UDK: 624.073/.074:519.673
COSATI: 20-12

Minimum Weight Design of Thin-Walled Structures Modeled by Shell Finite Elements

Dejan Malobabić, BSc (Eng)[1)]

This work deals with optimal design of thin-plate type structures. There are essentially two motivating factors for developing structural optimization capabilities. The more obvious of the two is to increase the productivity of a structural designer by automating for him the otherwise tedious, repetitive, and time-consuming manual design changes towards a design that is often only satisfactory, but not necessarily optimal. This work considers minimum weight design of structural components modeled by finite elements. To solve an optimization problem, optimality criteria methods [1-3] are used. The potential strength of the method is that the number of iterations needed to converge to an optimum is virtually independent of the number of structural members [2]. This property makes this method well suited for the optimum sizing of large practical structures. In order to illustrate the efficiency of this optimization procedure, minimum weight design of aircraft structural components is analyzed.

Key words: **thin-walled structures, thin-walled shells, optimal design, minimum weight design, finite elements method.**

Introduction

ATTENTION in this work is focused on practical sizing optimization problems. An efficient computation procedure is considered for an optimum design of plate and shell structures, when the design variables are continuous or discrete. The design variables can be sizing or shape variables. An optimum design of structures can be cosidered as a mathematical programming problem in which the objective function reflecting the weight or cost is minimized while the design constraints are satisfied. The objective and the constraints are expressed in terms of the design variables. Examples of the design variables are thickness and cross sectional dimensions. The design constraints are bounds on stresses, displacements, etc.

Mathematically, an optimization problem can be stated as follows:

$$\min imize \qquad F(X) \tag{1}$$

$$\text{subject to;} \quad g_j(X) \le 0 \quad j = 1, m \tag{2}$$

$$X_i^l \le X_i \le X_i^u \quad i = 1, n \tag{3}$$

where $F(X)$ and $g_j(X)$ are the objective function and constraints, respectively, and X is the vector of design variables. X_i^l, X_i^u are the lower and upper bounds on the design variable X_i. m is the number of constraints and n is the number of variables.

The problem given by equations (1-3) is, in general, a non linear programming problem (NMP) or optimality criterion (OC) and there are various techniques to solve this optimization problem [1-7]. Most optimization algorithms require that an initial set of design variables, X^0, be specified. Beginning from this starting point, the design is updated iteratively. The most common form of this iterative procedure is given

$$X^k = X^{k-1} + \alpha S^k \tag{4}$$

where k is the iteration number and S is the vector of search direction. The scalar quantity α defines the distance that we wish to move in the direction S to improve the design. There is a wide variety of methods for determining the search direction, S, as well as for finding the value of α . In numerical optimization techniques, these methods require evaluations of the objective and constraint functions as well as their gradients. As the overall iteration process is iterative, thus to reach the optimum solution, we often require hundreds of function evaluations and gradient calculations. Dealing with large scale optimization problems, a great number of finite elements analyses of the structure is required to complete the process, thereby making the process very inefficient.

In order to solve the problem efficiently, an attempt should be made to create a high quality approximation to the design problem and solve the approximate problem completely, without actually performing any finite element analyses. Because it is an approximation, it must be repeated so that at least a few detailed finite element analyses will be needed. The key to efficiency is the creation of high quality approximation, thus reducing the number of structural analyses. In the past, attempts have been made to reduce the computational burden by introducing some approximation concepts [2,7]. The number of the design variables was reduced by linking. This idea is reasonable as in practice some of the variables are the same. The number of constraints was also reduced by considering only the critical or the near critical constraints at

1) Military Technical Institute (VTI), Ratka Resanovića 1, 11132 Belgrade, SERBIA

each iteration [2,6,7]. A first order Taylor series expansion was used to generate the approximation forms of the constraints in terms of design variables (intermediate variables) or their reciprocals. The reason for using the reciprocal variables is due to the fact that structural response quantities such as stress and displacement are approximately linear with respect to the reciprocal variables. A second generation approximation techniques, by which the highest quality approximation can be achieved. The implicit structural responses such as forces, displacements, frequencies, etc., appearing in the optimization problem, are first approximated. By substituting these approximate functions into the original problem, a nonlinear explicit problem is created, the solution of which often requires less than ten analyses of the structure. This method is very robust and efficient for large structures, where the computational cost of the analysis is high. This work deals with structural analysis and minimum weight design of structural components modeled by shell finite elements.

