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The crack Growth Analysis on a Real Structure Using the X-FEM and EFG Methods

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The finite element method is the most universal and the best known one among numerical methods. However, for solving problems with specific conditions, such as singularities in the case of fracture mechanics and the crack growth, the EFG (Element-Free Galerkin) and the X-FEM (Extended Finite Element Method) are the methods that have advantages in comparison to the standard FEM. In this paper, we have shown the numerical simulation of the crack growth in the steam turbine housing using standard FEM, X-FEM and EFG method. For calculation of the stress intensity factors (SIFs) we used the J-integral. In this paper equivalent domain integral (EDI) method for evaluation of the J-integral is presented. The J-EDI method for determining SIFs in the standard FE, the X-FE and the EFG framework is used. Using the developed software, the stress intensity factors of the steam turbine housing were calculated and compared with the corresponding results obtained with conventional FEM software.

Key words: **fracture mechanics, crack, crack growth, finite element method, element free galerkin method (EFG), X-FEM method, J-integral.**

Introduction

THE finite element method is widely used in industrial design applications and many different software packages based on FEM techniques have been developed. It has proved to be very well suited for the study of crack initiation and crack growth [1].

Over the past few decades, several approaches have been proposed to model crack problems: method based on quarter-point finite elements [2]. To avoid the re-meshing step in crack modeling, diverse techniques were proposed: the incorporation of a discontinuous mode on the element level [3], a moving mesh technique [4], and an enrichment technique based on a partition-of unity X-FEM.

The essential idea in the extended finite element method is to add discontinuous enrichment functions to the finite element approximation using the partition of unity. An overview of the developments of the X-FEM method has been given by Karihaloo and Xiao [5].

In the X-FEM, the enrichment functions are added to the finite element approximation for representing the inter-element discontinuous field. The basis of the X-FEM method is presented in [6, 7]. The approximation space is extended to contain an additional family of functions that, in some cases, are able to represent the discontinuity of the solution. The X-FEM is based on the Partition of Unity (PU), a class of methods where a direct modification of the displacement approximation is involved. A Near Tip (NT) function, and the Heaviside function are used to enrich the finite element approximation in the X-FEM. We used 6x6 Gauss quadrature for the integration of enriched elements.

The EFG method has been applied to fracture mechanics problems, i.e. to quasi-static and dynamic fracture. The basis of the EFG method is presented in [9, 10, 11].

In this paper the equivalent domain integral (EDI) method for evaluation of the J-integral is presented as well as a procedure for calculation of the stress intensity factors (SIFs). The J-EDI numerical method is very useful for defining the SIFs parameters. This method could be also applied for post-processing in the FE framework as well as in the EFG method and the X-FEM approach. The results of the stress intensity factor calculations and fatigue life estimations using the X-FEM are compared with singular quarter-point (QP) elements [16, 17, 20] and good agreements are obtained.

Determination of the SIF usingthe J-EDI method

The contour J-integral [12] is not in the best-suited form for finite element calculations. Therefore, we transform the contour integral into an equivalent domain form. The equivalent domain integral method (EDI) is an alternative way to obtain the J-integral. The EDI approach has the advantage that the effect of body forces can be included very easily. The contour integral is replaced by an integral over a finite-size domain [13, 14, 15]:

$$J_1 = \int_A \left(\sigma_{ij} u_{i,1} - W \delta_{1j} \right) q_{,j} dA \qquad i, j = 1, 2 . \tag{1}$$

where W is the strain energy density given by:

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$$W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} = \frac{1}{2}C_{ijkl}\varepsilon_{kl}\varepsilon_{ij} \tag{2}$$

and n_j is the component of the outward normal vector to the contour integration Γ_S around the crack tip, σ_{ij} is the stress tensor, ε_{ij} is the strain tensor, C_{ijkl} is the constitutive tensor, u_i are the components of the displacement vector, where the q_j is the derivate of the weight function per coordinates x.

With the isoparametric finite element formulation the distribution of q within the elements is determined by a standard interpolation scheme with the use of the shape functions hi:

$$q = \sum_{i=1}^{m} h_i Q_i \tag{3}$$

where Q_i are the values of the weight function at the nodal points, and m is the number of nodes. The spatial derivatives of q can be found using the usual procedures for isoparametric elements.

