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The Stability of Linear Discrete Descriptor Systems Over the Finite Time Interval: An Overview

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This paper gives a detailed overview of the work and the results of many authors in the area of Non-Lyapunov (finite time stability, technical stability, practical stability, final stability) of a particular class of linear discrete descriptor systems. The geometric description of consistent initial conditions that generate tractable solutions to such problems and the construction of non-Lyapunov stability theory to bound rates of decay of such solutions are also investigated. The stabilty robusteness problem has been also treated and presented. This survey covers the period since 1985 up to nowadays and has a strong intention to present the main concepts and contributions that have been derived during the mentioned period throughout the world, published in respectable international journals or presented on workshops or prestigious conferences.

Key words: **linear system, discrete system, descriptor system, system stability, Non-Lyapunov stability, finite time stability.**

Introduction

IT should be noticed that in some systems we must consider their character of dynamic and static state at the same time. *Linear discrete descriptor systems* (LDDS) (also, referred to as degenerate, singular generalized, difference - algebraic or semi – state) are those the dynamics of which is governed by a mixture of *algebraic* and *difference* equations. Recently many scholars have paid much attention to singular systems and have obtained many significant consequences. The complex nature of singular systems causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a need for their control.

In practice one is not only interested in system stability (e.g. in sense of Lyapunov), but also in bounds of system trajectories. A system could be stable but completely useless because it possesses undesirable transient performances. Thus, it may be useful to consider the stability of such systems with respect to certain sub-sets of state-space, which are *a priori* defined in a given problem. Besides that, it is of particular significance to take into consideration the behavior of dynamical systems only over a finite time interval.

These bound properties of system responses, i.e. the solution of system models, are very important from an engineering point of view. Realizing this fact, numerous definitions of the so-called technical and practical stability were introduced.

Roughly speaking, these definitions are essentially based on the predefined boundaries for the perturbation of initial conditions and allowable perturbation of system response. In engineering applications of control systems, this fact becomes very important and sometimes crucial, for the purpose of characterizing in advance, in quantitative manner, possible deviations of a system response.

Thus, the analysis of these particular bound properties of solutions is an important step, which precedes the design of control signals, when finite time or practical stability control is concerned. In the context of practical stability for linear discrete descriptor systems, various results were first obtained in *Debeljkovic*, (1985) and *Owens, Debeljkovic* (1986).

Motivated by the *brief discussion* on practical stability in the monograph of *La Salle and Lefschetz* (1961), *Weiss and Infante* (1965, 1967) have introduced various notations of stability over the finite time interval, for continual-time systems and constant set trajectory bounds.

Further development of these results was due to many other authors, *Michel* (1970), *Grujic* (1971), *Lashirer*, *Story* (1972).

A type of practical stability for discrete-time systems defined over an infinite time interval was studied by *Hurt* (1967) in connection with error analysis in numerical computation.

The application of the concept of finite time stability to discrete-time systems was first considered by *Michel*, *Wu* (1969). Practical stability (or "set stability") allowing a quantitative estimation of the trajectory behaviour over a finite or infinite time interval was treated by *Heinen* (1970), who first obtained necessary as well as sufficient conditions in terms of existence of discrete Lyapunov functions.

Further results were presented by *Weiss* (1972) and

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Weiss, *Lam* (1973). Final stability of discrete-time systems with respect to time-varying sets was considered by *Lam*, *Weiss* (1974).

Grippo, *Lampariello* (1976) generalized previous results and gave necessary and sufficient conditions of a different type of practical stability of discrete-time systems based on the definitions of stability and instability formerly introduced by *Heinen* (1970).

The practical stability with settling time for discrete-time systems was considered by *Debeljkovic* (1979, 1983) in connection with analysing different classes of linear systems generally enough to include unforced systems, systems operating under the influence of disturbing forces, time-invariant and time-varying systems. Some questions of system instability were solved and discrete version of very well known Bellman-Gronwall *Lemma* is also presented.

Some linear systems not possessing a state variable representation can admit semistate (descriptor) equations.

These systems were treated by several authors from different points of view, *Rosenbrock* (1970, 1974.a, 1974.b), *Godbout and Jordan* (1975). Using Drazin inverse *Campbell et al.* (1974) derived a closed form of solutions, when differential equations have unique solutions for consistent initial conditions.The analogous results were presented also for a discrete-time case.

The notion of a descriptor system was introduced by *Luenberger* (1977).The structural characteristics of linear time-invariant discrete-descriptor systems were also investigated by *Luenberger* (1978). A significance contribution was due to *Campbell* (1980) where many previous results, both for continual and discrete as well as for time-varying singular systems, were generalised and presented.

Nonlinear time-variable semi state circuits were considered by *Newcomb* (1981) and *Campbell* 1981). Solvability, controllability and observability of continues descriptor systems were treated in a paper of *Yip and Sincovec* (1981).

Allowable equivalence transformations of such singular systems were specified in *Verghes et al.* (1981), with many new results concerning different characteristics usually defined for regular systems, with particular interest focused on impulsive solutions. Further results concerning behaviour of nonlinear semi state circuits are due to *Sadahmed and Zaghloul* (1982). Eigenvalue assignment problem in singular systems was solved and presented by *Paraskevopoulos* (1983) the same as an efficient algorithm for computation of the transfer function matrix for descriptor systems (1984).

Debeljkovic and Owens (1985) derived some new results in the area of practical and finite time stability for time-invariant, continuous linear singular systems These results represent the sufficient condition for stability of such systems and are based on Lyapunov-like functions and their properties on sub-space of consistent initial conditions. In particular these functions need not to have properties of positivity in the whole state space and negative derivatives along system trajectories.

The Lyapunov stability theory for both continuous and discrete-time linear singular systems was also investigated by the same authors (1985). The results are expressed directly in terms of the matrices E and A naturally occurring in the model and avoid the need to introduce algebraic transformations into statement of the theorems. It is expected that the geometric approach will give more insight into structural properties of singular systems and problems of consistency of initial conditions as well as to enable a basis-free description of dynamic properties.

In this paper some results that were developed in the area of non-Liapunov stability theory are extended to linear, time-invariant discrete descriptor systems. Some of them are mostly analogous to those derived in *Debeljkovic*, *Owens* (1985) for a continual-time case.

Basic notation

$\mathbb{R}$	– Real vector space
$\mathbb{C}$	– Complex vector space
C	Complex plane
I	Unit matrix
$F = (f_{ij}) \in \mathbb{R}^{n\times n}$	Real matrix
F^T	Transpose of matrix F
$F > 0$	Positive definite matrix
$F \geq 0$	Positive semi definite matrix
$\Re(F)$	Range of matrix F
$\mathbb{N}(F)$	Null space (kernel) of matrix F
$\lambda(F)$	Eigenvalue of matrix F
$\sigma_{(\ )}(F)$	Singular values of matrix F
$\sigma\{F\}$	Spectrum of matrix F
$\|F\|$	Euclidean matrix norm $\| F \| = \sqrt{\lambda_{\max}\left(A^T A\right)}$
F^D	Drazin inverse of matrix F
W_q	Subspace of consistent initial conditions
$\Rightarrow$	Follows
$\mapsto$	Such that

Preliminaries

Consider the system of n first order difference equations, represented in a vector form by:

$$E\mathbf{x}(k+1) = A\mathbf{x}(k),\quad \mathbf{x}(k_0) = \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

defined over time interval $K = \{k_0,\ \ (k_0 + k_N)\}$, where quantity k_N may be either a positive real number or symbol $+\infty$, so that *practical* and *finite time* stability can be treated simultaneously.

