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Probabilistic Approach to Component Useful Life Prediction

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Life estimation is widely used to set the useful/operational time of components and equipment. This paper gives an approach to mean life and useful life estimation. The maintenance policy is based on useful life determination. Practical application is illustrated by reliability testing of jet engine blades. The new blades as well as the blades after 400, 800 and 1200 operating hours were cyclic-tested to failure and the recorded data were analyzed. The lognormal distribution was applied for data analysis. The useful life for new components and the expired life for components from exploitation were determined. The regression line plotted on reliability testing data for new and aged jet engine blades gave the point of hundred per cent life for predetermined reliability. To ensure safe engine operation, blade replacement and engine overhaul have to be planned before the point of hundred per cent required reliability life is reached.

Key words: **technical maintenance planning, reliability testing, reliability analysis, useful life, blade.**

Introduction

A component fails when the applied load exceeds the component material strength. Taking load and strength as random variables, reliability is a probability that strength is higher than load, or in symbols: $R = P\ \{ S > L \}$. Material strength decreases over time due to fatigue, wear, corrosion, etc. In general, material strength degradation may be described as a continuous random process, and load is a random process as well. However, in this paper, we will consider the applied load as static random variables. The strength degradation brings closer the stress-strength interference increasing the failure probability function. Reliability decreases over time as a result of the strength degradation process, or $R\ (\ t\ ) = P\ \{\ S\ (\ t\ ) > L\ \}$ [1, 2].

The maintenance policy should take into consideration the material strength degradation and the failure probability increase over time. So, the question exists: what is the optimal component replacement time? One way of component replacement time determination may be by a cumulative probability of failure or safety criteria. In this case the replacement time is obtained from the reliability time dependent function.

In this paper, the instantaneous probabilities of failure and the reliability time functions are considered. The replacement policy is based on safety-reliability criteria. As an example, the jet engine blades are reliability tested [Vukoje 1995.]. The data analysis gives the indication of strength degradation, instantaneous probability of failure and reliability functions evaluation. The time to blade replacement is derived from the reliability limit.

Component life estimation

Life estimation is videly used to set the useful/operational time of components and equipement. We can point out that mechanical components under highly variable dynamic loads have stress amplitudes going into the plastic region. The problem of high dynamic loads, fatigue and crash of material may be extreme for components with sharp geometric discontinuty and stress concentration. Material fatigue and crash may be caused by normal operational loads. However, the problem is more significant for components exposed to frequent unregular operations, sudden shutdown, overloads, causing high plastic deformation and material failure. There are a lot of mechanical components exposed to high dynamic loads in regular operations like: gears, shafts, bearings, power transmitters. Let us mention engin components, especially aircraf jet engin components where operational stress levels are designed to be very high in order to decrease mass and increase specific power. These parts are discs, blades, chambers and many other components under high mechanical and temparature variable loads causing fatigue and component crack propagation. For these components the useful life is restricted to avoid a high probability of failure.

In engineering practice, we are interested in application of component life testing and useful operational life determination. For component reliability consideration, two life concepts are common: concept of failure and concept of restricted damage.

Before component life testing, it is necessary to define the basic elements:

- Criteria for useful life determination,
- Testing conditions (similar to the operational conditions),
- Testing plan (sample size, expected number of failures, etc.)

The criteria for component useful life determination can be different depending on the system application and the effects of component failure on the system, environment and human safety. The national techical standards may give

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recommendations for criteria (usually for systems and equipement - leading to component specification). Three forms of criteria are presented [8]:

1. The useful life T_V is determined by a simple relation between the measured mean time to failure $\overline{T}$ and the standard deviation s_T as:

$$T_V = \overline{T} - k \cdot s_T \tag{1}$$

where the coefficient k is to be chosen adequately for each element type [1,4].

This approach may be suitable while testing large size samples.

2. The useful life T_V is found from the reliability requirements R or the probability of failure requirements Q. The probability relations are:

$$R = P(T \ge T_V) = 1 - \alpha\,, \tag{2}$$

or,

$$P(T < T_V) = \alpha\,. \tag{3}$$

Like the coefficient k in the previous consideration, the risk level α is chosen adequately for each element type. For example, the helicopter structure reliability against fatigue failure is′six nanes′ [5,6].

