

# Numerical Model for the Critical Stress Determination in Spur Gears

Daniela Ristić, MSc (Eng)<sup>1)</sup>

Gear carrying capacity and stress state depends to a large extent on main gear profile configuration parameters. To support this analysis, gear kinematics, carrying capacity, strength, production and some other characteristics are being actively investigated a tooth root form and the fillet radius. Great influence on gear tooth root strength that is one of the primary subjects of this paper. A special attention is given to the analysis of the impact of the gear tooth fillet radius at the critical cross section on stress value and distribution. The stress intensity factor and the gear working life depend directly on the tooth root stress. Real gears are statically undetermined systems and tooth root stress concentration depends on many parameters. Determining an optimal gear form relative to stress concentration is one of the main problems of gears design. It is well known that a first initial crack appears at the gear tooth affected the most by root stress concentration. Hence, this paper's research topic is focused on finding the optimal fillet tooth root radius to minimize the root stress intensity. This paper provides the results achieved by the application of numerical methods: finite element method (FEM) and real working conditions simulation. The results are analyzed in order to form an effective numerical model for tooth root geometrical discontinuity phenomena at static loading. The results of this analysis are presented in this paper as figures and tables of Von Mises stresses as well as charts against different values of tooth root fillet radius  $\rho_F$ .

*Key words:* gear, spur gear, critical stress, stress concentration, finite element method.

## Introduction

GEAR load capacity and stress state depends to a large extent on main gear profile configuration parameters, which made it one of the key areas of interest for scientific analysis. To support this analysis, gear kinematics, carrying capacity, strength, production and some other characteristics are being actively investigated. A great deal of gear tooth root load capacity has a tooth root form, i.e. tooth root fillet radius. It has a direct influence on the stress intensity factor as well as on a gear service life.

Real gear transmitters are multiple statically undetermined systems and stress concentration in a gear tooth root depends on many parameters. An optimal gear form and meshing parameters discovery relative to stress concentration is one of the main problems of gears design. It is well known that a first initial crack appears at the gear tooth affected the most by root stress concentration.

A tooth root critical cross section, according to the ISO recommendation, is determined by a tooth fillet radius tangent that is positioned at  $30^\circ$  against the gear symmetry line, and its dimensions are the critical cross section width  $s_{Fn}$  and the face width  $b$ .

The tooth root critical cross section is exposed to a pressure load under the radial component, and under the tangential component it is exposed to bending and shearing. It all shows a complex load state in the tooth root critical cross section. Accumulation of normal stresses gives maximum stress to the under pressured side of the gear tooth ( $\sigma_s + \sigma_p$ ), while, in the same time, the resulting normal stress ( $\sigma_s - \sigma_p$ ) on the tensile side is considerably lower than on the pressured side of the gear. However, unwanted phenomena such as plastic deformations, crack initiation,

even a gear failure appear on the tensile side of the gear tooth. Plastic deformations make material strength, they give the elasticity threshold appearance (Bauschinger effect) which gives a crack initialization. According to that, the rectangular tooth root cross section is deformed and changes in to a trapeze form, which leads to changes of the position of the neutral axe.

Because of that all, the stress values on the tensile side of the gear tooth are important for the determination of applied stresses.

## Determination of the gear load capacity according to the tooth root strength

The load applied on a tooth root and caused the normal force  $F_{bn}$  acts in the contact points on the meshed teeth profiles. For analyzing the stress conditions at the gear tooth root, the tooth is approximated with the console shaped mechanical model, embedded in the gear body, at the end of which the load acts in the direction of the teeth profile pressure line.

The normal force can be divided into two components:

- radial  $F_r = F_{bn} \sin \alpha_{Fan}$  and
- tangential  $F_t = F_{bn} \cos \alpha_{Fan}$ ,

where the gear is exposed to a pressure load under the radial component, and under the tangential component, with the force arm  $h_{Fa}$ , it is exposed to bending and shearing. According to that, the complex stress condition in a tooth root characterizes pressures and bending normal stresses and shear tangential stresses.

The bending normal stress in a critical cross section is [1, 2, 6]:

<sup>1)</sup> Advance School of Engineering, TEHNIKUM TAURUNUM, Nade Dimić 4, 11080 Belgrade, SERBIA

$$\sigma_s = \frac{M}{W} = \frac{F_{bn} \cos \alpha_{Fan} h_{Fa}}{b s_{Fn}^2 / 6} \quad (1)$$

Knowing that the normal force is:

$$F_{bn1} = \frac{F_{t1}}{\cos \alpha_{wn} \cos \beta_w} = \frac{F_{t1}}{\cos \alpha_{wt} \cos \beta_b} \quad (2)$$

and after some transformations we come to the normal stress equation:

$$\sigma_{Fn} = \frac{F_t}{b m_n} \frac{6(h_{Fa} / m_n) \cos \alpha_{Fan}}{(s_{Fn} / m_n)^2 \cos \alpha} = \frac{F_t}{b m_n} Y_{Fa} \quad (3)$$

where:

$Y_{Fa}$  – shape factor

$\alpha_{Fan}$  – angle between the normal force  $F_{bn}$  and the horizontal line

The maximum normal stress of the cylindrical spur gears in a tooth root cross section, according to the stress concentration factor, is:

$$\sigma_{\max} = \sigma_{Fn} Y_s K_F \quad (4)$$

where

$$K_F = K_A K_V K_{F\alpha} K_{F\beta} \quad (5)$$

is the total load factor which considers the disagreement between real and theoretical working conditions ( $K_A$ -application factor,  $K_V$ -dynamic factor,  $K_{F\alpha}$ -unequal load distribution factor,  $K_{F\beta}$ - load distribution factor).

The tooth root fillet radius  $\rho_F$  is given in the form of:

$$\rho_F = \frac{c_p}{(1 - \sin \alpha_0)} \quad (6)$$

where  $c_p$  presents the gear tip clearance ( $c_p = (0.1 \div 0.3)m$ ,  $m$  - module).