$E,\ A \in \mathbb{R}^{n\times n}$ are constant matrices, with E singular, $\mathbf{x}(k) \in \mathbb{R}^n$ is the phase vector (i.e. generalized state-space vector), $\mathbf{x}_0$ is consistent initial condition.

In the discrete case the concept of smoothness has little meaning but the idea of consistent initial conditions, being these initial conditions $\mathbf{x}_0$ that generate solution sequence $\{\mathbf{x}(k), k \geq 0\}$, has a physical meaning.

The difference equation (1) is said to be tractable [23] if the initial value problem has a unique solution for each consistent initial condition $\mathbf{x}_0$.

So we need:

$$\det(cE - A) \neq 0 \tag{2}$$

to quarantee uniqueness (tractability) of solutions, where c is any real scalar.

It is very well known that the solution

sequence $\{\mathbf{x}(k)\}, k \geq 0$, of equation does not exist for all initial conditions. The subspace of consistent initial conditions, generating tractable solutions will be denoted by W_q *Campbell et al.* [20,23] has shown that subspace W_q is the set of vectors satisfying

$$\mathbf{x}_0 \in \Re\left(\hat{E}^q\right), \tag{3}$$

where $q = Ind(E)$ and $\hat{E} = (cE - A)^{-1} E$.

The range and kernel of the matrix F is denoted by $\Re(F)$ and $\mathbb{N}(F)$, respectively.

The smallest nonnegative integer q such that:

$$\text{rank}\left(F^q\right) = \text{rank}\left(F^{q+1}\right) \tag{4}$$

is called the index of the matrix F.

A geometric treatment *Owens* and *Debeljkovic* (1985) also yields W_q as the limit of the subspace algorithm:

$$\begin{aligned} W_0 &= \mathbb{R}^n \\ &\vdots \\ W_{j+1} &= A^{-1}\left(EW_j\right), \quad j = 0,1,2... \end{aligned} \tag{5}$$

where $A^{-1}(\cdot)$ denotes the inverse image of $(\cdot)$ under the operator A.

An important property of W_q is that:

$$W_q \cap \mathbb{N}(E) = 0, \tag{6}$$

and hence:

$$V(\mathbf{x}(k)) = \mathbf{x}^T(k) E^T P E \mathbf{x}(k), \tag{7}$$

is a positive-definite quadratic form on W_q if $P = P^T > 0$,

That is:

$$\begin{aligned} &V(\mathbf{x}) > 0, \\ &\forall \mathbf{x}(k) \in W_q \setminus \{0\}. \end{aligned} \tag{8}$$

It is obvious, because $\mathbf{x}^T(k) E^T P E \mathbf{x}(k)$ can be equal to zero if and only if $E\mathbf{x}(k) = \mathbf{0}$ or $\mathbf{x}(k) \in \mathbb{N}(E)$.

On the other side $\mathbf{x}_0$ must be in the subspace W_q and as a consequence, the tractable solution sequence $\mathbf{x}(k)$ must be also in this subspace, see Fig.1.

$V(\mathbf{x}(k))$ can hence be used as a *Lyapunov-like function*, to represent the growth of solution as $V^{1/2}(\mathbf{x})$ is generalized norm on W_q.

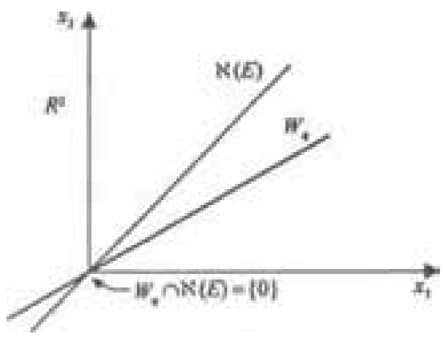

Figure 1. Graphical illustration of regularity condition

Let index β stands for the set of all allowable states of the system and index α for the set of all initial states $\mathbf{x}_0$ of the system, such that the set $S_\alpha \subseteq S_\beta$

Sets are assumed to be open, connected and bounded and defined by

$$S_\rho = \left\{\mathbf{x}(k) \in \mathbb{R}^n : \ \|\mathbf{x}(k)\|_G^2 < \rho, \ \forall \mathbf{x}(k) \in W_q \setminus \{0\}\right\}, \tag{9}$$

where G is symmetric, positive semi-definite matrix $\|\mathbf{x}\|_{(\cdot)}$ is called generalized Euclidean norm. W_q denotes the subspace of consistent initial conditions.

In particular case we use $G \geq 0$.

Proposition 1.1. If $f(\mathbf{x}(k)) = \mathbf{x}^{\mathrm{T}}(k) Q \mathbf{x}(k)$ is quadratic form on $\mathbb{R}^n$ then it follows that there exist numbers $\lambda(Q)$ and $\Lambda(Q)$, satisfying

$$-\infty < \lambda(Q) \leq \Lambda(Q) + \infty, \tag{10}$$

such that

$$\begin{aligned} &\lambda(Q) \leq \frac{\mathbf{x}^{\mathrm{T}}(k) Q \mathbf{x}(k)}{V(\mathbf{x}(k))} \leq \Lambda(Q), \\ &\forall \mathbf{x} \in W_q \setminus \{0\}. \end{aligned} \tag{11}$$

If $Q = Q^{\mathrm{T}}$ and $\mathbf{x}^{\mathrm{T}}(k) Q \mathbf{x}(k) > 0, \ \forall \mathbf{x} \in W_q \setminus \{0\}$, the $\lambda(Q) > 0$, and $\Lambda(Q) > 0$, where $\lambda(Q)$ and $\Lambda(Q)$ are defined in such way:

$$\lambda(Q) = \min_x \left\{\mathbf{x}^T Q \mathbf{x} : \mathbf{x} \in W_q \setminus \{0\}, \quad \mathbf{x}^T E^T P E \mathbf{x} = 1\right\}. \tag{12}$$

$$\Lambda(Q) = \max_x \left\{\mathbf{x}^T Q \mathbf{x} : \mathbf{x} \in W_q \setminus \{0\}, \ \mathbf{x}^T E^T P E \mathbf{x} = 1\right\}. \tag{13}$$

Consider (LDDS) (**1**).

For the needs of investigation the irregular (LDDS) we present the following discussion.

It is assumed that the matrix E is in the form $E = \text{diag}\left\{I_{n1}, \ 0_{n2}\right\}$, where I_p and 0_p stand for the $p \times p$ identity matrix and the $p \times p$ null matrix, respectively.