3. The useful life T_V is found from the reliability requirements R or the probability of failure requirements Q, stated for the specified confidence level $CL = 1 - \gamma$. The probability relation is:

$$P\,[P_f(T \le T_V) = \alpha] = CL\,. \tag{4}$$

This approach may be applied when testing small size samples, or for high safety requirements [6].

4. The percent of the expired life is:

$$P_T^i = \frac{T_i}{T_V} \cdot 100\%\,, \tag{5}$$

where: T_i - expired life (cumulative operating time), and T_V - life of new components.

The percent of the expired life can be expreced by a number of cycles as:

$$P_T^i = \frac{N_V^0 - N_V^i}{N_V^0} \text{ x } 100\ [\%], \tag{6}$$

Where: N_V^0 - Life expressed in a number of cycles for new components, and N_V^i - Number of cycles to failure at the component age T_i.

Strength degradation process

Strength degradation over time is due to wear, fatigue, crack growth caused by dynamic loads, corrosion, ageing, etc. In most engineering situations, the prime interest is to define material fatigue under high dynamic stress. A component design objective may be mass minimization to increase product performances (like air vehicle, special equipment, etc.) To achieve this objective, components and structures must be designed for higher stress levels and must be more sensitive to fatigue. Dynamic stresses cause strength degradation. Both strength and stress are random variables. So, as time functions, there are the random strength degradation process and the random stress process, Fig.1.

The strength degradation process is $S(t)$. A way to model degradation is by statistical distribution with time varying parameters $S_p\{\alpha\,(t),\ \beta(t),\ \gamma\,(t),.\}$. If the underlying strength distribution is normal, then the Gaussian process is:

$$S\,(t) \sim N\,(\mu_S(t),\ \sigma_S(t)), \quad t \in R^+. \tag{7}$$

The stress process is $L(t)$. For some applications, it is reasonable to take the stress distribution as:

$$L(t) = L(\mu_L,\ \sigma_L),\ \ t \in R^+. \tag{8}$$

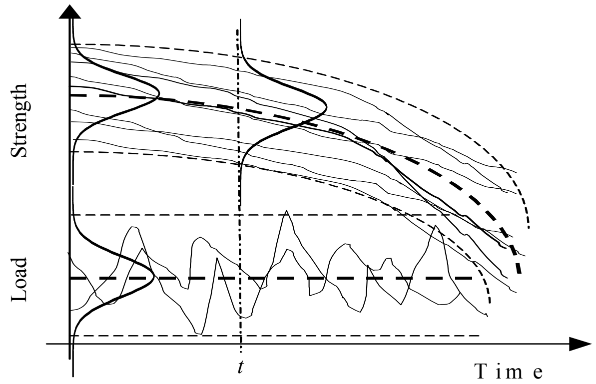

Figure 1. Random processes: Strength degradation and Stress

For any time instant t_1, the stress-strength overlapping area, Fig.2, indicates the probability of failure:

$$P_f(t_1) = P(S < L : t_1) = \int_0^\infty \left\{ S_{t_1}(x) \int_x^\infty L(y)dy \right\} dx \tag{9}$$

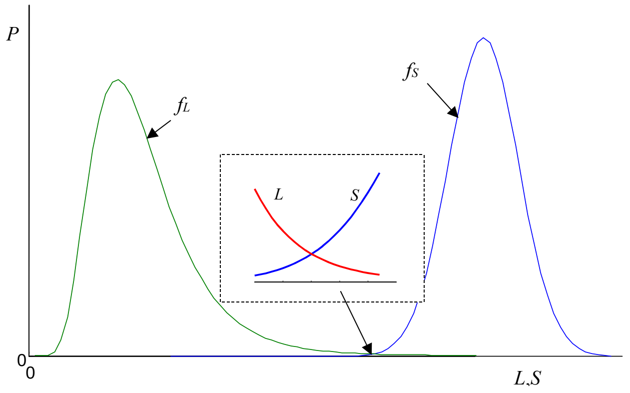

Figure 2. The stress-strength overlapping area

The reliability for time $t = t_1$ is:

$$R(t_1) = P(S > L : t \in [0, t_1]\,) \tag{10}$$

Taking the time increment Δt, after t_1, reduce the reliability by the level of probability of failure in that increment:

$$\begin{aligned} R(t_1 + \Delta t) &= P(S > L : t \in [0, t_1 + \Delta t]\,) \\ &= P(S > L : t \in [0, t_1]\,) \cdot P(S > L : t \in [t_1.t_1 + \Delta t]\,) \\ &= R(t_1) \cdot \left(1 - P(S < L : t \in [t_1, t_1 + \Delta t]\,)\right) \end{aligned} \tag{11}$$

So, reliability is decreasing over time as a function of time and the strength degradation process, or the increase of stress-strength interference.

High dynamic loads cause material strength degradation, fatigue and failure. The maintenance policy should prevent possibility of component failure under operating conditions [7, 9]. The time for component replacement should be selected by considering the objective requirements. If the components are critical to system safety, the components replacement time (useful life T_L) should be derived from the safety/reliability requirement R_r, Fig.3.

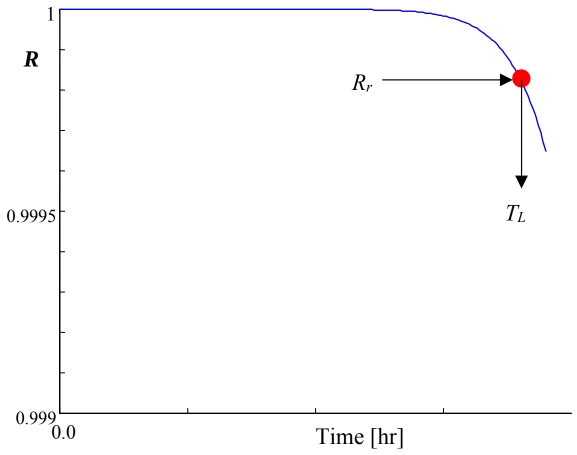

Figure 3. Component replacement time as the function of safety requirement

To be on the safe side, component replacement should be scheduled before the end of useful life ($T_P < T_L$) to prevent the component from entering into the region of high probability of failure.

Jet engine blades

The testing of jet engine blades was performed on new units, as well as for the units after 400, 800 and 1200 operating hours. Six units were tested to failure in each testing sample at the applied cycling stress of 473 N/mm^2. The testing results are listed in Table 1, [Vukoje, 1995]. It was requested to determine the life of new blades as well as the useful life with the success probability of $R=P(T \geq T_L)=0.99997$.

a) Percent survival method

The two-parameter lognormal underlying distributions are selected and the parameters of these fitted distributions ($\bar{x}$, s_x) are listed in Table 1, Fig.4.

Table 1. Number of cycles to failure for blades at 0, 400, 800 and 1200 operating hours

$T_0 = 0.0$ hr N_{i0}x10^6 [cycles]	$T_1 = 400$ hr N_{i1}x10^6 [cycles]	$T_2 = 800$ hr N_{i2}x10^6 [cycles]	$T_3 = 1200$ hr N_{i3}x10^6 [cycles]
4.137	2.2449	1.1549	0.6489
6.813	2.4089	1.8756	1.5210
8.225	4.9335	2.6389	1.5258
8.774	6.6605	4.0857	2.1292
18.275	8.1555	5.9250	3.6807
34.543	12.6343	7.6586	6.3403
$\bar{N}_0$=1.3461x10^7	$\bar{N}_1$=0.6173x10^7	$\bar{N}_2$=0.3890x10^7	$\bar{N}_3$=0.2641x10^7
$\bar{x}_0 = 16.1598$, $s_{x0} = 0.7578$	$\bar{x}_1 = 15.4514$, $s_{x1} = 0.6858$	$\bar{x}_2 = 14.9765$, $s_{x2} = 0.7158$	$\bar{x}_3 = 14.5347$, $s_{x3} = 0.7897$
$N_V^0 = 503\ 100$			

The K-S test is performed and each set of data fits the distribution.