Approximation concepts

A few approximate concepts are presented here [2-7]. The concept of choosing the functions to be approximated and the intermediate variables to be used to create a high quality approximation is fundamental to the overall efficiency and reliability of the optimization process. Consider a simple rectangular beam element of width B and height H. These are the physical design variables that are to be determined in an optimization problem. The maximum stress used in evaluating a stress constraint is

$$\sigma = \frac{M \cdot c}{I} \pm \frac{P}{A} \tag{5}$$

where $c=H/2$, $I=BH^3/12$ and $A=BH$ are simple functions of B and H. M is the bending moment and P is the axial force. A traditional linearization would be to create a Taylor series approximation to stress as;

$$\bar{\sigma} = \sigma^0 + \frac{\partial \sigma}{\partial X}(X - X^0) \tag{6}$$

where $X^T=[B,H]$. However, it is clear that the stress is higly nonlinear in the design variable, B and H, so the approximation of the stress given by equation (6) is not accurate and a very small move limit would be necessary during the solution of the approximate problem.

Consider how the stress might be better approximated. First A and I are considered as intermediate variables. Next, the gradients of M and P (intermediate responses) are calculated with respect to A and I, and M and P are approximated as;

$$\bar{M} = M^0 + \frac{\partial M}{\partial A}(A - A^0) + \frac{\partial M}{\partial I}(I - I^0) \tag{7}$$

$$\bar{P} = P^0 + \frac{\partial P}{\partial A}(A - A^0) + \frac{\partial P}{\partial I}(I - I^0) \tag{8}$$

When we need the value of stress, first A and I are calculated explicitly as functions of B and H. Then, the approximate member forces, $\bar{M}$ and $\bar{P}$ are evaluated. Finally, the stress and constraint are recovered in the usual manner.

Now the same approximate strategy is applied to the optimum design of plate and shell structures. Considering a four noded plate element with 6 degrees of freedom per node (3 translations and 3 rotations). Thus we have 24 nodal forces in the element. Only the 6 components of the forces at the centre of the element (element forces) are approximated. Then the approximate stresses can be calculated using these approximate forces. At the starting point of each iteration, the nodal displacments of the structure are known from the finite element analysis. The vector of element forces P^0 at the starting point is determined as;

$$P^0 = CSu^e \tag{9}$$

where C is the element material matrix in the element coordinate system, S is the strain-displacement matrix and u^e is the nodal displacement vector of the element. We are now able to approximate the element forces using a Taylor series expansion in the intermediate design variables. The intermediate design variables for plate structures are the shape design variables, the plate thickness t, and bending stiffnesses D, where $D=t^3/12$.

Finally the approximate principle, maximum shear, and VonMises stresses can be calculated. For example, the approximate Von Mises stress is

$$\sigma_{VM} = \sqrt{\bar{\sigma}_x^2 + \bar{\sigma}_y^2 - \bar{\sigma}_x\bar{\sigma}_y + 3\bar{\tau}_{xy}^2} \tag{10}$$

This can be used to establish the approximate stress constraint as;

$$\bar{g}(X) = \frac{\sigma_{VM} - \sigma_a}{\sigma_a} \le 0 \tag{11}$$

where σ_a is the allowable stress.

Gradient calculation

In this approach, the gradients of element forces with respect to intermediate variables are required. First the gradients of the nodal displacments are evaluated by using equation

$$KU = F \tag{12}$$

where K is the global stiffnes matrix, U is the displacement vector and F is the external load vector. Differentiation of this equation with respect to the intermediate variables, Y, yields

$$\frac{\partial u}{\partial Y_i} = K^{-1}\left[\frac{\partial F}{\partial Y_i} - \frac{\partial K}{\partial Y_i}u\right] \tag{13}$$

Now the gradients of the element forces with respect to Y, are easily calculated from the relationship

$$P^e = CSu^e = Mu^e \tag{14}$$

as

$$\frac{\partial P^e}{\partial Y_i} = \frac{\partial M}{\partial Y_i}u^e + M\frac{\partial u^e}{\partial Y_i} \tag{15}$$

where M is a known matrix and its derivatives can be evaluated and $\partial u^e / \partial Y_i$ are the gradients of the nodal displacements associated with this element and are recovered from equation (13).

