

# Gradient Based Model Probability Interacting Multiple Model Algorithm

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Parallel bank of non-adaptive estimators is used for adapting to target dynamics change. The discrete set of models describes all possible maneuver modes of the target. The state estimation is based on mixing/switching between filters in the set. Based on the gradient of the best model, a new adaptive model probability algorithm for the Interacting Multiple Model (Gradient based Model Probability Interacting Multiple Model - GMPIMM) is proposed in this article. The model probability coefficients are corrected in each IMM (Interacting Multiple Model) recursive loop, giving Minimum Mean-Square Error (MMSE) estimation. Experimental results show that the proposed GMPIMM gradient algorithm enhances global efficiency of the IMM algorithm in highly maneuvering target tracking applications.

*Key words:* target tracking, air target, interacting mode, interacting multiple model, gradient method.

## Introduction

TO adapt to changing target dynamics, tracking systems either estimate the unknown maneuver parameters, based on measurements, or apply a bank of parallel non-adaptive estimators where each estimator is tuned to a different operating condition and their outputs combined into a single weighted average estimation based on the apparent performance of each filter [1]. The Kalman filter (KF) with a single motion model is limited for such problems due to ineffective response to dynamics changes as the target maneuvers. The information about mode changes is contained in the model likelihood functions and the posterior model probabilities, both provided by the IMM filter [2]. The likelihood, while being more sensitive to model changes, is more prone to random fluctuations and false alarms than the model probabilities [3]. Multiple Model (MM) estimation is described by a stochastic hybrid system. The set of system modes reflects the possible system dynamics. The process of the system mode switching is usually modeled as a finite state *Markov* chain. It is assumed that any mode will be active at any time and mode switching is governed by the Transition Probability Matrix (TPM), [4, 5]. In target tracking applications the discrete set of modes is used to describe all possible maneuvers of the target. The IMM algorithm is generally considered to be one of the most cost-effective approaches of MM estimation, where filters run in parallel. Each of the filters is coupled with an appropriately weighted combination of mixed state estimates. In order to cope with highly maneuvers of the target, a modification of the IMM based transition probability Markov chain matrix is proposed in this article.

The paper is organized as follows. Section 2 formulates the problem of maneuvering target tracking. Section 3 describes the IMM concept. Section 4 discusses the GMPIMM approach and gives the algorithm. Section 5 commences with testing the initialization and iterative GMPIMM algorithm compared with the IMM algorithm.

## Problem formulation

### *Basis assumptions*

The maneuvering target tracking is an estimation problem, where the target motion is modeled by the various motion models (Singer, constant velocity, coordinated turns, etc). Consider a discrete-time stochastic linear hybrid system [6,7]:

$$\mathbf{x}(k) = \mathbf{F}[k, m(k)]\mathbf{x}(k-1) + \mathbf{G}[k, m(k)]\mathbf{u}[k-1, m(k)] + \mathbf{v}(k-1) \quad (1)$$

$$\mathbf{y}(k) = \mathbf{H}[k, m(k)]\mathbf{x}(k) + \mathbf{w}(k) \quad (2)$$

where  $\mathbf{x}(k)$  is the state vector in effect at the sample time  $k$  with the transition matrix  $\mathbf{F}[k, m(k)]$ ,  $\mathbf{y}(k)$  is the vector of observations in effect at the time  $k$ , with the measurement matrix  $\mathbf{H}$ ,  $\mathbf{u}[k-1, m(k)]$  is a known input vector,  $\mathbf{v}(k-1)$  is a process noise vector,  $\mathbf{w}(k)$  is a measurement noise vector. The sequences of process and measurement noise vectors are assumed to be mutually independent zero-mean, white and *Gaussian* process, with covariance matrices  $\mathbf{Q}(k)$  and  $\mathbf{R}(k)$  respectively.