If the matrix E is not in this form, then in many cases it can be transformed to the required form via left multiplication by a nonsingular matrix T, and such transformation will not alter the original phase variables $\mathbf{y}(k)$. At the expense of changing the original phase variables of the system (1) we can use right multiplication by a nonsingular matrix Q to achieve the same. However, a much broader class of systems can be brought into a suitable form using the transformation $E \to TEQ$, where T and Q are nonsingular matrices *Dai* (1989.a).

The resulting (LDDS) model will thus be given as

$$\mathbf{x}_1(k+1) = A_1 \mathbf{x}_1(k) + A_2 \mathbf{x}_2(k), \tag{14.a}$$

$$\mathbf{0} = A_3 \mathbf{x}_1(k) + A_4 \mathbf{x}_2(k), \tag{14.b}$$

where

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \end{bmatrix} \in \mathbb{R}^n, \tag{15}$$

with $\mathbf{x}_1(k)\in\mathbb{R}^{n1}$ and $\mathbf{x}_2(k)=\mathbb{R}^{n2}$ and $n=n_1+n_2$; here A_1, A_2, A_3 and A_4, are real matrices of dimension $n_1\times n_1, n_1\times n_2, n_2\times n_1$, and $n_2\times n_2$ respectively.

Relating the practical stability to the time interval K it will be possible to treat simultaneously the finite and infinite time practical stability.

The form (14) for a (LDDS) is also known as the second equivalent form, *Dai* (1989.a).

For the system (1), $\det E = 0$. As the system considered is time-invariant, it is sufficient to consider its solutions $\mathbf{x}(t)$ *as* functions of only the current discrete moment k and initial value $\mathbf{x}_0$ at the initial moment k_0.

Another reason to justify this is that k_0 is fixed.

Hence let $\mathbf{x}(k)=(k,\mathbf{x}_0)$ denote the value of a solution $\mathbf{x}(k)$ of (14) in the moment $k\in K$ which emanated from $\mathbf{x}_0$ at $k=k_0$.

In an abbreviated form, the value of solution $\mathbf{x}$ at the moment k will be denoted by $\mathbf{x}(k)$.

Time invariant regular

Discrete descriptor systems

Stability definitions

Definition 1. The system (1) is practically stable with respect to $\{K,\alpha,\beta,G,W_q\}$, if and only if a *consistent initial condition*, satisfying

$$\|\mathbf{x}_0\|_G^2<\alpha,\quad G=E^TPE,$$

implies

$$\|\mathbf{x}(k)\|_G^2<\beta,\quad \forall k\in K,$$

G is chosen to represent physical constraints on the system variables and it is assumed, as before, to satisfy:

$$\begin{aligned}&G=G^T,\\ &\mathbf{x}^T(k)G\mathbf{x}(k)>0,,\\ &\forall\mathbf{x}(k)\in W_q\setminus\{0\}\end{aligned}\tag{16}$$

Debeljkovic (1985, 1986), *Debeljkovic, Owens* (1986), *Owens, Debeljkovic,* (1986).

Definition 2. The system (1) is practically unstable with respect to $\{K,\alpha,\beta,G,W_q\}$, if and only if a *consistent initial condition*, satisfying

$$\begin{aligned}&\|\mathbf{x}_0\|_G^2<\alpha,\\ &G=E^TPE,\end{aligned}$$

and there exists discrete moment $k^*\in K$, such that the next condition is fullfiled

$$\left\|x(k^*)\right\|_G^2>\beta,\qquad \text{for some } k^*\in K,$$

Debeljkovic, Owens (1986), *Owens, Debeljkovic*, (1986).

Definition 3. System (14) is $\{K,\alpha,\beta,G\}$*-practically stable* if $\mathbf{x}_0\in\mathcal{W}_{q^*}\cap \mathrm{S}_G(\alpha)$ implies $\mathbf{x}(k,\mathbf{x}_0)\in \mathrm{S}_G(\beta)$ for all $k\in K$, *Bajic et al.* (1998).

Definition 4. System (14) is $\{K,\alpha,\beta,G\}$*-practically unstable* if there is $\mathbf{x}_0\in\mathcal{W}_{q^*}\cap \mathrm{S}_G(\alpha)$ and a solution $\mathbf{x}(k)=\mathbf{x}(k,\mathbf{x}_0)$ such that $\mathbf{x}(k^*,\mathbf{x}_0)\notin \mathrm{S}_G(\beta)$ for some $k^*\in K$, *Bajic et al.* (1998).

Remark 1. *Definitions* 3 and 4 were introduced in the context of analysis of a regular (LDDS).

In order to provide for simultaneous treatment of both a *regular* and an *irregular* (LDDS), we need the following definitions, based on *Bajic* (1995) and *Debeljkovic et al.* (1995).

Definition 5. The solutions $\mathbf{x}(k)$ of the system (14) are $\{K,\alpha,\beta,G\}$*-bounded* if $\mathbf{x}_0\in\mathcal{M}_1\cap \mathrm{S}_G(\alpha)$ implies $\mathbf{x}(k,\mathbf{x}_0)\in \mathrm{S}_G(\beta)$ for all $k\in K$, *Bajic et al.* (1998).

Definition 6. A solution $\mathbf{x}(k)=\mathbf{x}(k,\mathbf{x}_0)$ of the system (14), with $\mathbf{x}_0\in\mathcal{M}_1\cap \mathrm{S}_G(\alpha)$, is $\{K,\alpha,\beta,G\}$*-unbounded* if $\mathbf{x}(k^*,\mathbf{x}_0)\notin \mathrm{S}_G(\beta)$ for some $k^*\in K$, *Bajic et al.* (1998).

Any specific form of the matrix G can be assumed.

For example, a convenient one is $G=E^{\mathrm{T}}PE$, where P is an arbitrary symmetric pd matrix.

For the purpose of a more convenient analysis (since the matrix E of the system (14) is of a special structure) it is useful to reformulate *Definitions* 5 and 6 slightly as follows, using the notation

$$\mathcal{B}_j(\rho)=\left\{\mathbf{x}(k)\in\mathbb{R}^n:\ \|\mathbf{x}_j(k)\|^2<\rho\right\},(j=1,2),\tag{17}$$

where $\|(\cdot)\|$ denotes the Euclidean norm of a vector or the corresponding induced matrix norm.

Definition 7. The solutions $\mathbf{x}(k)=\mathbf{x}(k,\mathbf{x}_0)$ of the system (14) are $\{K,\alpha,\beta_1,\beta_2\}$ - *bounded* if $\mathbf{x}_0\in\mathcal{M}_1\cap\mathcal{B}_1(\alpha)\cap\mathcal{B}_2(\alpha\beta_2/\beta_1)$ implies $\mathbf{x}(k,\mathbf{x}_0)\in\mathcal{B}_1(\beta_1)\cap\mathcal{B}_2(\beta_2)$ for all $k\in K$, *Bajic et al.* (1998).

