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Analysis of Crack Propagation Using the Strain Energy Density Method

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In this paper, the prediction of the crack growth life under uniaxial stress condition is studied using the energy concept. The model is based on the strain energy density generated ahead of a fatigue crack. Mathematical relations are expressed in terms of low cycle parameters. The wing skin was analyzed as a damaged aircraft structural component. In order to calculate the corrective function (which includes the geometry of a structural element) important for a fatigue crack estimation, the analytical approach is used to determine the stress intensity factor. The accuracy of the modeling for cracked wing skin is validated with experimental results. A good agreement was found between the predicted and the experimental results for the cracked structural component.

Key words: **material fatigue, low cyclic fatigue, crack, crack propagation, numerical simulation, energy density method, aircraft wing, aircraft wing skin.**

Introduction

ONE of the main concerns with aircraft components is structural damage due to fatigue. Fatigue is caused by cyclic loading and as a complex process can be divided into: a) initiation phase and b) crack growth propagation phase. Due to the complexity of the fatigue phenomena, a large number of investigations for fatigue life predictions have been proposed. Many investigations have focused their attention on the fatigue crack growth propagation phase since the failure appears in that phase. Paris and Endogan [1] were apparently the first to determine the power law relationship which describes fatigue crack growth behavior. Based on Paris' law other models were formulated which included: the R-ratio effect, the thresholds value of the stress intensity factor range ΔK_{th} and fracture toughness of the material K_c [2]. All these models based on Paris' law fall under conventional methods. On the other hand, some experts have considered models based on the energy concept. These models generally take into consideration the main physical mechanisms that are governing the local elastic-plastic deformation and damage behavior. In these energy-based models the starting point is the fact that failure appears when energy within the material at the crack tip reaches a critical value. Some of these energy-based models are Weertmab [3], later McEvily and Johnston [4] as well as the model recommended by Chand and Garg [3]. Additionally, with the model recommended by Chand and Garg [5], Rice′s superposition method for cyclic loading was used. In addition to the fact that for the energy-based model Rice′s superposition method could be included, it is necessary to add adequate concept(s) to define absorbed energy within the material at the crack tip. There are various concepts that adequately define energy absorbed within the material at the crack tip. For engineering applications the most interesting are the concepts based on low cyclic fatigue properties [6-8], such as, for example, a strain energy density concept. By formulating and applying models based on low cyclic fatigue properties, fatigue life prediction for a crack propagation phase could be significantly simplified.

The main purpose of this paper is to formulate a complete computational model based on the strain energy density concept for the determination of the crack growth life. In this computational model low cyclic fatigue properties were used. The proposed model is applied on the wing skin subjected to uniaxial loading. In addition, new corrective functions (as a polynomial expression) used in the computational analysis have been evaluated.

Crack growth model based on the Strain Energy Density Method

Since the crack growth propagation is a complex process, it is necessary to carefully analyze the region near the crack tip. Depending on the behavior of the region near the crack tip during the cyclic loading three regions could be included in the analysis. The first region near the crack tip is the process zone, then follows the region between the process zone and the cyclic plastic zone and finally the region between the cyclic plastic zone and the monotonic plastic zone. The region outside the monotonic plastic zone and the cyclic plastic zone is considered to be the elastic region. For that

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elastic region, the stress at the point (r, θ) ahead of a crack [9] can be expressed as:

$$\sigma_{hi} = \frac{K}{\sqrt{2\pi r}}\, f_{hi}\left(\theta\right) \tag{1}$$

where the subscripts h and i refer to the component of stress, $f\left(\theta\right)$ depends on the angular disposition of r and K is the stress intensity factor.

On the other hand, the region right near the crack tip (the process zone) and the region between the process zone and the cyclic plastic zone are regions with a dominant effect of plastic deformations. In these cases the HRR relations [10] are valid and as a result relation (1) for the stress field [Skelton/19-21] has to be replaced with the equation:

$$\sigma_{hi} = \left(\frac{J\left(k'\right)^{\frac{1}{n'}}}{I_{n'} r}\right)^{\frac{n'}{1+n'}} \tag{2}$$

where k' is the cyclic strength coefficient, n' - the cyclic strain hardening exponent and I_n' is a constant depending on the value of n'.