The probability of failure 0.00003 corresponds to the point $\bar{x}_V - 4\ s_V$. The useful life of new blades is $N_V^0 = 503100$ cycles. The expired life determined by $P_T^i = \dfrac{N_V^0 - N_V^i}{N_V^0}$ [x100%] and the regression line are presented in Fig.5. The coefficient correlation for the pair (P_T^i, T) is very high, $c = 0.9922$.

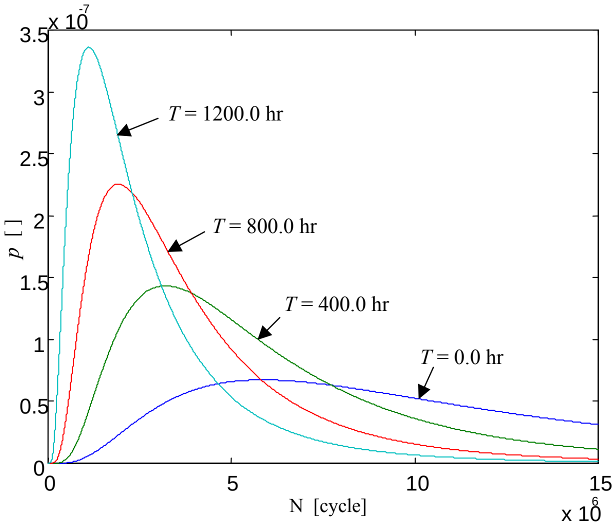

Figure 4. The lognormal distributions

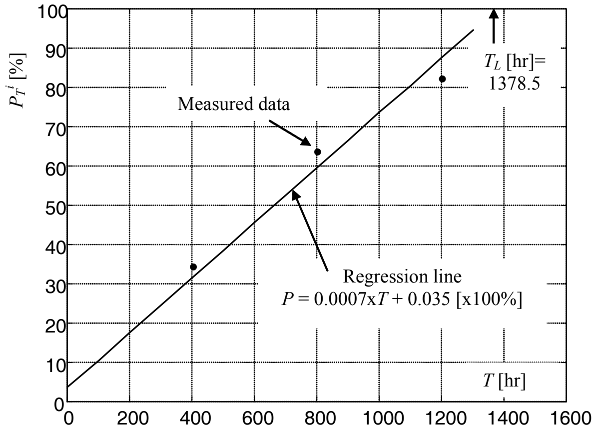

Figure 5. Percent of expired life and regression line for jet engine blades

The regression line points out that:

- The time point T_L = 1378.5 hr represents 100 % of blades useful life for the given probability of failure P_f=0.00003.
- Having the blades in operation beyond the age T_L=1378.5 hr would increase the probability of failure above predetermined probability limit, decreasing the engine safety. To be on the safe side, the planned blades replacement (preventive maintanance) should be scheduled before the component age of T_L = 1378.5 hr.

So, the classical percent survival method indicates the preventive blade replacement before reaching the age of T_L=1378.5hr.

b) Reliability methods based on strength degradation represented by the stochastic process model

The three-parameter lognormal probability density functions with the estimated location parameters (N_1 =-625000; N_2 = -10000; N_3 = -50000; N_4 = -35000) are plotted in Fig.6. For each case, the mean cycles of failure are pointed and the regression line fitted. The coefficient correlation for the mean cycles of failure is c = 0.9343.

It is obvious from Fig.6 that the mean strength (presented by the cycles to failure) decreases over the time. The estimated line indicates the mean (strength) degradation as 7.146772748841 % for 100 flying hours.

Through the optimal location parameter N_i the best line fit is plotted in Fig.7, and the zero cycle is found for T=1016.2 hr. The coefficient correlation for the location parameter N_i is, c = 0.7517.