### *Model Probability Density Function*

Let us consider a continuous stochastic time variable  $\theta(t)$ ,  $0 \leq t < +\infty$ , on the interval  $0 \leq \theta(t) \leq 1$ , which represents the realization of some event at the time  $t$ . It denominates the probability density function of the variable  $t$ . Then the probability  $\mu_i$  that  $\theta(t) \leq \theta(t_i)$  is given by the following:

$$\mu_i = P[\theta \leq \theta(t_i)] = \int_0^{t_i} \theta(t) dt, \quad 0 \leq \mu_i \leq 1 \quad (3)$$

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Next, consider the new continuous variable  $x(t, \mu_i)$ , which is defined as the compound functions of the two variables: the time  $t$  and the probability of event  $\mu_i(t)$ . If the conditions of the differentiability at the overall domain are satisfied, the gradient of the function  $x(t, \mu_i)$  must be calculated from the collection of the partial derivatives by the following equation:

$$\nabla x(t, \mu_i) = \frac{\partial x(t, \mu_i)}{\partial t} + \frac{\partial x(t, \mu_i)}{\partial \mu_i} \frac{d\mu_i(t)}{dt} = dx_t + dx_{\mu_i} \Delta \mu_i(t) \quad (4)$$

The second member of the above equation may be written as:

$$dx_{\mu_i} = \lim_{\Delta t \rightarrow 0} \frac{x[t, \mu_i(t + \Delta t)] - x[t, \mu_i(t)]}{\mu_i(t + \Delta t) - \mu_i(t)} \quad (5)$$

According to the total probability theorem, the probability density function (pdf) of  $x(t, \mu_i)$  is given by:

$$p[x(t, \mu_i)] = p[x(t, \mu_i) | \mu_i, y(t)] P[\mu_i | y(t)] \quad (6)$$

where  $y(t)$  is the input measurement. The function  $x(t)$  is given by the following integral:

$$x(t) = \int p[x(t, \mu_i) | \mu_i, y(t)] P[\mu_i | y(t)] d\mu_i \quad (7)$$

If the probability of the event is sampled at the discrete time interval  $T$ :  $\mu_i(t) \rightarrow \mu_i(k), k = 0, 1, 2, \dots$ , then the discrete represent of the continuous variable  $x[t, \mu_i(t)]$ , is  $x[k, \mu_i(k)]$ . Here,  $\mu_i(k)$  is the one of the  $N$  previously known possible events. Let us consider  $m(k)$  by set of the modes,  $m(k) \in \mathbf{M} = \{M_1, M_2, \dots, M_N\}$ , given by the Markov chain. Now, the initial model probability is  $\mu_i(0) = P[m(0) = M_i]$  and  $N$  is a number of elemental Kaman filters. Consequently, a *posteriori* model probability  $j$  in force, conditioned on the measurement history  $\mathbf{Y}^k$ , including the sample time  $k$  by [6], is given by:

$$\mu_j(k) = P[m(k) = M_j | \mathbf{Y}^k], \quad j = 1, 2, \dots, N \quad (8)$$

where  $P(\cdot)$  refers to the probability of a discrete event,  $N$  is total number of models,  $m_i(k)$  stands for the event  $m(k) = M_i$ . Hence, the event  $M_j$  is defined without any time sample argument, to represent the condition of that dynamics model in force. Theoretically, the transition probability matrix (TPM) gives the jump mode probability, from all to all modes. The transition probability does not depend on the measurement history and is given by the:

$$\begin{aligned} \pi_{ij} &= P[m(k) = M_i | m(k-1) = M_j, \mathbf{Y}^{k-1}] = \\ &= P[m_j(k) | m_i(k-1)] \quad i, j = 1, 2, \dots, N \end{aligned} \quad (9)$$

The sum of each TPM row must be equal to one ( $\sum_{j=1}^N \pi_{ij}(k) = 1, i = 1, 2, \dots, N$ ). The pdf of the discrete variable  $x[k, m(k)]$ , given by the pdf conditioned by the event  $m_i(k)$ , is  $p\{x[k, m(k)] | m_i(k), \mathbf{Y}^k\}$ , as following:

$$p\{x[k, m(k)]\} = \sum_{i=1}^N p\{x[k, m(k)] | m_i(k), \mathbf{Y}^k\} P[m_i(k) | \mathbf{Y}^k] \quad (10)$$

and vector  $x[k, m(k)]$  is to be presented by the following relation:

$$x[k, m(k)] = \sum_{i=1}^N x_i[k, m(k)] \mu_i(k) \quad (11)$$

Finally, let us define the Mean Square Error (MSE) as:

$$\begin{aligned} MSE\{x[k, \mu_i(k)]\} &= E\{[x(k, \mu_i(k)) - \hat{x}_i(k, \mu_i(k))] \cdot \\ &\cdot [x(k, \mu_i(k)) - \hat{x}_i(k, \mu_i(k))]^T | \mathbf{Y}^{k-1}\}, \quad i = 1, \dots, N \end{aligned} \quad (12)$$

where  $E\{\cdot\}$  denotes mathematical expectations and  $\hat{x}[k, \mu_i(k)] = E\{x[k, \mu_i(k)]\}$ .