In addition to defining the stress field, it is necessary to calculate the stress-strain distribution ahead of a crack in an elastic-plastic material. The function between stress and strain, as recommended by Ramberg-Osgood, provides a good description of elastic-plastic behavior of material and may be expressed as:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2k'}\right)^{\frac{1}{n'}} \tag{3}$$

where E is the modulus of elasticity, $\Delta\varepsilon/2$ is the strain amplitude and $\Delta\sigma/2$ is the stress amplitude. eq.(3) enables the calculation of the stress-strain distribution by knowing low cyclic fatigue properties. Ramberg-Osgood′s cyclic stress-strain curve is presented in Fig.1.

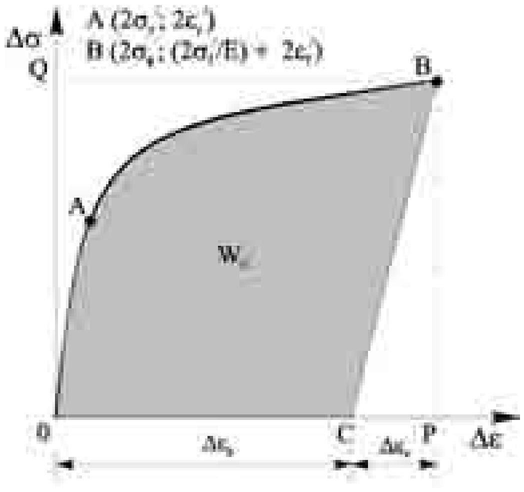

Figure 1. Cyclic stress – strain curve

As stated above, in this paper fatigue crack growth propagation based on the energy concept is considered and then it is necessary to determine the energy absorbed till failure. This energy can be calculated by using the cyclic stress-strain curve. The energy absorbed till failure can be calculated if the areas OABFO and the area OBEC are subtracted from the area OFBEO (Fig.1). As a result the energy absorbed till failure becomes:

$$W_c = \frac{4}{1+n'}\sigma'_f \varepsilon'_f \tag{4}$$

where σ'_f is the cyclic yield strength and ε'_f - the fatigue ductility coefficient.

Given the fact that the strain energy density method is considered, the energy absorbed till failure must be determined after the energy concept is based on the following fact: The energy absorbed per unit growth of a crack is equal to the plastic energy dissipated within the process zone per cycle. This energy concept is expressed by:

$$W_c \delta a = \omega_p \, , \tag{5}$$

where W_c is the energy absorbed till failure, ω_p - the plastic energy and a - the crack length.

In eq.(5) it is necessary just to determine the plastic energy dissipated in the process zone ω_p. By integration of the equation for the cyclic plastic strain energy density in the units of Joule per cycle per unit volume [11] from zero to the length of the process zone ahead of crack tip d* it is possible to determine the plastic energy dissipated in the process zone ω_p. After integration, the relation of the plastic energy dissipated in the process zone becomes:

$$\omega_p = \left(\frac{1-n'}{1+n'}\right)\frac{\Delta K_I^2 \,\psi}{E\, I_{n'}} \tag{6}$$

where ΔK_I is the range of stress intensity factor, ψ - the constant depending on the strain hardening exponent n', $I_{n'}$ - the non-dimensional parameter depending on n'.

The fatigue crack growth rate can be obtained by substituting eq.(4) and eq.(6) in eq.(5):

$$\frac{da}{dN} = \frac{(1-n')\psi}{4E\, I_{n'}\sigma'_f \,\varepsilon'_f}\left(\Delta K_I - \Delta K_{th}\right)^2 , \tag{7}$$

where ΔK_{th} is the range of the threshold stress intensity factor and is a function of the stress ratio i.e.

$$\Delta K_{th} = \Delta K_{th0}\left(1-R\right)^y , \tag{8}$$

ΔK_{th0} is the range of the threshold stress intensity factor for the stress ratio $R=0$ and y is the coefficient (usually y = 0.71).

