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Evaluating Fracture Mechanical Parameters of a Thermally Loaded Structures

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The Finite Element Method (FEM) is used to determine thermally induced stresses in cracked structural components. These stresses can cause initiation of cracks and their propagation. Particularly, bimaterial interface cracks are an area of interest in evaluating Fracture Mechanical Parameters. The values of induced fields of stresses and displacements are related to the complex Stress Intensity Factors (SIFs). For determining Stress Intesity Factors (SIFs) in this paper, the J-integral approach is used. It is an energy based concept in which the J-integral, J, can be considered as a non-linear elastic equivalent of the energy release rate, G. This statement is valid only in the domain of Linear Elastic Fracture Mechanics (LEFM). The Stress Intesity Factors derived from the J-integral approach is less sensitive than that from the displacement extrapolation method. To illustrate the accuracy and efficency of the J-integral approach in fracture mechanics computations various numerical examples are included in this text.

Key words: **fracture mechanics, thermal stress, multilayered structure, thermal load, thermal fatigue, crack, finite element method, J-integral.**

Introduction

CRACKS and flaws occurring in many structures and components, may cause catastrophic failures. The engineering field of fracture mechanics was established to develop a basic understanding of such crack propagation problems.

Crack propagation behavior is a major issue in a variety of industries. Aerospace structures, gas turbine engines, pressure vessels and pipelines are examples where failure could lead to catastrophic consequences and loss of life.

Fracture mechanics deals with the study of how a crack or flow in a structure propagates under applied loads. It involves correlating analytical predictions of crack propagation and failure with experimental results. The analytical predictions are made by calculating fracture parameters such as stress intensity factors (SIF), which can be used to estimate crack growth rates. The concepts of the linear elastic fracture mechanics, which lead to the plain strain fracture toughness property, K_{IC}, have already been used in engineering applications. The use of the J-integral and its critical value J_C as a fracture criterion has also been developed into elasto-plastic and fully plastic regimes.

Mechanical loading is not the only factor considered in the design structures or structural components. Thus, environmental conditions such as temperature or extensive exposure to irradiation can affect the fracture propensity of a given material [2].

Thermal effects

Although thermally induced stresses have been known for a long time, extensive interest in this subject did not develop until thirty years ago. Today, many high-performance structures and high-accuracy instruments have to consider thermal effects as a critical factor.

Thermal effects on structures can be grouped into the three principal categories:

1. Changes in mechanical properties of materials (elastic modulus, fracture toughness and yield strength),
2. Creep phenomenon associated with time to hold,
3. Stresses arising from temperature change.

A concern about the effects of elevated temperatures under service conditions, in engines, furnaces and in chemical processing facilities has led to a number of problems. These problems, including creep-fatigue interaction, thermal fatigue and stable creep crack growth have been selected as topics of analytical and experimental research investigations.

Another problem area is the effect of thermal stress on the event of crack extension. In the operation of gas turbine engines, thermal stresses can be as high as, or higher than, centrifugal stresses. The worst condition of the combinations of thermal, centrifugal and gas bending stresses at elevated temperatures result in high local stresses which can lead to cracking of turbine blades and rotor disks. Thus, thermal effects should not be ignored [14].

Evaluating stress intensity factors from finite element results

In fracture mechanics, under the linear elastic fracture mechanics (LEFM) assumptions, the stress, strain and displacement fields in the near crack-tip region are determined by the stress intensity factors (SIFs). Therefore, fundamental to the use of the finite element method for LEFM is the extraction of accurate SIFs from the finite element results. A large number of different techniques for extracting SIFs have been presented in literature. In this

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paper the J-integral approach is introduced.

The J-integral is a well-known nonlinear fracture mechanics parameter. Under linear elastic materials assumptions, the J-integral can be interpreted as being equivalent to the energy release rate, G. In its original formulation, it relates the energy release rate of a two-dimensional body to a contour integral. Using a crack coordinate system where the x_1 axis is tangential to the crack and the x_2 axis is perpendicular to the crack the J-integral is defined as:

$$J = \int_{\Gamma} \left(W dx_2 - \sigma_{ij} \frac{\partial u_i}{\partial x_1} n_j dS \right) \tag{1}$$

where W is the strain energy density and the path Γ on which the integral is taken is an arbitrarily chosen contour beginning at any point on the lower crack face leading anticlockwise around the crack tip to the upper face (Fig.1).

Unfortunately, the J-integral is restricted to two-dimensional bodies with external loading. The J-integral is path dependent for cases that include residual, inertial or thermal stress terms or loading along the crack face. It cannot be used for three-dimensional structures or non-homogeneous materials in the direction of crack advance [6].