### The interacting multiple model filter

The IMM algorithm is a suboptimal method of solving the problem of state estimation of a Markov Jump-Linear System (MJLS). A *Markov* chain transition matrix is used to specify the probability that the target is one of the models of operations. The IMM algorithm runs filters in parallel, each with an appropriately weighted combination of state estimates as mixed initial conditions [2]. The basic computational flow diagram of the IMM is given in Fig. 1.

At the beginning, the model-conditioned re-initialization is performed. A prediction of the model probability is calculated by the following [3]:

$$\begin{aligned} \mu_i(k | k-1) &= P[m_i(k) | \mathbf{Y}^{k-1}] = \\ &= \sum_{j=1}^N P[m_i(k) | m_j(k-1), \mathbf{Y}^{k-1}] \cdot P[m_j(k-1) | \mathbf{Y}^{k-1}] = \\ &= \sum_{j=1}^N \pi_{ij} \mu_j(k-1) \end{aligned} \quad (13)$$

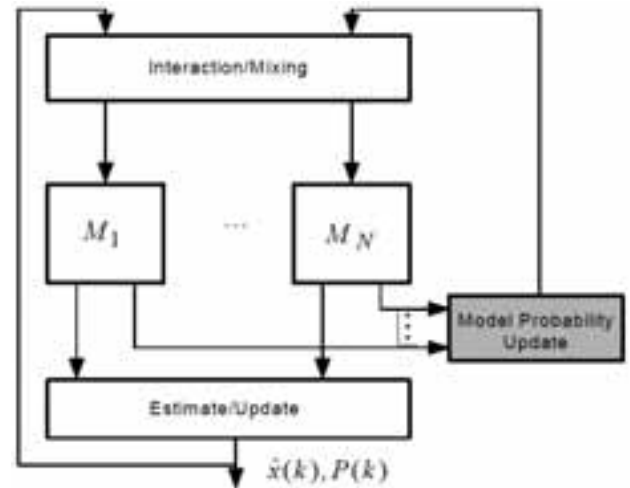


Figure 1. Computational flow of the IMM algorithm.

Likewise, the mixing weight of probability is given by:

$$\begin{aligned} \mu_{ij}(k-1) &= P[m_j(k-1) | m_i(k), \mathbf{Y}^{k-1}] = \\ &= \frac{\pi_{ij} \mu_j(k-1)}{\sum_{j=1}^N \pi_{ij} \mu_j(k-1)} \end{aligned} \quad (14)$$

Assume that the trajectory of the target is well modeled and give the measurement history  $Y^k$  by the  $N$  estimates. From the previous processing cycle, there is approximated a single *Gaussian* density  $\Lambda_i(k) \approx N\{\mathbf{x}(k); \hat{\mathbf{x}}^i(k), \mathbf{P}^i(k)\}$ , where the mean and covariance of the Gaussian are given:

$$\begin{aligned} \hat{\mathbf{x}}^i(k-1|k-1) &= E\{\mathbf{x}(k-1)|m_i(k), Y^{k-1}\} = \\ &= \sum_{j=1}^N \hat{\mathbf{x}}_j(k-1|k-1) \mu_{ij}(k-1|k-1) \end{aligned} \quad (15)$$

$$\begin{aligned} P^i(k-1|k-1) &= \sum_{j=1}^N \{P_j^i(k-1|k-1) + [\hat{\mathbf{x}}_j(k-1|k-1) - \hat{\mathbf{x}}^i(k-1|k-1)] \cdot \\ &[\hat{\mathbf{x}}_j(k-1|k-1) - \hat{\mathbf{x}}^i(k-1|k-1)]^T\} \cdot \mu_{ij}(k-1) \end{aligned} \quad (16)$$

Here,  $\hat{\mathbf{x}}_j(k), \mathbf{P}_j(k)$  refer to the filter estimate at the output of the previous processing cycle, while  $\hat{\mathbf{x}}^i(k), \mathbf{P}^i(k)$  represents the mixed estimates to be provided at the input to the next processing cycle. After the model conditioning filtering step, the mode probability update is performed. A mode probability is given by:

$$\mu_i(k) = \frac{\mu_i(k|k-1) \Lambda_i(k)}{\sum_{j=1}^N \mu_j(k|k-1) \Lambda_j(k)} \quad (17)$$