The equivalent domain integral in 2D can be calculated as a sum of the discretized values of eqs.(1), [13, 14, 15, 16]:

$$J_k = \sum_{\substack{elements \\ in\ A}} \sum_{p=1}^{P} \left[\left(\sigma_{ij}\frac{\partial u_i}{\partial X_k} - W\delta_{kj} \right) \frac{\partial q}{\partial X_j} \det\left(\frac{\partial X_m}{\partial \eta_n} \right) \right]_p w_p \tag{4}$$

The terms within $[\cdot]_p$ are evaluated at the Gauss points with the use of the Gauss weight factors for each point are w_p. The present formulation is for a structure of homogeneous material in which no body forces are present.

The J-integral evaluation in this paper is used for calculating SIFs in FEM, EFG and X-FEM.

The displacement approximation in the X-FEM

The displacement approximation $\mathbf{u}(\mathbf{x})$ in the X-FEM is decomposed into a continuous and an enrichment part, as:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}_{con}(\mathbf{x}) + \mathbf{u}_{enrh}(\mathbf{x}) \,, \tag{5}$$

where: the continuous displacement approximation $\mathbf{u}^h_{con}(\mathbf{x}) = \sum N_I(\mathbf{x})\mathbf{u}_I$ is the standard approximation in the FEM, and $\mathbf{u}_{enrh}(\mathbf{x})$ is the enrichment part of displacement approximation near the crack, (see Fig.1).

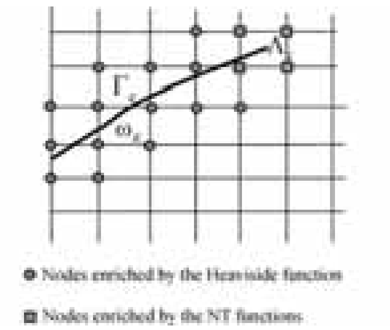


Figure 1. Enrichment nodes near the crack

In the particular instance of 2D crack modeling, the enriched displacement approximation is written as [6, 7, 16]:

$$\mathbf{u}^h_{enrh}(\mathbf{x}) = \sum_{I \in \mathcal{N}_a} N_I(\mathbf{x})H(\mathbf{x})\mathbf{a}_I + \sum_{I \in \mathcal{N}_b} N_I(\mathbf{x})F_\alpha(\mathbf{x})\mathbf{b}_I^\alpha ,\ \alpha{=}1.4 \tag{6}$$

where N_I, I=(1, N) are the finite element shape functions; $H(\mathbf{x})$ is the Heaviside function, and $F_\alpha(\mathbf{x})$, α=(1.4) are the Westergaard asymptotic Near-Tip (NT) functions:

$$F_1(r,\theta) = \sqrt{r}\sin\frac{\theta}{2},\ \ F_2(r,\theta) = \sqrt{r}\cos\frac{\theta}{2},$$
$$F_3(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}\sin\theta,\ \ F_4(r,\theta) = \sqrt{r}\cos\frac{\theta}{2}\sin\theta, \tag{7}$$

where are: $r(\mathbf{x})$ and $\theta(\mathbf{x})$ polar coordinates of the point $\mathbf{x}$. The polar coordinate system is attached to the crack tip.

In the equations (6) $\mathbf{a}_I$ are additional degrees of freedom associated with the Heaviside (discontinuous) function, $\mathbf{b}_I^\alpha$ are additional degrees of freedom associated with the Westergaard asymptotic crack-tip functions, $\mathcal{N}_a$ is the number of nodes per elements enriched by the Heaviside function and $\mathcal{N}_b$ is the number of nodes per elements enriched by the NT functions.