At the end it is very important that in a practical design problem, there is a great number of constraints involved and a large percentage of these constraints may never be critical during a design cycle, and so could be excluded from the constraint set in that cycle. This is done to decrease the cost of sensitivity analysis (gradient calculation) and to reduce the size of the approximate optimization problem. In addition, if a number of constraints are active in one region of the structure, say near a stress concentration, only a small number of most critical constraints in that region is retained.

Numerical examples

To illustrate an optimal design process of a shell type structure several examples are included here. This example represents an FE Model of a plate with a circular reinforcement hole, at one edge clamped and on the oposite edge with applied continuous tension load (400 daN). The geometry and the FE model are shown in Fig.1. The plate is made of aluminium alloy. The mechanical properties of this material are given in Table 1.

Table 1. Material Properties

Isotropic Aluminium Alloy – Dural	
Mass Density, ρ	2.77E-6 kg/mm3
Young's Modulus, E	7490 daN/mm2
Poisson's Ratio, ν	0.29
Limit Stress	
Tens/Compress, σ_a	29 daN/mm2
Shear, τ_a	17.5 daN/mm2

The purpose of these examples is to present an efficient FE method with adequate optimization algorithms to optimization process. An optimum design structure in this case is subjected to minimizing the weight, while the design constraints are satisfied. The design variables are the thicknesses and the design constraints are bound on stresses. The *NASTRAN* and *FEMAP* softwares [8] are used for this optimization process.

Case 1: Reinforcement hole (D_1=12.7mm; D_2=18mm; t_1=1mm, t_2=1mm)

Nastran's optimization process is an analysis process by default (Fig.2). The stress distribution static analysis is almost the same for all cases, because initial thicknesses for all properties are the same. The stress concentration factor of the axial loading case of a finite width plate, k_t=3.0, is expected.

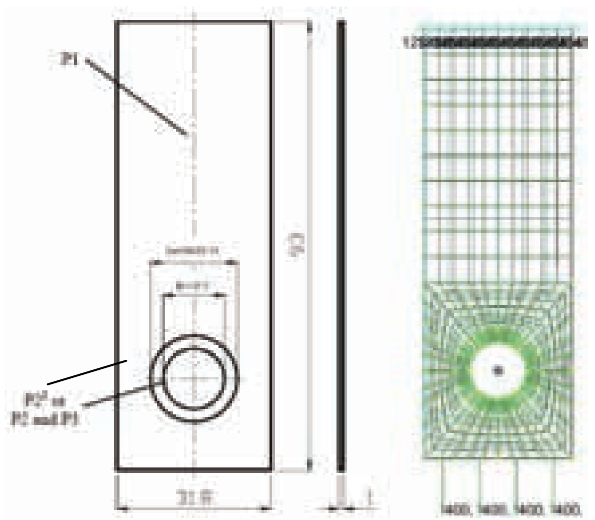


Figure 1. Plate geometry and model

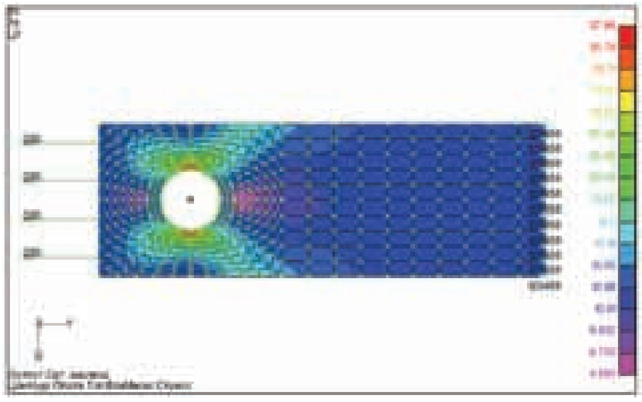
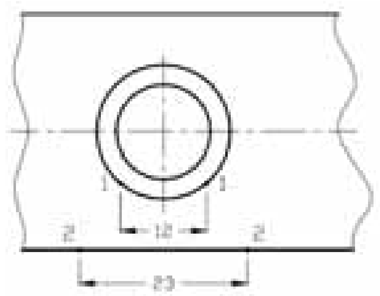


Figure 2. Stress distribution Static Analysis (all cases)

After six iterations the optimization process stopped, and all design constraints are satisfied, Fig.4. Using [4] and new geometry data after the optimization process, VonMises stresses at point 1 and point 2 are determined (Fig.3). The result of this comparasion indicates model and optimization process verification. The weight and property history diagram is shown in Fig.5.



