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Application of the Effective Strain Energy Density Factor in the Estimation of the Fatigue Life of Notched Specimens

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Life estimation and behavior of machine elements with initial cracks under cyclic loads is one of very important engineering problems. It is possible to considered structural elements with various shapes and crack dispositions as well as with various crack propagation models. The calculation of the stress intensity factors *K*, crack propagation and life of structural elements are of utmost importance in this analysis. The estimation of the life of structural elements is carried out under cyclic loads with a constant amplitude. This paper includes the comparisons of the numerical finite element method (FEM), an analytical one and the already available test results for the cylindrical thin-walled pipe of the radius *R* and the thickness *t*. The pipe is loaded by the internal pressure p_c. Critical pressure is determined using the strain energy density theory (SEDT) [1]. The additional feature of the S_c theory is that a single parameter can simultaneously determine the fracture toughness of the material and the direction of the crack growth initiation. Since a fatigue load cycle is characterized by the mean stress and stress range, the Paris rule may not be adequate to analyse the crack growth behavior because it only takes into account one loading parameter. To solve this problem a modified version SEDF based on a so-called effective strain energy density factor [2] will be used. It is intended to verify the applicability of empirical models based in experimental evidence.

Key words: **material fatigue, crack, crack propagation, life, energy density theory, finite element method (FEM).**

Introduction

ONE of important problems in engineering structures is the stress analysis of the problem, when intensity and direction of the stresses and strains at various points of the structure are known. The next step is to select a criterion of failure which determines the type of material to be used for each element of the structure [3]. Traditional design criteria make no attempt to account for the failure mode which is characteristic of a flawed, frangible structure. Laboratory test results for the critical values of K_c, the stress intensity factor, are limited to specimens where loads are applied symmetrically with respect to the crack plane. The evolution of a structure due to the influence of a crack will in general involve a mixed or combined load situation where the crack will grow in a curved manner. This necessitates the application of the energy density factor *S*, which has a geometrically controlled direction of the crack growth initiation, θ, which can also serve as a measure of material toughness.

The *S* theory is applied to determine crack initiation and direction for circumferentially cracked thin-walled pipes. The strain energy density factor *S* is a function of the stress intensity factors. The strain energy density theory provides a more general treatment of fracture mechanics problems by virtue of its ability in describing the multiscale feature of material damage and in dealing with mixed mode crack propagation problems.

Formulation of the SIF based on SED

Strain Energy Density Fracture Criterion

The K_c-theory in fracture mechanics is analogous to the maximum stress criterion applied to a simple tension specimen and cannot be used in combined load situations. For this reason, it is primarily restricted to laboratory use and has limited application in structure design. It should be emphasized that the effect of misalignment between the crack and applied load, if ignored, could lead to serious errors in the predictions of failure load.

The K_c-theory is based on the assumption that the crack must always be in a normal position with applied tension and hence the direction of crack propagation is a priori known. But in most of the structural components the flaws of cracks are seldom aligned perpendicularly to the direction of loading. That misalignment invalidates the K_c-theory. The crack tip stress state in a structural component must be described by at least two parameters [2], K_{I} and K_{II}, where $K_I = K/\sqrt{\pi}$ and where K_{II} stands for the amplitude of the skew-symmetric portion of the stress state. If the stress state is tri-axial in nature, a third parameter K_{III} is usually required and the fracture criterion must be modified accordingly.

The crack will generally follow a curved path in a structural component and the direction of crack propagation

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will depend on the energy state and material property in a region ahead of the crack tip. Therefore, it is very important to predict the direction of crack initiation [1]. Material in the immediate vicinity of the crack tip will behave differently from that of the bulk (Fig.1). The mechanical properties of the material next to the crack tip, where the stresses are exceedingly high, are not known. Attention is focused on elements in the regions away from local disturbances and the amount of energy stored in one of these elements can be computed from:

$$\frac{dW}{dA} = \frac{1}{r}\left(a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2\right) + ... \quad (1)$$

in which the coefficients $a_{11}, a_{12}, ...$ for plane strain are given by:

