dfrac{\partial x_f}{\partial y_i} & \dfrac{\partial x_f}{\partial h_i} \\ \dfrac{\partial y_f}{\partial x_i} & \dfrac{\partial y_f}{\partial y_i} & \dfrac{\partial y_f}{\partial h_i} \end{bmatrix}_{\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_i, t_i} \tag{8}$$

$$\frac{\partial \mathbf{r}_F}{\partial \mathbf{v}_i} = \begin{bmatrix} \dfrac{\partial x_f}{\partial V_i} \\ \dfrac{\partial y_f}{\partial V_i} \end{bmatrix}_{\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_i, t_i} \tag{9}$$

$$\frac{\partial \mathbf{r}_F}{\partial \mathbf{\Gamma}_c} = \begin{bmatrix} \dfrac{\partial x_f}{\partial \gamma_i} & \dfrac{\partial x_f}{\partial \chi_i} \\ \dfrac{\partial y_f}{\partial \gamma_i} & \dfrac{\partial y_f}{\partial \chi_i} \end{bmatrix}_{\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_i, t_i} \tag{10}$$

$$\frac{\partial \mathbf{r}_F}{\partial t_i} = \begin{bmatrix} \dfrac{\partial x_f}{\partial t_i} \\ \dfrac{\partial y_f}{\partial t_i} \end{bmatrix}_{\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_i, t_i} \tag{11}$$

$$\Delta \mathbf{r}_i = \mathbf{r}_i' - \mathbf{r}_i = \begin{bmatrix} x_i' - x_i \\ y_i' - y_i \\ h_i' - h_i \end{bmatrix} = \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta h_i \end{bmatrix} \tag{12}$$

$$\Delta \mathbf{\Gamma}_i = \mathbf{\Gamma}_i' - \mathbf{\Gamma}_i = \begin{bmatrix} \gamma_i' - \gamma_i \\ \chi_i' - \chi_i \end{bmatrix} = \begin{bmatrix} \Delta \gamma_i \\ \Delta \chi_i \end{bmatrix} \tag{13}$$

$$\Delta \mathbf{v}_i = \mathbf{v}_i' - \mathbf{v}_i = \left[V_i' - V_i\right] = \left[\Delta V_i\right] \tag{14}$$

$$\mathbf{r}_T - \mathbf{r}_F\left(\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_i, t_i\right) = 0 \tag{15}$$

Partial derivatives can be determined by the method of differential corrections. This method is related to the numerical determination of the first derivative of the trajectory coordinates relative to the parameters which defines ballistic flight trajectory from the point M_i to the target

$$\begin{aligned}
\frac{\partial a}{\partial x_i} &= \frac{1}{\Delta x}\Big[a\left(x_i + \Delta x, y_i, h_i, V_i, \gamma_c, \chi_c, t_i\right) - \\ &\quad - a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c, t_i\right)\Big] \\
\frac{\partial a}{\partial y_i} &= \frac{1}{\Delta y}\Big[a\left(x_i, y_i + \Delta y, h_i, V_i, \gamma_c, \chi_c, t_i\right) - \\ &\quad - a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c, t_i\right)\Big] \\
\frac{\partial a}{\partial h_i} &= \frac{1}{\Delta h}\Big[a\left(x_i, y_i, h_i + \Delta h, V_i, \gamma_c, \chi_c, t_i\right) - \\ &\quad - a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c, t_i\right)\Big] \\
\frac{\partial a}{\partial V_i} &= \frac{1}{\Delta V}\Big[a\left(x_i, y_i, h_i, V_i + \Delta V, \gamma_c, \chi_c, t_i\right) - \\ &\quad - a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c, t_i\right)\Big] \\
\frac{\partial a}{\partial \gamma_i} &= \frac{1}{\Delta \gamma}\Big[a\left(x_i, y_i, h_i, V_i, \gamma_c + \Delta\gamma, \chi_c, t_i\right) - \\ &\quad - a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c, t_i\right)\Big] \\
\frac{\partial a}{\partial \chi_i} &= \frac{1}{\Delta \chi}\Big[a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c + \Delta\chi, t_i\right) - \\ &\quad - a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c, t_i\right)\Big] \\
\frac{\partial a}{\partial t_i} &= \frac{1}{\Delta t}\Big[a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c, t_i + \Delta t\right) - \\ &\quad - a\left(x_i, y_i, h_i, V_i, \gamma_c, \chi_c, t_i\right)\Big]
\end{aligned} \tag{16}$$

where $a = x_f$ for the range and $a = y_f$ for lateral deviation.