### Characteristics of the analyzed gears

All analytical and numerical research is carried out on a real construction with real condition simulation. The considered gears are the part of the structural elements of a high power planetary transmitter (big 2x550kW excavator) with the following characteristics:

- module  $m = 24\text{mm}$ ,
- profile movement coefficient  $x_1 = 0$  и  $x_2 = 0.326$ ,
- face width  $b = 350\text{mm}$ ,
- rotation moment  $T = 2528.8\text{kNm}$ ,
- number of rotations  $n = 4.1596\text{min}^{-1}$ ,
- number of teeth  $z_1 = 20$  и  $z_2 = 96$ ,
- gear material is carburized steel 17CrNiMo6 (according to DIN) with Young's modulus  $E = 2.1 \times 10^5 \text{MPa}$  and Poisson's ratio  $\nu = 0.3$ .

As a consequence of working conditions and irregular by formed tooth root shapes, appearance of failures is often noticed during exploitation [2, 3, 4, 5]. Because of that, the topic of this work is the determination of exact tooth root stress and an optimum tooth root shape.

According to the theory, at the contact points B and D, the double mesh follow transforms to a single one and reversely. At these points the  $F_{bn}$  force acts in its total value, while at the points A and E, the  $F_{bn}$  force acts with a half of its total value. Hence, the most important loads for stress concentration appearance are the loads in the contact points E and D for the driving gear, and the contact points

A and B at the driven gear. Loads in other mesh contact points have no influence on critical cross section stress concentration and they do not cause failures and crack initiation in a tooth root. This work will represent only the results for the driving gear and its E and D contact points.

These researches are deduced to stress determination for a less-driving and larger-driven gear of the observed planetary transmitter according to the various tooth root fillet radius  $\rho_F$ . The tooth root fillet radius  $\rho_F$  has the most important influence on tooth root stress concentration, so that is the topic of this work. It is analyzed for eight different values of the tooth root fillet radius  $\rho_F$ . The lowest value is  $\rho_F = 4.56\text{mm}$  and it is incrementally increased to the value of  $\rho_F = 10.94\text{mm}$ . Fig.1 shows dimensions of the critical section for both gears.

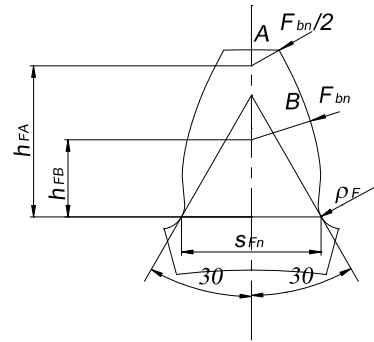


Figure 1. Dimensions of the critical section for the driving gear

### Analysis of the results by FEM

The numerical analysis is deduced to the determination of Von Mises stresses and normal stresses for two meshed gears of the planetary transmitter according to the tooth root fillet radius  $\rho_F$ . The finite element method (FEM) is used for the numerical analysis here. For that purpose the finite element package FEMAP v.9.3 [10] is used. On the basis of the gear data, the 3D finite element model is made. One driving tooth has 12489 elements and 14301 nodes. The mesh is refined in the tooth fillet region in order to show the best possible way the stress condition in that section. Only 50mm width layer of the tooth is analyzed according to a supposition that load is equally distributed along the instantaneous tooth side contact line. The gear tooth is loaded with the normal force  $F_{bn}$  which is acting at the outer point of the single tooth contact (D) and the inner point of double contact (E). Fig.2 shows a 3D finite element model for the analyzed gears.

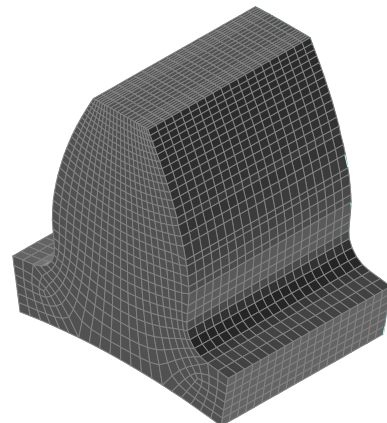


Figure 2. 3D finite element model for the driving and the driven gear tooth

The outer load, i.e. the normal force  $F_{bn}$  that is equally distributed along the instantaneous tooth side contact line is changed with a concentrated force at nodes along that line (21 nodes along the width  $b_i$ ). It is possible to get equal force distribution along the tooth width if it is possible to have more layers of finite elements and that reduce the influence of a concentrated force in the instant contact point.

The developed FEM (finite element method) of stress determination in a gear tooth root allows not only a good picture of the stress distribution but even defines the exact position of the maximum stress value point. In the same time, the processes of stress concentration are nominated which is very important in engineering.

**Table 1.** Von Mises  $\sigma_e$  stresses at the critical section of the gear

Fillet radius $\rho_F$ [mm]	Tensile side $\sigma_e$ [MPa]		Pressured side $\sigma_e$ [MPa]	
	E	D	E	D
4.56	212.3445	277.6251	254.0958	323.4744
5.47	203.1780	261.8667	242.2392	303.9231
6.38	190.8302	247.4392	231.2011	288.7467
7.29	179.6996	233.5653	215.6862	271.2920
8.22	171.5908	222.7844	206.2833	259.0119
9.12	164.0627	213.8899	197.2313	248.1438
10.03	156.4730	204.7536	188.8939	237.4938
10.94	151.7939	198.0450	184.6548	229.6540

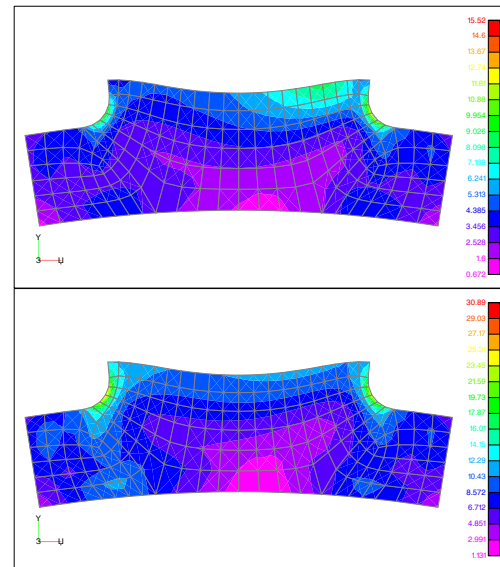
**Table 2.** Normal stresses  $\sigma_y$  at the critical section of the gear