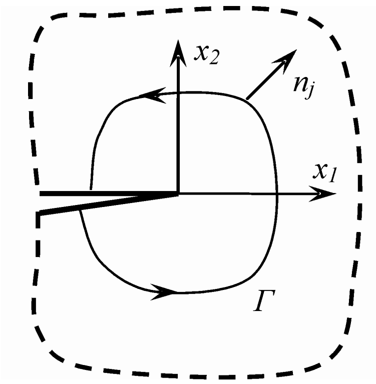


Figure 1.

In many cases, the crack tip stress intensity factors must be determined for cracked structures subjected to thermal load. The stress intensity factor of the interaction of mechanical and thermal loads is given as:

$$K_I = (K_I)_M + (K_I)_T \tag{2}$$

where $(K_I)_M$ and $(K_I)_T$ are the stress intensity factors due to mechanical and thermal loads, respectively. The J-integral formulated by Rice (eq.1) is one of the best methods for analyzing fractured structures. The J – integral is modified for the cases which include elastoplasticity or thermal stress retaining path independence in these cases.

Blackburn [18] proposed the following form of the J^*-integral for three-dimensional cases:

$$J^* = \lim_{r \to 0} \int_{r} \left(\frac{1}{2} \sigma_{ij} \frac{\partial u_i}{\partial x_j} dx_2 - \sigma_{ij} \frac{\partial u_i}{\partial X_1} n_j dS \right) \tag{3}$$

The integral (3) can be evaluated by applying the Greens theorem, to give

$$\begin{aligned} J^* = & \int_{\Gamma} \left(W dX_2 - \sigma_{ij} \frac{\partial u_i}{\partial X_1} n_j dS \right) + \\ & + \lim_{r \to 0} \int_{A_0} \left(W \frac{\partial u_i}{\partial X_1} - \frac{1}{2} \frac{\partial \sigma_{ij}}{\partial X_1} \frac{\partial u_i}{\partial X_1} - \frac{\partial}{\partial X_3} \left| \sigma_{i3} \frac{\partial u_i}{\partial X_1} \right| \right) dA \end{aligned} \tag{4}$$

where A_0 is the area inside any contour away from the crack tip region.

Wilson's and Ainsworth's proposed the two-dimensional thermal J^*-integral for linear thermo elastic materials and can be expressed in a simple form:

$$J^* = \int_{\Gamma} \left(W^* dX_2 - \sigma_{ij} \frac{\partial u_i}{\partial X_1} n_j dS \right) + \frac{E\alpha}{1-2\nu} \int \varepsilon_{ii} \frac{\partial \theta}{\partial X_1} dA \tag{5}$$

$$W^* = W - \frac{E\alpha\theta}{2(1-2\nu)} \varepsilon_{ii} \tag{6}$$

$$\sigma_{ij} = \lambda \varepsilon_{ii} \delta_{ij} + 2\mu \varepsilon_{ij} - \frac{E\alpha}{1-2\nu} \theta \delta_{ij} \tag{7}$$

$$W = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} \tag{8}$$

where μ and λ are the Lame constants, θ is the temperature and α is the coefficient of thermal expansion. The physical interpretation of J is as the energy release rate, and hence in the case of a linear thermo elastic material

$$J^* = \left(1-\nu^2\right) K_I^2 / E \tag{9}$$

FEM applications together with the J^*-integral approach in thermo elastic fracture mechanics, including behavior of cracked structure subjected to transient thermal singularities, are illustrated in references [16, 17].

Numerical examples

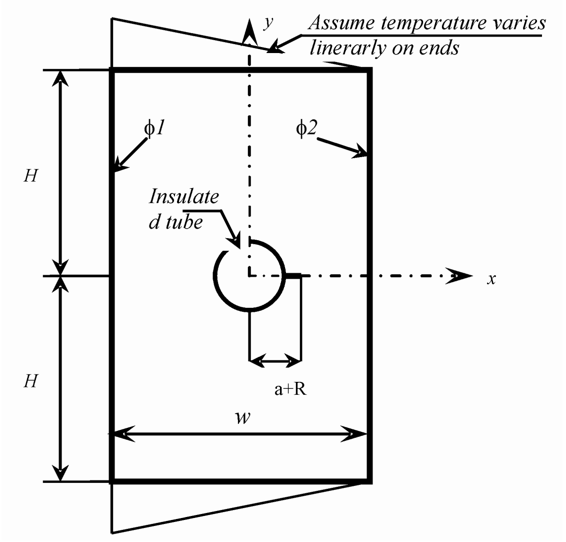
Example 1

In this paper, a numerical investigation of the temperature effect for a finite sheet made of aluminum alloy (Young's modulus 7400 daN/mm^2 and Poisson ratio 0.33), with traction-free boundaries was conducted. The finite element analysis incorporates a feature that thermo-mechanical properties of material are described as a function of temperature.