The increment of the flight path angles due to disturbances can be determined from equation (7) since the first two terms $\mathbf{r}_T$ and $\mathbf{r}_F\left(\mathbf{r}_i, \mathbf{v}_i, \mathbf{\Gamma}_i, t_0\right)$ are aqual

$$\Delta \mathbf{\Gamma}_c = -\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{\Gamma}_c}\right)^{-1}\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{r}_i}\right)\Delta \mathbf{r}_i - \left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{\Gamma}_c}\right)^{-1}\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{v}_i}\right)\Delta \mathbf{v}_i - \\ -\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{\Gamma}_c}\right)^{-1}\left(\frac{\partial \mathbf{r}_F}{\partial t_i}\right)\Delta t_i \tag{17}$$

Equation (17) can be simplified by using a new notation of the matrix:

$$\frac{\partial \mathbf{\Gamma}_c}{\partial \mathbf{r}_i} = -\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{\Gamma}_c}\right)^{-1}\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{r}_i}\right) \tag{18}$$

$$\frac{\partial \mathbf{\Gamma}_c}{\partial \mathbf{v}_i} = -\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{\Gamma}_c}\right)^{-1}\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{v}_i}\right) \tag{19}$$

$$\frac{\partial \mathbf{\Gamma}_c}{\partial t_i} = -\left(\frac{\partial \mathbf{r}_F}{\partial \mathbf{\Gamma}_c}\right)^{-1}\left(\frac{\partial \mathbf{r}_F}{\partial t_i}\right) \tag{20}$$

Substituting (18), (19) and (20) into equation (17) the relation between the flight path angles increament $\Delta\mathbf{\Gamma}_c$ and disturbances $\Delta\mathbf{r}_i$, $\Delta\mathbf{v}_i$, Δt_i can be written in a simplified form

$$\Delta\mathbf{\Gamma}_c = -\left(\frac{\partial\mathbf{\Gamma}_c}{\partial\mathbf{r}_i}\right)\Delta\mathbf{r}_i - \left(\frac{\partial\mathbf{\Gamma}_c}{\partial\mathbf{v}_i}\right)\Delta\mathbf{v}_i - \left(\frac{\partial\mathbf{\Gamma}_c}{\partial t_i}\right)\Delta t_i \qquad (21)$$

where

$$\Delta\mathbf{\Gamma}_c = \begin{bmatrix}\Delta\gamma_c\\ \Delta\chi_c\end{bmatrix} \qquad (22)$$

Flight Path Correction in the Vertical and Horizontal Plane

In general, there are coupling effects between the disturbances in the vertical and horizontal plane and the flight path correction in these two planes. In order to analyze these cross influences the matrix equation (21) can be written in a developed form.

$$\begin{bmatrix}\Delta\gamma_c\\ \Delta\chi_c\end{bmatrix} = -\begin{bmatrix}\frac{\partial\gamma_c}{\partial x_i} & \frac{\partial\gamma_c}{\partial y_i} & \frac{\partial\gamma_c}{\partial h_i}\\ \frac{\partial\chi_c}{\partial x_i} & \frac{\partial\chi_c}{\partial y_i} & \frac{\partial\chi_c}{\partial h_i}\end{bmatrix}\begin{bmatrix}\Delta x_i\\ \Delta y_i\\ \Delta h_i\end{bmatrix} - \begin{bmatrix}\frac{\partial\gamma_c}{\partial V_i}\\ \frac{\partial\chi_c}{\partial V_i}\end{bmatrix}\Delta V_i - \begin{bmatrix}\frac{\partial\gamma_c}{\partial t_i}\\ \frac{\partial\chi_c}{\partial t_i}\end{bmatrix}\Delta t_i \qquad (23)$$

Based on the analysis of the rockets ballisitc flight, the variation of the flight path angle in the vertical plane ($\Delta\gamma_i$) creates only the variation of the range (Δx_f) and the variation of the flight path angle in the horizontal plane ($\Delta\chi_i$) creates only the deviation of the rocket trajectory (Δy_f). It can be taken that the sensitivity coefficients of the range variation due to flight path angle variation in horizontal plane is equal to zero $\left(\frac{\partial x_f}{\partial\chi_i}\approx 0\right)$. The horizontal deviation of the rocket due to the flight path angle variation in the vertical plane is also eqal to zero $\left(\frac{\partial y_f}{\partial\gamma_i}\approx 0\right)$.

Since the deviation of the trajectory in the vertical plane (Δx_i, Δh_i) at an arbitrary point provoques the miss of the target along the range, the partial derivative $\frac{\partial x_f}{\partial y_i}$ is equal to zero $\left(\frac{\partial x_f}{\partial y_i}=0\right)$.

Only the lateral deviation of the real trajectory relative to the nominal trajectory creates a lateral miss of the target. As the consequence, the range and height deviations at an arbitrary point have no influence on the lateral miss of the target $\left(\frac{\partial y_f}{\partial x_i}=0, \frac{\partial y_f}{\partial h_i}=0\right)$.

In the passive phase of the rocket flight, the impact point dispersions do not depend on the current time estimation $\left(\frac{\partial x_f}{\partial t_i}=0, \frac{\partial y_f}{\partial t_i}=0\right)$ because the mass and the inertial characteristics of the rocket are constant. Since the sensitive coefficients $\frac{\partial x_f}{\partial t_i}$ and $\frac{\partial y_f}{\partial t_i}$ are equal to zero, the variations of the flight path angles due to time variations are equal to zero $\left(\frac{\partial\gamma}{\partial t_i}=\frac{\partial\chi}{\partial t_i}=0\right)$.