Fillet radius $\rho_F$ [mm]	Tensile side $\sigma_y$ [MPa]		Pressured side $\sigma_y$ [MPa]	
	E	D	E	D
4.56	195.6763	246.4310	234.0091	286.6890
5.47	194.9251	244.8961	231.7459	282.9113
6.38	178.2365	224.7813	216.8373	262.7499
7.29	163.1245	205.0326	195.5132	237.3590
8.22	153.4023	192.2681	183.8581	222.4591
9.12	144.0161	181.0111	172.7297	209.1441
10.03	135.1255	170.2814	162.9967	196.9606
10.94	129.6790	162.6787	158.2093	188.7827

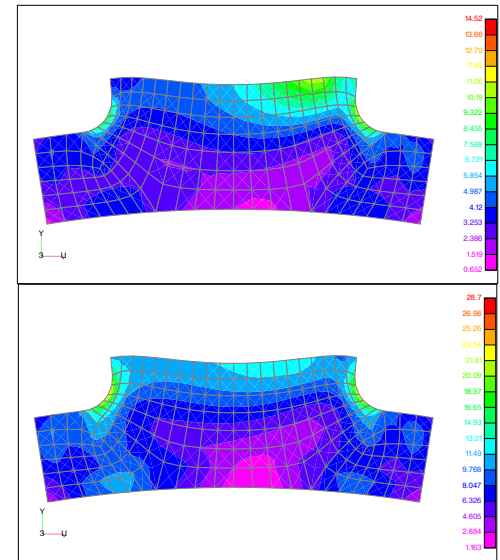
The results of such numerical computations for Von Mises stresses at the critical section are presented in Table 1 and in Table 2 normal stresses are presented. For research in this paper, the most important stress values are those in the critical section nodes of the tensile side of the gear because these stresses cause damage and failures of the gear at the end [7, 8, 9]. But because of the whole stress state presentation in this paper the critical stress values on the both side of the gear tooth root are shown.

It may be seen from these tables that Von Mises  $\sigma_e$  and normal  $\sigma_y$  stresses become lower as the fillet radius  $\rho_F$  grows and it is in agreement with the analytical results (Eq.3). So, there is a recommendation for higher values of the fillet radius, but only in the allowed boundaries (Eq.6).

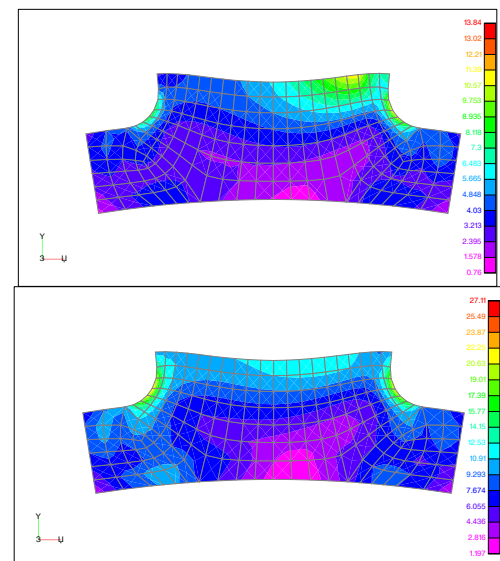
Fig.3 show the results of the numerical computations for Von Mises stresses, and Fig.4 show the results for normal stresses both for the load acting in the first two contact points (E and D). The results in Tables 2 are taken from Fig.3 and results in Table 3 are taken from Fig.4.