Definition 8. A solution $\mathbf{x}(k,\mathbf{x}_0)$ of the system (14), with $\mathbf{x}_0\in\mathcal{M}_1\cap\mathcal{B}_1(\alpha)\cap\mathcal{B}_2(\alpha\beta_2/\beta_1)$, is $\{K,\alpha,\beta_1,\beta_2\}$ - *unbounded* if there exists $k^*\in K$ such that $\mathbf{x}(k^*,\mathbf{x}_0)\notin\mathcal{B}_1(a_1)\cap\mathcal{B}_2(a_2)$, *Bajic et al.* (1998).

Two comments are necessary at this stage.

First, if all solutions starting from all points of $\mathcal{M}_1\cap \mathrm{S}_G(\alpha)$ are $\{K,\alpha,\beta,G\}$ - bounded, then the system considered is $\{K,\alpha,\beta,G\}$ - practically stable.

The second comment is that if there is any one solution which is $\{K,\alpha,\beta,G\}$-unbounded, the system considered is $\{K,\alpha,\beta,G\}$ - practically unstable.

Definitions 3 - 8 can be obtained as special cases of a generic qualitative concept from *Bajic* (1992.b).

Stability theorems

Theorem 1. The system (1) is practically stable with respect to $\{K,\alpha,\beta\}$, $\beta>\alpha$, if the following condition is satisfied

$$\Lambda^k(Q)<\beta/\alpha,\quad \forall k\in K.\tag{18}$$

where $\Lambda(Q)$ is defined by

$$\Lambda(Q)=\max_{\mathbf{x}}\left\{\begin{matrix}\mathbf{x}^{\mathrm{T}}(k)A^{\mathrm{T}}PA\mathbf{x}(k):\mathbf{x}(k)\in W_q\setminus\{0\}\\ \mathbf{x}^{\mathrm{T}}(k)E^{\mathrm{T}}PE\mathbf{x}(k)=1\end{matrix}\right\}. \quad (19)$$

with matrix $P=P^T>0$, *Debeljkovic*, *Owens* (1986).

Proof. Let $\mathbf{x}_0$ be an arbitry consistent initial condition and $\mathbf{x}(k)$ resulting solution sequence (system trajectory).

Then $\mathbf{x}(k)\in W_q\setminus\{0\},\quad \forall k\geq 0$.

It is very well known that in the discrete-time systems it is more convenient to use the agregation function of the system in such way:

$$V(\mathbf{x}(k))=\ln\mathbf{x}^T(k)E^TPE\mathbf{x}(k), \quad (19)$$

instead of

$$V(\mathbf{x}(k))=\mathbf{x}^T(k)E^TPE\mathbf{x}(k). \quad (20)$$

Of course, (20) can be used too, but many mathematical problems have to be solved.

Using (19) instead of (20) we overcome these problems .

Finding:

$$\Delta V(\mathbf{x}(k))=\ln\frac{\mathbf{x}^T(k)A^TPA\mathbf{x}(k)}{\mathbf{x}^T(k)E^TPE\mathbf{x}(k)}, \quad (21)$$

and using the idea of eq. (13) one can write:

$$\Delta V(\mathbf{x}(k))\leq\ln\Lambda(Q), \quad (22)$$

$\Lambda(Q)$ defined by (19).

If one forms a sum $\sum_{k_0}^{k_0+k-1}$ of the previous inequality for any $k\in K$, we get:

$$\begin{aligned}&\ln\mathbf{x}^T(k_0+k)E^TPE\mathbf{x}(k_0+k)=\\&=\sum_{k_0}^{k_0+k-1}\ln\Lambda(Q)+\ln\mathbf{x}^T(k_0)E^TPE\mathbf{x}(k_0)\end{aligned} \quad (23)$$

Taking into account the first condition of *Definition* 1 and (18) we get the following result

$$\ln\mathbf{x}^T(k_0+k)E^TPE\mathbf{x}(k_0+k)<a,\quad \forall k\in K, \quad (23)$$

that proves the *Theorem* Q.E.D.

It is clear that it was used:

$$\sum_{k_0}^{k_0+k-1}\ln\Lambda(Q)=\ln\prod_{k_0}^{k_0+k-1}\Lambda(Q)=\Lambda^k(Q), \quad (24)$$

Debeljkovic (1986), *Debeljkovic*, *Owens* (1986).

Theorem 2. The system (1) is practically unstable with respect to $\{K,\alpha,\beta\}$, $\beta>\alpha$, if there exists a positive scalar $\gamma\in\left]0,\alpha\right[$ and a discrete moment k^*, $\exists\left(k^*>k_0\right)\in K$ such that the following condition is satisfied

$$\lambda^{k^*}(Q)>\beta/\gamma,\quad \text{for some } k^*\in K. \quad (25)$$

where $\lambda^k(Q)$ being defined by (12), *Debeljkovic*, *Owens* (1986).

Theorem 3. The system (**1**) is practically stable with respect to $\{K,\alpha,\beta\}$, $\beta>\alpha$, if the following condition is satisfied

$$\left\|\Psi(k)\right\|<\beta/\alpha,\quad \forall k\in K. \quad (26)$$

where $\Psi(k)=\left(\hat{E}^D\hat{A}\right)^k$ and $\hat{E}=(cE-A)^{-1}E$, $\hat{A}=(cE-A)^{-1}A$, *Debeljkovic* (1986).

For the needs of the following presentation we define the smallest (respectively largest) eigenvalues of a matrix $R=R^T$ w.r.t. W_q and matrix G,

$$\lambda(R,G,W_q)=\min\left\{\begin{matrix}\mathbf{x}^T(t)R\mathbf{x}(t):\mathbf{x}(t)\in W_q\\ \mathbf{x}^T(t)G\mathbf{x}(t)=1\end{matrix}\right\}, \quad (27)$$

$$\Lambda(R,G,W_q)=\max\left\{\begin{matrix}\mathbf{x}^T(t)R\mathbf{x}(t):\mathbf{x}(t)\in W_q\\ \mathbf{x}^T(t)G\mathbf{x}(t)=1\end{matrix}\right\}, \quad (28)$$

and note that $0<\lambda(\ )$ if $R=R^T>0$.

Theorem 4. System (1) is practically stable w.r.t. $\{K,\alpha,\beta,G\}$ if

$$\beta/\alpha\geq\Lambda^k\left(A^TPA,G,W_q\right),\qquad \forall k\in K \quad (29)$$

Owens, Debeljkovic (1986).

Theorem 5. System (**1**) is practically unstable w.r.t. $\{K,\alpha,\beta,G\}$ if $\exists d$, $0<d<\alpha$ and $k^*\in K$ such that

$$\beta/d<\lambda^{k^*}\left(A^TPA,G,W_q\right), \quad (30)$$

Owens, Debeljkovic (1986).

Note 1. Here $G=E^TPE$ where $P=P^T>0$ is an arbitrarily specified matrix.

Note that (6) implies that $\|\mathbf{x}(t)\|_G=\sqrt{\left(\mathbf{x}^T(t)G\mathbf{x}(t)\right)}$ is a norm on W_q.

