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Dynamic Analysis of Hybrid Aluminum/Composite Shafts

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The aim of modern mechanisms is to achieve the highest operating speeds as possible, so a precise dynamic analysis of stability of shafts is very important. The analysis of the vibratory characteristics (fundamental natural frequencies, critical speed and critical torque) of a composite shaft from a real construction, subject to orientation of fibres and geometrical measures of shaft, is presented in the paper. Owing to a high value of specific stiffness, the shafts made of composite materials have considerably higher values of fundamental natural frequencies compared to steel shafts. An important advantage of composite materials for manufacturing shafts is that their application enables the increase of shaft length that would lead to the occurrence of bending resonance in case other materials were used. It is thus suitable to make shafts for automobiles, trucks and other relevant systems out of composite materials in the future.

Key words: **shaft, composite materials, aluminium, dynamic analysis, vibrations, vibration stability.**

Introduction

MODERN machine constructions, apart from new technologies having been introduced in their design and manufacture, demand application of new materials that may have higher specific strength and hardness compared to traditional materials. Composite materials meet these criteria. Their values of impact tenacity, strength and hardness and especially of resistance to fatigue and vibratory and acoustical loads may be enhanced in relation to known metals and alloys. Distinctiveness of composite materials is in conjunction with different features in order to gain materials with better characteristics in relation to components. These features may be varied, according to needs, by selection of components, their quantity, allocation and orientation within the material. Due to their good characteristics, composite materials are increasingly used for manufacture of transmission shafts today.

The shell theory is most frequently used for the analysis of vibratory characteristics of composite shafts. Most of the studies for analysis of shafts are based on the classic theory of thin shells or on the theory of thick shells.

Zinberg and *Symonds* (1970) have analysed critical speeds of rotational anisotropic shaft. Their experiment has confirmed the advantages of composite shafts in relation to aluminum alloy shafts. *Reis* and *Goldman* [9] (1987) have applied the Finite Element Method (FME) to analyse the critical speed of thin walled laminate composite shafts and emphasised great resistance of composite shafts under the action of dynamic loads. *Lim* and *Darlow* [7] (1986) have presented an optimal calculation of composite drive shafts with a variation of laminate diameters. *Lam* and *Loy* [5] (1994) have analyzed vibrations of laminate thin walled shafts, using the Loves' approximate theory and applying the shell theory. They have also analyzed the critical speeds of shafts using *Donnell's*, *Flügge's*, *Loves' and Sanders'* theory of shells. Those results have shown that *Donnell's* theory may be applied only to shafts having a small ratio L/R (L being the length and R being the radius of the shaft) and small thickness of the walls. For huge ratios L/R, Loves' theory, simpler and more accurate, is used.

A beam model is also used for the analysis of dynamic characteristics of composite shafts. *Singh* and *Gupta* [10] (1996) have researched the vibratory characteristics of composite cylindrical shafts using *Timoshenko's* beam theory and the results achieved have shown good accordance with the shell theory for L/R ratio in the range of 12÷600. For large L/R ratios, *Timoshenko*'s theory should be verified in the course of dynamic analysis of composite shafts.

In 2001, *Song* has applied the beam model to the analysis of fundamental natural frequencies and stability of composite shafts, examining the influence of the laminate angle on the axial carrying capacity of the shaft. This model presented the foundation for future development of the shell theory for analysis of stress-deformation state of the composite shaft.

Design of the aluminum/composite drive shaft

A two-piece steel shaft in automobiles and trucks with rear drive may be replaced with one-piece hybrid shaft made of aluminum and carbon fibres/epoxy composites. Layouts of a two-piece steel Cardan shaft and a corresponding one-piece aluminum/composite shaft are presented in Fig.1.

The torque T_t ([3], [4]) transmitted by the hybrid drive shaft is the sum of the torque T_{tal} transmitted by the aluminum tube, and T_{tco} by the composite layer:

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$$T_t = T_{tal} + T_{tco} \tag{1}$$

The torque transmitted by the aluminum tube is calculated as follows:

$$T_{tal} = \frac{G_{al} J_{al}}{G_{al} J_{al} + G_{co} J_{co}} T_t \tag{2}$$

where: G is the shear modulus, J is the polar moment of inertia, and subscripts “*al*” and “*co*” represent the aluminum tube and the composite layer, respectively.