The EFG interpolation of thedisplacement field

In the EFG method, due to the application of the MLS (moving least-square) approximation, displacement $u^h(x,y,z) = u^h(\mathbf{x})$ is [18]:

$$u^h(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x})\, a_j(\mathbf{x}) = \mathbf{p}^T(\mathbf{x})\,\mathbf{a}(\mathbf{x}) \tag{8}$$

where $p_j(\mathbf{x})$ are the basic functions of the coordinates of free points, and $a_j(\mathbf{x})$ are the coefficients, which are the functions of the spatial coordinates $\mathbf{x}$. In general case, the basis functions for two-dimensional problems (in this paper, we have used the linear base m =3):

$$\mathbf{p}^T(\mathbf{x}) = [1, x, y] \tag{9}$$

The coefficients $\mathbf{a}(\mathbf{x})$ in (8) for every point $\mathbf{x}$ have been obtained by minimization of the middling form:

$$\chi(\mathbf{x}) = \sum_{I=1}^{n} w_I(\mathbf{x})[\mathbf{p}^T(\mathbf{x}_I)\,\mathbf{a}(\mathbf{x}) - \tilde{u}^I]^2 \tag{10}$$

and have the value:

$$\mathbf{a}(\mathbf{x}) = \mathbf{A}^{-1}(\mathbf{x})\,\mathbf{P}^T\mathbf{W}(\mathbf{x})\,\tilde{\mathbf{u}} \tag{11}$$

where: $\tilde{u}^I$ is the displacement of the free point I, $w_I(\mathbf{x})$ is the weight function of the free point I, and n is the number of free points which influence the integration point, Fig.2. Matrix $\mathbf{A}$ has been defined in the following way:

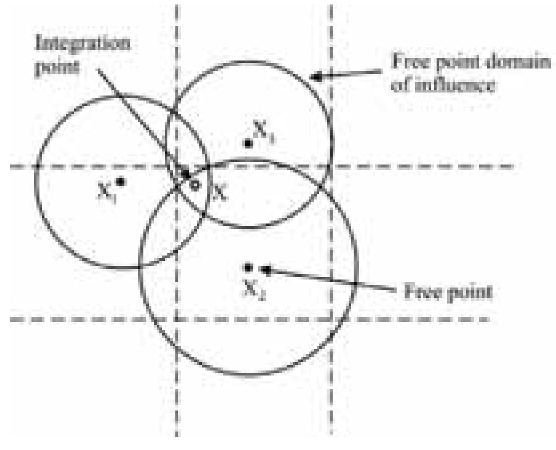


Figure 2. Free points domain in relation to the integration point

$$\mathbf{A}(\mathbf{x}) = \mathbf{P}^T \mathbf{W}(\mathbf{x})\,\mathbf{P} \tag{12}$$

where:

$$w_{IJ} = w_J(\mathbf{x} - \mathbf{x}_J)\,\delta_{IJ} \tag{13}$$

and

$$\mathbf{P}_I = \mathbf{p}_I^T \tag{14}$$

In the EFG method, the weight functions $w_I(\mathbf{x})$ are generally monotonously falling functions of the distance $\| \mathbf{x} - \mathbf{x}_I \|$. These functions influence to the displacement $u^h(\mathbf{x})$. This is obvious when $\mathbf{a}(\mathbf{x})$ from (13) is replaced in (8). In this paper, we used the following form of the weight function [4]:

$$w_I(d_I) = \begin{Bmatrix} \dfrac{e^{-\left(\frac{d_I}{c}\right)^{2k_I}} - e^{-\left(\frac{d_{\max I}}{c}\right)^{2k_I}}}{1 - e^{-\left(\frac{d_{\max I}}{c}\right)^{2k_I}}} & d_I \le d_{\max I} \\ 0 & d_I > d_{\max I} \end{Bmatrix} \tag{15}$$

where $d_I = \|\mathbf{x} - \mathbf{x}_I\|$ is the distance from free the point $\mathbf{x}_I$ to the interpolated point $\mathbf{x}$, and $d_{\max I}$ defines the maximal area of weight function influence for each free point - influence radius. The coefficient c has been differently defined in literature. In this paper, we have used a definition according to [19]:

$$c = \alpha\, d_{\max I} \tag{16}$$

In this case, the recommendation for the value of α is 0.4. According to [4] we have adopted a value for the coefficient $K_I = 1$.