The lateral deviations of the missile impact points are not influenced by the dispersion of the actual rocket velocity relative to the nominal velocity $\left(\frac{\partial y_f}{\partial V_i}=0\right)$. As the consequence of the zero value of the coefficient $\frac{\partial y_f}{\partial V_i}$, the variation of the flight path angle in the horizontal plane due to the rocket velocity variation is eqal to zero $\left(\frac{\partial\chi_c}{\partial V_i}=0\right)$.

Taking into consideration zero values of the analyzed sensitive coefficients, equation (23) can be written in the form of two separate equations. One equation gives the relation between the flight path angle increment in the vertical plane $(\Delta\gamma_c)$ and the disturbances $\Delta x_i, \Delta h_i, \Delta V_i$. The second one gives the relation between the flight path angle variation in the horizontal plane $(\Delta\chi_c)$ and the displacement of the rocket relative to the nominal trajectory in the horizontal plane (Δy_i)

$$\Delta\gamma_c = -\frac{\partial\gamma_c}{\partial x_i}\Delta x_i - \frac{\partial\gamma_c}{\partial h_i}\Delta h_i - \frac{\partial\gamma_c}{\partial V_i}\Delta V_i \qquad (24)$$

$$\Delta\chi_c = -\frac{\partial\chi_c}{\partial y_i}\Delta y_i \qquad (25)$$

The total time of the rocket flight depends on the total impulse of the rocket engine. The diagrams of the velocities of the hypotetical rocket in function of time are given in Fig.2 and the diagrams of the same velocities in function of the range are given in Fig.3. There are three curves related to the three values of the corrective factores of the total impulse $K_{I_{tot}}$ = 1.0, 0.96 and 1.04. The corrective factor of the total impulse is a ratio between the actual total impulse and the nominal value of the total impulse ($K_{I_{tot}} = I_{tot}/I_{totN}$).

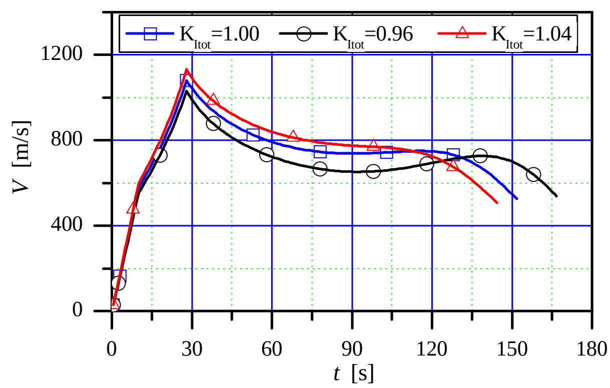

Figure 2. Rocket velocity in a function of time

The nominal trajectory parameters (x_i, y_i, h_i, V_i) and the sensitive coefficients $\left(\frac{\partial\gamma_c}{\partial x_i}, \frac{\partial\gamma_c}{\partial h_i}, \frac{\partial\gamma_c}{\partial V_i}, \frac{\partial\chi_c}{\partial y_i}\right)$ are calculated at the control points before lauching the rocket. During the rocket flight the difference between parameters of the actual trajectory and the parameters of the nominal trajectory are calculated. Based on these differences the corrections of the flight path angles are determined by applyng the formula (24) and (25).

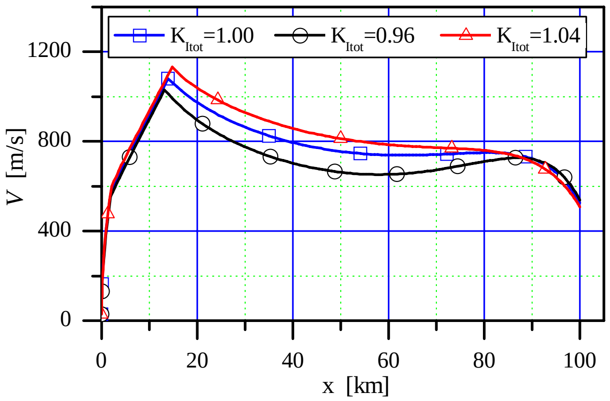

Figure 3. Rocket velocity in a function of range

If the nominal trajectory parameters (x_i , y_i , h_i , V_i) and the sensitive coefficients $\left(\frac{\partial\gamma_c}{\partial x_i}, \frac{\partial\gamma_c}{\partial h_i}, \frac{\partial\gamma_c}{\partial V_i}, \frac{\partial\chi_c}{\partial y_i}\right)$ are given in a function of time it is necessary to take into consideration the time correction because the total time of flight, in general case, is not equal to the nominal value.

If the nominal trajectory parameters and the sensitive coefficients are given in a function of the range, the flight path corrections ($\Delta\gamma_c, \Delta\chi_c$) are not dependent on the current time estimation.