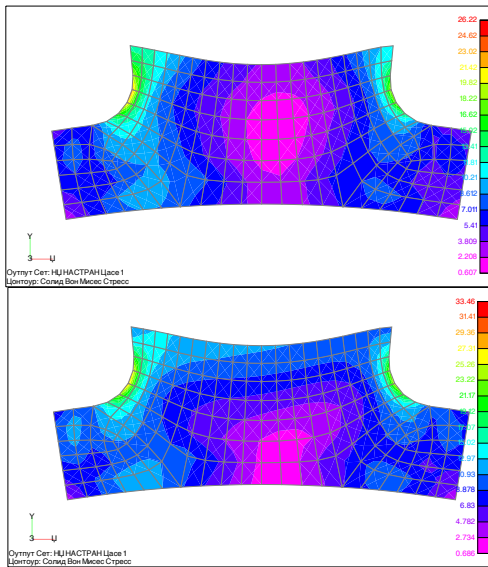
$\rho_F = 4,56\text{mm}$



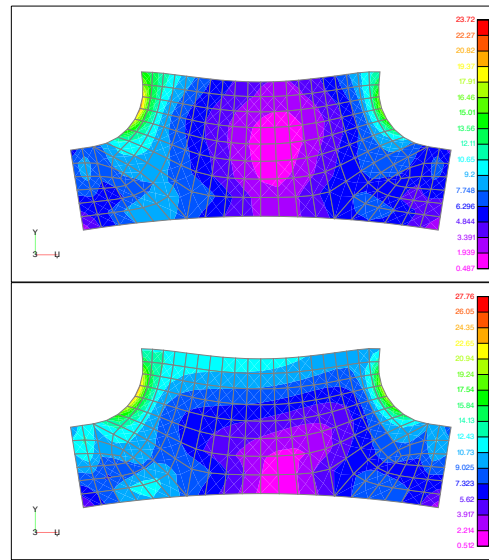
$\rho_F = 5,47\text{mm}$



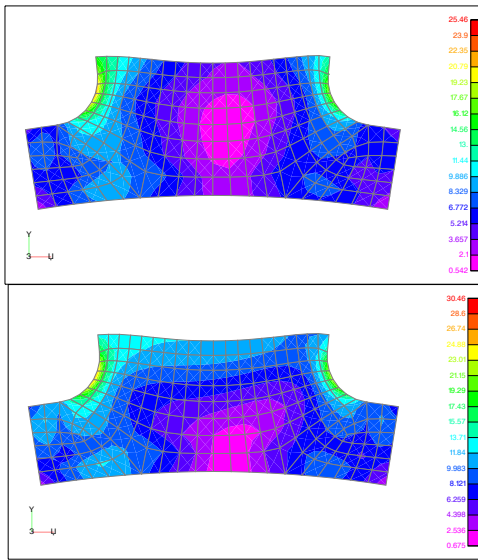
$\rho_F = 6,38\text{mm}$



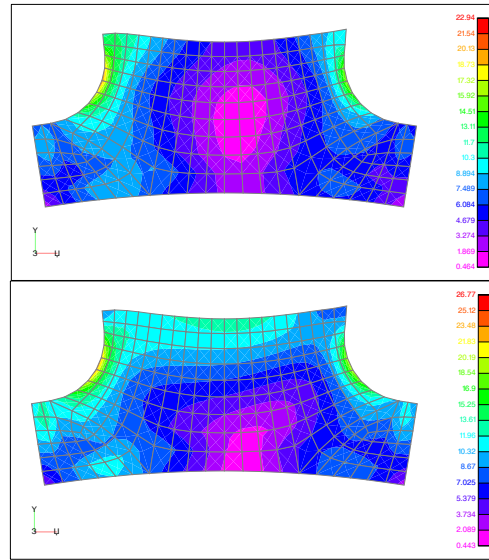
$\rho_F = 7,29\text{mm}$



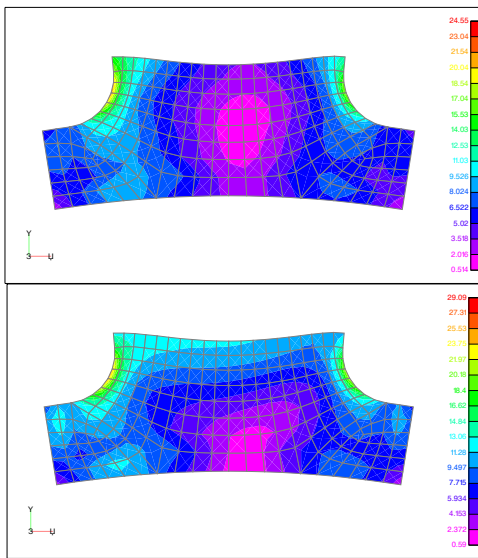
$\rho_F = 10,031\text{mm}$



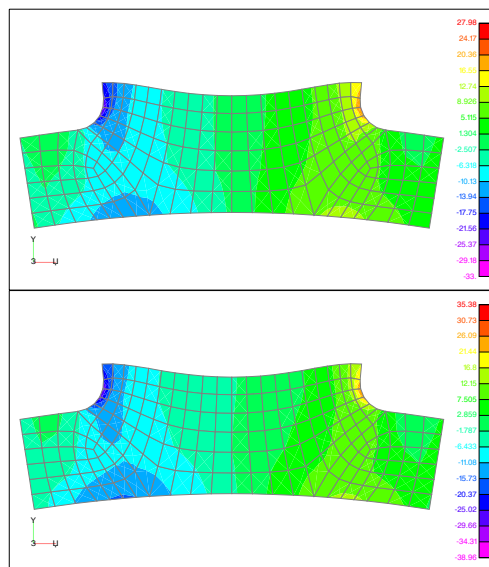
$\rho_F = 8,22\text{mm}$



$\rho_F = 10,934\text{mm}$   
(a) contact point E (b) contact point D

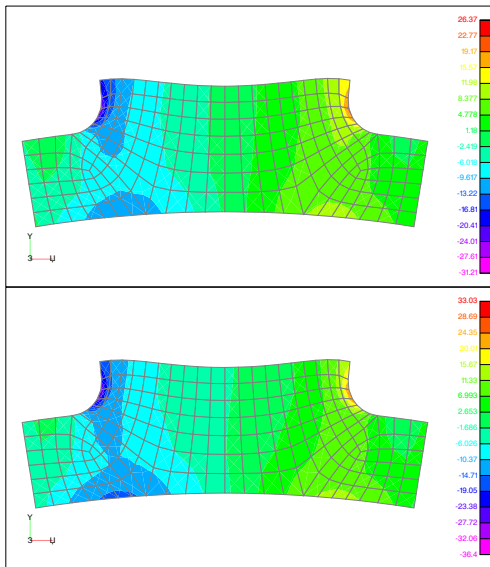


$\rho_F = 9,12\text{mm}$

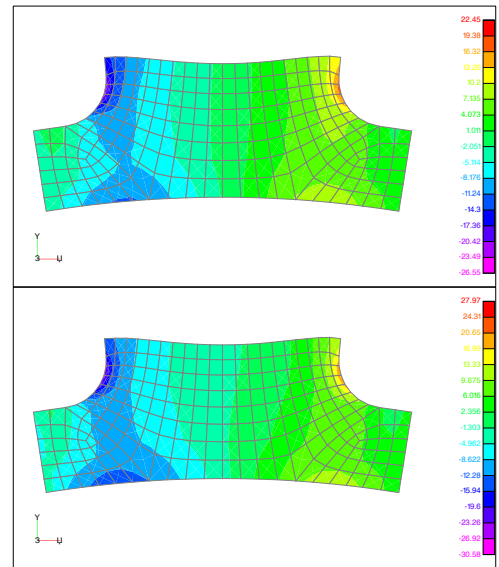


$\rho_F = 4,56\text{mm}$

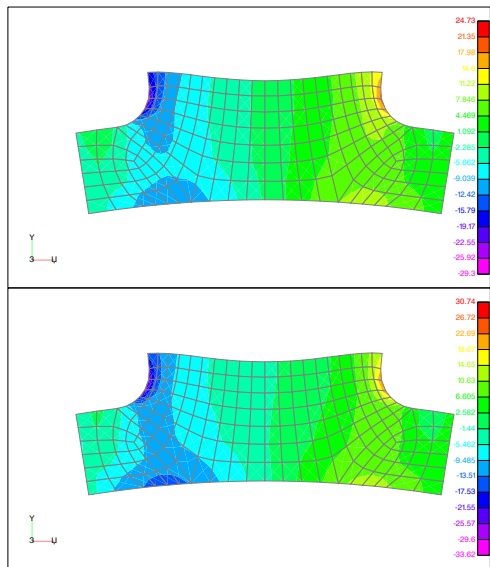
**Figure 3.** Von Mises stresses in the E and D contact points of the gear for various fillet radius values



