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Dynamic of Biped Movement on a Mobile Platform in the Presence Elasticity Elements

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This work is concerned with the modeling and analysis of a complex humanoid robotic system walking on a mobile platform. For this purpose, a software package was synthesized allowing the selection of configurations of both the humanoid and the platform. Each joint of the biped and the platform can be defined by the user via the motor state (active or locked) and the gear type (rigid or elastic). In the moment when the biped steps on the platform, the latter, by its dynamics, acts on the biped dynamics and the biped on the other hand, by its characteristics, influences the dynamics of the platform motion. These two complex contacting systems form a more complex system, the mathematical model of which has to encompass all the elements of coupling between the humanoid joints and the platform joints. It has been shown that coupling is more influenced when elasticity elements are included into the configuration. Reference trajectory of each joint can be defined so as to encompass or not encompass elastic deformations as well as known or unknown characteristics of coupling between the humanoid and the platform. The control structure for the biped walking on the platform should be defined so that it satisfies the requirement for the ZMP (*Zero-Moment Point*) to be within the given boundaries in every sampling instant, which guarantees a dynamic balance of the locomotion mechanism in the real regime. The synthesized new software FLEXI makes it possible to choose a robotic configuration. The analysis of simulation results of the humanoid robot motion on an immobile platform gives evidence for all the complexity of this system and shows how much system parameters (choice of trajectory, configuration, geometry, elasticity characteristics, motor, etc.) influence stabilization of its humanoid motion. All research in humanoid robotics has an aim to create a robot similar to a human that would be his servant, worker or soldier and that would replace him in all dangerous situations.

Key words: **robotics, humanoid robot, movement dynamics, mobile platform, modeling, coupling, joint elasticity, programmed trajectory, software package.**

Introduction

MODELING of the locomotion mechanisms of a rigid anthropomorphic structure is certainly a very interesting problem.

During the last four decades we have witnessed a strong development of a new class of mechanisms capable of performing diverse artificial motions of a locomotion-manipulation type [7-11]. The paper [12] considers the control of a robot with elastic joint in contact with dynamic environment.

For a long time already, researchers have been thinking about the implementation of elastic joints into an already complex model of a locomotion mechanism.

Let us review in brief some of the works dealing with robotic mechanisms that involve elasticity at joint.

The method proposed in [5] provides a general approach to the problem of defining mechanical configuration models and kineto-ealstodynamic effects. In [6], the author discusses the problem of force control of a manipulator with an elastic joint. Using the same principles defined by Spong [5] still in 1978, in this work we introduce elastic joints into a mathematical model. The generated new software FLEXI defines a mathematical model of the robotic locomotion system. The analysis of simulation results was carried out on a 10-DOFs (degrees of freedom) humanoid robot and a 3-DOFs platform. In the present research phase the robot's motion on a mobile platform is analyzed.

Modeling of a Humanoid Robotic System

Kinematics and dynamics of a humanoid locomotion robotic system walking on a mobile platform are modeled. Since elasticity elements are introduced, it is necessary to explain the kinematics of these systems.

Kinematics

In the presence of elasticity elements, the notion of a joint (DOF) requires a new meaning, which is necessary to expand and explain. In its configuration, each joint may contain:

– motor and/or

– gearing system.

The motor may be

* A*ctive*, if it realizes the motion whereby the motor deflection angle $\overline{\theta}_i \neq const$ or

* L*ocked*, if it is fixed at a certain position, and its speed and acceleration are zero, then $\overline{\theta}_i = const$.

Behind any motor there may be implemented a gearing system, which may be:

~ E*lastic*, if the deformation is elastic, then the joint deflection angle $\xi_i \neq 0$.

~ R*igid*, if elastic deformation is $\xi_i = 0$.

This means that a joint can be defined in dependence of the working state of the motor and a type of gearing. There

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are four types of joints. Therefore, this work analyzes the behavior of a humanoid robotic system which may contain joints of the types AE, LE, AR and LR, as defined in [16]. We can define all kinematic characteristics: the Denavit-Hartenberg parameters, rotation matrix, the overall transformation matrix and the Jacobi matrix.