The equation for the flight path correction can be written in a function of the range

$$\Delta\gamma_c(x) = -\frac{\partial\gamma_c}{\partial h}(x)\,\Delta h(x) - \frac{\partial\gamma_c}{\partial V}(x)\,\Delta V(x) \tag{26}$$

$$\Delta\chi_c(x) = -\frac{\partial\chi_c}{\partial y}(x)\,\Delta y(x) \tag{27}$$

The desired flight path angle in the vertical plane (γ_c) is obtained by adding the corrections $\Delta\gamma_c$ to the nominal value of the flight path angle (γ_i)

$$\gamma_c(x) = \gamma_i(x) + \Delta\gamma_c(x) \tag{28}$$

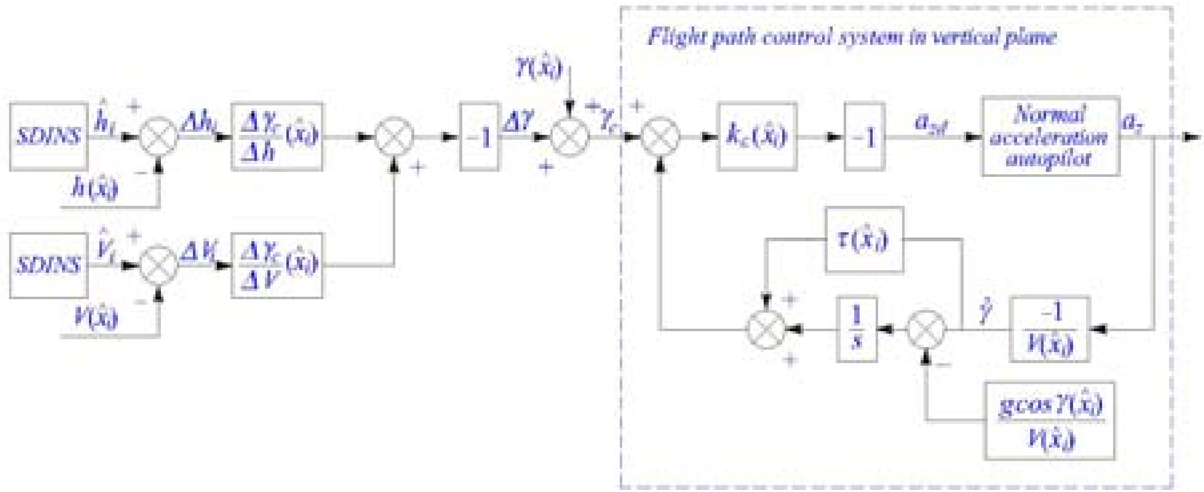

Figure 4. Guidance loop in the vertical plane

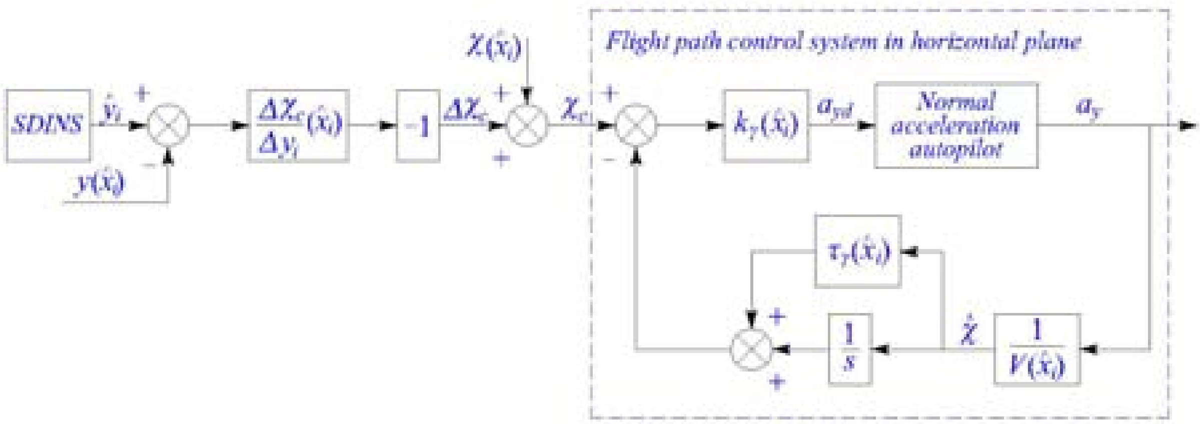

Figure 5. Guidance loop in the horizontal plane

Numerical Simulation

In order to verify the efficiency of the flight path guidance system, a computer program for rocket flight with a complete guidance and control system is built. The flight path guidance system with synthetic pitch autopilots with accelerometers and rate gyros are included in the program.

The flight path guidance system is used to generate the demanded acceleration a_{zd} to eliminate the difference between the estimated ($\tilde{\gamma}$) and desired (γ_c) values of the flight path angle [4].

$$a_{zd} = V\, k_\gamma \left[\gamma_c - (\tilde{\gamma} + \tau_\gamma \dot{\tilde{\gamma}}) \right] \tag{29}$$

The block diagram of the flight path angle correction algorithm with the closed guidance loop in the vertical and horizontal plane is given in Figures 4 and 5 respectively.