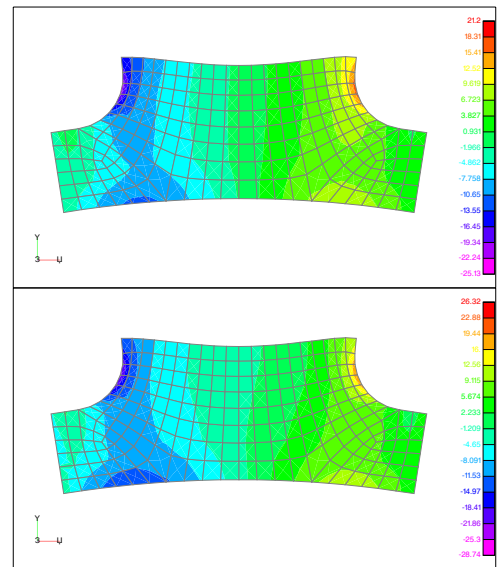
$\rho_F=5,47\text{mm}$



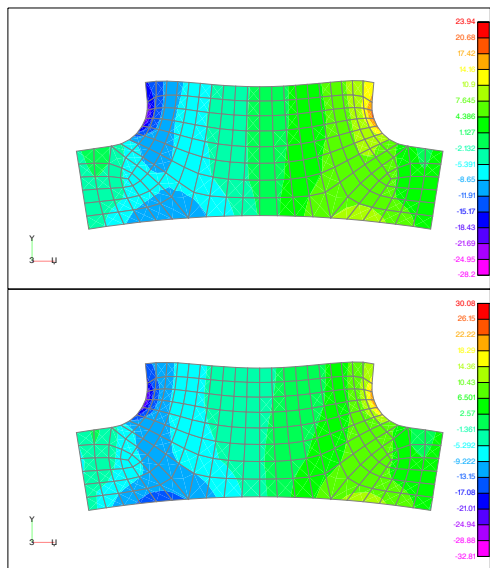
$\rho_F=8,22\text{mm}$



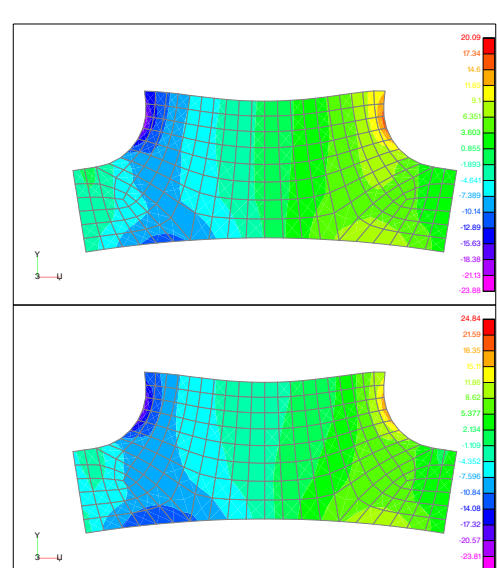
$\rho_F=6,38\text{mm}$



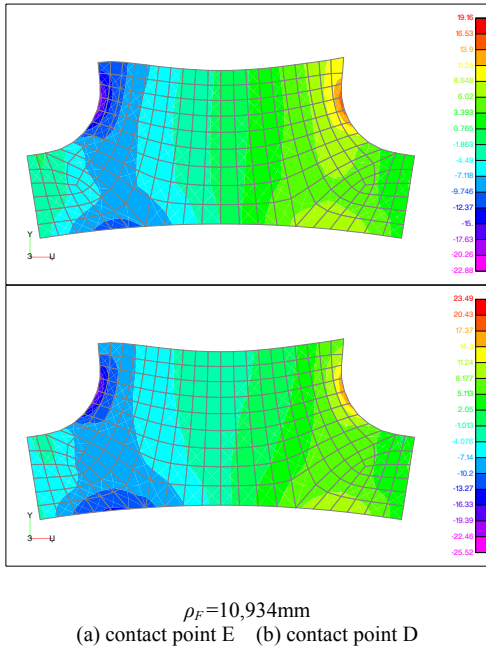
$\rho_F=9,12\text{mm}$



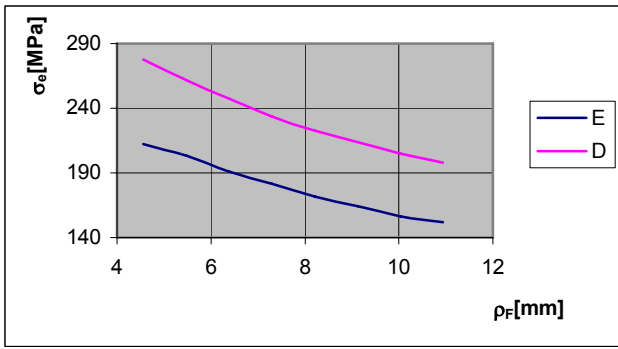
$\rho_F=7,29\text{mm}$



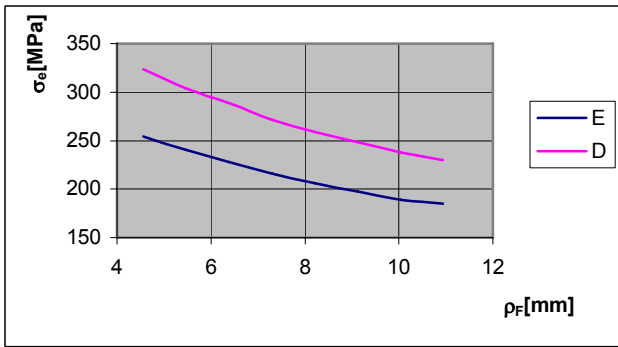
$\rho_F=10,031\text{mm}$



**Figure 4.** Normal stresses in the E and D contact points of the gear for various fillet radius values



(a)