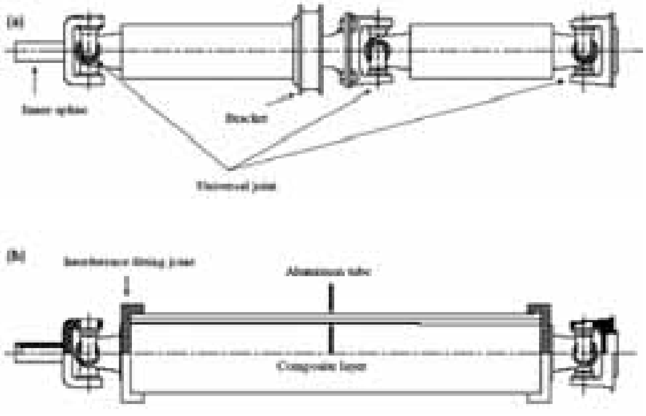


Figure 1. Layouts of drive shafts: (a) two-piece steel drive shaft and (b) one-piece aluminum/composite hybrid drive shaft

Considering that the product of the sheer modulus and the polar moment of inertia for the aluminum shaft is considerably higher than for the composite layer, the resistance of the aluminum shaft to the torque is almost the same as of the hybrid aluminum/composite shaft. Now, the values of the static torque and the critical torque that lead to the instability of aluminum/composite drive shaft may be calculated by neglecting the composite layer as follows [6]:

$$T_{tstat} = 2\pi r_{sr}^2 t_{al} S_{s,al} \tag{3}$$

$$T_{tbuck} = \frac{\pi\sqrt{2}E_{al}}{3\left(1-\nu_{al}^2\right)^{0,75}}\sqrt{r_{sr} t_{al}^5} \tag{4}$$

where: T_{tstat} and T_{tbuck} are the static and buckling torque capabilities of the hybrid aluminum/composite shaft, respectively, r_{sr} is the average radius of the aluminum tube, t_{al} is the thickness of the aluminum tube, E_{al} is the elastic modulus of aluminum, $S_{s,al}$ is the shear strength of aluminum and ν_{al} is the Poisson`s ratio of aluminum.

If the value of the aluminum shaft shear strength of 210 MPa is considered, a diagram of dependence between the static torque and the thickness and the outer diameter of the aluminum tube (Fig.2) may be constructed using the expression (3). In addition, a diagram of dependence between the buckling torque and the same quantities (Fig.3) may be constructed using the expression (4).

The fundamental bending natural frequency of the drive shaft was calculated with the simply supported boundary condition on the both ends, with the following equation:

$$f_n = \frac{9,869}{L^2}\sqrt{\frac{E_{al} I_{al} + E_{co} I_{co}}{\rho_{al} + \rho_{co}}} \tag{5}$$

where: E is the elastic modulus in the axial direction of the drive shaft, I is the sectional moment of inertia, ρ is the mass per unit length, L is the length of the drive shaft.