Finally, the number of cycles till failure can be determined by the integration of the relation for the fatigue crack growth rate:

$$N = B\int_{a_0}^{a_c}\frac{da}{\left(\Delta K_I - \Delta K_{th}\right)^2}\, , \;\; B = \frac{4\, E\, I_{n'}\sigma'_f\, \varepsilon'_f}{\left(1-n'\right)\psi} \tag{9}$$

and

$$\Delta K_I = Y\, S\, \sqrt{\pi\, a}\, , \tag{10}$$

Eq.(9) enables the determination of the crack growth life of different structural components. A very important fact is that eq.(9) is easy for application since low cyclic properties (n', δ'_f , ε_f) available in literature are used as parameters. The only important point is the stress intensity factor which, depending on the geometry complexity and the type of loading, could be determined by using analytical and/or numerical approaches.

Corrective function and the stress intensity factor

Geometry has a significant impact on the length of service life of structural elements. The relation that introduces the geometry of a structural element in estimation is a corrective function and it is usually a function dependent on: a – length of crack, w – width of specimen, R_1 – hole radius, etc. A corrective function could be determined for simple geometry using data available in literature (graphics and tables) and from them it is possible to formulate needed analytical relations. However, for more complicated geometry and loading cases, it is necessary to use numerical methods.

In this paper, a wing skin with a central crack and a wing skin with a circular hole and one crack as well as two cracks were analyzed and it is necessary to formulate relations for corrective functions when using the analytical approach.

So, for the wing skin with a central crack subjected to axial loading, the corrective function can be expressed as:

$$Y = \left(1 + 0.256\left(\frac{a}{w}\right) - 1.152\left(\frac{a}{w}\right)^2 + 12.2\left(\frac{a}{w}\right)^5\right) \quad (11)$$

where w is the width of the structural element; a - the length of the crack.

On the other hand, the corrective function for the configuration with one opposing crack extending from the edge of the hole along the line perpendicular to the direction of axial stress S can be expressed as:

$$Y = z\, Y_w\, Y_{b1} \quad (12)$$

and

$$z = \sqrt{\frac{1}{\cos\left(2R_1 \frac{\pi^2 w}{180}\right)}}\,;$$

$$Y_w = \sqrt{\frac{1}{\cos\left(\frac{\pi}{2}\frac{(2R_1 + a)}{(w-a)}\right)}}\,; \quad (13)$$

$$Y_{b1} = 0.70833 + 1.29275\, e^{\frac{-(a/w)}{0.17197}} + 0.29223\, e^{\frac{-(a/w)}{4.81617}} + 1.10057\, e^{\frac{-(a/w)}{1.04267}}$$

where R_1 is the radius of the hole and a - the length of the crack.

Additionally, the corrective function for the configuration of the wing skin with circular hole and two cracks extending from the edge of the hole along the line perpendicular to the direction of axial stress S has been considered by Bowie [13]. Based on Bowie's graphics, the relation for the corrective function in the case of axial loading became:

$$Y = -0.28659 + 1.36339\left(1 - e^{\frac{-((a/\eta)-0.96104)}{0.08936}}\right)^{0.78013} e^{\frac{-((a/\eta)-0.96104)}{43.4814}} \quad (14)$$

One of relations (11), (12) (with (13)) or (14) for the corrective functions could be used in eq.(9), i.e. eq.(10) for the determination of the number of cycles till failure N.

Numerical examples

The above formulated procedure for fatigue crack growth estimation on damaged structural components with different geometry is applied in this section. These damaged structural components are subjected to uniaxial loading with constant amplitude. The stress intensity factors were calculated using the analytical approach. In this paper, therefore, new corrective functions needed for the determination of stress intensity factors were evaluated.

