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Stability Analysis of Thin-Walled Structures Subjected to Thermal and Mechanical Loads

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An attention in this work is focused on stability analysis of the thin-walled isotropic and composite structures. A finite element model that accounts for both geometric and material nonlinearities is included. An inelastic material model for isotropic plates and shells is considered. The effects of material or thotropy on geometric nonlinear behavior are analyzed too. For this purpose high-quality 4-node shell finite elements are used. Present finite element solutions are compared with available analytic, numerical and experimental results. Numerical examples demonstrate the validity of the computation procedure in domains geometric and material nonlinear problems.

Key words: **thin-walled structures, isotropic materials, composite materials, thermal loading, mechanical loading, structure stability, stability analysis, finite elements, plates, shells.**

Introduction

THIN-WALLED composite structures are being increasingly used in aeronautical and aerospace construction. Their components are often subjected to combinations of mechanical and thermal loading. In fact, many structures are subjected to high load levels resulting in nonlinear load-deflection relationships due to large deformations of the shells. One of the important problems deserving special attention is the study of their nonlinear response to large deflections and postbuckling.

Many studies according to classical plate/shell theory for the large deflection of multilayered composite plates subjected to mechanical or thermal loading are available in literature [1,2]. Numerous studies involving the application of the shear deformation plate theory to nonlinear bending analysis can be found in Refs. [3-5]. In contrast, there have been fewer investigations for the thermal postbuckling of composite laminated plates [6,7]. The analysis of the buckling and postbuckling behaviour of isotropic or composite laminated shells is a topic of considerable technical importance in number of branches of engineering. Such behaviour may result from mechanical loading or from thermal loading or from a combination of the two, i.e. from thermomechanical loading. A book edited by Turvey and Marshall [8] contains much information on available methods, particularly as related to flat, rectangular plates, and includes details of several hundred pertinent references dating up until the mid-1990s. A large part of this literature, however, is naturally concerned with buckling under mechanical loading. Less information exists on the buckling of shells under thermal loading. The problem of buckling under thermomechanical loading has been conidered by few investigators and some details are in Ref. [9-11].

In this paper the particular concern is with the nonlinear analysis under thermo-mechanical loading of isotropic / composite shell type structures.

The present paper describes the use of the finite element method (FEM) in predicting the buckling and postbuckling response of isotropic / laminated plates when subjected to thermal or mechanical loading or combined thermo-mechanical loading.

Nonlinear analysis

The nonlinearity in structural problems can be of two types: (1) the geometric nonlinearity, which is associated with the changing geometry of a structure, and (2) material nonlinearity which is associated with the nonlinear behavior (stress-strain relations). Often a combination of the two is present when materials are loaded to their ultimate loads. The phenomena of plasticity, creep or other complex constitutive relations come under the class of material nonlinearity. The geometric nonlinearity enters the equations of equilibrium via the strain-displacement relations and the govering equations. Since the finite element method has proved to be a powerful tool for analyzing elastic structural problems, involving complex geometries, variety of loading and boundary conditions, it is quite natural to extend this technique to the solution of nonlinear problems.

The governing equation can be used to study the linear / nonlinear static and eigenvalue buckling analysis. Governing equation for the deformation of the shell can be written as [4,11]

$$\begin{aligned}&\left[[K]-[K_T]+[K_G]+\frac{1}{2}[N_1(\delta)]+\frac{1}{3}[N_2(\delta)]\right]\{\delta\}=\\&=\{F_M\}+\{F_T\}\end{aligned}\qquad(1)$$

where $[K]$ is the linear stiffness matrix, $[N_1]$ and $[N_2]$ are nonlinear stiffness matrices linearly and quadratically dependent on the field variables, respectively, $[K_T]$ and $[K_G]$ are the geometric stiffness matrices due to thermal and initial stress resultants. $\{F_M\}$ and $\{F_T\}$ are mechanical and thermal load vectors, δ is the vector of degrees of freedom

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associated to the displacement field in a finite element discretisation.