	MODEL	ESDU	
Point 1	29.05	30.6	daN/mm^2
Point 2	21.3	21.2	daN/mm^2

Figure 3. Stress Comparison

Vary-Design Variables	Object	Limit-Design Constraints
0.5<Thickness<2 (20%)t_1 [mm]	property 1	1<σ_{VM} [daN/mm^2]<29
0.5<Thickness<4 (20%)t_2 [mm]	property 2	1<σ_{VM} [daN/mm^2]<29

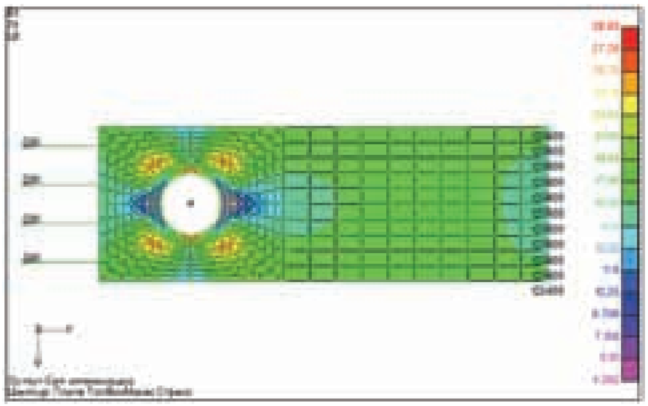


Figure 4. Stress distribution optimization (Case 1)

Table 2. Weight and property history data (Case 1)

Iteration [N]	0	1	2	3	4	5	6
W [g]	7.8	9.4	9.6	8.0	6.6	6.2	6.2
t1 [mm]	1.0	1.2	1.21	1.0	0.8	0.73	0.73
t2 [mm]	1.0	1.2	1.4	1.7	1.93	1.94	1.94

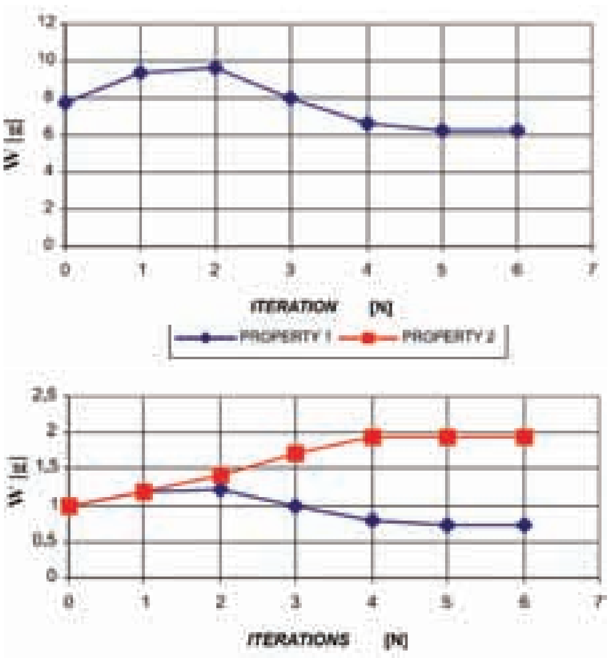


Figure 5. Weight and property history diagram (Case 1)

Case 2: Gradual Reinforcement hole (D_1=12.7mm; D_2=18mm; t_1=1m; t_2=1mm; t_3=1mm)

The difference between Case 1 and Case 2 is because in Case 2 reinforcement is divided in two concentric groups of elements, two properties, around the hole. Property 3 is near the edge of the hole. This approach provided better optimization results.

Table 3. Design Variables and Design Constraints (Case 2)

Vary-Design Variables	Object	Limit-Design Constraints
0.5<Thickness<2 (20%); t_1 [mm]	property 1	1<σ_{VM} [daN/mm^2]<29
0.5<Thickness<3 (20%); t_2 [mm]	property 2	1<σ_{VM} [daN/mm^2]<29
0.5<Thickness<4 (20%); t_3 [mm]	property 3	1<σ_{VM} [daN/mm^2]<29

Table 4. Weight and property history data (Case 2)