Finally, the overall estimate and covariance are given by:

$$\hat{\mathbf{x}}(k|k) = \sum_{i=1}^N \hat{\mathbf{x}}^i(k|k) \mu_i(k) \quad (18)$$

$$\begin{aligned} P(k|k) &= \sum_{i=1}^N \{P^i(k|k) + [\hat{\mathbf{x}}^i(k|k) - \hat{\mathbf{x}}(k|k)] \cdot \\ &= [\hat{\mathbf{x}}^i(k|k) - \hat{\mathbf{x}}(k|k)]^T\} \cdot \mu_i(k) \end{aligned} \quad (19)$$

### The gradient based model probability IMM algorithm

A new generation of MM estimators assumes that the TPM governing the mode jumps is known, but, it is practically unknown. The determination of the TPM elements to identify Markov transition law is very difficult [9, 10]. The decreasing of the position RMSE (Root Mean Square Errors) could be obtained over an extended number of filter models. Here, a new adaptive IMM method, denominated as GMPIMM, using the same number of known parameters (transition matrix, covariance matrix, measurement noise matrix, etc.) is proposed. The adaptation is based on the gradient of the model probabilities. The GMPIMM algorithm consists of  $N$  filters designed for various target motion modes. During the tracking process, which is a discrete time process, the number of filters is kept constant. The largest transition probability is to remain to the same model. It implies the diagonal elements of the *Markov* chain matrix are the largest. Therefore, our goal is position error decreasing, during the target maneuver. The proposed modification is imported into the *update model probabilities* step of the IMM.

#### One cycle of the GMPIMM algorithm

As per the preceding multiple model techniques, the combined estimates are calculated at each processing cycle

to give the output of the estimator. Assume that  $\Pi = \{\pi_{ij}\}$  is unknown but the time-invariant and symmetric matrix with some given prior distribution of valid transition probability matrix GMPIMM recursive process must be obtained by the following procedure: at each sample time  $k$ , a modified model probability  $\Psi_i^k$  is calculated by the SD method. A new model probability does not affect the likelihood function, apropos posterior pdf. Hence, the fourth step of the IMM algorithm is varied, such as a new model probability (it is used only for output purpose), while the IMM method procedure is not changed (Fig.2). One cycle of the IMM recursive estimator consists of the following five steps [10]:

- (i) *Calculation of mixing probabilities.* The conditional model probabilities  $\mu_{ij}(k-1)$  or the posteriori model probability is calculated according to the *Bayes rule*, using the following equations:

$$\mu_{ij}(k-1) = \frac{\pi_{ij} \mu_j(k-1)}{\sum_{j=1}^N \pi_{ij} \mu_j(k-1)}, i, j = 1, \dots, N \quad (20)$$

The final state estimates and covariance matrix  $\hat{\mathbf{x}}_j^0(k-1|k-1), P_j^0(k-1|k-1)$  are the goal of the calculations.

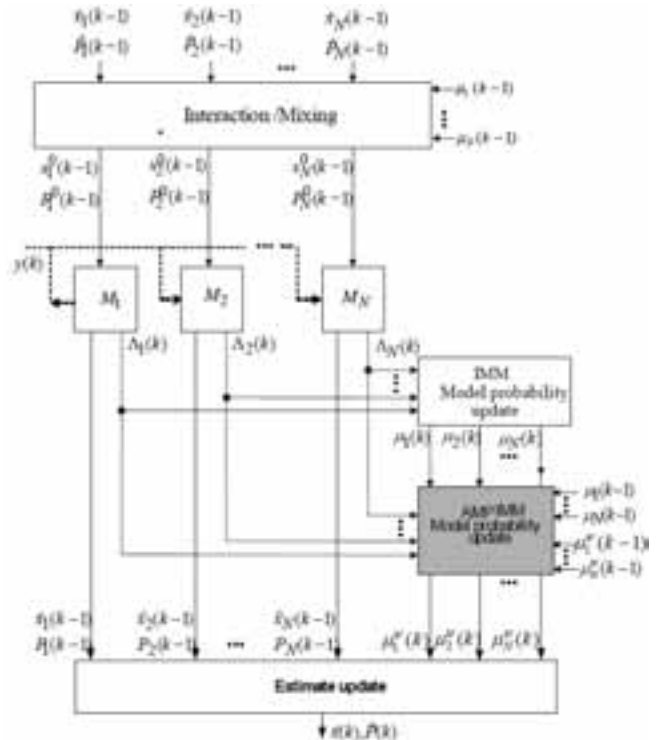


Figure 2. Block diagram of the GMPIMM algorithm

- (ii) *Calculation of the mixed initial condition.* The previous state estimates  $\hat{\mathbf{x}}^i(k-1|k-1)$  and corresponding covariance matrices  $\mathbf{P}^i(k-1|k-1)$  are the onset of this step. The mixed initial condition for the filter  $M_j$  and the corresponding covariance matrix are calculated by the following equations:

$$\hat{\mathbf{x}}_j^0(k-1|k-1) = \sum_{i=1}^N \hat{\mathbf{x}}_i(k-1|k-1) \cdot \mu_{ij}(k-1) \quad (21)$$

$$\mathbf{P}_j^0(k-1|k-1) = \sum_{i=1}^N \{ \mathbf{P}_i(k-1|k-1) + [\hat{\mathbf{x}}_i(k-1|k-1) - \hat{\mathbf{x}}_i^0(k-1|k-1)] \cdot [\hat{\mathbf{x}}_i(k-1|k-1) - \hat{\mathbf{x}}_i^0(k-1|k-1)]^T \} \mu_{ij}(k-1) \quad (22)$$

Each of the  $N$  different filters operates in the parallel mode and a result of this calculation is a new state estimate.

(iii) Performing of the mode matching filtering and calculation of the likelihood function:

$$\Lambda_j(k) = P\{\mathbf{y}(k) | \mathbf{Y}^{k-1}, \mathbf{x}(k), M_j(k)\} = \frac{1}{(2\pi)^{n/2} |S(j)|^{1/2}} \exp\left[-\frac{1}{2} r_j^T(k) S_j^{-1}(k) r_j(k)\right] \quad (23)$$

for each of the  $N$  Kalman filters. The results of the previous step  $\hat{\mathbf{x}}_j^0(k-1|k-1)$ ,  $\mathbf{P}_j^0(k-1|k-1)$  are the input for this step, where  $r_j(k)$  is a corresponding innovation for the filter  $j$  and  $S_j(k)$  is the covariance matrix associated with  $r_j(k)$ .

(iv) IMM step (updating of model probabilities for each model) is performed, by the following equation:

$$\boldsymbol{\mu}(k) = \begin{bmatrix} \mu_1(k) \\ \vdots \\ \mu_i(k) \\ \vdots \\ \mu_N(k) \end{bmatrix} = \begin{bmatrix} \frac{\mu_1(k|k-1)\Lambda_1(k)}{\sum_{j=1}^N \mu_j(k|k-1)\Lambda_j(k)} \\ \vdots \\ \frac{\mu_i(k|k-1)\Lambda_i(k)}{\sum_{j=1}^N \mu_j(k|k-1)\Lambda_j(k)} \\ \vdots \\ \frac{\mu_N(k|k-1)\Lambda_N(k)}{\sum_{j=1}^N \mu_j(k|k-1)\Lambda_j(k)} \end{bmatrix} \quad (24)$$

The results of the Steepest Descent Gradient (SDG) algorithm always give a new model probabilities vector  $\boldsymbol{\mu}^{\Psi^k}$ , used at the next GMPIMM cycle, for the output purpose only, by the following equation:

$$\boldsymbol{\mu}^{\Psi^k} = \boldsymbol{\mu}^{\Psi^{k-1}} + \alpha_i^k \boldsymbol{\rho}_j = \boldsymbol{\mu}^{\Psi^{k-1}} + \frac{[\boldsymbol{\mu}(k) - \Pi \boldsymbol{\mu}^{\Psi^{k-1}}]^T [\boldsymbol{\mu}(k) - \Pi \boldsymbol{\mu}^{\Psi^{k-1}}]}{[\boldsymbol{\mu}(k) - \Pi \boldsymbol{\mu}^{\Psi^{k-1}}]^T \Pi [\boldsymbol{\mu}(k) - \Pi \boldsymbol{\mu}^{\Psi^{k-1}}]} \cdot [\boldsymbol{\mu}(k) - \Pi \boldsymbol{\mu}^{\Psi^{k-1}}] \quad (25)$$

where  $i=1, \dots, N$ . The adaptive model probabilities are expressed by:

$$\boldsymbol{\mu}^{\Psi}(k) = \left[ \boldsymbol{\mu}^{\Psi^1} \quad \boldsymbol{\mu}^{\Psi^2} \quad \dots \quad \boldsymbol{\mu}^{\Psi^N} \right]^T \quad (26)$$