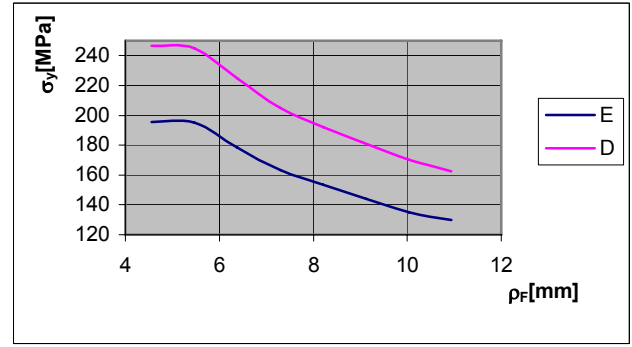


(b)

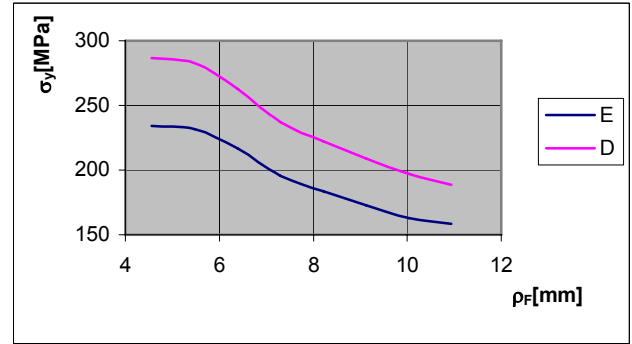
**Figure 5.** Equivalent stress  $\sigma_e$  in function of fillet radius  $\rho_F$ : (a) tensile and (b) pressured side of a gear.

Fig.5 shows the functional relationship between equivalent stresses and various values of the fillet radius  $\rho_F$  for the analyzed gear, and Fig.6 the relationship between normal stresses and the fillet radius  $\rho_F$ . These graphs show again a fact that tooth root stresses become lower as the fillet radius  $\rho_F$  grows.

All values of equivalent stresses in a function of the notch parameter  $s_{Fn}/2\rho_F$  are presented by graphs in Fig.7 and the values of normal stresses in Fig.8. Notch parameter becomes lower as the fillet radius grows, and that cause, again, lower values of tooth root stresses.

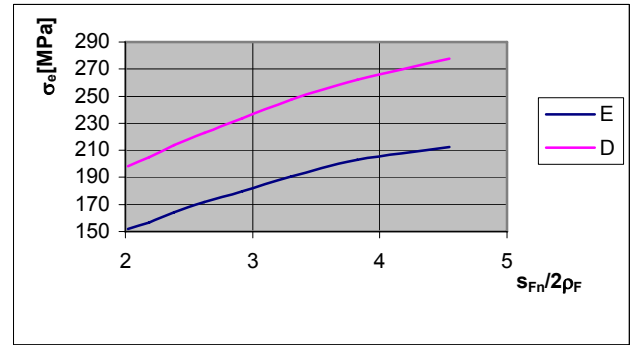


(a)

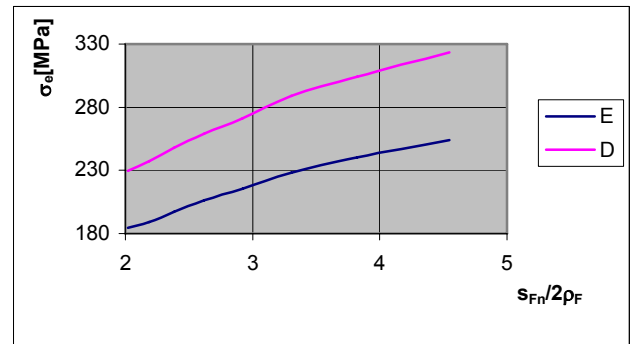


(b)

**Figure 6.** Normal stress  $\sigma_e$  in function of fillet radius  $\rho_F$ : (a) tensile and (b) pressured side of a gear.



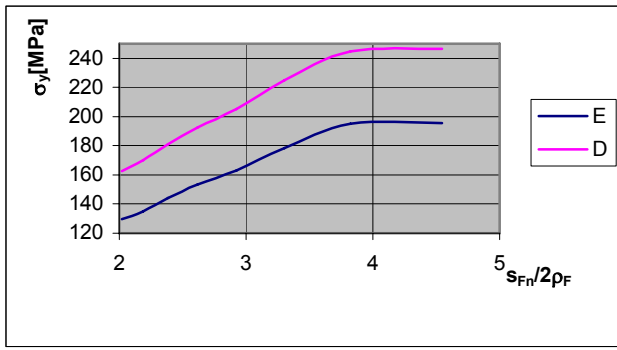
(a)



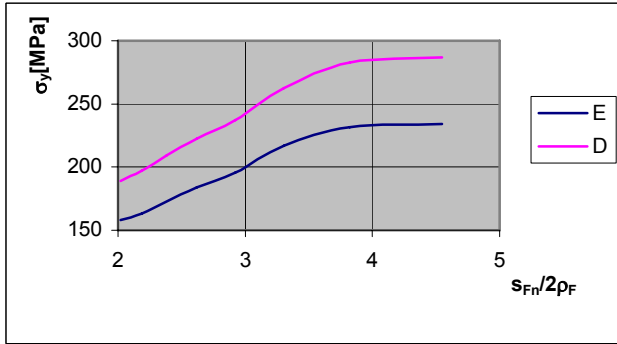
(b)

**Figure 7.** Equivalent stress  $\sigma_e$  in a function of the notch parameter  $s_{Fn}/2\rho_F$  for: (a) tensile and (b) pressured side of a gear.

The greatest values of normal and equivalent stresses in the tooth root appear in the moment when the contact between two meshed gears is in the outer point of the single mesh. It is supposed that tooth root stresses change linearly in exact range along the contact line (A-B, B-D, D-E), so the diagrams of stresses in this paper show good agreement with that supposition.



(a)



(b)

**Figure 8.** Normal stress  $\sigma_e$  in a function of the notch parameter  $s_{Fn}/2\rho_F$  for: (a) tensile and (b) pressured side of a gear.

There is a conclusion, according to the analysis in this paper, that a notch parameter has a significant influence on tooth root load capacity. In the same time, as the notch parameter grows the stresses, and reversely [11, 12]. The critical section width  $s_{Fn}$  grows as the tooth root radius increases and that cause lower stresses. Therefore, critical section stresses become lower as the fillet radius increases.