$$\begin{aligned} a_{11} &= \frac{1}{16\mu}\left[(3-4\nu-\cos\theta)(1+\cos\theta)\right] \\ a_{12} &= \frac{1}{16\mu}(2\sin\theta)\left[\cos\theta-(1-2\nu)\right] \end{aligned} \quad (2)$$

$$a_{22} = \frac{1}{16\mu}\left[4(1-\nu)(1-\cos\theta)+(1+\cos\theta)(3\cos\theta-1)\right]$$

$$a_{33} = \frac{1}{4\mu}$$

where ν is Poisson's ratio and μ the shear modulus of elasticity. It can be noticed that dW/dA becomes exceedingly large as r becomes smaller and smaller reaching a limit on the boundary of the core region $r = r_0$. The intensity of this energy field which varies along the periphery of the circle $r = r_0$ will be denoted by S and referred to as the strain-energy density factor

$$S = a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2 \quad (3)$$

This factor depends on θ marking the position of the element ΔA through the coefficients $a_{11}, a_{12}, ...$ and therefore describes the variation of the local energy density around the region where fracture will be initiated.

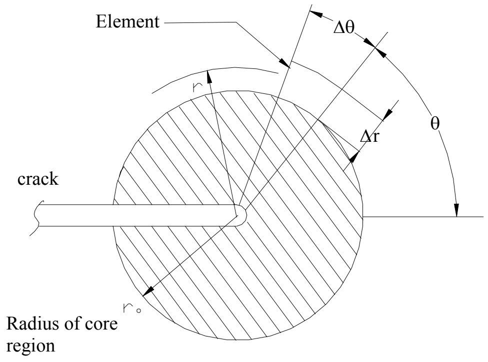

Figure 1. Crack tip element

The fundamental hypotheses on unstable crack growth in the Sih theory are as follows:

a) Crack initiation takes place in a direction determined by the stationary value of the strain energy density factor:

$$\frac{dS}{d\theta} = 0, \text{ at } \theta = \theta_0. \quad (4)$$

b) Crack extension occurs when the strain energy density factor reaches a critical value:

$$S_c = S\,(K_{I}, K_{II}, K_{III}), \text{ for } \theta = \theta_0. \quad (5)$$

The difference between S and S_c is analogous to the difference between K and K_c and thus S_c is also a measure of the resistance of a material against fracture [3]. The additional feature of the S_c theory is that the single parameter can simultaneously determine the fracture toughness of the material and the direction of crack initiation. The validity of the S_c theory can be checked by imposing different loading conditions on the crack specimens made of the same material and show that S_c indeed remains constant.

The angle θ_0 in equations (4) and (5) is zero when the crack is orientated normal to the direction of applied tension and the crack extends in a self similar manner. In this case $k_2 = k_3 = 0$, $a_{11} = (1-2\nu)4\mu$ and equation (3) simplifies to:

$$S = \frac{1-2\nu}{4\mu}k_1^2 \quad (6)$$

Here, k_1 is associated with the same crack tip stress field as K in equations:

$$\begin{aligned} \sigma_x &= \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) + ... \\ \sigma_y &= \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) + ... \\ \tau_{xy} &= \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} + ... \end{aligned} \quad (7)$$

They differ only by a factor of $\sqrt{\pi}$, $k_1 = \frac{K}{\sqrt{\pi}}$. At the point of crack instability, S_c can be obtained by a simple tension test since:

$$S_c = \frac{1-2\nu}{4\pi\mu} = K_c^2 = \frac{(1+\nu)(1-2\nu)}{2\pi E}K_c^2 \quad (9)$$

Once the conventional elastic properties ν and μ or ν and E are known [8], S_c can be computed from K_c for any other material.

Application of the S_c theory

The S_c theory provides a simple procedure for defining the worst crack at a given stress level without causing fracture [4, 5]. The basic idea is to keep the worst crack below certain size so that design can proceed on the basis of a known allowable stress. This approach is particularly useful in dealing with high strength materials which are more vulnerable to sudden catastrophic fracture.