The finite element mesh used in the analysis of a specimen is shown in Fig.2. Around the crack tip there are 16 quadrilateral, eight-node plane strain elements modified into degenerated triangular quarter point elements, which serve to model the singular stress behavior at the crack tip more accurately. The remainders of elements are quadrilateral, eight-node, plain strain elements. The analysis consists of a steady state thermal finite element analysis to determine the temperature distribution followed by a quasi-static finite element analysis to determine the displacements and stresses. These results are used to calculate the J-integral value and SIFs (K_I).

An apparent temperature gradient, which is defined as a temperature gradient across the plate width, in the absence of the hole and crack, is used for convenience of describing the temperature distribution. Since the problem is linear, only two different temperature gradients are necessary to obtain the K_I – temperature gradient relationship. The crack length measured from the center of the hole to the crack tip remains 0.25w for each size of the hole.

Fig.3 shows the results of stress intensity factors versus temperature gradients for two different sizes of the hole. The stress intensity factors are derived from the values of the J-integral approach (eq.9) and the crack opening displacement method. The values of stress intensity factors evaluated by two different techniques agree well. Thus, for the condition a+R=0.25w=constant, the values of SIFs are smaller for higher values of the hole size.



Temperatures $\phi 1$ *i* $\phi 2$ *are constants and* $\phi 1 > \phi 2$

Figure 2.

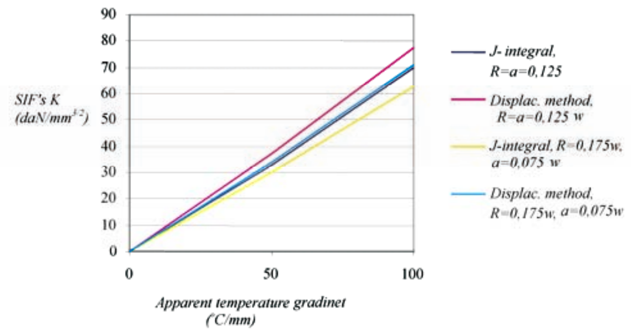


Figure 3. SIFs versus temperature gradients for traction-free boundary conditions

Example 2.

In practice, biomaterial structures are common in use. The use of ceramic coatings on metal components to provide thermal insulation is an example of such structures. During operational life, these structures with different coefficients of thermal expansion lead to thermally induced stresses, which involve fracture of coating and loss of thermal protection.

In this example, a thermally loaded ceramic coating with an interface crack is introduced. The stress intensity factors are evaluated for the specimen shown in Fig.4. The finite element mesh used in the analysis is shown in Fig.5. The two upper rows of elements constitute the ceramic coating (ZrO2), and the elements below this make the substrate material (AlSI 304 stainless steel). These materials are modeled as linear elastic and isotropic the properties of which are shown in Table 1. The termomechanical properties are described as functions of temperature and derived from a number of sources shown in Fig.6.

The crack extends 7 mm from the left side of the mesh (a/W=0.41). The boundary conditions for the thermal analysis are the prescribed temperature of 1200 K on the upper surface and 500 K on the lower surface. The sides of the mesh are insulated and there is no heat flux across them. The heat transfer across the crack is not modeled in this analysis. The resulting nodal temperatures correspond to a steady state condition achieved after the imposed boundary conditions are applied and the materials reach thermal equilibrium. For the stress analysis, the specimen was constrained in the y direction along its bottom surface and pinned at its lower left corner. The other surfaces are all traction free, so that the stresses near the crack tip are caused primarily by mismatch of thermal expansion coefficients.