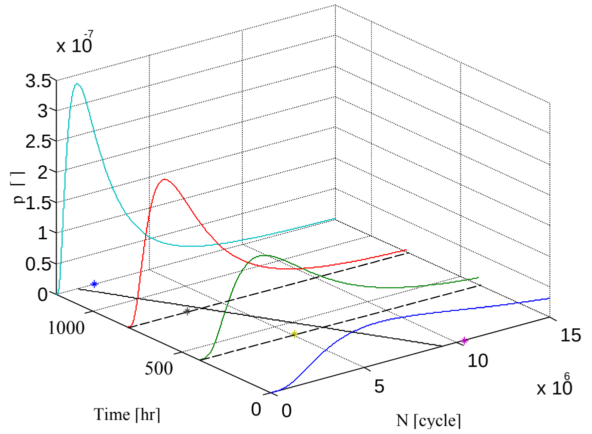

Figure 6. The three-parameter lognormal pdf at time 0, 400, 800, 1200 hr, the means of the measured cycles and the regression line of the means

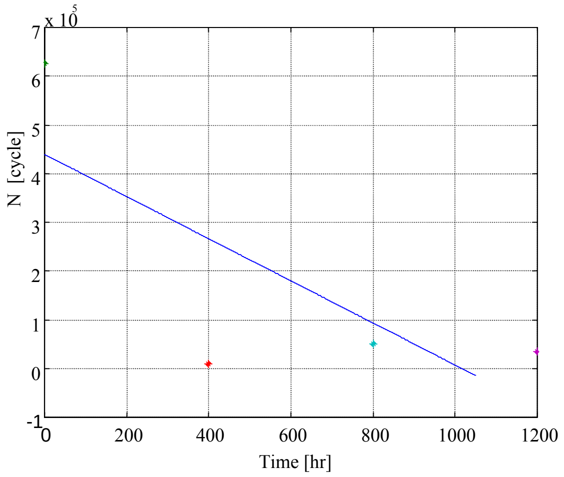

Figure 7. The regression line for the location parameter of the three-parameter lognormal distribution

So, the strength degradation process is described by the cycles to failure three-parameter lognormal pdf with the parameters:

$\mu(t) = \ln(1.1753e{+}007 - 8685.9 \cdot t)$ – mean
$\sigma = 0.7373$ – std
$\gamma = 4.3950e{+}005 - 432.5 \cdot t$ – location

The probability of failure at any instant of time $p_f(t)$, as a function of time, is determined from this process, Fig.8. Also, the cumulative probability of failure $Q(t)$ is calculated as a function of time.

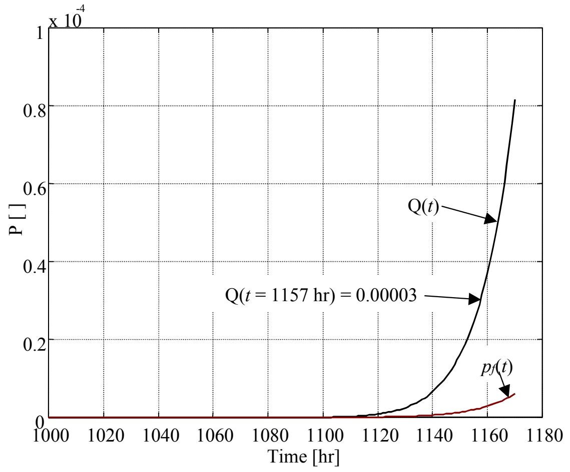

Figure 8. The instantenious and cumulative probability of failure $p_f(T)$, $Q(t)$, $p_f > 0$ for $T > 1016.2$ hr

The required cumulative probability of failure is 0.00003, so the estimation time to meet this objective is 1158 hr, Fig.7.

Comparing the percent survival method and the stochastic process method, it can be seen:

- Both methods give relatively close replacement times (1378.5 hr and 1157 hr)
- A little lower value of replacement time, 1157 hr, was generated by the stochastic process method.
- To get more confidence in the replacement time generated by the stochastic process method, more test data are suggested. This will lead to lognormal parameter estimation with a higher precision and confidence level.

Conclusions

Maintenance has a very large influence on system reliability and safety. The material strength degrades over time due to fatigue, wear, corrosion, etc, causing a component to weaken and fail under operational loads. The strength degradation process increases the stress-strength overlapping area and the probability of failure. The maintenance policy should be scheduled in a way to prevent the component entering into the region of higher probability of failure.