Numerical examples

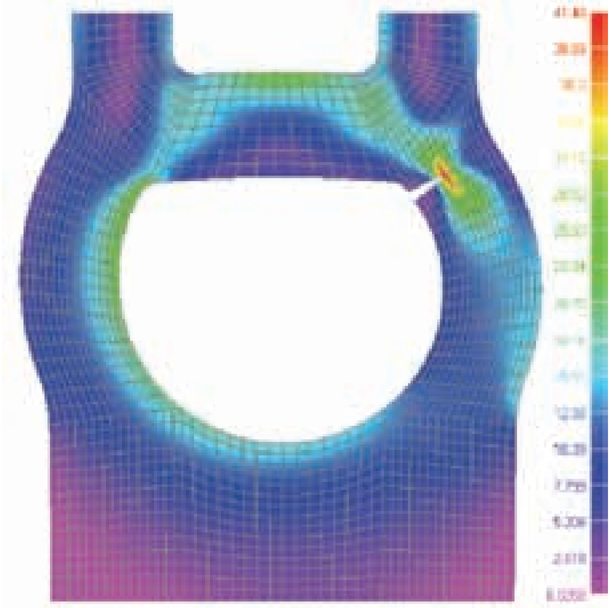
Detailed validations of the X-FEM method in fracture mechanics and comparissons with quarter-point (QP) singular finite elements are given in references [16, 17].

This X-FEM methodology is used and implemented in the PAK software [8] based on the standard finite element approximation. In this example the stress intensity factor of the crack located in a steam turbine housing is calculated. Due to the fact that there is no analytical solution for this example, the numerical results obtained with EFG and X-FEM were compared with the corresponding ones obtained using the standard FEM.

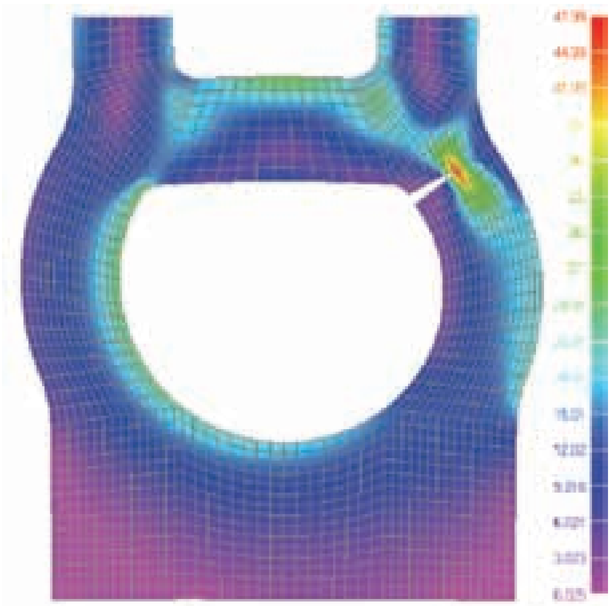
The first step was generating a 2-D FE model of the lower housing part with insulation. After that, following steps were carrying out:

- Calculate the temperature field in the nominal regime as well as the corresponding stress field;
- Calculate the stress and strain fields of the turbine for different crack lengths (20–60 mm);
- Analyze influence of the crack length on the corresponding stress field as well as on the stress intensity factor;

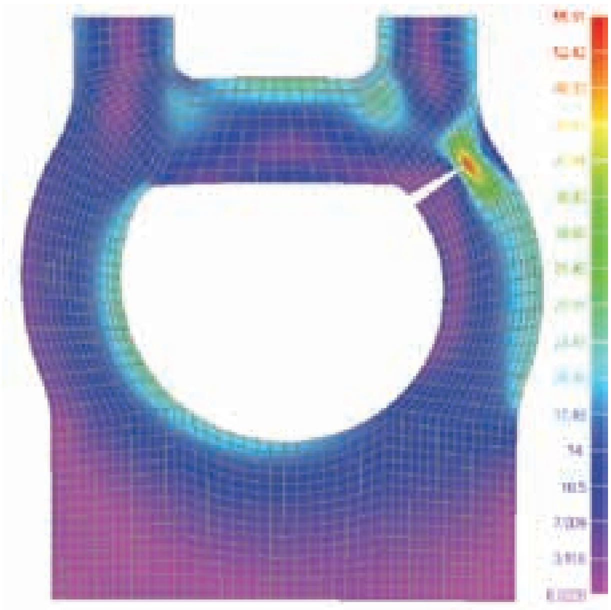
The calculation of the SIF for the crack in the steam turbine housing is performed using the standard FEM, EFG method and X-FEM. In the standard FEM and EFG method, a 2D mesh with eight nodes per element is used. The number of Gauss' integration points in both methods is 2x2. In the X-FEM, linear four-node elements are used and 6x6 Gauss quadrature only in the part of the domain with enriched n.