Iteration [N]	0	1	2	3	4	5	6	7	8
W [g]	7.8	9.4	9.5	8	6.5	6	5.85	5.8	5.8
t_1 [mm]	1.0	1.2	1.204	0.98	0.79	0.72	0.71	0.7	0.7
t_2 [mm]	1.0	1.2	1.44	1.73	1.58	1.26	1.01	0.83	0.83
t_3 [mm]	1.0	1.2	1.436	1.72	2.07	2.2	2.27	2.28	2.28

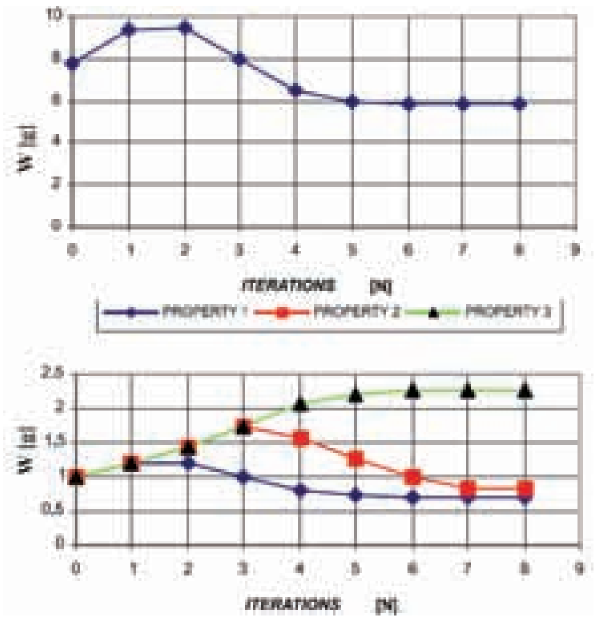


Figure 6. Weight and property history diagram (Case 2)

Case 3: Reinforcement hole (D_1=12.7mm; D_2=20.7mm; t_1=1m; t_2=1mm)

Table 5. Design Variables and Design Constraints (Case 3)

Vary-Design Variables	Object	Limit-Design Constraints
0.5<Thickness<2 (20%); t_1 [mm]	property 1	1<σ_{VM} [daN/mm^2]<29
0.5<Thickness<4 (20%); t_2 [mm]	property 2	1<σ_{VM} [daN/mm^2]<29

Table 6. Weight and property history data (Case 3)

Iteration [N]	0	1	2	3	4	5
W [g]	7.8	9.4	7.8	6.4	5.8	5.8
t_1 [mm]	1.0	1.2	0.96	0.77	0.68	0.68
t_2 [mm]	1.0	1.2	1.4	1.5	1.55	1.55

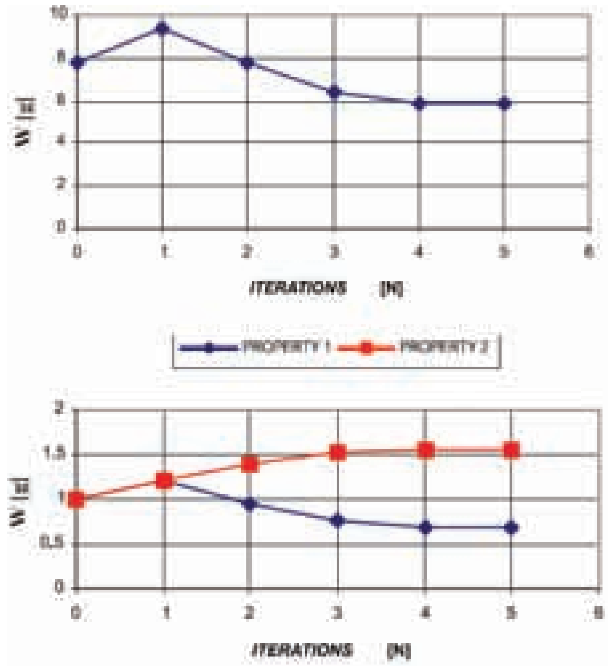


Figure7. Weight and property history diagram (Case 3)

Case 4: Gradual Reinforcement hole (D_1=12.7mm; D_2=20.7mm; t_1=1m; t_2=1mm; t_3=1mm)

Table 7. Design Variables and Design Constraints (Case 4)