Example 1. Crack growth estimation for the specimen with a central crack

This example considered fatigue crack growth prediction. The structural element is a central cracked plate made of 2219 T851 Al Alloy. External loading is axial with constant amplitude. The material characteristics of Al Alloy under cyclic loading are: $\sigma'_f = 613$ MPa ; $\varepsilon'_f = 0.35$; $n' = 0.121$; $k' = 710$ MPa ; $S_y = 334$ MPa ; $E = 7.1 \cdot 10^4$ MPa ; $K_{IC} = 120$ MPa m$^{1/2}$; $I'_n = 3.067$; $\psi = 0.95152$ and $\Delta K_{th0} = 8$ MPa m$^{1/2}$. The geometry characteristics for a central cracked plate are: $w = 152.4$ mm ; $a_0 = 7.6$ mm .

With calculation, the stress intensity factor K_I (using eq. (10)) was first determined and after that the number of cycles to failure, N, using the strain energy density method.

The number of cycles predicted and presented in Fig.2 is obtained using eq.(9) and using eq.(8) for ΔK_{th} . In the case of the relation for the corrective function, eq. (11) was used, since in this example the central cracked plate specimen was considered.

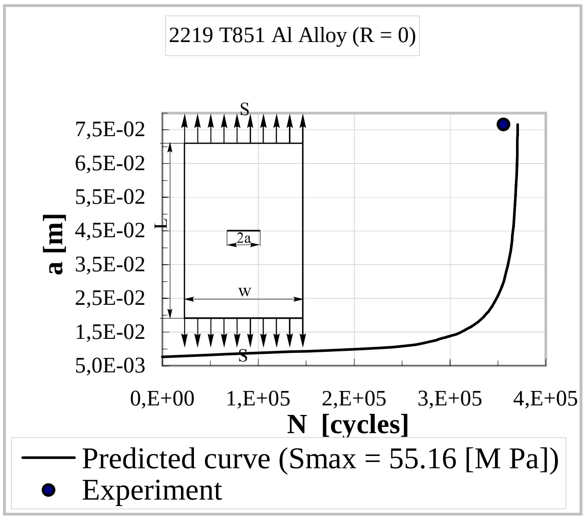

Figure 2. Crack length versus the number of cycles to failure *N*

The crack propagation experiments were carried out for the stress ratios of R = 0 and the maximum nominal stress $S_{mac} = 55.16\,[\text{MPa}]$. The central cracked plate was subjected to external loading up to failure. As shown in Fig.2, the predictions were compared with experimental data and a good agreement was obtained.

Example 2. Crack growth prediction of the wing skin with a circular hole and one crack

As in the previous example, the fatigue life estimation of the wing skin was carried out. The considered wing skin has one opposing crack extending from the edge of the hole along the line perpendicular to the direction of the axial stress *S*. The structural element was subjected to axial loading. The material used in this example is the same as previous (2219 T851 Al alloy). The geometry characteristics for the wing skin with a circular hole and one crack are: $w = 0.0256$ m, $R_1 = 0.0064$ m; $a_0 = 0.00032$ m, $t = 0.0768$ m and $L = 0.1$ m (t is the thickness of the wing skin).

Since, in Example 1, the formulated procedure for the estimation of fatigue life was verified, now it is possible to start with the determination of the number of cycles for the crack propagation phase. Prior to the estimation of the number of cycles till failure, it is necessary to determine the stress intensity factor.