Dynamics

Coupling between the biped and the platform

In order to model the biped motion on a mobile platform it is necessary to define first the characteristics of the coupling between these two complex systems. To analyze the coupling characteristics we will use first the rigid configurations of the biped and the platform. The coupling characteristics are very important, and therefore their explanation should not be blurred by some other effects such as, for example, elasticity.

Biped rigid, platform rigid

The complex robotic mechanism consisting of the humanoid and the platform being in contact at the point “A” is sketched in Fig.1a. The system's mathematical model is of the form:

$$\tau = H(q)\ddot{q} + h(q,\dot{q}) \tag{1}$$

The inertia matrix $H(q)$ is a full matrix, having also elements outside the diagonal, characterizing the coupling between the present DOFs. The presence of coupling is also reflected on the vector $h(q,\dot{q})$ through the Coriolis' and centrifugal forces. The control vector is τ .

The complex system shown in Fig.1a is divided, first hypothetically, in two parts (Fig.1b), whereby the upper system can move independently of the lower one. The upper system is the biped and the lower one is the platform. The motion dynamics of the complex biped mechanism (the upper part in Fig.1b) is described by the equation:

$$\tau_b = H_b(q_b)\ddot{q}_b + h_b(q_b,\dot{q}_b) \tag{2}$$

The motion dynamics of the complex platform mechanism (the lower part of Fig.1b) is described by the equation:

$$\tau_p = H_p(q_p)\ddot{q}_p + h_p(q_p,\dot{q}_p) \tag{3}$$

When separated, these two systems perform independent motions. As we want to analyze the motion dynamics of the biped and the platform in contact, we presume exactly such a situation and because of that it is necessary to connect these two subsystems. Their linking gives a system shown in Fig.1c. Let us compare Fig.1a and 1c. They stand for the similar robotic configurations. It has been already mentioned that between particular DOFs in Fig. 1a there exists strong coupling, and because of that it is logical to suppose that the same strong coupling exists between the particular biped DOFs and the particular platform DOFs from Fig.1c.

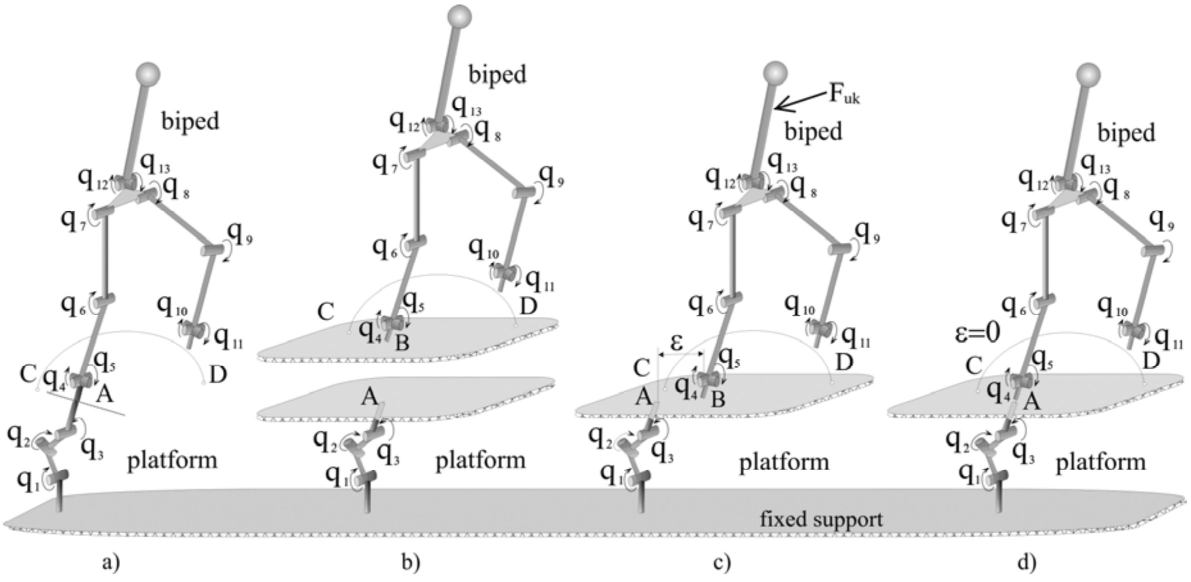

Figure 1. Complex robotic mechanism biped-platform.