Vary-Design Variables	Object	Limit-Design Constraints
0.5<Thickness<2 (20%); t_1 [mm]	property 1	1<σ_{VM} [daN/mm^2]<29
0.5<Thickness<3 (20%); t_2 [mm]	property 2	1<σ_{VM} [daN/mm^2]<29
0.5<Thickness<4 (20%); t_3 [mm]	property 3	1<σ_{VM} [daN/mm^2]<29

Table 8. Weight and property history data (Case 4)

Iteration [N]	0	1	2	3	4	5
W [g]	7.8	9.4	7.7	6.3	5.4	5.4
t_1 [mm]	1.0	1.2	0.96	0.77	0.65	0.65
t_2 [mm]	1.0	1.2	1.25	1.0	0.8	0.8
T_3 [mm]	1.0	1.2	1.44	1.67	1.9	1.9

Table 9. Summary weight history data (Case 1,2,3,4)

Iteration [N]	0	1	2	3	4	5	6	7	8
W_{case1} [g]	7.8	9.4	9.6	8	6.6	6.2	6.2		
W_{case2} [g]	7.8	9.4	9.5	8	6.5	6	5.85	5.8	5.8
W_{case3} [g]	7.8	9.4	7.8	6.4	5.8	5.8			
W_{case4} [g]	7.8	9.4	7.7	6.3	5.4	5.4			

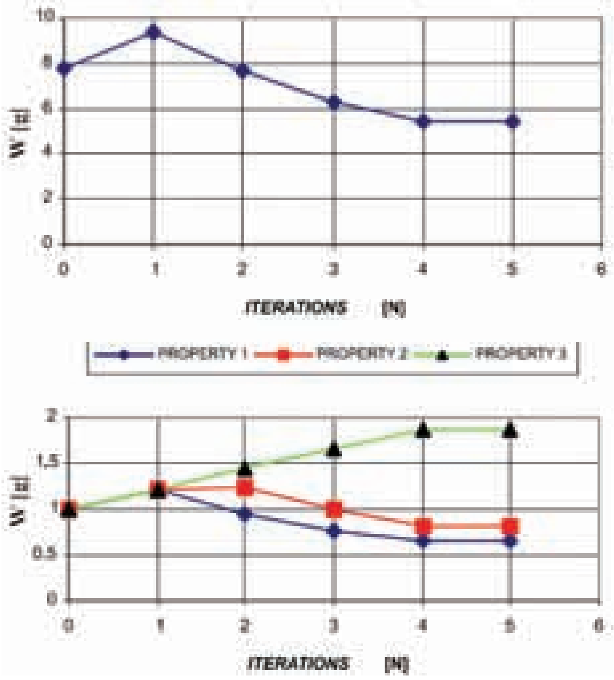


Figure 8. Weight and property history diagram (Case 4)

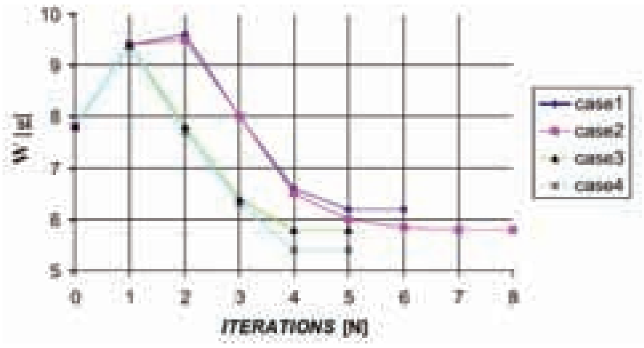


Figure 9. Summary weight history diagram (Case 1,2,3,4)

Conclusion

Structural optimization problems considered in this work consist of minimizing some objective function subject to nonlinear constraints and bounds insuring the feasibility of the structural design. Attention is focused on the minimum weight design of characteristic aircraft structural components such as plate with reinforced holes. An efficient optimization procedure combining finite element analysis and approximation concepts can be effectively used for this purpose. The numerical examples have indicated that the approximation concepts approach is a powerful and a rather general and practical approach to structural optimization of aircraft structural components. Convergence to feasible near optimum design has been obtained after 5-8 actual analyzes, depending on the problem and the move limit. It has been shown that the introduction of high quality explicit approximation for the objective function and the behavior constraints improves the convergence characteristics of structural optimization problems.

References

[1] VANDERPLAATS,G.N., THOMAS,H.L.: *An improved approximation for stresses constraints in plate structures*, Structural Optimization, 1993, Vol.6, No.1, pp.1-7.