The synthesis of the guidance loop and synthetic pitch autopilot is done for the 120 km range hypothetic missile. The synthesis procedure is out of the scope of this paper [4], [5], [6], [7].

The flight path corrections (26) and (27) require the determination of the difference between the actual trajectory and the nominal trajectory and the difference between the actual rocket velocity and the nominal velocity. In order to solve this problem the SDINS is incorporated in the program for numerical simulation. The influence of the SDINS noise on the trajectory estimation is not analyzed in this paper.

The efficiency of the proposed trajectory correction of the ground-to-ground rockets is verified for the unrealistic deviation of the total impulse of ±4% relative to the total impulse which is used for the calculation of the nominal trajectory. Since the total impulse is directly related to the thrust, the variation of the total impulse is obtained by multiplying the nominal values of the thrust with the corrective factors $K_{I_{tot}}$

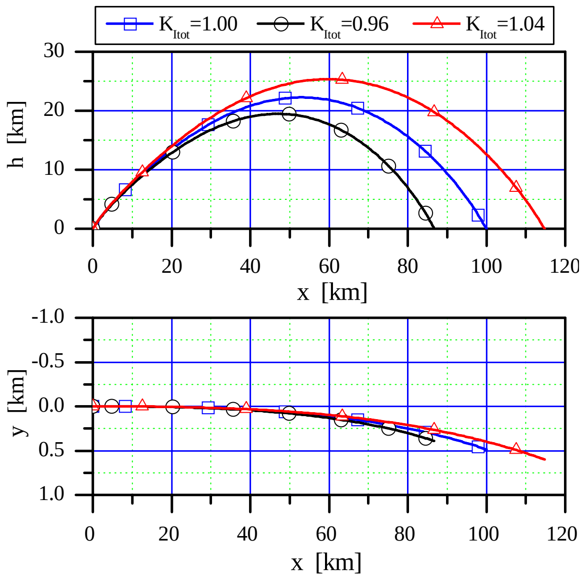

Figure 6. Influence of the total impulse variation on the ballisitc rocket range

The ballistic trajectories for three values of the total impulses are given in Fig.6. One trajectory is calculated for nominal values of the total impulse ($K_{I_{tot}}$ = 1.0). The other two trajectories are obtained for the thrust 4% higher and 4% lower than the nominal values. The corrective factors of the total impulse for these two cases are equal $K_{I_{tot}}$ = 1.04 and $K_{I_{tot}}$ = 0.96. The miss distances at the impact point due to these total impulse variations are approximately 15 km.

On the basis of these three ballistic trajectories, it can be concluded that the total impulse deviation has a significant influence on the achieved range.

The absolute values of the miss distance in a function of percentages of the total impulse deviation for the ballistic flight of the rockets are given in Fig.7. This diagram is obtained by the Monte Carlo simulation. The constant elevation angle is chosen to hit the target 100 km away from the launching place. One hundred runs are done with the standard deviation of the thrust $\sigma_F = 2.0\,\%$.

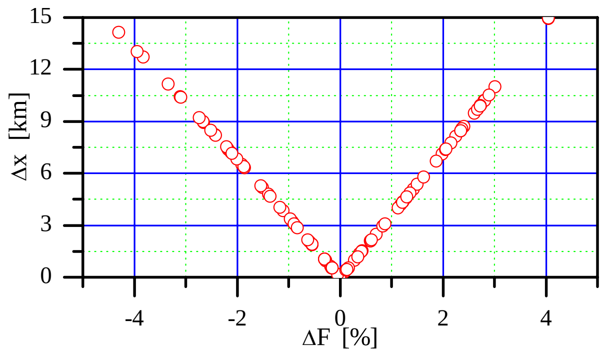

Figure 7. Miss distance due tothe total impulse deviation

The trajectories of the rockets guided by the flight path guidance are given in Fig.8. There are two phases of the rocket flight. After the ballistic flight in the first phase, one second after the launch, the guidance with trajectory correction starts. In all analyses, the rocket is guided to the end of the flight (impact into surface). Aerodynamic fins are used for the rear control of the rocket.

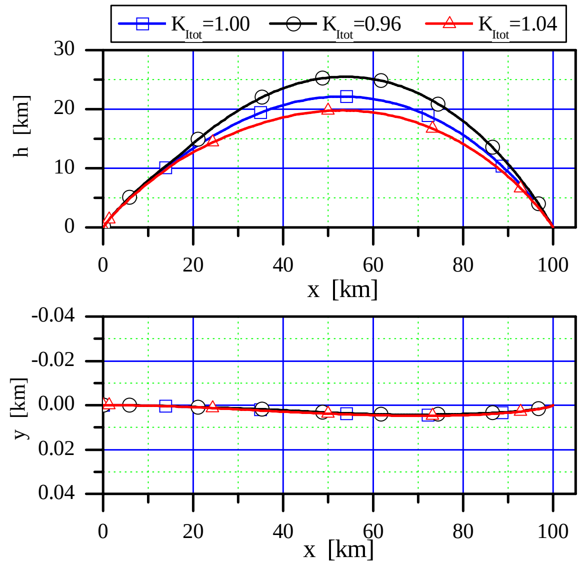

Figure 8. Influence of the total impulse variation on the guided rocket range

There are three trajectories (Fig.8) related to the variation of the total impulse ($K_{I_{tot}}$ = 1.0, 0.96 and 1.04).