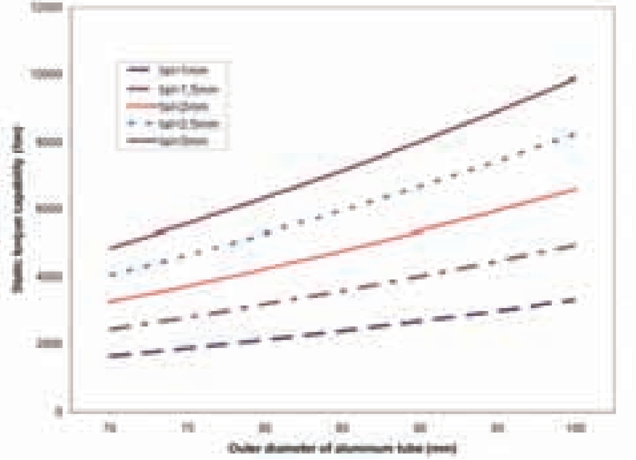


Figure 2. Static torque capability of the aluminum tube

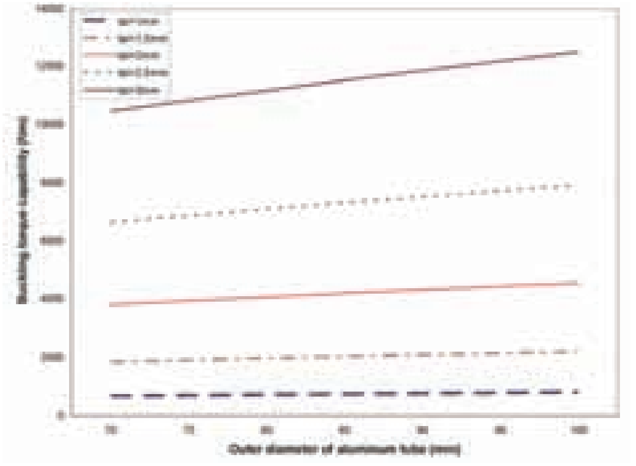


Figure 3. Buckling torque capability of the aluminum tube

The analysis has established that the fundamental natural frequencies of the shaft depend on the number of composite layers. Thus, if more than three layers of carbon fibres/epoxy layers are applied, natural frequencies of the aluminum composite shaft may reach 9200 min^{-1}. In addition, the values of orientation angles of fibres have considerable influence on the values of natural frequencies. For low values of layer orientation angles, high values of shaft fundamental natural frequencies are gained.

Numeric analysis of the fundamental natural frequencies of the aluminum/composite shaft

A real shaft of TURBO ZETA 85.14B truck made of composite obtained by the combination of aluminum and carbon fibres/epoxy resin, instead of being made of steel, is analysed in the paper. The basic shaft measures are: length - 1,35 m, mean radius -0,041 m, thickness of annular cross section wall -0,003 m. The shaft is subjected to the maximum torque of 5000 Nm.

Basic characteristics of the composite material (carbon fibres/epoxy composite), most frequently used for manufacturing shafts and applied in the paper, are given in Table 1.

Table 1. Mechanical properties of composite materials

Material	Carbon fiber epoxi composite (USN150)
E_1 (Gpa)	131.6
E_2, E_3 (Gpa)	8.20
G_{23} (Gpa)	3.5

G_{12} (Gpa)	4.5
v	0.281
S_1^t (Mpa)	2000
S_1^c (Mpa)	-1400
S_2^t (Mpa)	61
S_2^c (Mpa)	-130
S_{23} (Mpa)	40
S_{13} (Mpa)	70
ρ (kg/m^3)	1550
t_{s1} (mm)	0.125

where: E_1 - longitudinal modulus; E_2, E_3 - transverse modulus; G_{12}, G_{23} - shear modulus; v - Poisson's ratio; S_1^t, S_1^c - longitudinal tensile and compressive strength; S_2^t, S_2^c - transverse tensile and compressive strength; S_{13}, S_{23} - shear strength; ρ - density; t_{s1} - thickness of composite.

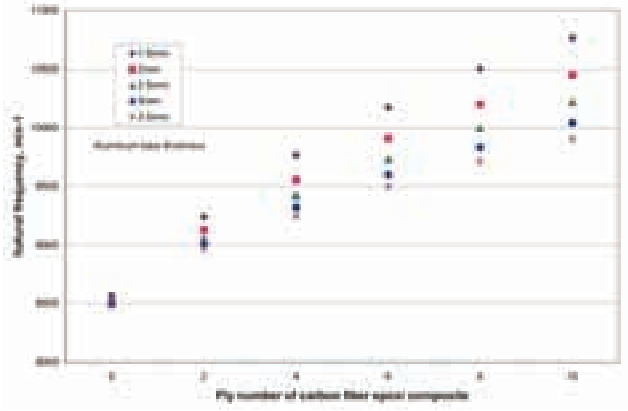
The characteristics of the aluminum shaft that, combined with composite, is used for manufacturing hybrid aluminum/composite shafts are given in Table 2.