(v) Calculation of the final state estimates  $\hat{\mathbf{x}}(k)$  and corresponding covariance  $\mathbf{P}(k)$  (for the output purpose only). The modification is applied to the step (iv) of the IMM by combining the model conditioned estimates and the covariance which leads to the final form of (v) of the GMPIMM for the new output estimate and the corresponding covariance matrix:

$$\hat{\mathbf{x}}^\delta(k|k) = \hat{\mathbf{x}}(k) \cdot \boldsymbol{\mu}^\delta(k) = \sum_{j=1}^N \hat{\mathbf{x}}_j(k|k) \mu_j^\delta(k) \quad (27)$$

$$\begin{aligned} \mathbf{P}^\delta(k) &= \boldsymbol{\mu}^\delta(k) \cdot [\mathbf{P}(k) + E\{\hat{\mathbf{x}}^\delta(k)\}] = \\ &= \sum_{j=1}^N \mu_j^\delta(k) [\mathbf{P}_j(k) + (\hat{\mathbf{x}}_j(k|k) - \hat{\mathbf{x}}^\delta(k|k)) (\hat{\mathbf{x}}_j(k|k) - \hat{\mathbf{x}}^\delta(k|k))^T] \end{aligned} \quad (28)$$

## Simulation Results

The implemented GMPIMM and the corresponding IMM algorithms are evaluated by Monte Carlo (MC) simulations over a representative test trajectory. The measures of the examined performance are the RMSE (Root Mean Square Errors). The Cartesian coordinates  $(\xi, \eta)$  are combined at each scan, for the position RMSE  $(k)$ , and calculated according to [10]:

$$RMSE(k) = \sqrt{\frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} (\hat{\xi}^i(k) - \xi^{true}(k))^2 + (\hat{\eta}^i(k) - \eta^{true}(k))^2}$$

where  $\hat{\xi}^i(k)$ ,  $\hat{\eta}^i(k)$  are the filter position estimates at the discrete time  $k$ , in  $i^{th}$  run and  $N_{MC}$  is the number of Monte Carlo runnings.

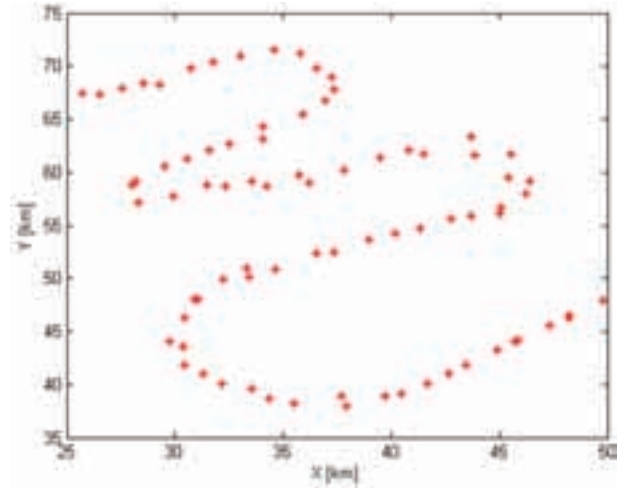


Figure 3. Motion scenario

A target motion scenario (Fig.3) includes a series of non-maneuvering and maneuvering flights modes. The target performs full turn maneuvers with intensity  $g$ ,  $2g$ ,  $5g$  and  $2g$  during the scans 10-28, 37-45, 55-58, and 65-73, respectively. The speed is constant and equal to 311 m/s. The sampling period of the radar sensor is  $T = 5s$ . The duration of the scenario is 80 scans. The constant velocity (CV) model and the coordinate turn (CT) model provide an adequate and self contained model set for tracking purposes. Hence, the model set for the IMM filters has been selected as follows:  $M_1=CV$ ,  $M_2=CT$ . Both models have the same state variables and this greatly simplifies the re-initialization operation of the IMM filter. The system input is modeled as follows: the vector state  $\mathbf{x}(k) = [x \dot{x} y \dot{y}]^T$  where  $x, y$  are the Cartesian coordinates of the target position, and  $\dot{x}, \dot{y}$  are the appropriate velocities, the initial target state  $\mathbf{x}_0(k) = [0 \ 0 \ 0 \ 120]^T$ , and the transition matrices are given by:

$$\mathbf{F}_1 = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 1 & \sin wT/w & 0 & \frac{\cos wT - 1}{w} \\ 0 & \cos wT & 0 & -\sin wT \\ 0 & \frac{1 - \cos wT}{w} & 1 & \frac{\sin wT}{w} \\ 0 & \sin wT & 0 & \cos wT \end{bmatrix},$$

process noise matrices are given by:

$$Q(k) = q \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix},$$

respectively, where  $q=q_1=0.005^2$  is a maneuver coefficient for the model  $M_1$  and  $q=q_2=0.05^2$  is a maneuver coefficient for the model  $M_2$ . The measurement model is given by the matrices  $H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$  and