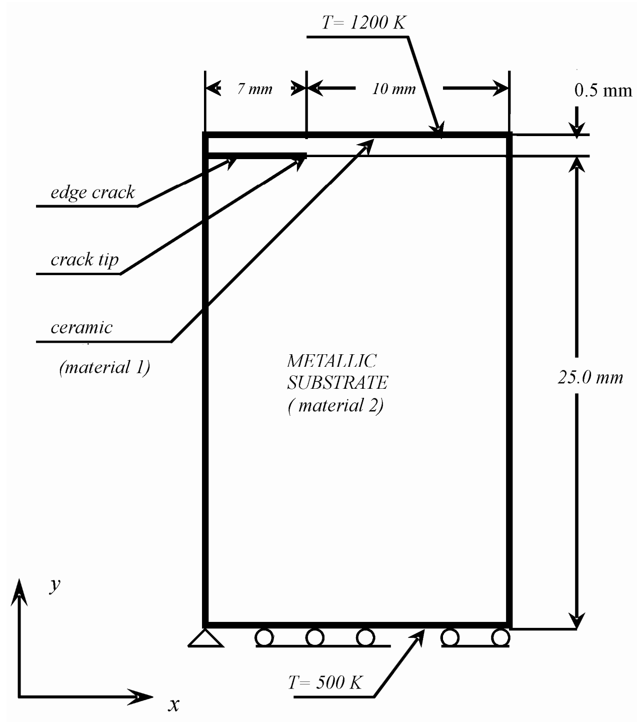


Figure 4.

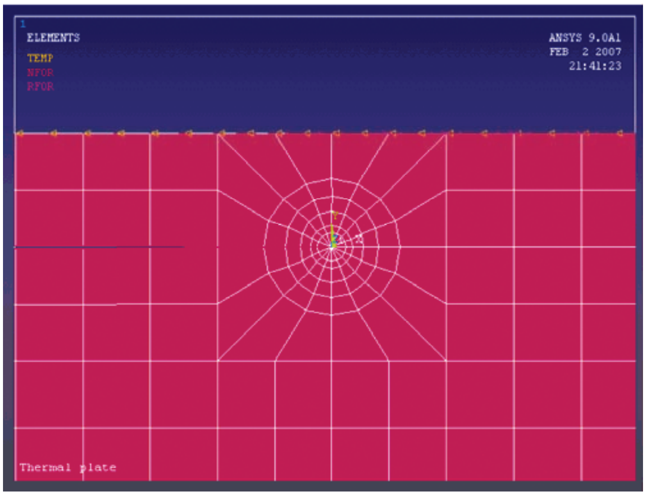


Figure 5. Finite element mesh, close up view around the crack tip

Table 1.

Property	ZrO2	AISI 304 steel
Density (kg/m^3)	5200	8000
Young's modulus (GPa)	200	193
Poisson ratio	0.23	0.25

In this numerical example, the stress intensity factor is evaluated from the numerical data generated by the finite element analysis and the J-integral approach. Using this procedure, the stress intensity factor is calculated to be 8.24 MPa$\sqrt{m}$. This value of the stress intensity factor could be compared with the result 9.10 MPa$\sqrt{m}$, evaluated by T.C.Miller and R.Chona in their paper "Finite Element Analysis of a thermally loaded interface crack in a ceramic coating" [21]. Comparing stress intensity factors, small difference in values resulted in not taking into consideration heat transfer across the crack.

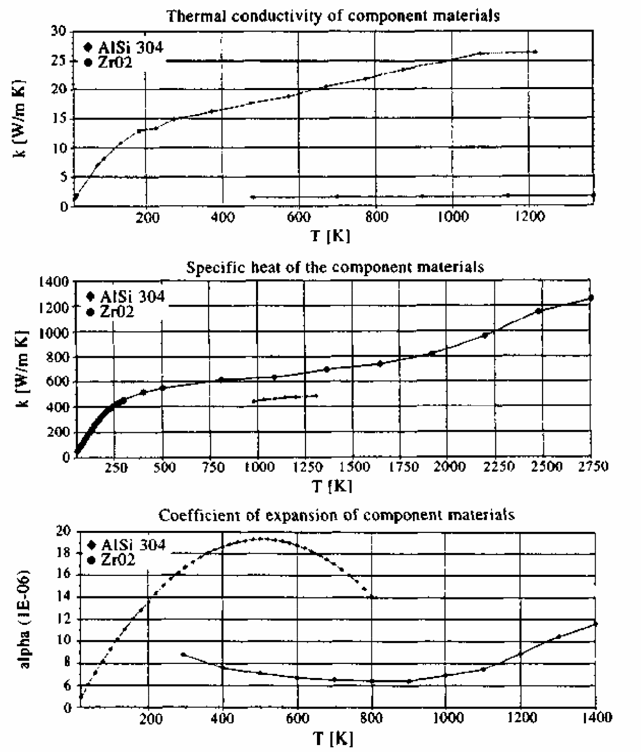


Figure 6. Temperature dependant thermo-mechanical properties of the component materials

Conclusion

In this paper two different thermally loaded structures are investigated. In the first example, the influence of a simple supported structure with a different size of a hole on the stress intensity factor can be seen in Fig.3. The results are plotted when the temperature along the right edge is maintained at the initial or reference temperature. It is seen that values of stress intensity factors are smaller for higher values of the hole size.