The governing equation (1) can be used to study the linear/nonlinear static and eigenvalue buckling analysis by neglecting the appropriate terms as:

a) Linear static analysis:

$$[K]\{\delta\}=\{F_M\}+\{F_T\} \tag{2}$$

b) Nonlinear static analysis:

$$\left[[K]-[K_T]+\frac{1}{2}[N_1(\delta)]+\frac{1}{3}[N_2(\delta)]\right]\{\delta\}=\{F_M\}+\{F_T\} \tag{3}$$

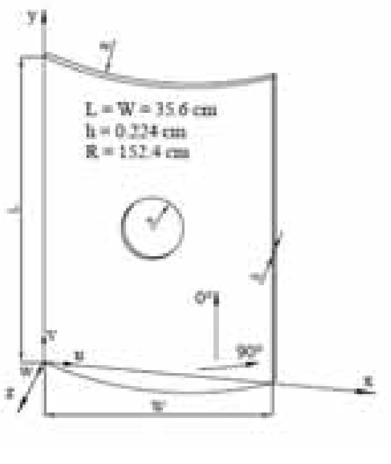
c) Eigenvalue buckling analysis:

$$[K]\{\delta\}=\Delta T[K_G^*] \tag{4}$$

where $[K_G^*]$ is the geometric stiffness due to initial state of stress developed because of unit uniform temperature rise and ΔT is the temperature rise. It may be noted here that for the purpose of evaluating $[K_G^*]$, firstly the static analysis of the shell using eq.(2) for unit temperature rise is carried out. The resulting deformation filed is used to calculate the initial state of stress resultants using Mindlin formulation [1,4] for the displacements of plate and in turn, for evaluating the $[K_G^*]$ matrix. The nonlinear prebuckling axisymmetric deformation followed by the postbuckling equilibrium path (a symmetric deformation) is traced by solving eq.(3) using Newton–Raphson iteration procedure coupled with the displacement control method [12]. The equilibrium is achieved for each load/displacement step until the convergence criteria suggested by Bergan and Clough [13] are satisfied within the specific tolerance limit of less than 1%.

Numerical examples

To illustrate the effect of temperature on the nonlinear behavior of isotropic and laminated composite structures, some numerical examples are included.



Material properties:

Material 1: Isotropic

E=200 GPa
v=0.33
α= 9.0x10^{-6}/°C

Material 2: Composite

E_L=130.3 GPa
E_T=9.37 GPa
G_{LT}=4.502 GPa
v_{LT}=0.33
α_L= 0.139x10^{-6}/°C
α_T= 9.0x10^{-6}/°C

Figure 1. Geometry and material properties of a curved panel with a hole

Nonlinear behavior of an isotropic curved panel with a hole

This example considers the response of a thermal loaded, thin, isotropic, curved panel with a hole (Fig.1). The geometry and material properties (Material 1) are also defined in Fig.1. The stress – strain curve is given in Fig.2. The stress distributions for linear, geometric nonlinear and combined geometric and material nonlinearity under thermal loads (ΔT=1100^{o}C) are given in Fig.3. The effect of geometric and material nonlinear behavior is evident.

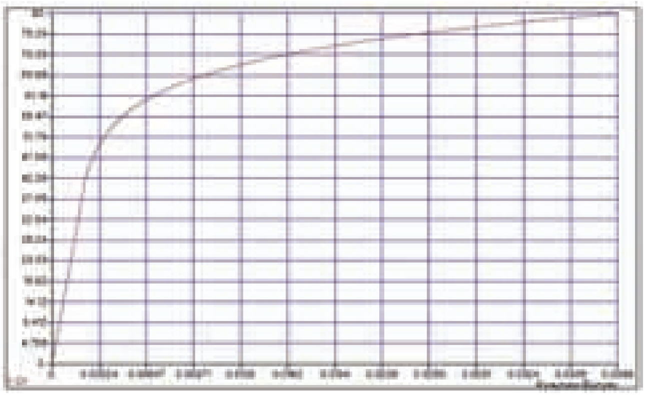


Figure 2. Elastic-plastic material behavior

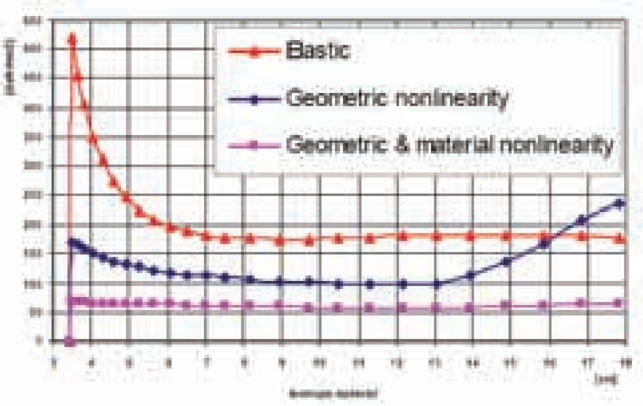


Figure 3. The stress distributions for a curved isotropic panel under thermal loading for linear, geometric nonlinear and combining geometric/material nonlinearity