When the total impulse is higher than the nominal total impulse the flight path correction formula (26) decreases the flight path angle relative to the nominal flight path angle and the trajectory is below the nominal trajectory. When the total impulse is lower than the nominal, required flight path angle is greater than the nominal one and the trajectory is above the nominal in order to hit the same target.

The flight path angles in a function of time are given in Fig.9 for three analyzed values of the total impulse corrective factors. The decrease of the flight path angles for the thrust greater than the nominal is evident. For the thrust value lower than the nominal one, the increase of the flight path angles is required to hit the same target.

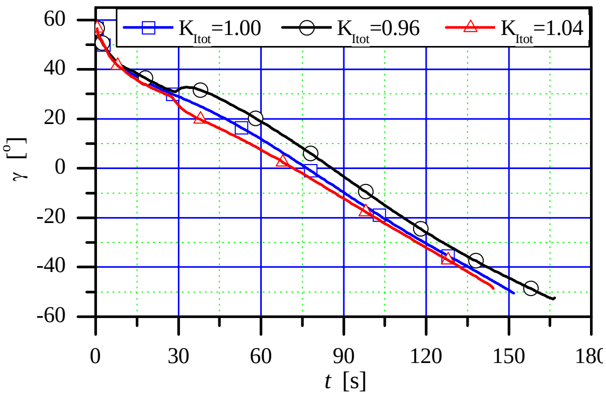

Figure 9. Flight path angle

The diagrams of the control fin deflections in the pitch and yaw planes are given in Fig.10. At the engine cut off moment the difference between the actual and nominal rocket velocity (Fig.3) is evident. As a result, there is a sharp deflection of the fins in order to compensate for the differences between the actual velocity and the nominal one.

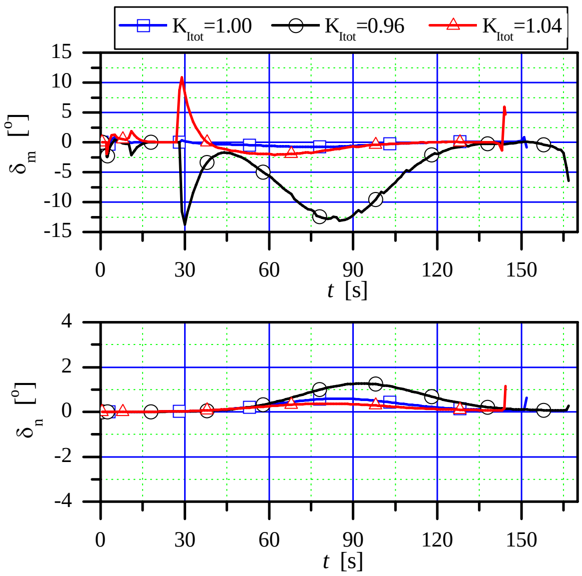

Figure 10. Fins deflection in the pitch and yaw plane

The values of the angles of attack and the sideslip angles are related to the control fins deflection. This relation is determined by the aerodynamic configuration of the rocket. The diagrams of the angles of attack and the sideslip angles in a function of time are given in Fig.11. The angles of attack are not greater than 5° in order to compensate for the maximum variation of the thrust (±4%).

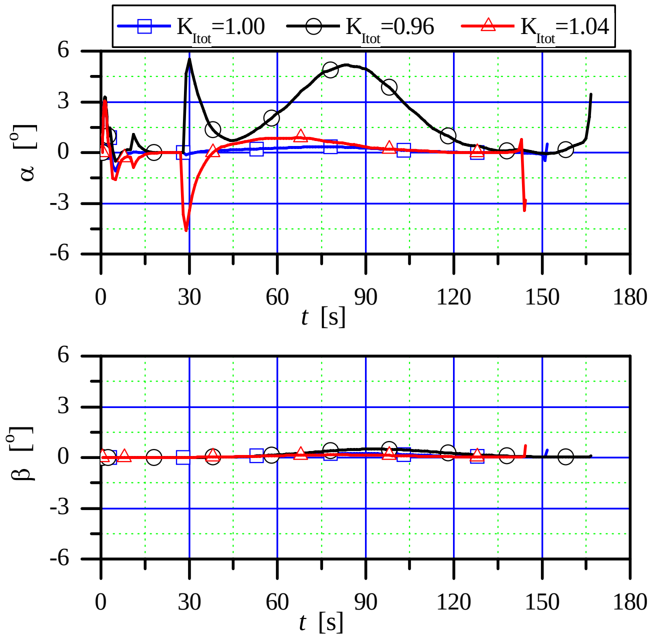

Figure 11. Angles of attack and sideslip angles

The achieved ranges of the guided rockets in a function of the total impulse correction factor (K_{Itot}) are given in Table 1. It can be seen that the miss distance of the rokets with the nominal total impulse is $\Delta x = 2$ m. Since in the inner loop of the lateral acceleration autopilot there is the pitch rate loop (Appendix), the realized deflection of the control fins is not equal to zero even for the zero demanded acceleration because the pitch rate is not equal to zero due to gravity.