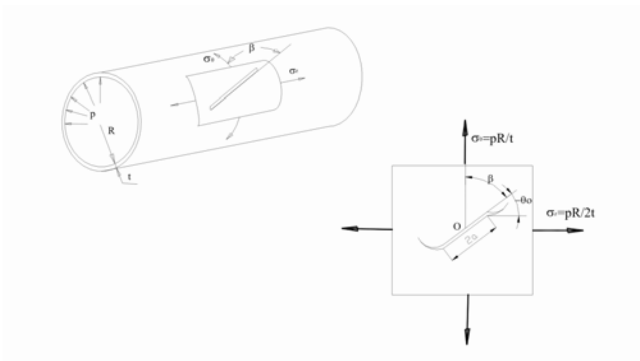

Figure 2. Thin-walled pipe pressurized internally

Consider a thin-walled pressure pipe made of 4340 steel [3] with K_c = 44.8MPa$\sqrt{m}$ and σ_u = 2109MPa. The vessel contains a small crack of length $2a = 0.508\,\text{cm}$ orientated at an angle β with the direction of the circumferential stress σ_0 in Fig.3. The problem is to determine the allowable internal pressure p or the limiting pipe dimensions R/t where R is the vessel radius and t the thickness. The hoop stress σ_θ and the longitudinal stress σ_z can be easily found from the σ stress analysis and they are

$$\sigma_0 = \frac{pR}{t}\ ,\ \sigma_z = \frac{pR}{2t} \tag{10}$$

The crack does not coincide with any of the planes of the principal stress, so its direction of propagation is not obvious and must be determined from the S_c theory. The stress intensity factors for this problem are:

$$K_I = \frac{pR}{2t}\sqrt{a}\left(1+\sin^2\beta\right),\ K_{II} = \frac{pR}{2t}\sqrt{a}\sin\beta\cos\beta \tag{11}$$

If we put these two equations into equation (3) we will get the strain energy density factor as following:

$$S = \left(\frac{pR}{2t}\right)^2 aF\left(\beta,\theta\right) \tag{12}$$

The function $F(\beta,\theta)$ stands for

$$F\left(\beta,\theta\right) = a_{11}\left(1+\sin^2\beta\right)^2 + \\ +a_{12}\left(1+\sin^2\beta\right)\sin 2\beta + a_{22}\sin^2\beta\cos^2\beta$$

where the coefficients a_{11}, a_{12}, a_{22} are given by eq. (2). If we put equation (12) into (4), the directions of crack propagation defined by the angles $-\theta_0$ are obtained for all crack positions described by β = 10°, 20°,..., 90°. The results are tabulated in Table 1 for ν = 0.25 [7]. When we insert the fracture angles $-\theta_0$ into equation (12), S becomes a real material parameter S_c:

$$S_c = \left(\frac{p_c R}{2t}\right)^2 aF\left(\beta,+\theta_o\right)$$

which can be rearranged to solve the quantity

$$\frac{p_c R}{2t}\sqrt{a} = \sqrt{\frac{S_c}{F\left(\beta,+\theta_o\right)}} \tag{13}$$

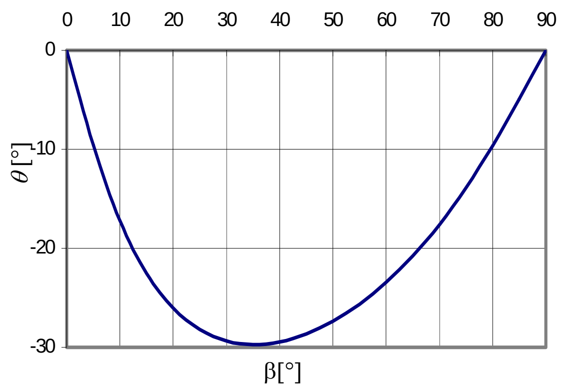

Figure 3. Fracture angle *vs* crack angle

The numerical values of equation (13) are given in Table 2 for σ Poisson's ratio of ν = 0.25. Fig.2 depicts a circumferential pipe under internal pressure p_c = 0.5 daN/mm^2 with relative dimensions. Some important dimensions for the pipe should be noted: R = 50mm, l = 300 mm, t = 2.5 mm and 2θ = 36°.