For the driving gear there is some stress value disagreement of the linear distribution for the fillet radius smaller than  $\rho_F = 5.47\text{mm}$  (Fig.5 to Fig.8). For that reason, these values of the fillet radius should be boundary ( $5.47\text{mm} < \rho_F < 10.03\text{mm}$ ).

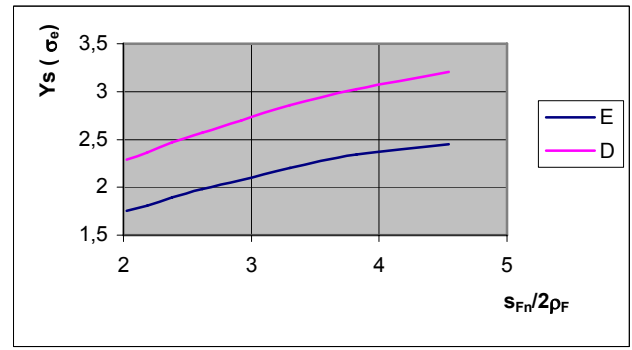
The graphs in this research show that an appropriate selection of the fillet radius should reduce critical stresses for 30% and that is important information for designers.

The stress concentration factor  $Y_s$  may be determined on the basis of the expression:

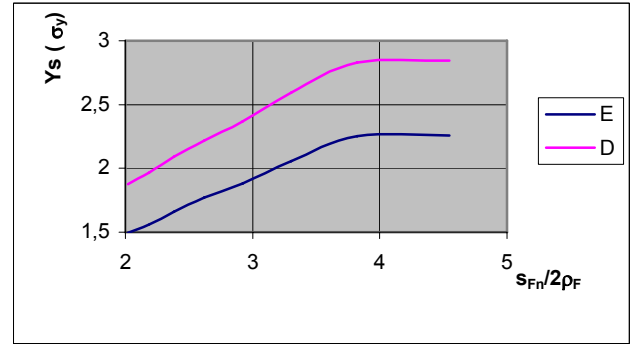
$$Y_s = \frac{\sigma_{\max}}{\sigma_{Fn}} \quad (7)$$

**Table 3.** Parameters that determine the stress concentration factor  $Y_s$  on the tensile side of a gear

Fillet radius $\rho_F$ [mm]	$s_{Fn}/2\rho_F$	E	D	E	D
		$Y_s$ ( $\sigma_e \max$ )		$Y_s$ ( $\sigma_y \max$ )	
4.56	4.5448	2.4512	3.2048	2.2588	2.8447
5.47	3.8235	2.3454	3.0229	2.2501	2.8270
6.38	3.0931	2.2028	2.8563	2.0575	2.5948
7.29	2.9248	2.0744	2.6961	1.8830	2.3668
8.22	2.6182	1.9808	2.5717	1.7708	2.2194
9.12	2.3817	1.8939	2.4690	1.6624	2.0895
10.03	2.1864	1.8062	2.3658	1.5598	1.9656
10.94	2.0242	1.7522	2.2861	1.4969	1.8779



(a)



(b)

**Figure 9.** Diagrams of the stress concentration factor  $Y_s$  in a function of the notch parameter  $s_{Fn}/2\rho_F$  for the: equivalent stresses normal stresses.

In this expression  $\sigma_{\max}$  is determined experimentally or by some numerical methods (Table 1 and Table 2). Whereas there is a complex strain in the tooth root, the maximum stress is determined by the FEM in this paper. The nominal stress  $\sigma_{Fn}$  may be expressed as:

$$\sigma_{Fn} = \frac{F_t}{b \cdot m} \quad (8)$$

The radial component of the  $F_{bn}$  force is ignored at the nominal stress  $\sigma_{Fn}$  determination, as the pressure and share stresses in the tooth root are as well.

The parameters that determine the stress concentration factor  $Y_s$  are given in Table 3. According to the results of the research in this paper, the diagrams of the stress concentration factor  $Y_s$  in a function of the notch parameter  $s_{Fn}/2\rho_F$ , for both stresses (equivalent and normal) are constructed in Fig.9 and Fig.10.

The stress concentration factor  $Y_s$  values from these diagrams may be used for gear stresses determination.

The created model represents the basis for further development of the calculation of tooth root strength from the aspect of geometry, load distribution and the teeth manufacturing accuracy and for optimization of gear trains.

## Conclusions

Topic of this paper is forming an efficient and reliable numerical model for the determination of tooth root phenomena caused by geometrical discontinuity under static and cyclic loads. The results in this research show good agreement with data in literature and practice, so this methodology may be applied in practice.

Critical tooth root stress concentration is caused, firstly, by inner contact point load of a single mesh follow (B) and then by an outer contact point of a double mesh follow (A). These loads are main cause of initial cracks appearance and only they are researched in this paper.

The most important tooth root stresses are those caused by load in the inner contact point of the mesh, because they are the greatest and the most dangerous for failure appearance.

It can be concluded, analyzing the results of this research, that stresses reduce as the tooth fillet radius increases. The reduction of stress concentration acts directly on gear service life elongation because it deflects danger of initial cracks appearance and increases the safety factor SF on that place. Although the stresses on the pressured side are higher than on the tensile, the stress concentration on the tensile side is more important for initial crack appearance. But for a better tooth root stress consideration, stresses on both tooth root sides are presented in this paper.

In this research Von Mises and normal tooth root stresses are presented. All results show that Von Mises stresses have higher values than normal stresses.

This paper shows that an appropriate fillet radius selection can increase tooth root stresses in its critical section even by 30%. It was the intention of this research, because in that case it could be followed by better tooth root load capacity and service life elongation.

We still do not have acceptable data for the parameters that describe tooth root phenomena, so these analyses are still of importance. Accordingly, further research is of great interest for science development.