Now we turn our attention to the **forced** linear discrete descriptor systems of the form

$$E\mathbf{x}(k+1)=A\mathbf{x}(k)+\mathbf{u}(k),\quad \mathbf{x}^*(k_0)=\mathbf{x}_0^*, \quad (31)$$

with $\mathbf{u}(k)\in\mathbb{R}^n$.

It is necessary to underline that the consistent initial conditions for systems governed by (1) and (31) may be, in general, distinct.

Definition 9. System (20) is finite time stable w.r.t. $\{K,\alpha,\beta,\varepsilon\}$, $\alpha<\beta$, if and only if a consistent initial condition, $\mathbf{x}_0^*\in W_q$ satisfying

$$\left\|\mathbf{x}^*(k_0)\right\|<\alpha, \quad (32)$$

implies

$$\left\|\mathbf{x}^*(k)\right\|<\beta,\quad k\in K, \quad (33)$$

whenever

$$\|\hat{\mathbf{u}}(j)\| < \varepsilon,\ \forall j = 0,1,...k-1, \tag{34}$$

where:

$$\hat{\mathbf{u}}(i) = (cE - A)^{-1}\mathbf{u}(i), \tag{35}$$

and:

$$p = \operatorname{Ind}(\hat{E}), \tag{36}$$

$\hat{E}$ being defined with:

$$\hat{E} = (cE - A)^{-1} E, \tag{37}$$

Debeljkovic et al. (1998).

Theorem 6. System (20) is finite time stable w.r.t. $\{K, \alpha, \beta, \varepsilon\}$, $\alpha < \beta$ if the following condition is satisfied:

$$\| \Psi^k \| \cdot \| \mathrm{E} \| + \varepsilon_0 \cdot \| \hat{E}^D \| \cdot \sum_{j=0}^{k-1} \| \Psi^{k-j-1} \| < \beta / \alpha, \tag{38}$$

where:

$$\Psi = \hat{E}^D \hat{A}, \qquad \mathrm{E} = \hat{E}\,\hat{E}^D, \tag{39}$$

and

$$\varepsilon_0 = \varepsilon / \alpha, \tag{40}$$

Debeljkovic et al. (1998).

Time invariant irregular

Discrete descriptor systems

Now we turn our attention to a particular class of (LDDS) described by (14).

Some preliminaries are needed for the results to be presented.

Namely, when the matrix pencil $\{cE - A : c \in \mathbb{C}\}$ is regular, i.e. when there exists c such that $\det(cE - A) \neq 0,\ c \in \mathbb{C}$, then solutions of (1) exist and they are unique for the so-called consistent initial values $\mathbf{x}_0$.

Moreover, a closed form of the solutions exists *Campbell* (1980). If matrix A_4 is non-singular, then the *regularity condition* for the system (14) considerably simplifies and reduces to

$$\begin{aligned} &\det\left(cI_{n_1} - A_1\right)\det\left(-A_4 - A_3\left(cI_{n_1} - A_1\right)^{-1} A_2\right) \\ &= (-1)^{n2} \det A_4 \det\left(\left(cI_{n_1} - A_1\right) + A_2 A_4^{-1} A_3\right) \neq 0 \end{aligned} \tag{41}$$

It was proved in *Owens*, *Debeljkovic* (1985) that, under the conditions of an appropriate *Lemma*, $\mathbf{x}_0$ is a consistent initial condition for (1) if $\mathbf{x}_0 \in W_{q^*}$, where is the subspace of consistent initial conditions.

Moreover, $\mathbf{x}_0$ generates a discrete solution sequence $(\mathbf{x}(k) : k \geq 0)$ (in this case, $k_0 = 0$ and $k_{\text{fin}} = \infty$), such that $\mathbf{x}(k) \in W_{q^*}$ for all $k \geq 0$.

The subspace is equivalent to $\mathbf{N}\left(I - \hat{E}\hat{E}^D\right)$, where $\hat{E}^D$ is the so-called Drazin inverse of $\hat{E}$, with $\hat{E}$ defined by $\hat{E} = (cE - A)^{-1} E$.

The following discussion on the consistent initial values is taken from *Bajic* (1995).

Let us denote the set of the consistent initial values of (14) by $\mathcal{M}_1$.

Consider the manifold $\mathcal{M} \in \mathbb{R}^n$ determined by (14.b) as $\mathcal{M} = \left\{\mathbf{x} \in \mathbb{R}^n : \mathbf{0} = A_3\mathbf{x}_1 + A_4\mathbf{x}_2\right\}$.

For system (14), in the general case, $\mathcal{M}_1 \subseteq \mathcal{M}$

Thus a consistent value $\mathbf{x}_0$ has to satisfy $\mathbf{0} = A_3\mathbf{x}_{10} + A_4\mathbf{x}_{20}$, or equivalently

$$\mathbf{x}_0 \in \mathcal{M}_1 \subseteq \mathcal{M} \equiv \mathbb{N}\left(\left(A_3,\ A_4\right)\right). \tag{42}$$

However, if

$$rank\left(A_3,\ A_4\right) = rank\, A_4, \tag{43}$$

then *Bajic* (1995) $\mathcal{M}_1 = \mathcal{M} = \mathbb{N}\left(\left(A_3,\ A_4\right)\right)$, and the determination of $\mathcal{M}_1$ requires no additional computation, except to convert (1) into the form (14).

To show the last relation let us assume that $rank\, A_4 = r \leq n_2$.

Then if follows, when $\mathbf{x}_0 \in \mathbb{N}\left(\left(A_3,\ A_4\right)\right)$ and when (43) holds, that $n_1 + n_2 - r$ components of the vector $\mathbf{x}_0$ can be chosen arbitrarily while maintaining the consistency of the initial conditions of the system governed by (14). In particular, since $0 \leq r \leq n_2$, in (14.b) we can always freely select the whole vector $\mathbf{x}_1$, as well as $n_2 - r$ components of the vector $\mathbf{x}_2$.

Let $\mathbf{x}_2^{\mathrm{f}} \in \mathbb{R}^{n2-r}$ represent the components of the vector $\mathbf{x}_2$ that are chosen as free, while $\mathbf{x}_2^{\mathrm{d}} \in \mathbb{R}^r$ represents those components of $\mathbf{x}_2$ that are dependent (so that $\mathbf{x}_2$ is a splice of $\mathbf{x}_2^{\mathrm{f}}$ and $\mathbf{x}_2^{\mathrm{d}}$).

Then we can write $\mathbf{x}_2^{\mathrm{d}}(k) = F_1\mathbf{x}_1(k) + F_2\mathbf{x}_2^{\mathrm{f}}(k)$, where F_1 and F_2 are appropriate matrices.

If $\mathrm{rank}A_4 = r = n_2$, and if the vector $\mathbf{x}_1$ is chosen as free, then there are no components of $\mathbf{x}_2$ that can be chosen freely, and $\mathbf{x}_2^{\mathrm{d}}(k) \equiv \mathbf{x}_2(k)$ so that $\mathbf{x}_2(k) = F_1\mathbf{x}_1(k)$, where $F_1 = -A_4^{-1}A_3$.