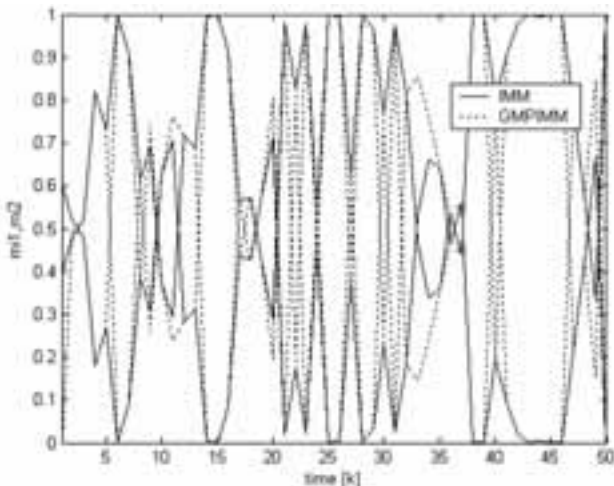
$R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$ . The simulation process is governed by a

Markov chain with TPM  $\Pi = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$ ,  $N_{MC} = 100$  Monte Carlo realizations were processed.

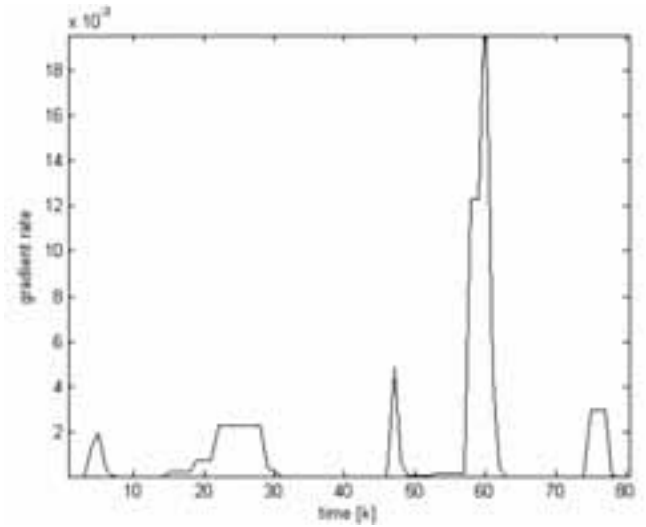
**Table 1.** RMSE position and velocity

Man.	GMPIMM		IMM	
	RMSE Pos.[m]	RMSE Vel.[m/s]	RMSE Pos.[m]	RMSE Vel.[m/s]
10-28	135.9	59.3	142.0	61.5
37-45	166.4	60.9	170.4	62.7
55-58	171.5	62.4	181.5	64.0
65-73	181.8	58.8	196.5	60.3
Overall	172.5	61.6	183	62.8

GMPIMM model probabilities are shown and the gradient is given in Figures 4a and 4b, respectively. Comparative RMSE positions of the GMPIMM and IMM algorithms are shown in Fig.5. The resulted RMSE show that the overall results of the GMPIMM are better than those of the IMM. The model probability presentations give a faster growth of the GMPIMM actual model probability which is maneuver tracking designed (model in force), during the maneuver. It directly relieves RMSE position during the maneuver. A high oscillation of the IMM probability coefficient, after a quick maneuver, is now smooth in the GMPIMM.