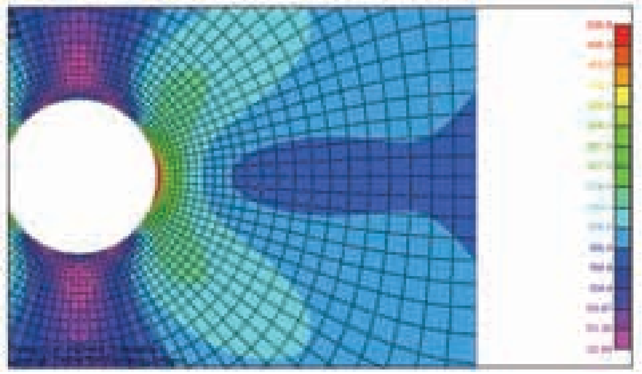


Figure 4. Linear stress distributions for a curved panel with a hole

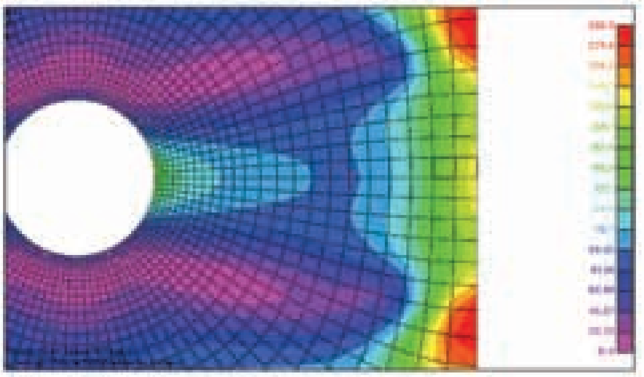


Figure 5. Stress distributions for a curved panel with a hole using the geometric nonlinear FEA

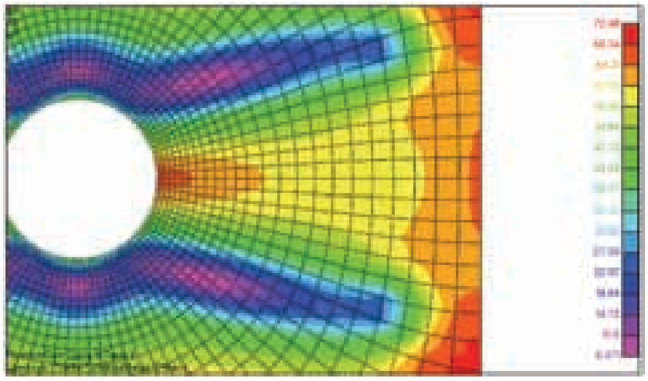


Figure 6. Stress distributions for a curved panel with a hole using the geometric/material nonlinear FEA under thermal loading

Thermal buckling and postbuckling of a curved laminated panel with a hole

The second validation problem concerns the buckling and postbuckling response of a curved laminated panel with a hole subjected to uniform temperature. In design processes where thermal loading is a concern, this type of analysis is important for the identification of the effects of various laminate stacking sequences, fiber orientation, number of layers and aspect ratio of the panels on their stability. This curved panel subjected to uniform temperature change is considered in order to examine the effect of a hole size and radius of curvature on panel stability. The geometry and material properties (Material 2) associated with each lamina are given in Fig.1 the planform geometry is defined by L and W, and both are equal to 356 mm. The panel thickness, h, is 2.24 mm, resulting from a laminate lay-up of $[\pm45^o/0^o/90]_{2S}$. The boundary conditions imposed along the panel edges are:

Along the vertical edges (x=0, x=W); $w=w_{,y}=0$ (without edge restraints)