[2] MAKSIMOVIĆ,S.: *Some computational and experimental aspects of optimal design process of composite structures*, J. Composite Structures, Vol. 17, 1990, 237-258.

[3] ZHANG,W.H., FLEURY,C.: *Selection of appropriate approximation schemes in engineering optimization*, LTAS internal report oa-29, 1993.

[4] ESDU, *Engineering Sciences Data Structures*, Subseries - Volume 7, Stress Concentrations

[5] BERKE,L AND VENKAYYA: *Review of Optimality Criteria Approachs to Structural Optimization,* ASME Structural Optimization Symposium AMD , Vol. 7, 1974, 23-34.

[6] MAKSIMOVIĆ,S.: *On Multilevel Optimization Method with Applications on Aircraft Landing Gears*, WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS, Issue 2, Volume 1, 2006, pp 231-238

[7] MAKSIMOVIĆ,S., ZELJKOVIĆ,V.: *Multilevel Optimization Approach Applied to Aircraft Structures*, Scientific Technical Review, 2006, Vol.LVI, No.2, pp.17-23.

[8] MSC/NASTRAN: *Theoretical Mannuals.*

Received: 17.11.2008.

Minimizacija mase strukture tipa tankih ploča modeliranih primenom metode konačnih elemenata

Ovaj rad bavi se optimizacijom strukture tipa tankih ploča. Postoje dva suštinska motivaciona faktora za razvoj optimizacionih modela. Najznačajniji su obezbeđenje efikasnog procesa projektovanja i smanjenje vremena rada za postizanje optimalnih rešenja. Ovaj rad razmatra minimizaciju mase strukture uz primenu metode konačnih elemenata. Pri rešavanju optimizacionog problema korišćeni su kriterijumi optimalnosti [1-3]. Primarni potencijal korišćene metode je da broj iteracija do same konvergencije rešenja ne zavisi od broja strukturalnih elemenata [2]. Ova karakteristika čini metod veoma pogodnim za optimizaciju velikih strukturalnih problema. U cilju ilustracije efikasnosti optimizacione procedure ovde je razmatrana minimizacija mase komponenti avionske strukture.

Ključne reči: **tankozidna struktura, tanka ljuska, optimalno projektovanje, minimizacija mase, metoda konačnih elemenata.**

Минимизация масы структуры типа тонких плит моделированных применением метода конечных элементов

В настоящей работе представлена оптимизация структуры типа тонких плит. Существуют два существенных мотивирующих фактора для развития оптимизационных моделей, а самыми важными являются обеспечивание эффективного процесса проектирования и уменьшение времени работы для достижения оптимальных решений. В настоящей работе рассматривается минимизация масы структуры с применением метода конечных элементов. При решении оптимизационной проблемы пользованы критерии оптимальности Ш1-3Я. Первычный потенциал использованого метода - число итераций (шагов) до самой сходимости решения не зависит от числа структуральных элементов Ш2Я. Эта характеристика делает метод очень удобным для оптимизации больших структуральных проблем. С целью растолковывания эффективности оптимизационной процедуры здесь рассматривана минимизация масы составных частей структуры самолёта.

Ключевые слова: тонкостеная конструкция, тонкая оболочка, оптимальное проектирование, минимизация масы, метод конечных элементов.

Minimisation de la masse des constructions à parois minces modelées par les éléments finis du type coque

Ce papier s'occupe de l'optimisation de la structure des plaques minces. Il y a deux types essentiels de facteurs de motivation pour le développement des modèles d'optimisation. Les plus importants sont ceux qui assurent un procès efficace dans l'élaboration du projet et diminuent le temps de travail pour la réalisation des solutions optimales. Ce travail considère la minimisation de la masse de structure par l'application de la méthode des éléments finis. Pour résoudre le problème de l'optimisation on a utilisé les critères d'optimalité [1-3]. Le potentiel primaire de la méthode utilisée est dans le fait que le nombre d'itération jusqu'à la convergence de la solution ne dépend pas du nombre des éléments structuraux [2]. Cette caractéristique rend la méthode très propice pour l'optimisation de grands problèmes structuraux. Pour illustrer l'efficacité du procès d'optimisation on a considéré dans ce travail la minimisation de la masse des composantes chez la structure d'avion.

Mots clés: **construction aux parois minces, coque mince, conception optimale, minimisation de la masse, méthode des éléments finis**