In this case, system (14) reduces to $\mathbf{x}_1(k+1) = \left(A_1 - A_2A_4^{-1}A_3\right)\mathbf{x}_1(k)$, and existence of its solutions is guaranteed.

In the case when rank $rank\, A_4 = r < n_2$, the condition (43) implies that we can express $\mathbf{x}_2^{\mathrm{d}}(k) = F_1\mathbf{x}_1(k) + F_2\mathbf{x}_2^{\mathrm{f}}(k)$.

Since $\mathbf{x}_2^{\mathrm{f}}(k)$ can be chosen freely, we take it to be equal to the zero vector, so $\mathbf{x}_2^{\mathrm{d}}(k) = F_1\mathbf{x}_1(k)$.

Then $\mathbf{x}_2(k) = \begin{bmatrix} F\mathbf{x}_1(k) \\ 0 \end{bmatrix}$ and, after substitution into (14.a), we get

$$\mathbf{x}_1(k+1) = A_1\mathbf{x}_1(k) - A_2\begin{bmatrix} F_1\mathbf{x}_1(k) \\ 0 \end{bmatrix} = A_0\mathbf{x}_1(k). \quad (44)$$

Thus, in both of these cases, (14) is reducible to a lower-order normal form system with $\mathbf{x}_1(k)$ as the state variable. Consequently the existence of solutions of (14) is guaranteed under the condition (43).

Since $\mathbf{x}_0$ was an arbitrary point in $\mathbb{N}((A_3,\ A_4))$, we have $\mathcal{M}_{\mathrm{I}} = \mathcal{M} = \mathbb{N}((A_3,\ A_4))$.

Note that the uniqueness of solutions is not guaranteed when rank $rank\,A_4 = r < n_2$.

Boundedness properties of solutions $\mathbf{x}(k)$ of (1) can be expressed in the equivalent form as constraints on $\mathbf{x}(k)$ of (14) if the transformation from (1) to (14) is done via non-singular matrices.

Thus, we will present our problems for a (LDDS) in the form (14). Our primary interest is to investigate boundedness properties and, in this connection, *the potential* (weak) domain of *practical stability.*

However, we will use the term *potential* or *weak* domain because, for each $\mathbf{x}_0$ in this domain, we will guarantee only that there exists at least one solution with the specific *practical-stability* characterization.

We will not prove that all solutions emanating from the concerned points $\mathbf{x}_0$ possess the required practical stability property.

In this case given *Definitions* 3 – 8 are of particular interest.

The *potential* (or *weak)* domain of $\{K,\,\alpha,\,\beta,\,G\}$-practical stability of (14) is defined by

$$\mathcal{P} = \left\{ \begin{matrix} \mathbf{x}_0 \in \mathcal{M}_{\mathrm{I}} \cap \mathrm{S}_G(\alpha) : (\exists \mathbf{x}(k_0,\mathbf{x}_0)), \\ (\forall k \in K)\,\mathbf{x}(k,\mathbf{x}_0) \in \mathrm{S}_G(\beta) \end{matrix} \right\}. \quad (45)$$

The potential domain of $\{K,\,\alpha,\,\beta_1,\,\beta_2\}$ - practical stability is defined in an analogous manner as

$$\boldsymbol{\mathcal{B}} = \left\{ \begin{matrix} \mathbf{x}_0 \in \mathcal{M}_{\mathrm{I}} \cap \mathcal{B}_1(\alpha) \cap \mathcal{B}_2(\alpha\beta_2/\beta_1): \\ (\exists \mathbf{x}(k_0,\mathbf{x}_0)) \\ (\forall k \in K)\,\mathbf{x}(k,\mathbf{x}_0) \in \mathcal{B}_1(\beta_1) \cap \mathcal{B}_2(\beta_2) \end{matrix} \right\}. \quad (46)$$

Our task is to estimate the sets $\mathcal{P}$ and $\boldsymbol{\mathcal{B}}$.

We will use the Lyapunov's direct method to obtain underestimates $\mathcal{P}_{\mathrm{u}}$ of $\mathcal{P}$ and $\boldsymbol{\mathcal{B}}_u$ of $\boldsymbol{\mathcal{B}}$ (i.e. $\mathcal{P}_{\mathrm{u}} \subseteq \mathcal{P}$ and $\boldsymbol{\mathcal{B}}_{\mathrm{u}} \subseteq \boldsymbol{\mathcal{B}}$).

As it will be seen, our development will not require the regularity condition of the matrix pencil $\{(cE - A): c \in \mathbb{C}\}$.

We assume that condition (43) holds, which implies $\mathcal{M}_{\mathrm{I}} = \mathbb{N}((A_3,\ A_4))$ for the system (14).

Consequently there exists a matrix L which satisfies the matrix equation

$$0 = A_3 + A_4 L, \quad (47)$$

where 0 is the null matrix of the same dimensions as A_3.

From (43) and (47) follow that, if solutions of (14) exist, then there will be solutions $\mathbf{x}(k)$ whose components are tied by

$$\mathbf{x}_2(k) = L\mathbf{x}_1(k) \quad \forall k \in K. \quad (48)$$

Under the rank condition (43), it follows *Bajic* (1995) that $\mathbb{N}((L - I_{n2})) \subseteq \mathbb{N}((A_3 \quad A_4))$. To show this, consider an arbitrary $\mathbf{x}^* = \mathbb{N}((L,\,-I_{n2}))$, i.e. $\mathbf{x}_2^*(k) = L\mathbf{x}_1^*(k)$, where L is any matrix that satisfies (47).

Then multiplying (47) from the right by $\mathbf{x}_1^*(k)$ and using (48), one gets $0 = A_3\mathbf{x}_1^*(k) + A_4L\mathbf{x}_1^*(k) = A_3\mathbf{x}_1^*(k) + A_4\mathbf{x}_2^*(k)$, which shows that $\mathbf{x}^*(k) \in \mathbb{N}((A_3,\ A_4))$.

Hence $\mathbb{N}((L, -I_{n2})) \subseteq \mathbb{N}((A_3,\ A_4))$.

Consequently those solutions of (14) that satisfy (48) also have to satisfy the constraints (14.b).

For all solutions of (14) for which (47) holds, the following also hold:

1. The solutions of (14) have trajectories within the set $\mathbb{N}((L - I_{n2}))$.
2. If, under the rank condition (43), the existence of a solution $\mathbf{x}(k)$ of (14) which satisfies (47) and is $\{K,\,\alpha,\,\beta,\,G\}$ - bounded is proved, then the potential domain of $\{K,\,\alpha,\,\beta,\,G\}$-practical stability of (14) can be underestimated by

$$\mathcal{P}_{\mathrm{u}} = \left\{\mathbf{x}(k) \in \mathbb{R}^n : \mathbf{x}(k) \in \mathrm{S}_G(\alpha) \cap \mathbb{N}\left(\left(L - I_{n_2}\right)\right)\right\} \subseteq \mathcal{P} \quad (49)$$

The last fact will be shown in *Theorem* 9 for the case of $\{K,\,\alpha,\,\beta_1,\,\beta_2\}$ -boundedness and associated practical stability.