Table 1. Fracture angle *vs* crack angle

β[°]	θ[°]	β[°]	θ[°]
0	0	50	-27.377
5	-9.577	55	-25.636
10	-17.201	60	-23.442
15	-22.532	65	-20.775
20	-26.04	70	-17.6
25	-28.205	75	-13.888
30	-29.363	80	-9.644
35	-29.735	85	-4.953
40	-29.47	90	0
45	-28.664		

Table 2. Critical internal pressure vs. angle β

β	$\frac{p_c R\sqrt{a}}{2t}$		β	$\frac{p_c R\sqrt{a}}{2t}$	
[°]	$\left[ksi\sqrt{in}\right]$	$\left[MPa\sqrt{m}\right]$	[°]	$\left[ksi\sqrt{in}\right]$	$\left[MPa\sqrt{m}\right]$
0	22.50	25.21	50	13.54	15.17
10	21.60	24.20	60	12.50	14.00
20	19.35	21.68	70	11.80	13.23
30	16.95	18.99	80	11.40	12.77
40	15.00	16.81	90	11.30	12.66

If we consider the maximum stress criterion without taking the crack into consideration, the pressure p in the vessel is assumed to reach the critical value p_c when the maximum principal stress $\sigma_\theta = \sigma_u$ and hence $p_c = 2109\ t/R$ MPa. The conventional theory is seen to differ from that of fracture mechanics by more than 270 per cent [1].

Analytical analysis. Critical internal pressure p_c

This text will consider influence of a thickness t and a crack length a on a value of a critical internal pressure p_c, and the stress intensity factor K_I [7]. Simple equations are used for this purpose.

The initial crack length a is very important for the critical pressure p_c values. Consider a pipe of the following dimensions: R = 500 mm, t = 10 mm and a = 2.54 mm. The characteristics of the material are: $K_c = 44.8MPa\sqrt{m}$, σ_u = 2109 MPa and E = 210900 MPa. The critical pressure p_c for the pipe with no initial crack is calculated by the equation $p_c = \dfrac{\sigma_u t}{R}$ by changing different values of the thickness t (Table 3).

Table 3. Critical pressure p_c *vs* pipe thickness t

		p = 42.18 [MPa]			
		Results by FE method		Results by equation (10)	
t [mm]	p_c [MPa]	σ_θ [MPa] ×10	σ_z [MPa] ×10	σ_θ [MPa] ×10	σ_z [MPa] ×10
10	42.18	215	107.5	210.9	104.45
12	50.616	179.18	89.57	210.9	104.45
14	59.052	153.55	76.77	210.9	104.45
16	67.488	134.32	67.15	210.9	104.45
18	75.924	119.39	59.78	210.9	104.45
20	84.36	107.43	53.78	210.9	104.45

The critical pressure p_c can be calculated by equation (13) by changing different values of the pipe thickness t for three values of the initial crack angle β, shown in Table 4 and Fig.4.

Table 4. Critical pressure p_c vs thickness t for three different values of the angle β

	β=25°	β=35°	β=60°
t [mm]	p_c [MPa]	p_c [MPa]	p_c [MPa]
6	9.66146	8.4822	6.66127
8	12.88194	11.3096	8.88169
10	16.10243	14.137	11.10211
12	19.32291	16.96441	13.32253
14	22.5434	19.79181	15.54296
16	25.76388	22.61921	17.76338
18	28.98437	25.44661	19.9838
20	32.20486	28.27401	22.20422

Fig.4 gives expected results, for the same values of the pipe thickness t as the fracture angle β is higher the critical pressure p_c is lower.

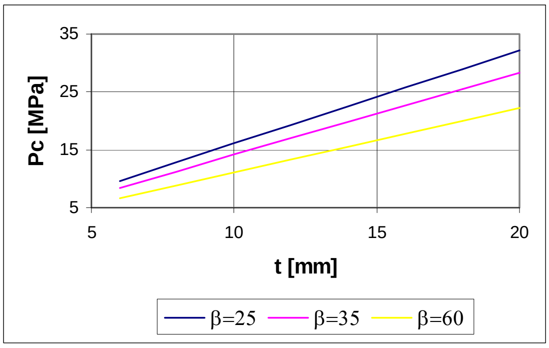

Figure 4. Critical pressure p_c *vs* pipe thickness t for three different angle β values

The critical pressure p_c can be calculated by eq. (13) by changing different values of the crack length a for three values of the initial crack angle β (Table 5). For the same values of the crack length a as the fracture angle β is higher the critical pressure p_c is lower (Fig.5).