2.a. Stress intensity factor calculation of the wing skin with a circular hole and one crack

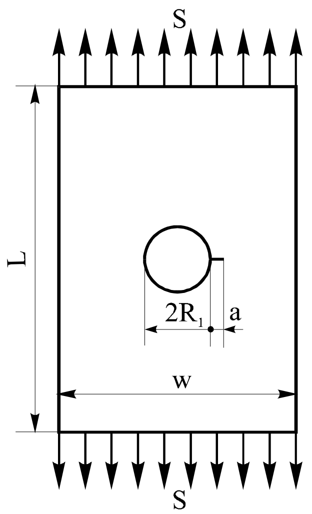

Figure 3. Geometry of the wing skin with a hole and one crack

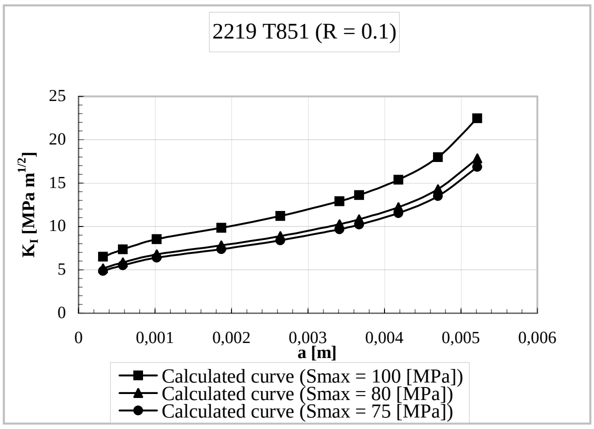

Figure 4. The stress intensity factor versus the crack length

The stress intensity factor can be determined using analytical and numerical approaches. The analytical approach was used in this example. As in this case geometry presented in Fig.3 is analyzed then evaluated the relation for the corrective function (Eqs.(12) and (13) together with the equation for stress intensity factor eq.(10)) must be used. All values for stress intensity factors obtained by application of these equations are presented in Fig.4.

2.b. Fatigue life prediction of the wing skin with a circular hole and one crack

Since the determined stress intensity factors are for different cases of axial loading, it is possible for each one of them to predict a number of cycles to failure. The configuration of the wing skin considered in this example is presented in Fig.3. For the determination of the number of cycles to failure it is necessary to use near eq.(9), the corective function (Eqs.(12) and (13)) and the relation of the stress intensity factor (eq.(10)). The obtained predictions for the number of cycles to failure for different nominal stresses of axial loading with the constant amplitude are illustrated in Fig.5.

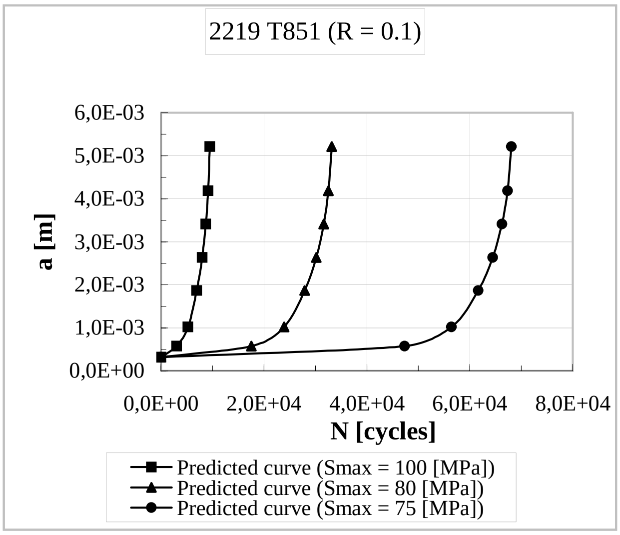

Figure 5. Crack growth analysis of the wing skin with a circular hole and one crack.

The presentation of all predicted curves in the Figure was done in order to perform the comparison of the number of cycles to failure for different nominal stress levels of loading and R = 0.1. Fig.5 shows that for the initial crack length a_c = 0.00032 m and the final crack length a_c = 0.0052144 m, the numbers of cycles to failure are:

N_f = 9516 cycles for S_{max} = 100 [MPa];

N_f = 33176 cycles for S_{max} = 80 [MPa];

N_f = 68077 cycles for S_{max} = 75 [MPa].