Table 1.

Correction factor of the total impulse K_{Itot}	Achieved range [m]
1.00	100002
0.96	99979
1.04	100016

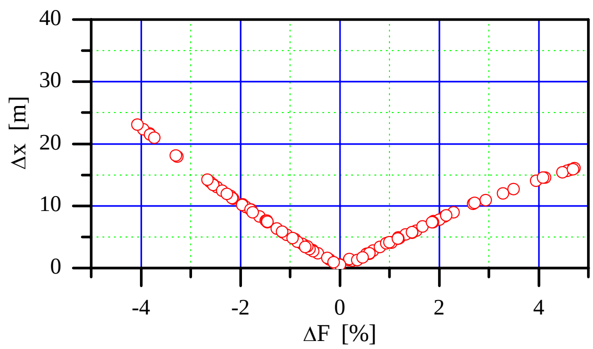

Figure 12. Absolute miss distance due to total impulse deviation

The absolute values of the range errors in a function of percentages of the total impulse deviation for the guided ground to ground rocket are given in Fig.12. This diagram is obtained by the Monte Carlo simulation of the guided flight. The absolute values of the range errors are obtained as absolute values of the differences between the achieved range and the range calculated for the rocket with the nominal total impulse (x=100002 m). It can be seen from the diagram that the maximum miss value is lower than 25 m.

Conclusion

The basic requirements for ground-to-ground rockets are to increase the range and to improve the accuracy. These contradictory requirements can be solved by the guidance system and a mathematical model for the compensation of disturbances.

The proposed algorithm for the flight path correction requires the calculation of the nominal trajectory and the bsensitivity coefficients in the control points. The sampling interval of the control points is chosen to fulfil the requirements that the position, velocity and sensitive coefficients between the control points can be obtained by the linear interpolation of the values in the control points.

The mathematical model of the flight path correction is explicitly solved by comparing the actual trajectory parameters with the nominal trajectory parameters. The nominal trajectory is obtained by the ballistic flight of the rocket to the target. The flight path corrections depend on the differences between the measured rocket velocity and the position and the calculated velocity and the position of the nominal trajectory.

The flight path guidance system with a synthetic pitch autopilot with an accelerometer and a rate gyro is used to steer the rocket to the target. The gains and time constant of the guidance system are designed for a hypothetical rocket and they are time varying in order to obtain the stability of the guidance and control loop in the whole region of the rocket flight.

The proposed algorithm of the flight path correction is analyzed for a hypothetical rocket with a maximum range of $120\ \text{km}$. The flight path correction algorithm is verified for the target located $100\,km$ from the launching site.

If the variations of the total impulses are $\pm 4\%$ relative to the nominal total impulse, the miss distances of the rocket ballistic flight are $\approx \pm 15\ \text{km}$.

The guidance of the same type of the rocket with the flight path correction to compensate the disturbances decreases the miss distance from $-23\ \text{m}$ to $15\ \text{m}$ due to the same total impulse variation ($\pm 4\%$). The miss distance of the rokets with the nominal total impulse is $\Delta x = 2\ \text{m}$ relative to the desired range. Since there is a pitch rate inner loop in the lateral acceleration autopilot, the deflections of control fins are not equal to zero for the zero demanded accelerations because the pitch rate is not equal to zero due to the influence of the gravity.

The efficiency of the given algorithm for the flight path correction should be verified with the noise due to measurements of the accelerations and the angular rates included in the SDINS system.

Appendix: Lateral Acceleration Autopilot

Having in mind the rocket pitch rate transfer function (A1) and the lateral acceleration transfer function (A2),

$$\frac{\Delta q}{\Delta \eta}(s) = \frac{\omega_n^2 K_q \left(T_q s + 1\right)}{s^2 + 2\varsigma_n \omega_n s + \omega_n^2} = \frac{K_q \left(T_q s + 1\right)}{T_n^2 s^2 + 2\varsigma_n T_n s + 1} \quad \text{(A1)}$$

$$\frac{\Delta a_z}{\Delta \eta}(s) = -U_k K_q \frac{\omega_n^2 \left(T_\gamma^2 s^2 + 2\varsigma_\gamma T_\gamma s + 1\right)}{s^2 + 2\varsigma_n \omega_n s + \omega_n^2} \quad \text{(A2)}$$

the transfer function for the acceleration ($\Delta a_z'$), measured by the accelerometer which is ahead of the center of gravity for $\Delta\ell$, can be derived using the rotation in Fig. A-1 [7].