Table 5. Critical pressure p_c vs crack length a for three different values of the angle β

	β=25°	β=35°	β=60°
a [m]	p_c[MPa]	p_c[MPa]	p_c[MPa]
0.0022	17.30202	15.19018	11.92919
0.0024	16.56543	14.54349	11.42133
0.0026	15.91555	13.97293	10.97326
0.0028	15.3366	13.46466	10.5741
0.003	14.81657	13.00809	10.21555
0.0032	14.34608	12.59503	9.89116
0.0034	13.91774	12.21898	9.59584
0.0036	13.52561	11.87471	9.32548
0.0038	13.16486	11.55799	9.07676
0.004	12.83152	11.26534	8.84693

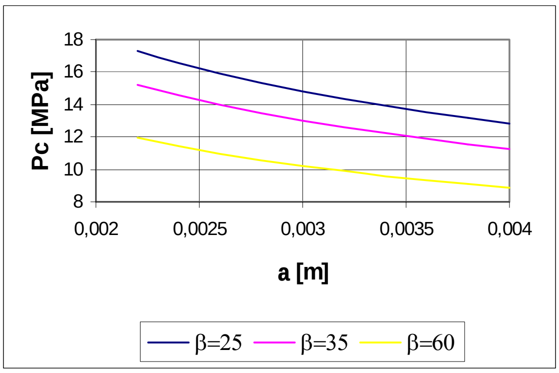

Figure 5. Critical pressure p_c vs crack length a for three different angle β values

Results of the finite element analysis for the stress intensity factor K_I

The finite element analysis, based on singular finite elements [5, 6] considers a circumferential pipe under internal pressure. The crack length is characterized by the half crack angle, θ. The analyses of the FE model for the considered pipe (Fig.2) were performed, using the FE program FEMAP v9.0.

Fig.2 depicts a circumferential pipe under the internal pressure p_c = 0.5 daN/mm^2 with relative dimensions. Some important dimensions for the FE model for the pipe should be noted: R = 500 mm, t = 10 mm and 2θ = 36°.

Internal pressure is applied as a distributed load to the inner surface of the FE model.

Delicateness of the finite element mesh as an influence on the stress intensity factor K_I values

The following section describes different aspects of the finite element analysis. Considering eq. (11) for the pipe (Fig.2) the stress intensity factor K_I can be found. For this relevant dimension, $K_\mathrm{I} = 19.92\ \mathrm{daN/mm^2}\sqrt{\mathrm{mm}}$.

The results for the same pipe for the FE analysis are shown in the next three examples. Different levels of delicateness of the FE mesh give varied results for the stress intensity factor K_I.

Fig.6 presents one typical mesh with about 2500 elements and nodes. In this case the stress intensity factor K_I is $14.69\ \mathrm{daN/mm^2}\sqrt{\mathrm{mm}}$. Comparing this and the analytical results gives a difference of nearly 26 per cent.