Example 3. Crack growth estimation of the wing skin with a circular hole and two cracks

In this example the crack growth rate and the fatigue life prediction of the wing skin with two opposing cracks extending from the edge of the hole along the line perpendicular to the direction of the axial stress S were considered. The wing skin was made of 2219 T851 Al alloy. The geometry characteristics for the wing skin with a circular hole and two opposing cracks are: w = 0.0256 m, R_1 = 0.0064 m; a_0 =0.00032 m, $t = 0.00768$ m and $L = 0.1$ m.

3.a. Stress intensity factor calculation of the wing skin with a circular hole and two cracks

In this example the stress intensity factor was calculated. However, as the wing skin with two opposing cracks was analyzed (see Fig.6) the following equations were used: for the stress intensity factor, eq.(10) and for the corrective function, a new evaluated eq.(14) as a polynomial expression.

The obtained values for the stress intensity factors using eq.(14) in the equation for the stress intensity factor (10) are presented in Fig.7.

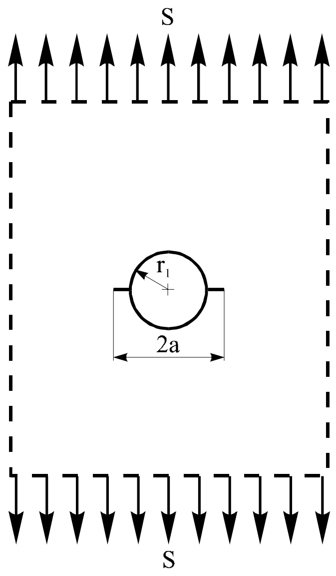

Figure 6. Geometry of the wing skin.

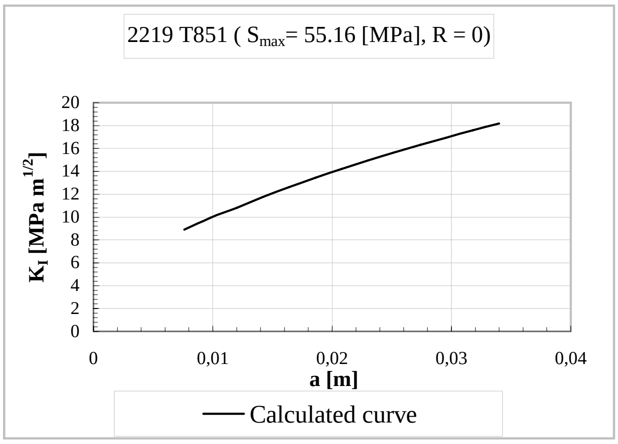

Figure 7. The stress intensity factor versus the crack length.

3.b. Fatigue life prediction of the wing skin with a circular hole and two equal cracks

Similarly to the previous example after the calculation of the stress intensity factors it is possible to determine the number of cycles to failure for the selected discrete values of the crack length a. For the prediction of the number of cycles to failure for the material 2219 T851 it is necessary to use low cyclic parameters (see Example 1).

By using eq.(9) with relation (10) or one of corrective functions (eq.(13) or eq.(14)) it is possible to find the number of cycles to failure. The obtained results for the number of cycles to failure are presented in Fig.8 for the nominal stress S_{max}= 55.16 MPa.

The analysis of these predicted curves and the results for two different geometries of the wing skin (a circular hole with one and two cracks) subjected to axially loading (see Fig.8) showed that in the case of the geometry with circular hole and one or two cracks the fatigue life to failure is significantly reduced.