In the second example, the finite element method has been used to model the thermally induced stresses in an interface crack between a ceramic coating and a metallic substrate. The stress intensity factor was determined using the J-integral approach. The J-integral approach is the easiest means of calculating the stress intensity factor $|K|$. This method is easy to use when software supports determination of the contour integral. The results are accurate even for coarse meshes because the contours can be chosen remotely from the near tip region. The disadvantage of this method is that it cannot determine the phase angle.

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Određivanje parametara mehanike loma

Termički naponi mogu uzrokovati pojavu naprslina i njihovo širenje u strukturalnim elementima. Često se metoda konačnih elemenata koristi za određivanje termičkih napona u strukturama sa naprslinama. Od naročite važnosti je određivanje parametara mehanike loma kod struktura sastavljenih od različitih materijala. Kod takvih struktura vrednosti polja napona i pomeranja su u relaciji sa kompleksnim faktorom intenziteta napona. Za određivanje faktora intenziteta napona u ovom radu korišćen je J-integral pristup. Ovaj pristup se bazira na energetskom principu u kome se J-integral može posmatrati kao nelinearan parametar mehanike loma koji je ekvivalentan stepenu promene energije, G. Ova pretpostavka je validna samo u domenu linearno elastične mehanike loma. Vrednosti faktora intenziteta napona sračunati koristeći J-integral pristup su manje osetljiv nego sračunati metodom ekstrapolacije pomeranja. Da bi pokazali tačnost i efikasnost J-integral pristup u izračunavanju parametara mehanike loma u ovom radu su korišćeni različiti primeri.

Ključne reči: **mehanika loma, termičko opterećenje, višeslojna struktura, termičko naprezanje, termički zamor, prskotina, metoda konačnih elemenata, J-integral.**

Определение параметров механики излома у структур подвержённых термическим нагрузкам

Термические напряжения могут вызывать трещины и их расширение в элементах структуры. Методом конечных элементов часто пользуются для определения термических напряжений в структурах с трещинами. Особое значение имеет определение параметров механики излома у структур составленных из различных материалов. У таких структур значения поля напряжения и сдвига находятся в отношении с комплексным фактором интенсивности напряжения. Для определения фактора интенсивности напряжения в этой работе пользован J-интеграл подход. Этот подход основывается на энергетическом принципе, в котором J-интеграл возможно рассматривать в роли нелинейного параметра механики излома, эквивалентного степени изменения энергии G. Это предположение бывает действительным только в области линейной эластичной механики излома. Значения фактора интенсивности напряжения, рассчитаны пользуя J-интеграл подход, меньше чувствительны чем рассчитаные методом экстраполяции сдвига. Чтобы показать точность и эффективность J-интеграл подхода во высчитывании параметров механики излома в настоящей работе пользованы различные примеры.

Ключевые слова: механика излома, термическая нагрузка, многослойная структура, термическое напряжение, термическая усталость, трещина, метод конечных элементов, J-интеграл.

Evaluation des paramètres de la mécanique de fracture chez les structures sous les charges thermiques

Les charges thermiques peuvent causer l'apparition des fissures et leur propagation dans les éléments de structure. La méthode des éléments finis est souvent utilisée pour l'évaluation des charges thermiques chez les structures à fissures. L'évaluation des paramètres de la mécanique de fracture est très importante chez les structures composées de différents matériaux. Chez ces structures les valeurs du champ de charge et les déplacements son en relation avec le facteur complexe de l'intensité de charge. Pour évaluer le facteur de l'intensité de charge on a employé l'approche avec l'intégrale J. Cette approche est basée sur le principe énergétique où l'intégrale J peut être considéré comme le paramètre non-linéaire de la mécanique de fracture qui est équivalent au degré du changement d'énergie G. Cette supposition est valable seulement dans le domaine de la mécanique élastique linéaire de fracture. Les valeurs du facteur de l'intensité de charge, calculées par l'intégrale J sont moins sensibles que celles obtenues par la méthode de l'extrapolation de déplacement. Pour démontrer l'exactitude et l'efficacité de l'approche par l'intégrale J dans l'évaluation de la mécanique de fracture on a employé divers exemples dans ce travail.

Mots clés: **mécanique de fracture, charge thermique, structure en couches, contrainte thermique, fatigue thermique, fissure, méthode des éléments finis, intégrale J.**