Table 2. Mechanical properties of aluminum (6061-T6)

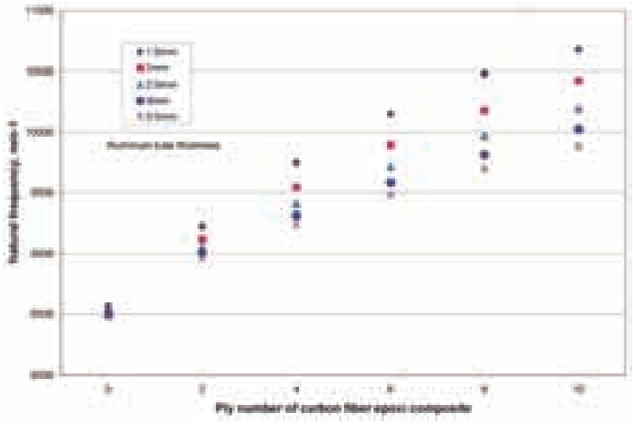
Tensile modulus, E	72000 MPa
Shear modulus, G	27000 MPa
Density, ρ	2695 kg/m^3
Tensile strength, R_m	350 MPa
Yielding strength, R_e	325 MPa
Shear strength	210 MPa
Aluminum tube thickness	2.5 mm

The shaft is modelled by the isoparametric tetragonal finite elements in the shape of multiple layer shells. The NeNastran 8.6 software is used for the analysis.

The eigenfrequencies of the shaft are analysed with respect to the number of layers of carbon/epoxy composites and the thickness of the aluminum tube and shown in Fig.4.



(a)



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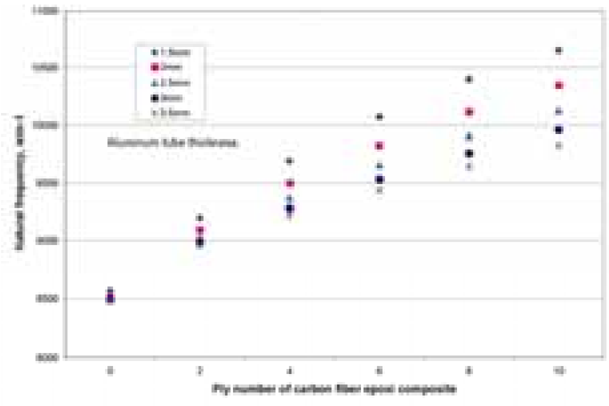
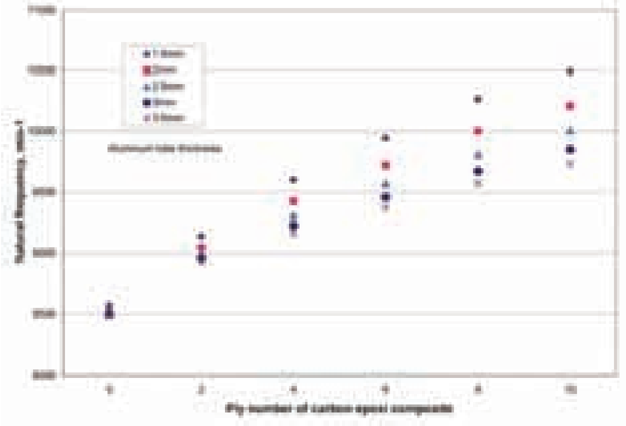


(c)



(d)

Figure 4. Fundamental natural frequency of the hybrid aluminum/composite shaft with respect to the thickness of the aluminum tube and the stacked ply number of the carbon epoxy composite: (a) Al/ [0_{USN}], (b) Al/ [$\pm 5_{USN}$], (c) Al/ [$\pm 10_{USN}$], (d) Al/ [$\pm 15_{USN}$]

From the figures above, it can be concluded that the reduction of the thickness of the aluminum tube leads to the increase of the fundamental natural frequencies of the shaft. In addition, it can be seen that fundamental natural frequencies have the largest value when the orientation angle of carbon fibres is 0°, while the increase of the orientation angle of fibres leads to reduction of fundamental natural frequency values.

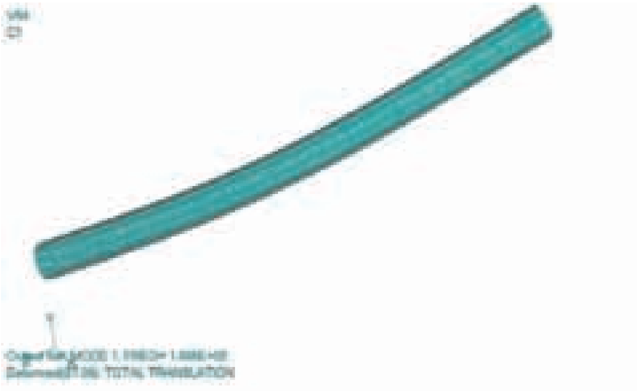


Figure 5. Fundamental natural frequency of the hybrid aluminum/composite shaft Al/ [$0_{USN,8}$]

The numerically determined values of fundamental natural frequencies of the hybrid shaft made of 2.5 mm thick aluminum tube and of eight layers of carbon fibres/epoxy composites with orientation angles of fibres of 0°, are presented in Fig.5.