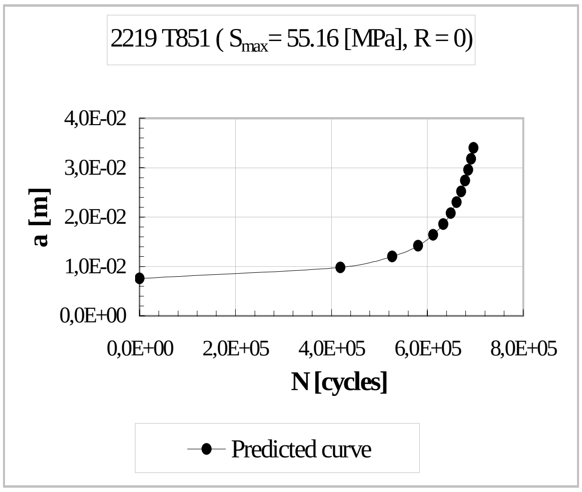

Figure 8. Crack length versus the number of cycles to failure *N*

Conclusion

This paper formulates a complete computational procedure/model based on the strain energy density concept for fatigue life prediction of the crack propagation phase. The verification of the computational model with the experimental data shows that the proposed model can be used in engineering applications. The additional convenience when using the Strain energy density method is that low cyclic fatigue properties are used and these are available in literature for different materials. Additionally in this paper, new empirical relations for corrective functions for a wing skin with a circular hole and one crack subjected to axial loading were analyzed. The most important fact is that the formulated computational procedure based on the strain energy density concept does not require any additional material costs for experimental research except for those needed for fatigue crack initial prediction. It is, therefore, possible to determine very easily the total fatigue life prediction of a considered structural component.

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Analiza širenja prskotine primenom metode gustine energije deformacije

U radu je razmatran problem proračunske analize širenja prskotine i procene veka za strukturalni element polja oplate sa prskotinom koristeći metod energije. Korišćen je metod gustine energije deformacije. Matematički model je formiran koristeći ciklične karakteristike materijala. Polje oplate krila aviona je razmatrana kao oštečeni strukturalni element. Za određivanje faktora intenziteta napona, odnosno korektivne funkcije, kod polja oplate sa otvorom i inicijalnom prskotinom korišćen je analitički metod. Tačnost rezultata koristeći prezentovan pristup je obezbeđen kroz poređenja sa eksperimentalnim rezultatima. Dobijeno je dobro slaganje između prezentovanih i eksperimentalnih rezultata.

Ključne reči: **zamor materijala, niskociklični zamor, prskotina, rast prskotine, numerička simulacija, metoda gustine energije, krilo aviona, oplata.**

Анализ расширения трещины с применением метода плотности энергии деформации

В настоящей работе рассматривана проблема расчётного анализа расширения трещины и оценки века для структурального элемента поля обшивки (обтекателя) с трещиной при пользовании метода энергии. Тоже использован и метод плотности энергии деформации. Математическая модель сформирована при пользовании циклической характеристики материала. Поле обшивки (обтекателя) крыла самолёта рассматривано в роли повреждённого структурального элемента. Для определения фактора интенсивности напряжения, т.е. корректировочной функции, у поля обшивки с отверстием и с начальной трещиной использован аналитический метод. Точность результатов при использовании представленного подхода обеспечена через сопоставления с экспериментальными результатами и так получено хорошее согласовывание между представленными и экспериментальными результатами.

Ключевые слова: **усталость материала, низкоциклическая усталость, трещина, рост трещины, усталостная трещина, цифровое моделирование, метод плотности энергии, крыло самолёта, обшивка (обтекатель).**

Analyse de la croissance de la fissure par la méthode de la densité d'énergie de déformation

Ce papier étudie le problème de l'analyse du calcul de la croissance de la fissure ainsi que l'estimation de la durée de vie pour l'élément structural du revêtement ayant la fissure par la méthode de la densité d'énergie. On a employé la méthode de la densité d'énergie de la déformation. Le modèle mathématique est formé à l'aide des caractéristiques cycliques des matériaux. Le champ du revêtement des ailes d'avion est considéré comme l'élément structural endommagé. Pour déterminer le facteur de l'intensité de tension, c'est-à-dire les fonctions correctives, chez le champ du revêtement à l'ouverture et à la fissure initiale, on a utilisé la méthode analytique. Par l'approche présentée on a assuré la validité des résultats expérimentaux. On a constaté bon accord entre les résultats présentés et les résultats d'essais.

Mots clés: **fatigue des matériaux, fatigue basse cyclique, fissure, croissance de la fissure, fissure de fatigue, simulation numérique, méthode de la densité d'énergie, aile, revêtement.**