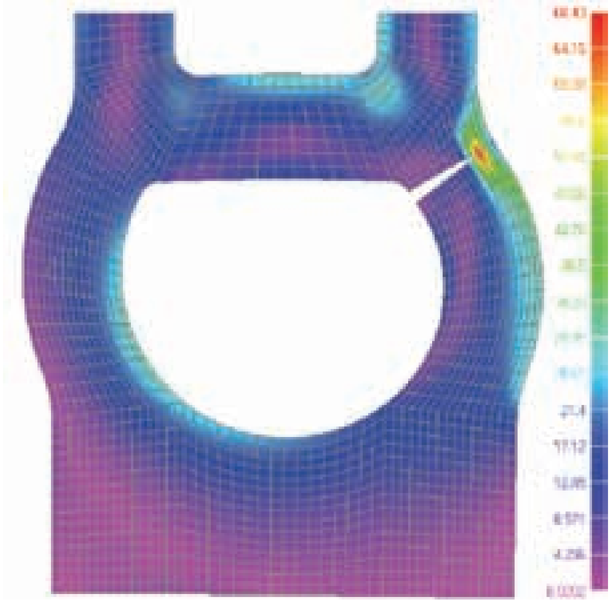
a) crack length of30 mm



b) crack length of 40 mm



c) crack length of 50 mm



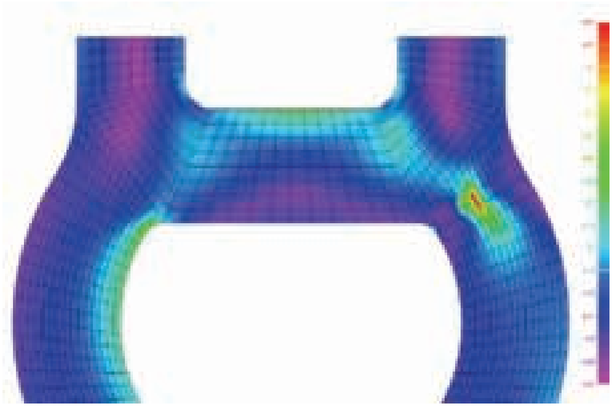
d) crack length of 60 mm

Figure 3. Effective stress field for a number of crack lengths by the FEM and EFGM

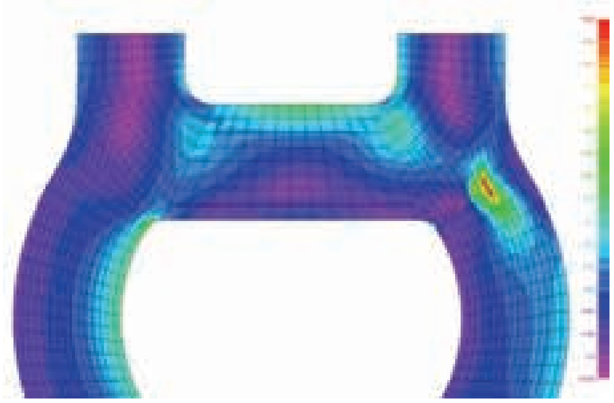
The effective stresses for the 2D turbine model without insulation, for a number of crack lengths using the FEM and the EFGM are shown in Fig.3.

Effective stress fields for the 2D turbine model without insulation, for a number of crack lengths using the X-FEM are shown in Fig.4.