The fundamental natural frequencies of the steel shaft, having the same dimensions as the composite shaft, are given in Fig.6, while the fundamental natural frequencies of the aluminum shaft are given in Fig.7.

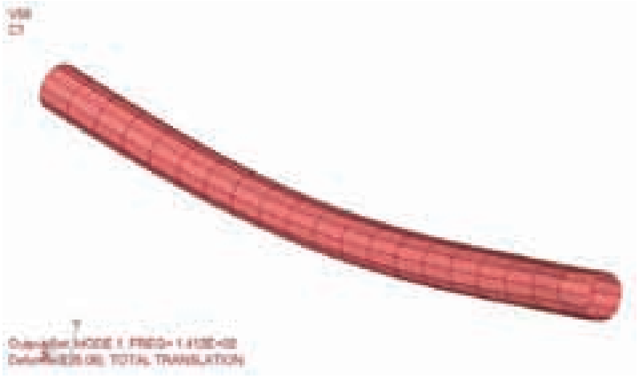


Figure 6. Fundamental natural frequency of the steel shaft

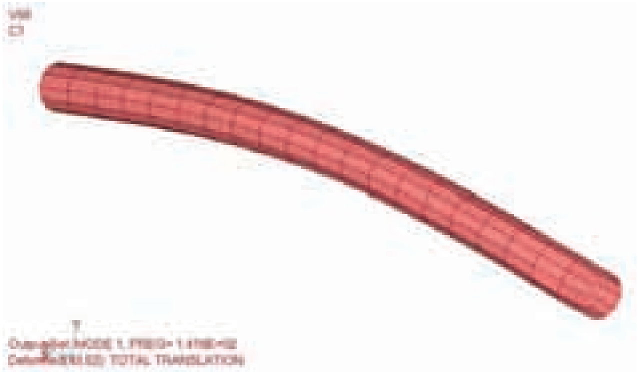


Figure 7. Fundamental natural frequency of the aluminum shaft

If the number of fundamental natural frequencies of the steel shafts or aluminum alloy shafts are close to the resonant regime or get out of the resonant regime, the geometric dimensions of the shaft should be changed. The same may be achieved by changing fibre orientation in each layer on composite shafts.

Based on the btained values of the fundamental natural frequencies in the first vibration mode, f_s, the critical rotational speed of the analyzed shafts may be obtained using the following expression:

$$n_{kr} = 60 \cdot f_s \tag{6}$$

The values of the critical speeds obtained in such way are presented in Table 3.

Table 3. Critical speed

Critical speed, *rpm*	
Al/USN carbon fiber epoxy composite Al/[0_8]	9996
Steel	8472
Aluminum	8496

Analysis of shaft stability

In the case of shafts subjected to the torque, certain values of the torque may lead to the loss of the shaft.stability. These are so-called critical values of the torque. The critical values of the torque may also be determined numerically, during a so-called "buckling analysis". In fact, the coefficient λ for the first buckling mode is determined numerically, and the critical torque has the value of:

$$T_{tcr} = T_t \cdot \lambda \tag{7}$$

In the case of the composite shaft obtained by combination of aluminum and composite material of USN carbon fibres/epoxy composites, with a fibre orientation angle of 0^o, the following values of the coefficient λ for the first buckling mode are obtained by the application of the Finite Element Method (Fig.8):

Figure 8. The first mode of torsional buckling of the hybrid aluminum/composite shaft Al/ [$0_{USN,8}$]

The determined value of the critical buckling torque in the case of the analyzed composite shaft loaded with the maximum torque of 5000 Nm is presented in Table 4.