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## Numerički model za određivanje kritičnih napona kod cilindričnih zupčanika

Nosivost zupca u velikoj meri zavisi od glavnih parametara koji određuju oblik profila zupca. U tom smislu posmatrali su se: kinematika, nosivost, čvrstoća, izrada i druge karakteristike zupčanika. Kad je reč o nosivosti podnožja zupca zupčanika, o čemu će se najviše govoriti u ovom radu, vrlo veliki uticaj ima oblik podnožja zupca, odnosno radijus prelaznog zaobljenja u korenu zupcu. Posebna pažnja usmerena je na radijus podnožnog zaobljenja korena zupca u kritičnom preseku. To direktno utiče na faktor koncentracije napona i sam radni vek zupčanika. Realni zupčasti prenosnici su u suštini višestruko statički neodređeni sistemi i koncentracija napona u podnožju zupca zupčanika zavisi od mnogo parametara. Nalaženje optimalnog oblika zubaca, odnosno parametara ozubljenja, sa aspekta koncentracije napona je jedan od ključnih problema kod zupčastih prenosnika. Poznato je da prva prslina u podnožju zupca zupčanika nastaje zbog koncentracije napona na tom mestu. Upravo zato, predmet daljeg istraživanja ovog rada je usmeren na iznalaženje optimalnog oblika prelaznog zaobljenja u podnožju zupca zupčanika sa ciljem da se dobije minimalna vrednost koncentracije napona. U cilju brzog i efikasnog rada u narednom ispitivanju će se primeniti metoda konačnih elemenata (MKE) pri simulaciji stvarnih radnih uslova. Predmet ovog rada je formiranje efikasnog i pouzdanog numeričkog modela za saznavanje fenomena koji nastaju usled geometrijskih diskontinuiteta u podnožju zupca zupčanika a pri statičkim i cikličnim opterećenjima. Ovde će biti prikazani ekvivalentni (Von Mises) naponi, kako na slikama tako i tabelarno, kao i dijagrami za različite vrednosti radijusa podnožnog zaobljenja  $\rho_F$ .

*Ključne reči:* zupčanik, cilindrični zupčanik, naponsko stanje, kritični napon, koncentracija napona, metoda konačnih elemenata.



## Цифровая модель для определения критического напряжения у цилиндрических зубчатых колёс

Несущая способность зубчатого колеса в значительной степени зависит от основных параметров, определяющих форму профиля зубьев. В том направлении и рассматриваны - кинематика, несущая способность, прочность, обработка и другие характеристики зубчатых колёс. Когда речь идёт о несущей способности ножки зубчатого колеса, о чём больше всего и говорит настоящая работа, большое влияние оказывает форма ножки зубчатого колеса, т.е. радиус переходного закругления в корне зуба. А особое внимание направлено на радиус закругления в ножке корня зубчатого колеса в критическом сечении. Это непосредственно влияет на фактор концентрации напряжений и на сам ресурс зубчатого колеса. Реальные зубчатые передаточные механизмы являются многократными статическими неопределёнными системами и концентрация напряжений в ножке зубчатого колеса зависит от множества параметров. Обнаружение оптимальной формы зубьев, т.е. параметров нарезания, со стороны концентрации напряжений представляет одну из главных проблем у зубчатых передаточных механизмов. Уже известно, что первая трещина в ножке зуба зубчатого колеса появляется из-за концентрации напряжений на этом месте. Именно из-за этого тема дальнейших исследований настоящей работы направлена к обнаруживанию оптимальной формы переходного закругления в ножке зуба зубчатого колеса, с целью получить минимальное значение концентрации напряжений. Ради быстрой и эффективной работы в будущих исследованиях будет использован метод конечных элементов (МКЭ) при имитационном моделировании действительных рабочих условий. Предметом настоящей работы является формирование эффективного и надёжного цифрового метода с целью узнавания феноменов, которые возникают из-за геометрических дисконтинуитетов в ножке зуба зубчатого колеса, а при статическими и циклическими нагрузками.

Здесь тоже будут показаны соответствующие (Вон Мисес) напряжения, как на рисунках, так и в таблицах, а в том числе и диаграммы  $d/f$  различных значений радиуса закругления в ножке зуба зубчатого колеса  $\rho_F$ .

*Ключевые слова:* Шестерня, цилиндрическое зубчатое колесо, напряжённое состояние, критическое напряжение, концентрация напряжений, метод конечных элементов.

## Modèle numérique pour la détermination des tensions critiques chez les engrenages cylindriques

La capacité de portance de la dent dépend beaucoup des paramètres principaux qui déterminent la forme du profil de cette dent. Dans ce sens on a examiné la cinématique, la portance, la solidité, la fabrication et les autres caractéristiques des engrenages. Quand il s'agit de la portance de la racine chez la dent d'engrenage, dont on parlera le plus dans cet article, une grande importance a la forme de la racine de la dent, c'est-à-dire le rayon de l'arrondi de cette dent. L'attention particulière est portée sur le rayon de l'arrondi de la racine de dent dans l'intersection critique. Cela influe directement sur le facteur de la concentration de tension et sur la durée de vie de l'engrenage. Les engrenages réels sont les systèmes non déterminés de point de vue statique et la concentration de la tension dans la racine de la dent d'engrenage dépend de nombreux paramètres. La découverte de la forme optimale pour les dents, c'est-à-dire les paramètres de la denture quant à la concentration de la tension, est l'un des problèmes essentiels chez les engrenages. Il est bien connu que la première fracture de la racine de la dent d'engrenage se produit à cause de la concentration de la tension à cet endroit. Pour cette raison le but des recherches dans ce travail était de trouver la forme optimale de l'arrondi de la racine de la dent d'engrenage afin d'obtenir la valeur minimale de la concentration de tension. Pour travailler vite et avec efficacité dans les futurs recherches, on utilisera la méthode des éléments finis (MKE) pendant la simulation des conditions réelles de travail. L'objet de ce travail est la formation d'un modèle numérique efficace et sûr pour connaître les phénomènes se produisant à cause de la discontinuité géométrique dans la racine de la dent d'engrenage lors de charges statiques et cycliques. On présentera ici les tensions équivalentes (Von Mises) à l'aide des photos, les tableaux et les diagrammes pour les différentes valeurs du rayon de la racine  $\rho_F$ .

*Mots clés:* engrenage, engrenage cylindrique, état de tension, tension critique, concentration de tension, méthode des éléments finis.