For the system of the form (14), the Lyapunov function can be selected as

$$V(\mathbf{x}(k)) = \mathbf{x}_1^{\mathrm{T}}(k)P\mathbf{x}_1(k), \quad (50)$$

where P is a symmetric pd matrix.

The expression

$$\Delta V(\mathbf{x}(k)) = V(\mathbf{x}(k+1)) - \rho V(\mathbf{x}(k)), \quad (51)$$

where $\rho \in \mathbb{R}$, calculated along the solutions of (14), is then

$$\begin{aligned} \Delta V(\mathbf{x}(k)) &= \wp_{11}(k) + \wp_{12}(k) + \wp_{21}(k) + \wp_{22}(k) \\ &\quad -\rho\,\mathbf{x}_1^{\mathrm{T}}(k)P\mathbf{x}_1(k) \end{aligned} \quad (52)$$

where: $\wp_{ij} = \mathbf{x}_i^{\mathrm{T}}(k)A_i^{\mathrm{T}}PA_j\mathbf{x}_j(k), \qquad i,j = 1,2$.

Employing (48) and (51) one obtains

$$\begin{aligned} \Delta V(\mathbf{x}(k)) &= \mathbf{x}_1^T(k)\left((A_1 + A_2L)^T P(A_1 + A_2L)\right)\mathbf{x}_1(k) \\ &\quad -\rho\,\mathbf{x}_1^T(k)P\mathbf{x}_1(k) = \mathbf{x}_1^T(k)\,\Xi\,\mathbf{x}_1(k) \end{aligned} \quad (53)$$

where

$$\Xi = (A_1 + A_2L)^T P(A_1 + A_2L) - \rho P. \quad (54)$$

We note that Ξ is a real symmetric matrix.

Let $\lambda_{\mathrm{M}}(X)$ and $\lambda_{\mathrm{m}}(X)$ denote the maximal and the minimal eigenvalue of a real symmetric matrix X respectively.

We are now in the position to state the following result.

Theorem 7. Let (43) hold.

Let P be a real symmetric pd matrix.

If L is any real matrix that satisfies (47), then the system (14) has solutions which are $\{K, \alpha, \beta_1, \beta_2\}$ - bounded, with $\alpha \le \beta_1$, if the following conditions are satisfied

(i) The matrix Ξ defined in (54) is nsd.

(ii) $\rho^k \alpha \lambda_{\mathrm{M}}(P)/\lambda_{\mathrm{m}}(P) < \beta_1 \quad \forall k \in K$. (55)

(iii) $\|L\|^2 \le \beta_2/\beta_1$, (56)

Bajic et al. (1998).

Remark 2. Some preliminary results on this mater have been derived in *Dihovicni et al.* (1996).

Theorem 8. Let (43) hold.

Let P be a real symmetric pd matrix.

Then the system governed by (14) has solutions which are $\{K, \alpha, \beta_1, \beta_2\}$ - unbounded with $\alpha \le \beta_1$, if there is some $\delta \in \{0, \alpha\}$ and some $k^* \in K$ such that:

(i) The matrix Ξ defined by (54) is psd.

(ii) $\rho^{k^*} \ge \beta_1 \lambda_{\mathrm{M}}(P)/\lambda_{\mathrm{m}}(P)\delta$, (57)

Bajic et al. (1998).

Theorem 9. Let the conditions of *Theorem* 7 hold.

Then the underestimate $\mathcal{B}_{\mathrm{u}}$ of the potential domain $\mathcal{B}$ of $\{K, \alpha, \beta_1, \beta_2\}$ - practical stability for system (14) can be determined by

$$\mathcal{B}_{\mathrm{u}} = \mathbb{N}\left(\left(L - I_{n_2}\right)\right) \cap \mathcal{B}_1(\alpha) \cap \mathcal{B}_2(\alpha\beta_2/\beta_1) \tag{58}$$

where $\mathcal{B}$ is defined by (46), *Bajic et al.* (1998).

Robustness of finite time (practical) stabilty

In the control and system theory, it is of great importance to preserve various system properties under large perturbations of the system model. Such perturbations of the system model may be caused by changes in the manufacturing process of components, variations of constructive elements, or changes of environmental conditions.

The insensitiveness of system properties is called *robustness* and it is an important field of investigation. The fact is that in many practical situations the parameters of system components are not known exactly. Usually, we only have some information on the intervals to which they belong.

Therefore, the robustness for any system property is an important theoretical and practical question.

In recent years, a considerable attention has been focused on the design of controllers for multivariable linear-systems so that certain system properties are preserved under various classes of perturbations occurring in the system. Such controllers are called *robust controllers,* and the resulting system is said to be robust in some context.

Dynamic system behavior in the presence of *small perturbations* is treated within the *sensitive theory.* The theory of robustness is related to the cases when perturbations are qiute significant.

For contemporary control systems, it is of particular importance to preserve not only the stability characteristics, but also the performances such as: controllability, observability, identificabilty etc. Therefore, the robustness can be assigned to any system feature.

Robustness, besides its theoretical significance has a very impressive practical meaning, since in many cases the exact values of system parameter components are not known very well, although some boundedness properties of system responses may be estimated.

Roughly speaking, some definitions of robustness are essentially based on the predefined boundaries for the perturbation of initial conditions and the allowable perturbation of the system response. In the engineering applications of control systems, this fact becomes very important and sometimes crucial, for the purpose of characterizing in advance, in quantitative manner, possible deviations of the system response.

Thus, the analysis of these particular boundedness properties of solutions is an important step, which precedes the design of control signals in all cases.

There are significant differences in applying this concept towards single input - single output systems (SISO) in comparison with multi input - multi output systems (MIMO). More detailed information regarding this problem can be found in the cited references.

Let a (LDDS) be described by the difference equation

$$E\mathbf{y}(k+1) = A\mathbf{y}(k) + A_{\mathrm{p}}\mathbf{y}(k) \qquad \mathbf{y}(k_0) = \mathbf{y}_0, \tag{59}$$

where A_{p} is a matrix representing perturbations in the system model.

To analyse *robustness of practical stability* properties of (59) let us consider (59) transformed to the form

$$\mathbf{x}_1(k+1) = (A_1 + B_1)\mathbf{x}_1(k) + (A_2 + B_2)\mathbf{x}_2(k), \tag{60.a}$$

$$0 = A_3\mathbf{x}_1(k) + A_4\mathbf{x}_2(k) + B_{34}\mathbf{x}(k), \tag{60.b}$$

where $\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \end{bmatrix}$ need not represent the original variables $\mathbf{y}(k)$ of the system (59).

To simplify formulation of the stability robustness results, we introduce the following assumption.