Table 4. Critical buckling torque T_{tcr}

Material	Buckling load factor λ	Critical buckling torque (Nm)
Al/[$0_{USN,8}$]	3.7263	18631

Conclusions

The values of fundamental natural frequencies of the shaft obtained by a combination of aluminum and composite material are analyzed in the paper depending on the number of carbon fibres layers and the thickness of the wall of the aluminum tube. It can be concluded, by the analysis of the obtained diagrams, that the reduction of the thickness of the aluminum tube wall leads to the increase of fundamental natural frequencies of the shaft. In addition, it may be seen that the fundamental natural frequencies have the largest values if the orientation angle of carbon fibres is 0^o, while the increase of the angles of orientation of fibres leads to the decrease of the fundamental natural frequencies values.

The comparison between the critical speeds of steel, aluminum and hybrid aluminum/carbon fibres/epoxy composite shafts leads to the conclusion that the advantage of the composite shaft over the classical metal shaft is in biased limits for the critical value of fundamental natural frequencies and the critical speed. This means that composite shaft may operate at higher speeds and at higher frequencies compared to steel shafts.

Replacement of classic materials for shaft manufacture with composite materials is recommended, based on what has been shown above.

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Dinamička analiza kombinovanog aluminijum/kompozitnog vratila

Težnja savremenih mehanizama je postizanje što veće brzine rada pa je otuda i precizna dinamička analiza stabilnosti vratila veoma bitna. U ovom radu je data analiza vibracionih karakteristika kompozitnog vratila jedne realne konstrukcije (sopstvenih frekvencija, kritične brzine, kritičnog momenta) u zavisnosti od orijentacije vlakana i geometrijskih mera vratila. Zahvaljujući velikoj vrednosti specifične krutosti vratila od kompozitnog materijala imaju znatno veće vrednosti sopstvenih frekvencija u odnosu na čelična vratila. Bitna prednost kompozitnih materijala za izradu vratila je da njihova primena omogućava povećanje dužine vratila koja bi, u slučaju drugih materijala, dovela do pojave fleksione rezonance. Otuda je celishodno vratila automobila, kamiona i drugih značajnih sistema u buduće raditi od kompozitnih materijala.

Ključne reči: **vratilo, kompozitni materijali, aluminijum, dinamička analiza, vibracije, vibraciona stabilnost.**

Динамический анализ комбинированного алюминий/композитного шпинделя

Стремление развития современных механизмов состоится в достижении всё большей их скорости работы, из-за чего очень важным является и точный динамический анализ устойчивости шпинделя. В настоящей работе представлен анализ вибрирующих характеристик композитного шпинделя одной реальной конструкции (собственных частот, критической скорости, критического момента) в зависимости от ориентации волокон и геометрических размеров шпинделя. Благодаря большому значению специфической жёсткости, у шпинделей из композитного материала значительно большие значения собственных частот по отношению к шпинделям из стали. Существенным преимуществом композитных материалов для выработки шпинделя является то, что их применение обеспечивает увеличение длины шпинделя, которая бы в случае применения других материалов привела к появлению флексооного резонанса. Оттуда целесообразно шпиндели автомашин, грузовиков и других важных систем в будущем вырабатывать из композитных материалов.

Ключевые слова: шпиндель, композитные материалы, алюминий, динамический анализ, колебания (вибрации), вибрирующая устойчивость.

Analyse dynamique de l'arbre hybride aluminium/composite

La tendance des mécanismes modernes est d'atteindre la plus grande vitesse possible de fonctionnement et pour cette raison l'analyse dynamique précise de la stabilité de l'arbre est très importante. Dans ce travail on a donné l'analyse des caractéristiques vibratoires de l'arbre composite d'une construction réelle (propres fréquences, vitesse critique, moment critique) en fonction du sens des fibres et des mesures géométriques de l'arbre. Grâce à la grande valeur de la rigidité spécifique les arbres des matériaux composites ont les valeurs de fréquences naturelles considérablement plus grandes par rapport aux arbres en acier. L'avantage essentiel des matériaux composites pour la fabrication des arbres est dans le fait que leur application permet l'augmentation de longueur de l'arbre ce qui, chez les autres matériaux amènerait à l'apparition de la résonance flexionnelle. Pour cela il convient de fabriquer dans le futur les arbres des autos, camions et d'autres systèmes importants en matériaux composites.

Mots clés: **arbre, matériaux composites, aluminium, analyse dynamique, vibrations, stabilité vibratoire.**