Along the horizontal edges (y=0, y=L); $v=w=w_{,x}=0$

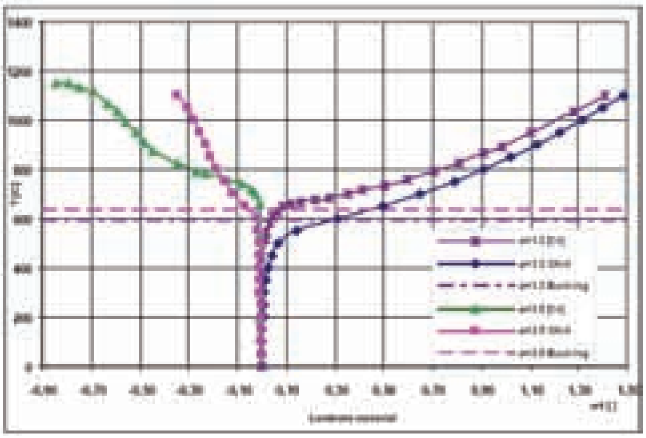


Figure 7. The thermal buckling and postbuckling responses of a curved panel with a hole

The complete buckling and postbuckling behaviors of a curved laminated panel with a hole under thermal loads are shown in Figures 7-11. The effect of thermal loads on buckling and postbuckling behavior is evident. The buckling temperature increases with the increasing hole size, Figures 7-9. The present finite element results are compared with available results [14]. Good agreement with the results in [14] is obtained, see Fig.7.

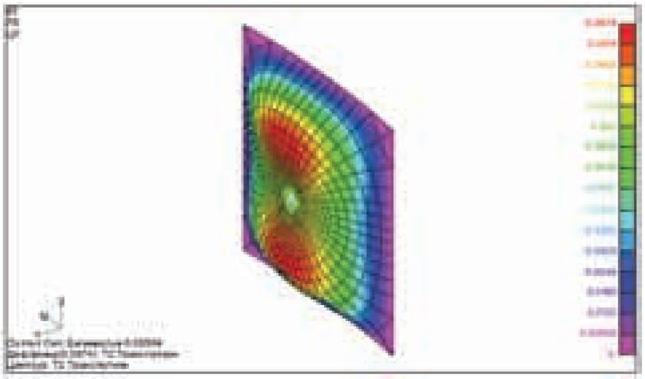


Figure 8. The first buckling mode using the linear Eigen-value method (T_{CR}= 588°C, a=10 mm)

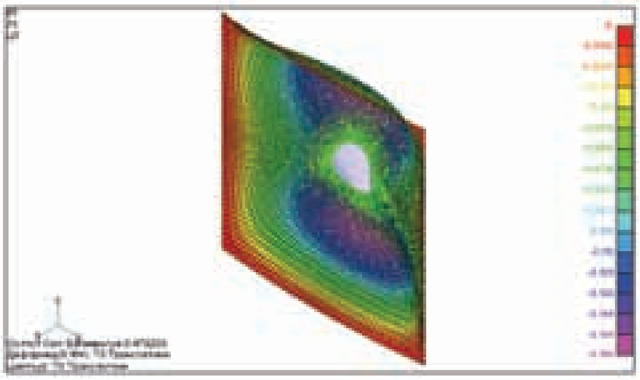
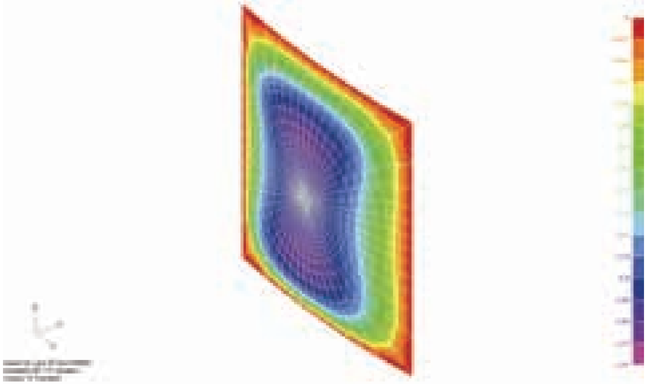
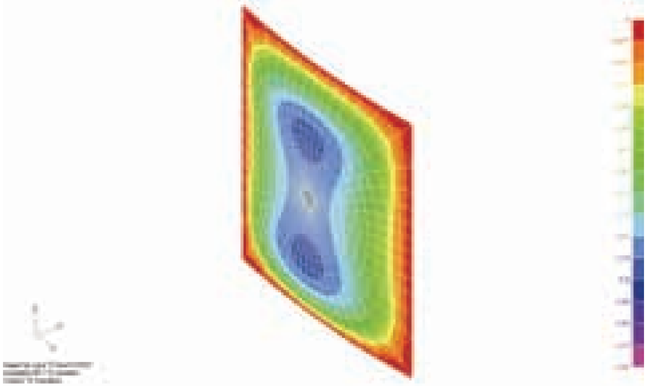