Assumption 1 The matrix B_{34} in (60.b) is a null matrix and $\|B_1\| \le \varepsilon_1$, $\|B_2\| \le \varepsilon_2$, and $\|L\| \le \varepsilon_3$.

To perform the analysis of robustness for system (60), we employ the Lyapunov function $V(\mathbf{x}(k))$ defined by (50).

Let the rank condition (43) hold.

Then, by taking into account (48) and (4.7), the expression ΔV given by (52) along the solutions of (60) is obtained as

$$\begin{aligned} \Delta V(\mathbf{x}(k)) &= \mathbf{x}_1^{\mathrm{T}}(k)\left((\Upsilon_1 + \Upsilon_2 L)^T P(\Upsilon_1 + \Upsilon_2 L)\right)\mathbf{x}_1(k) \\ &-\rho\,\mathbf{x}_1^{\mathrm{T}}(k)P\mathbf{x}_1(k) = \mathbf{x}_1^{\mathrm{T}}(k)\Xi_{\mathrm{p}}\mathbf{x}_1(k), \end{aligned} \tag{61}$$

$$\Xi_{\mathrm{p}} = (\Upsilon_1 + \Upsilon_2 L)^T P(\Upsilon_1 + \Upsilon_2 L) - \rho P, \tag{62}$$

$$\Upsilon_i = A_i + B_i, \quad (i = 1, 2). \tag{63}$$

Note that Ξ_{p} is a real symmetric matrix.

Now we are in a position to state the following result.

Theorem 10 Let the rank condition (43) and *Assumption* 1 hold.

Let P be a real symmetric pd matrix.

If L is any real matrix that satisfies (47), then there are

$\{K, \alpha, \beta_1, \beta_2\}$ - practically stable solutions of (60) and the underestimate $\mathcal{Q}_u$ of the potential domain of $\{K, \alpha, \beta_1, \beta_2\}$ - practical stability can be determined by (58) if

(i) $$\begin{aligned}&\lambda_M(\Xi) + \lambda_M(P)(\varepsilon_1 + \varepsilon_2\varepsilon_3)^2 \\ &+2\lambda_M(\Theta^T\Theta)(\varepsilon_1 + \varepsilon_2\varepsilon_3) \le 0\end{aligned}, \quad (64)$$

where Ξ is the matrix from (4.8) and $\Theta = P\left(A_1 + A_2 L\right)$

(ii) $\rho^k \alpha\, \lambda_M(P)/\lambda_m(P) < \beta_1, \quad \forall k \in K,$ (65)

(iii) $\| L \| \le \beta_2/\beta_1,$ (66)

Bajic et al. (1998).

Conclusion

The main features of finite tine stability have been extended to singular discrete-time linear systems. The derived result represents a sufficient condition for this kind of stability for investigated systems. The result is based on existing Lyapunov-like functions and theirs properties on sub-space of consistent initial conditions.

For a class of (LDDS), simple sufficient algebraic conditions for the existence of solutions with specific practical stability constraints and practical instability are derived. The estimate of a potential domain of practical stability is obtained. The results could serve as a basis for further development of a similar existence analysis for time-variable and nonlinear descriptor systems. The results are adapted to cater for the robustness of practical stability for a class of a perturbed (LDDS).

All results are presented in the chronological order to show the developtment of idea (concept) of finite time stabilty and its extension to linear discrete descriptor systems.

Some of the results have been used to analyse system stabilty robustness performances.

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Stabilnost linearnih diskretnih deskcriptivnih sistema na konačnom vremenskom intervalu: Pregled rezultata

U ovom radu je izložen detaljan pregled rezultata brojnih autora na polju proučavanja problematike neljapunovske stabilnosti (stabilnost na konačnom vremenskom intervalu, tehnička stabilnost, praktična stabilnost, krajnja stabilnost) za posebne klase linearnih diskretnih deskriptrivnih sistema.

Geometrisjki opis konzistentnih početnih uslova koji generišu plemenita rešenja za ovu klasu problema, kao i iznalaženje uslova pod kojima neljapunovski koncept stabilnosti garantuje ograničenost takvih rešenja jesu problemi koji su ovde, detaljno istraživani.

Problem robusnosti stabilnosti, takođe, su ovde razmatrani i izloženi.

Ovaj pregled radove pokriva period od 1985 godine sve do današnjih dana i ima neospornu nameru da prikaže osnovne koncepte i doprinose koji su bili ostvareni za vreme pomenutog perioda u toku svetskih zbivanja a koji su publikovani u respektibilnim međunarodnim časopisima ili saopšteni na prestižnim međunarodnim konferencijama.

Ključne reč: **linearni sistem, diskretni sistem, deskriptivni sistem, stabilnost sistema, neljapunovska stabilnost, stabilnost na konačnom vremenskom intervalu.**

Устойчивость линейных дискретных дескриптивных систем на конечном временном интервале: Обзор результатов

Настоящая работа даёт подробный обзор результатов многих авторов в области исследования неляпуновой устойчивости (устойчивость на конечном временном интервале, техническая устойчивость, практическая устойчивость, конечная устойчивость) особого класса линейных дискретных дескриптивных систем.

Геометрическое описание согласующихся начальных условий генерирующих послушные решения для таких проблем, а в том числе и разыскивание условий, под которыми неляпуновая концепция устойчивости гарантирует пределы таких решений, являются проблемами здесь подробно исследоваными.

Проблемма крепкости устойчивости тоже здесь рассматривана и растолкована.

Этот обзор результатов охватывают период с 1985-ого года до сих пор и у него выразительное намерение представить основные концепции и вклады в этой области созданые в целом мире в упомянутом периоде и опубликованные в передовых международных журналах или показаны и представлены на выдающихся международных конференциях.

Ключевые слова: линейная система, дискретная система, дескриптивная система, устойчивость системы, неляпуновая устойчивость, устойчивость Ляпунова, конечный временной интервал.

Stabilité des systèmes linéaires descriptifs discrets chez le délai temporel fini: compte-rendu des résultats

Dans cet article on a présenté un compte-rendu détaillé sur les résultats des recherches obtenus par un grand nombre d'auteurs dans le domaine de la stabilité non Lyapunov (stabilité chez le délai temporel fini, stabilité technique, stabilité pratique, stabilité finale) pour les classes particulières des systèmes linéaires discrets et descriptifs. La description géométrique des conditions initiales consistantes qui produisent des solutions précieuses pour cette classe de problèmes ainsi que la recherche des conditions sous lesquelles le concept non Lyapunov de stabilité garantit les limites de telles solutions sont des problèmes ici étudiés en détail Les problèmes de la robustesse de stabilité ont été considérés et exposés. Ce compte-rendu des travaux comprend la période de 1985 jusqu'à nos jours et a pour but de présenter les concepts basiques et les contributions réalisés dans le monde entier pour la période citée et qui ont été publiés dans les revues internationales renommées ou exposés lors des conférences internationales réputées.

Mots clés: **système linéaire, système discret, système descriptif, stabilité du système, stabilité de non Lyapunov, stabilité chez le délai temporel fini.**