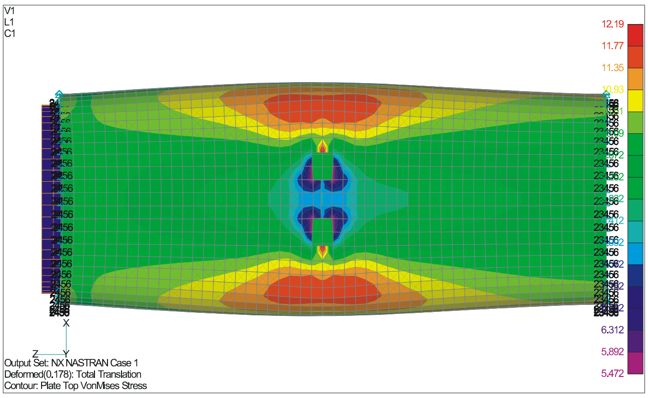

Figure 6. Von-Mises stresses for the first FE analysis

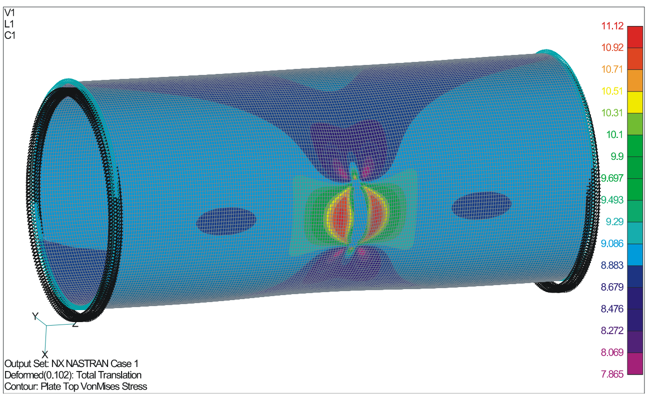

Figure 7. Von-Mises stresses for the second FE analysis

The number of elements and nodes in the second presented FE mesh are about 10000. It is much more refined than the first one presented. Therefore, the stress intensity factor K_I with this FE analysis is

$20.63\ \mathrm{daN/mm^2}\sqrt{\mathrm{mm}}$. The difference of the analytical result for this presentation is less than 4 per cent which is quite good. The FE data of this case have been best fitted and eq. (11) is proposed to determine the stress intensity factor K_I

In the third investigation the FE mesh is extremely refined with 39994 elements and 40220 nodes. Therefore, the stress intensity factor K_I in this analysis is $22.7\ \mathrm{daN/mm^2}\sqrt{\mathrm{mm}}$. If the results of the analytical method and the numerical one are compared, the difference of 13 percent is calculated. Similarity between the presented solutions and the analytical solutions provides confidence of the present FE analysis.

Conclusion

The strain energy density theory provides a simple procedure for defining the worst crack at a given stress level without causing fracture. In this investigation a simple method is applied for finding an approximate stress intensity factor. This paper also included the stress analysis for a pipe with different pressure values. It is also possible to predict the direction of the initiation of crack growth (θ) when we determine the direction of the maximum potential energy density. The critical intensity of the strain energy density factor S_c of this potential field governs the onset of crack propagation. The paper considers a cylindrical thin-walled pipe of the radius R and the thickness t much smaller than R, containing a crack of length $2R\theta$ along the circumference. It is loaded by the internal pressure p_c. The critical pressure is determined using the strain energy density theory. The additional feature of the S_c theory is that a single parameter can simultaneously determine the fracture toughness of the material and the direction of the initiation of crack growth. This investigation basically uses the Finite Element Method for stress analysis in correlation with appropriate failure criteria in determining a load level when initial failure occurs. The predictions of the proposed equations are compared with available numerical data. Simple formulas for stress intensity factors were derived showing a good approximation compared with the FEM results. This investigation assumed that the stress intensity factor is higher for more refined FE mesh solutions.

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Primena efektivne metode gustine energije deformacija pri određivanju zamornog veka uzoraka sa oštećenjem

Procena veka i određivanje ponašanja mašinskih elemenata sa prslinom pretstavljaju važne probleme u mašinstvu. Danas je moguće razmatrati mašinske elemente u prisustvu različitih oblika prslina i njihovog položaja, a kao i drugačijih oblika širenja prslina. Proračun faktora intenziteta napona K, širenje prsline i radni vek konstruktivnih elemenata zahtevaju veliku pažnju konstruktora. Određivanje veka elemenata se vrši pod dejstvom cikličnih opterećenja konstantne amplitude. U radu je izvršeno poređenje metode konačnih elemenata (MKE), odnosno rezultata dobijenih ovom metodom, rešenja dobijenih analitičkim putem i postojećih rezultata testiranja za tankozidni cilindrični rezervoar poluprečnika R i debljine t. Rezervoar je opterećen unutrašnjim pritiskom p_c. Kritični pritisak se određuje uz pomoć metode gustine energije deformacija (SEDT) [1]. Značaj S_c teorije je u tome što sam parametar S_c može da odredi lomnu žilavost materijala i pravac širenja prsline. Parisov zakon se ne može koristiti za analizu širenja prsline jer uzima u obzir samo jedan parametar opterećenja, dok se opterećenje pri zamoru karakteriše srednjim naponom. Da bi se rešio ovaj problem u radu je primenjena usavršena verzija ove metode, takozvana efektivna metoda gustine energije deformacija [2]. Namera ovog ispitivanja je bila da se potvrdi tačnost empirijskih modela baziranih na eksperimentalnim podacima.