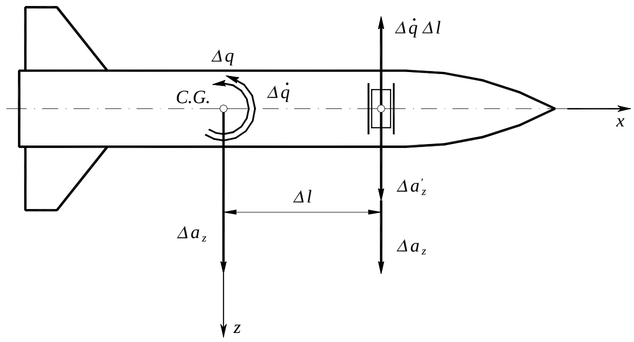

Figure A-1. Accelerometr position relative to the center of gravity

$$\frac{\Delta a_z'}{\Delta \eta}(s) = -U_k K_q \frac{T_\gamma'^2 s^2 + 2\varsigma_\gamma' T_\gamma' s + 1}{T^2 s^2 + 2\varsigma_n T s + 1} \quad \text{(A3)}$$

where

$$T_\gamma'^2 = T_\gamma^2 + \frac{T_q \Delta\ell}{U_k}$$
$$2T_\gamma' \varsigma_\gamma' = 2T_\gamma \varsigma_\gamma + \frac{\Delta\ell}{U_k} \quad \text{(A4)}$$

The lateral acceleration autopilot [7] with the pitch rate and the integrated pitch rate in the inner loop is given in Fig. A-2.

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Received: 19.10.2009.

Vođenje raketa zemlja-zemlja primenom metode kontrole ugla nagiba putanje

Balistički let nevođene rakete zemlja-zemlja osetljiv je na dejstvo poremećaja (rasturanje totalnog impulsa, ekscentritet sile potiska, vetar, itd.) Sa povećanjem dometa raketa zemlja-zemlja povećava se rasturanje pogodaka. Zahtev za povećanje dometa i minimizaciju rasturanja pogodaka može da se reši dodavanjem na raketu sistema za vođenje i upravljanje. U radu je dat algoritam određivanja uglova nagiba putanje rakete na osnovu razlika između stvarnih parametara leta rakete i izračunatih za nominalnu putanju. Efikasnost predloženog algoritma proveren je numeričkom simulacijom leta hipotetičke rakete sa uključenim sistemom za vođenje i upravljanje. Rezultati numeričke simulacije dobijeni su pod pretpostavkom da ne postoje greške merenja aksijalnih ubrzanja i ugaonih brzina rakete sa inercionom mernom jedinicom čvrsto vezanom za telo rakete.

Ključne reči: **višecevni raketni sistem, vođena raketa, vođenje rakete, raketa zemlja-zemlja, rasturanje pogodaka, algoritam vođenja.**

Улучшенный закон наведения для управляемых ракет земля-земля

Баллистический полёт неуправляемой ракеты земля-земля очень чувствителен на действия возмущений (рассеивание суммарного импульса, ветер, эксцентриситет силы тяги и.т.п.). С повышением дальности полёта ракеты многоствольных пусковых установок повышается и рассеивание выигрышей. По требованию повышение дальности полёта ракеты и минимизацию рассеивания выигрышей возможно решить добавлением к ракете системы наведения и управления. В настоящей работе приведён алгорифм определения углов уклона траектории ракеты на основании разниц между истинными параметрами полёта ракеты и высчитанными параметрами для номинальных траекторий. Действенность предложенного алгорифма испытана цифровым моделированием полёта предположительной ракеты со включеной системой наведения и управления. Результаты цифрового моделирования получены с предположением,что не существуют ошибки измерения аксиальных ускорений и угловых скоростей ракеты с инерционной единицей измерения, прочно связанной с телом ракеты.

Ключевые слова: **многоствольная ракетная система, управляемая ракета, наведение ракеты, ракета земля-земля, рассредоточение удара, алгорифм наведения.**

Le guidage du fusée sol-sol par la méthode du contrôle de l'angle de l'inclinaison de la trajectoire

Le vol balistique d'une fusée sol-sol non guidée est sensible à l'action du dérangement (dispersion de l'impulsion totale, l'excentricité de la force de poussée, le vent, etc.). En augmentant la portée des fusées chez le lance- fusée à tubes multiples on a augmenté aussi la dispersion des coups. L'exigence de l'augmentation de la portée et de la minimisation de la dispersion des coups peut être résolue par l'addition d'un système de guidage et de commande à la fusée. Dans ce papier on a donné l'algorithme pour la détermination des angles de l'inclinaison de la trajectoire, basé sur les différences entre les paramètres réels du vol de fusée et les paramètres calculés pour la trajectoire nominale. L'efficacité de l'algorithme proposé a été vérifié par la simulation numérique du vol d'une fusée hypothétique avec le système de guidage et de contrôle incorporés. Les résultats de la simulation numérique ont été obtenus sous la supposition que les erreurs de mesurage des accélérations axiales et les vitesses d'angle de la fusée à l'unité de mesure inertielle, lié étroitement au corps de fusée, n'existent pas.

Mots clés: **lance-fusée à tubes multiples, fusée sol-sol, missile guidé, guidage de missile, dispersion des coups, algorithme de guidage.**