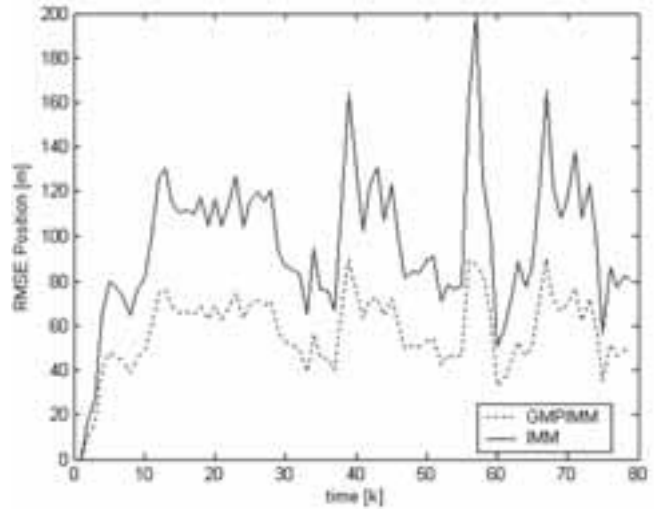


a)



b)

**Figure 4.** The GMPIMM a) model probabilities, b) gradient rate



**Figure 5.** RMSE position diagram

## Conclusions

Based on the multiple model estimation methodology, a gradient-based model probabilities algorithm has been designed (GMPIMM). A recursive procedure uses the Steepest Descent gradient method and modifies model probability, and new model probability is not affected by the likelihood function, apropos posterior pdf. Hence, the fourth step of the IMM algorithm is varied, such as a new model probability (which is used only for output purpose), while the IMM method procedure is not changed. The performance of algorithms is evaluated and compared over a maneuvering flight scenario. The simulation results have shown that during maneuvers the GMPIMM provides substantially better tracking characteristics compared with the IMM algorithm by two filters. However, the model choice is not obvious and requires additional requirements and experience. The proposed modification is a simple and straight-forward solution. The RMSE is efficiently decreased in case of adaptive modification of IMM model probabilities. The overall conclusion is that for the considered highly maneuvering target tracking problem, the GMPIMM approach proposes a powerful means of synthesis of new practically more efficient tracking

algorithms, and the oscillation of the IMM probability coefficient, after a quick maneuver, is now smooth. The fundamental limitations of the GMPIMM are reflected in a fixed number of models, imposing severe restriction on the achieved improvements [11].

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Received: 12.10.2008.

## Metoda interaktivnih višestrukih modela zasnovana na gradijentu verovatnoća modela

U radu je predložena nova metoda za praćenje manevrišućih ciljeva, zasnovana na poznatoj metodologiji interaktivnih višestrukih modela. Metoda je bazirana na forsiranju vodećeg modela, metodom gradijenta njegovove verovatnoće tokom manevra. Testirana je na manevrišućim putanjama u okruženju šuma i pokazala je bolje rezultate nego klasična metoda interaktivnih višestrukih modela. Pored toga, ova metoda ima nepromjenjene performanse kada je u pitanju računarska efikasnost, pa se kao takva preporučuje za praćenje pokretnih ciljeva u vazduhu.

*Ključne reči:* praćenje ciljeva, cilj u vazduhu, interaktivni režim rada, interaktivni višestruki model, gradijentna metoda.

## Метод взаимодействия многократных моделей, обоснованный на градиенте (уклоне) вероятности моделей

В настоящей работе предложен новый метод для преследования движущихся целей, обоснованный на уже известной методологии взаимодействующих многократных моделей. Метод базируется на форсировании передовой модели методом градиента (уклона) его вероятности в течении манёвра. Он испытыван на маневровыми орбитами в зоне около шума и показал лучшие результаты чем классический метод взаимодействующих многократных моделей. Кроме того, в этом методе существуют постоянные характеристики в случае компьютерной эффективности и таков рекомендуют для преследования движущихся целей в воздухе.

*Ключевые слова:* преследование цели, цель в воздухе, взаимодействующий режим работы, взаимодействующая многократная модель, градиентный метод.

## Méthode des multiples modèles interactifs basée sur le gradient de la probabilité du modèle

Ce papier propose une nouvelle méthode pour la poursuite des cibles manoeuvrantes, basée sur la méthodologie déjà connue des multiples modèles interactifs. Cette méthode se base sur le forçage du modèle principal à l'aide de la méthode de gradient de sa probabilité au cours de la manoeuvre. Elle a été testée sur les trajectoires manoeuvrantes dans l'ambiance forestière et a démontré meilleurs résultats que la méthode classique des multiples modèles interactifs. A part cela, cette méthode a de stables performances quant à l'efficacité numérique ce qui la recommande pour la poursuite des cibles mobiles en l'air.

*Mots clés:* poursuite de la cible, cible aérienne, régime du travail interactif, multiple modèle interactif, méthode de gradient