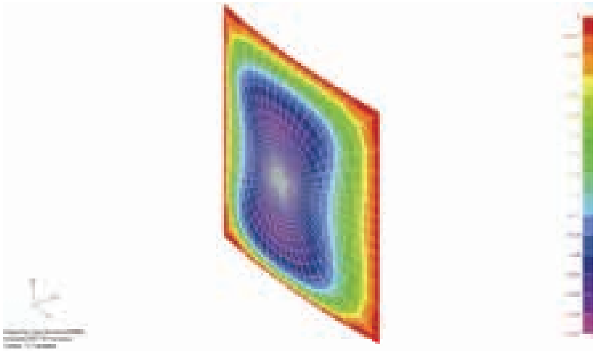
Figure 9. The first buckling mode using the linear Eigen-value method (T_{CR}= 623°C, a=35 mm)



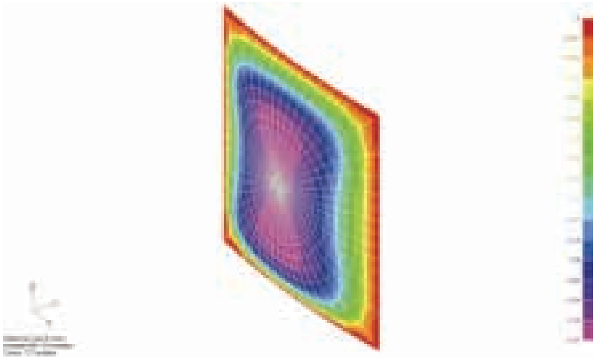
T = 600°C



T = 850°C

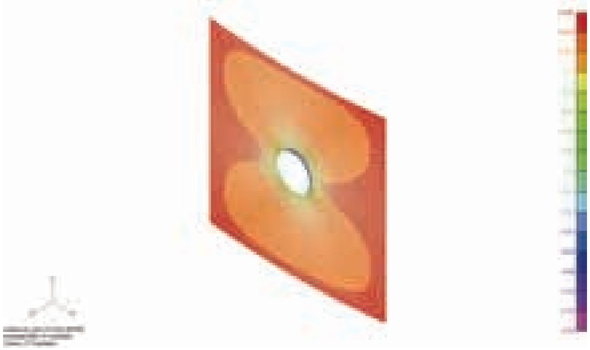


T = 1000°C

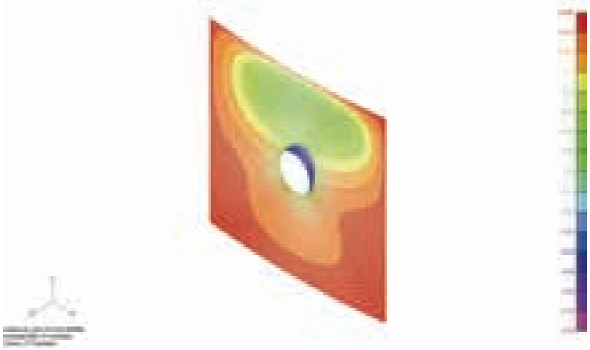


T = 1100°C

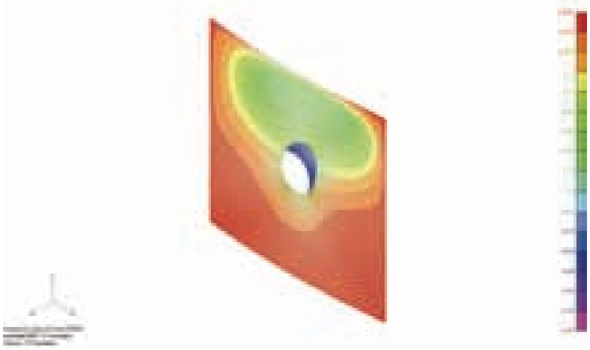
Figure 10. Typical deformation patterns for curved laminate with a hole under thermal loading (a=10 mm)



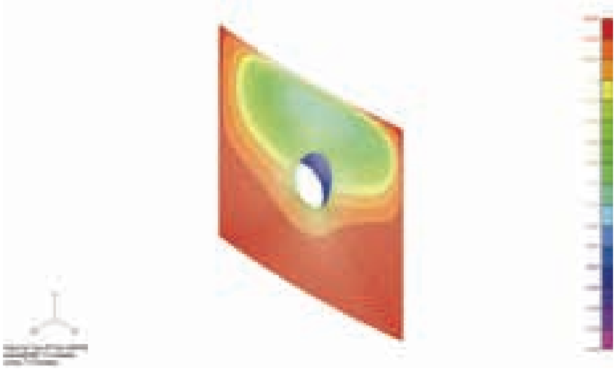
T = 600°C



T = 850°C



T = 1000°C



T = 1100°C

Figure 11. Typical deformation patterns for curved laminate with a hole under thermal loading (a=35 mm)

Conclusions

Finite element method capability has been considered for predicting the geometric nonlinear behavior including thebuckling and postbuckling response of composite laminated plates subjected to combined thermal and mechanical loads.