In this paper, we give the basic percent survival method, as well as the analytical background on the material strength degradation process, stress-strength interference, and instantaneous probability of failure and reliability calculations. This gives the basis for component useful life estimation and maintenance policy scheduling.

The jet engine blades reliability testing is performed on new, as well as the blades that have been used in operation for 400, 800 and 1200 hours. The results indicate the mean strength degradation. The component replacement time is estimated based on the reliability limit imposed.

The experience we gained indicates the possibility of applying both methods to establish maintenance policies, but for the strength degradation process method, it would be better to use a larger sample for reliability testing to obtain a high confidence level in analytical distribution parameters determination.

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Moguć prilaz za procenu korisnog životnog veka komponente

Procena životnog veka komponenata i opreme se široko koristi za uspostavljanje korisnog/operativog vremena rada. Ovaj rad daje prilaz za određivanje srednjeg i korisnog životnog veka. Politika održavanja se bazira na određivanju korisnog životnog veka. Praktična primena je ilustrovana na primeru testiranja pouzdanosti lopatica mlaznog motora. Nove lopatice, kao i lopatice koje su radile 400,800 i 1200 sati su testirane do otkaza i podaci zapisivani. Primenjena je lognormalna raspodela za analizu dobijenih podataka. Određen je koristan životni vek novih lopatica i preostali životni vek lopatica koje su uzete iz eksploatacije. Primenom regresione linije određen je životni vek lopatica za zahtevani nivo pouzdanosti. Zamena lopatica i remont motora treba planirati pre isteka određenog životni vek lopatica za zahtevani nivo pouzdanosti, da bi se osigurao bezbedan rad.

Ključne reči: **tehničko održavanje, ispitivanje pouzdanosti, analiza pouzdanosti, životni vek, lopatica.**

Пробабилистический подход для оценки полезного срока службы состовляющей

Оценка срока службы составляющей и оборудования широко пользуется для восстановления полезного/оперативного времени. Настоящая работа дает подход для определения среднего и полезного срока службы. Обслуживание базируется на определении полезного срока службы. Практическое применение иллюстрировано на примере испитания надежности лопаток реактивного двигателя. Новые лопатки, а в том числе и лопатки работавшие 400, 800 и 1200 часов испитаны до отказа и все данные записаны. Применено логнормальное распределение для анализа полученых данных. Тоже определен полезный срок службы новых лопаток и остаточный срок службы эксплуатационных лопаток. Применением регрессивной линии определен срок службы лопаток для запрошенного уровня надежности. Замену лопаток и капитальный ремонт двигателя нужно планировать раньше истечения определенного срока службы лопаток для запрошенного уровня надежности, чтобы обеспечить надежность работы.

Ключевые слова: **техническое обслуживание, испытание надежности, анализ надежности, срок службы, лопатка.**

Approche probabiliste pour l'évaluation de la durée de vie utile de la composante

L'estimation de la durée de vie des composantes et de l'équipement est largement utilisée pour établir le temps utile /opérationnel. Ce papier expose une approche pour le détermination de la moyenne durée de vie utile. La politique de l'entretien se base sur la détermination de la durée de vie utile. L'application pratique est donnée sur l'exemple de l'essai de fiabilité des lames du moteur à réaction. On a examiné les nouvelles lames ainsi que les lames après 400, 800 et 1200 heures de travail et cela jusqu'à la défaillance. Les données obtenues ont été notées et on a utilisé la distribution longnormale pour les analyser. On a déterminé la durée de vie de nouvelles lames et celle des lames déjà exploitées. En appliquant la ligne de régression, on a déterminé la durée de vie des lames pour le niveau exigé de fiabilité. Le remplacement des lames et la remise à l'état du moteur est à prévoir avant la fin de la durée de vie des lames pour assurer un fonctionnement sûr au niveau de la fiabilité exigée.

Mots clés: **entretien technique, essai de fiabilité, analyse de fiabilité, durée de vie, lame.**