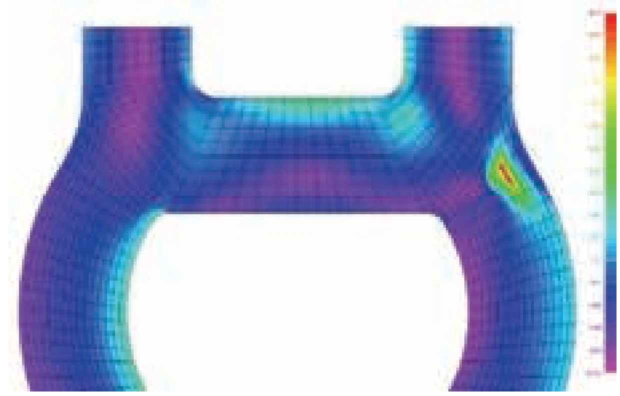
The crack path is independent of the mesh structure, as it is shown in Fig.4. The crack growth is considered in 8 steps as well as in reference [15].



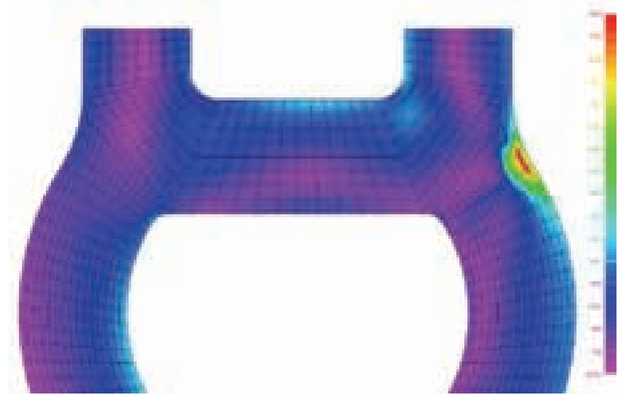
a) crack length of 30 mm



b) crack length of 40 mm



c) crack length of 50 mm



crack length of 60 mm

Figure 4. Effective stress fields for a number of crack lengths using the X-FEM

The results shown in Fig.5 were obtained using the standard FEM, EFG and X-FEM. The J-EDI approach for defining stress intensity factor was used in those methodologies.

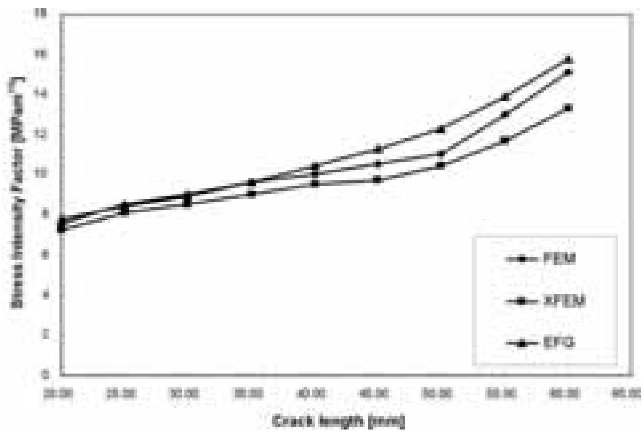


Figure 5. Relationship between the stress intensity factor K_I and the crack length

We have displayed only the opening mode stress intensity factor, K_I, because it is dominating in this example. In the Fig.5, the relationship between the stress intensity factor K_I and the crack length is shown. Increasing the crack length from 20 mm to 60 mm causes increasing of the stress intensity factor, as illustrated in Fig.5.

Conclusions

In order to investigate different methodologies and their influence on the result in calculation of a real construction, we have written the program in FORTRAN and integrated in PAK. The developed program is based on the standard FEM, EFGM and X-FEM.

As it can be seen in the shown example, the FEM and the EFGM give better results than the X-FEM, but generally, the results are very similar. Therefore, in practical usage, calculation of a real structure, any of these methods can be applied. The advantage of the X-FEM related to the standard FEM is feasibility to use the fixed finite element mesh, whereby the crack growth is independent of the mesh.