Thermal buckling and postbuckling analysis of graphite/epoxy laminated plates without temperature dependent material properties is carried out using the finite element method. The analysis reveals that the entire equilibrium path of symmetrically laminated plates under uniform temperature rise consists of three parts: the prebuckling path, the symmetric postbuckling path (after buckling) and the unsymmetrical postbuckling path (after secondary instability).

The considered applications have demonstrated that FEM is versatile and accurate. In general, a close comparison of the finite element results with those of other approaches has been shown. The emphasis here has been on demonstrating the validity of the procedure but clearly the capability could be used in parametric studies.

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Analiza gubitka stabilnosti tankozidnih konstrukcija pod dejstvom termičkih i mehaničkih opterećenja

Pažnja u radu usmerena je na analizu gubitka stabilnosti tankozidnih struktura od izotropnih i kompozitnih materijala. Za tu svrhu korišćen je metod konačnih elemenata koji uzima u obzir efekte geometrijske i materijalne nelinearnosti. Za izotropne materijale nelinearni materijalni model ponašanja je uključen. Efekti materijalne ortotropije na geometrijsku nelinearnost je takođe razmatrana. Za tu svrhu veoma kvalitetni 4-čvorni konačni element ljuski je korišćen. Prezentovana rešenja primene konačnih elemenata su upoređena sa raspoloživim analitičkim, numeričkim i eksperimentalnim rezultatima. Numerički primeri ilustruju valjanost prezentovane proračunske procedure pri rešavanju geometrijski i materijalno nelinearnih problema.

Ključne reči: **tankozidna struktura, izotropni materijali, kompozitni materijali, termičko opterećenje, mehaničko opterečenje, stabilnost srukture, analiza stabilnosti, konačni element, ploča, ljuska.**

Анализ потери устойчивости тонкостеных конструкций под влиянием термических и механических нагрузок

В настоящей работе внимательность направлена к анализе потер устойчивости тонкостеных структур из изотропных и композитных материалов. Таким образом предназначен метод конечных элементов, который учитывает эффекты геометрической и материальной нелинейности. Для изотропных материалов включена нелинейная материальная модель поведения. Тоже рассматриваны и эффекты материальной ортотропии на геометрическую нелинейность и с том целью использован очень качественный конечный элемент оболочки из 4-узлов. Представленые решения применения конечных элементов сопоставлены с аналитическими, цифровыми и экспериментальными результатами в распоряжении. Цифровые примеры растолковывают порядочность представленой расчётной процедуры при решении геометрических и материально нелинейных проблем.

Ключевые слова: тонкостеная конструкция, изотропные материалы, композитные материалы, термическая нагрузка, механическая нагрузка, устойчивость структуры, анализ устойчивости, конечный элемент, плита, оболочка .

Analyse de la perte de stabilité chez les construction à parois minces sous l'effet des charges thermique et mécaniques

Dans ce travail, l'attention principale est portée sur l'analyse de la perte de stabilité chez les structures aux parois mince faites des matériaux isotropes et composites. Dans ce but, on a utilisé la méthode des éléments finis qui considère les effets de la non linéarité géométrique et matérielle. Le modèle non linéaire matériel du comportement est inclus pour les matériaux isotropes. On a étudié aussi les effets de l'orthotropie matérielle quant à la non linéarité géométrique. On a utilisé à cet effet un excellent élément fini des coques à 4 nœuds. Les solutions présentées de l'application des éléments finis ont été comparées avec les disponibles résultats analytiques, numériques et expérimentaux. Les exemples numériques illustrent la validité du procédé numérique au cours de la résolution des problèmes géométriques et non linéaires matériellement.

Mots clés: **construction, matériaux isotropes, matériaux composites, charge thermique, charge mécanique, stabilité structurale, analyse de la stabilité, élément fini, plaque, coque.**