Ključne reči: **zamor materijala, prskotina, rast prskotine, vek trajanja, metoda gustine energije, metoda konačnih elemenata.**

Применение эффективного метода плотности энергии деформации при определении срока усталости образцов с повреждением

Оценка срока службы и определение поведения машинных элементов с трещиной представляют очень важные проблемы в машиностроительстве. Сегодня существует возможность рассматривать машинные элементы при наличии различных форм трещин и их положения, а в том числе и иных форм расширения трещин. Расчет фактора интенсивности напряжения *K*, расширение трещины и срок службы элементов конструкции требуют большого внимания конструкторов. Определение срока службы элементов делается под влиянием циклических нагрузок постоянной амплитуды. В настоящей работе сделано сравнивание метода конечных элементов (МКЭ), т.е. результатов полученых этим методом, решений полученных путем анализа и существующих результатов испытаний для тонкостенного цилиндрического бака радиуса *R* и толщины *t.* Бак нагружен (усилен) внутренним давлением *pc.* Критическое давление определяется при помошчи метода плотности энергии деформации (SEDT) [1]. Значительность *Sc* теории в том, что сам параметр *Sc* может определить излом живучести материала и направление расширения трещины. Закон Париса не может быть использован для анализа расширения трещины, ибо учитывает только один параметр нагрузки, пока усталостная нагрузка характеризуется средним напряжением. Чтобы решить эту проблему, в настоящей работе применена усавершенствованная версия этого метода, так называемый эффективний метод плотности энергии деформации [2]. Назначение этого исследования было – потвердить точность эмпирических моделей обоснованных на экспериментальных данных.

Ключевые слова: **усталость материала, трещина, рост трещины, срок службы, метод плотности энергии, метод конечных элементов.**

Application de la méthode effective de la densité d'énergie des déformations dans la détermination de la vie de fatigue des échantillons à encoche

L'estimation de la durée de vie et la détermination du comportement des éléments mécaniques à fissure représentent les problèmes importants dans la construction des machines. Aujourd'hui il est possible de considérer les éléments mécaniques en présence de différentes formes de fissures et de leurs positions ainsi que les différentes formes de croissance des fissures. Le calcul du facteur de l'intensité de tension *K*, la croissance de la fissure et la durée de vie des éléments constructifs demandent grande attention des constructeurs. La détermination de la durée de vie des éléments se fait sous l'action des charges cycliques de l'amplitude constante. Dans ce papier on a fait la comparaison de la méthode des éléments finis (MEF), c'est-à-dire des résultats obtenus par cette méthode, solutions obtenus par le moyen analytique et des résultats existants des essais pour le réservoir cylindrique aux parois minces du rayon *R* et l'épaisseur *t.* Le réservoir est chargé de la tension intérieure *pc.* La tension critique est déterminée par la méthode de la densité d'énergie des déformations (SEDT) [1]. L'importance de la théorie *Sc* est dans le fait que le paramètre même *Sc* peut déterminer la résistance du matériel à la fissure et la direction de croissance de la fissure. La loi de Paris ne peut s'appliquer pour analyser la croissance de la fissure car cette loi considère un seul paramètre de charge alors que la charge à la fatigue est caractérisée par la moyenne tension. Pour résoudre ce problème, on a employé la version perfectionnée de cette méthode appelée la méthode effective de la densité d'énergie des déformations [2]. Le but de la recherche était de confirmer la précision des modèles empiriques basés sur les données expérimentales.

Mots clés: **fatigue de matériel, fissure, croissance de la fissure, durée de vie, méthode de la densité d'énergie, méthode des éléments finis.**