Also, we can see, by the virtue of the obtained results, that the order of interpolation of the finite element has greater influence on the results than the order of the Gauss quadrature. The difference in numerical results could be addressed to a different order of element interpolation, which is used in the X-FEM and the EFG.

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Analiza širenja prsline kod realnih konstrukcija primenom X-FEM i EFG metodama

Metoda konačnih elemenata je univerzalna i jedna od najpoznatijih numeričkih metoda. Međutim, za rešavanje problema sa specifičnim uslovima, kao što su to singulariteti u slučajevima mehanike loma i kod širenja prsline EFG i X-FEM metode imaju prednosti u poređenju sa standardnom metodom konačnih elemenata (MKE). U ovom radu mi smo prezentovali numeričku simulaciju širenja prsline kod kućišta turbine koristeći standardnu MKE kao i poboljšane EFG i X-FEM metode. Za sračunavanje faktora intenziteta napona (FIN) mi smo koristili J-integral metodu. Za tu svrhu u radu je korišćen EDI metod pri sračunavanju J-integrala. J-EDI metod je korišćen za određivanje faktora intenziteta napona koristeći standardnu MKE kao i X-FEM i EFG metode. Koristeći razvijeni softver, faktori intenziteta napona su sračunati kod kućišta turbine i rezultati su upoređeni sa raspoloživim rezultatima dobijenih primenom konvencionalnog MKE softvera.

Ključne reči: **mehanika loma, prskotina, rast prskotine, metoda konačnih elemenata, Galerkinova metoda slobodnih elemenata, X-FEM metoda, J-integral.**

Анализ расширения трещин у реальных конструкций с применением X-FEM и EFG методов

Метод конечных элементов является универсальным и одним из самых известных цифровых методов. Но, для решений проблем со специфическими условиями в роде единственности в случаях механики излома и у расширении трещины EFG и X-FEM методы имеют преимущества в сравнении со стандартным методом конечных элементов (МКЭ). В настоящей работе мы представили цифровую симуляцию расширения трещины у диска турбины пользуясь стандартным МКЭ, а в том числе и улучшенными EFG и X-FEM методами. Для подсчитывания фактора интенсивности напряжения (ФИН) мы пользовались Й-интеграл методом. С этой целью в работе использован ЕДИ-метод при подсчитывании Й-интеграла. Й-ЕДИ метод использован для определения фактора интенсивности напряжения при использовании стандартного МКЭ, а в том числе и EFG и X-FEM методов. Пользуясь развитым программным обеспечением, факторы интенсивности напряжения подсчитаны у диска турбины и результаты сравниваны со результатами в распоряжении, полученными применением принятого конвенционального МКЭ програмного обеспечения.

Ключевые слова: механика жёсткого тела, механика излома, трещина, рост трещины, метод конечных элементов, метод Галеркина свободных элементов, X-FEM метод, Й-интеграл.

Analyse de la croissance de la fissure chez les constructions réelles par les méthodes X-FEM et EFG

La méthode des éléments finis est universelle et l'une des méthodes numériques les plus connues. Mais pour résoudre les problèmes avec les conditions spécifiques tels que les singularités dans les cas de la mécanique de fracture ou la croissance de la fissure, les méthodes EFG et X-FEM ont l'avantage sur la méthode des éléments finis. Dans ce travail nous avons présenté la simulation numérique de la croissance de fissure dans le logement de la turbine à l'aide de la méthode des éléments finis ainsi que par les méthodes EFG et X-FEM améliorées. Pour calculer le facteur de l'intensité de tension nous avons utilisé la méthode par intégrale J.A cet effet on a applique la méthode EDI pour calculer intégrale J. La méthode J-EDI a servi pour déterminer le facteur d'intensité de tension au moyen de la MKE ordinaire ainsi que par les méthodes X-FEM et EFG. Employant un logiciel développé, les facteurs d'intensité de tension ont été calculés pour le logement de la turbine. Les résultats obtenus ont été comparé avec les résultats réalisés par un logiciel MKE conventionnel.

Mots clés: **mécanique de fracture, fissure, croissance de la fissure, méthode des éléments finis, méthode des éléments libres,méthode X-FEM, intégrale J**