In general, the biped need not be in contact with the platform at the point "A" but at some other point, for example "B" (see Fig.1c). If the points "A" and "B" coincide, then the geometry of the overall complex mechanism is simpler. This would be a most favorable case. However, a much more realistic is the case when the points "A" and "B" do not coincide. Then there exists a quantity ε which participates in the dynamics of the overall system motion.

Full coupling between the particular DOFs of the biped and platform is an essential characteristic of this robotic system. In this sense the motion of the system shown in Fig. 1c should be modeled by the following equation, which encompasses the full coupling between all present DOFs.

$$\begin{bmatrix} \tau_{\bar{p}} \\ \tau_{\bar{b}} \end{bmatrix} = \begin{bmatrix} H_{\bar{p}} & H_{\bar{p}\bar{b}} \\ H_{\bar{b}\bar{p}} & H_{\bar{b}} \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_{\bar{p}} \\ \ddot{q}_{\bar{b}} \end{bmatrix} + \begin{bmatrix} h_{\bar{p}\bar{b}} \\ h_{\bar{b}\bar{p}} \end{bmatrix} \tag{4}$$

The inertiality matrix in the model (4) is a full matrix, and the Coriolis' and centrifugal forces encompass also the coupling between the biped and the platform.

If, apart from the platform, the biped is in contact with some other environment as in [13], then the action of that environment is defined by the dynamic force F_{uk}. In our example from Fig.1c the external force F_{uk} acts on the dynamics of the motion of both the biped and the platform, i.e. on the complex biped-platform system, via the system geometry, that is via the Jacobian matrix J. We thus define the overall dynamic model of the complex system biped-platform by the following equation:

$$\begin{bmatrix} \tau_{\bar{p}} \\ \tau_{\bar{b}} \end{bmatrix} = \begin{bmatrix} H_{\bar{p}} & H_{\bar{p}\bar{b}} \\ H_{\bar{b}\bar{p}} & H_{\bar{b}} \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_{\bar{p}} \\ \ddot{q}_{\bar{b}} \end{bmatrix} + \begin{bmatrix} h_{\bar{p}\bar{b}} \\ h_{\bar{b}\bar{p}} \end{bmatrix} + \begin{bmatrix} J^T_{\bar{p}} \\ J^T_{\bar{b}} \end{bmatrix} \cdot F_{uk} \tag{5}$$

In order to analyze the presence of coupling between the biped and the platform we adopted the configurations of the biped and the platform shown in Fig.1d. Let us assume that $\varepsilon = 0$. All DOFs of both the biped and the platform are rotational (the biped has 10, and platform 3 such joints), and they are defined as the joints of the type AR. the rotational joints of the platform are q_1, q_2, q_3.

The biped and the platform with elasticity elements

Now, when we explained the characteristic of the coupling between the biped and the platform as a rigid systems, it is logical to conclude that the additional elasticity in the system (elastic joints of the humanoid and/or elastic joints of the platform) could only increase the complexity of the coupling that takes place via the elasticity forces, as well as via the inertial (Coriolis' and centrifugal) forces.

To analyze the properties of the coupling between the biped and the platform in the presence of elasticity elements we consider the configuration from Fig.2. If we want to set some arbitrary joints of the AE, LE or AR type and the rest of the LR type, then the system structure changes as well as the structural matrix in it. In order to present the general structure of the mathematical model of the robotic mechanism with arbitrarily selected types of each joint, let us fix certain joints of the biped and the platform in the following way:

- $eo = 0$ selected joints of type AE,
- $ne = 2$ selected joints of type LE,
- $nr = 11$ selected joints of type AR,
- $nc = 15$ selected joints of type LR.

With the joints thus selected, the mathematical model of the overall mechanism defined in Fig.3 would be of the form:

$$0_{ne} = H_e \cdot \ddot{q} + h_e + C_n \cdot \xi_n + B_n \cdot \dot{\xi}_n + J_e^T \cdot F_{uk} \tag{6}$$

$$\tau_e = I_e \cdot \ddot{\bar{\theta}}_e + B_e \cdot \dot{\bar{\theta}}_e - S_e (C_n \cdot \xi_n + B_n \cdot \dot{\xi}_n) \tag{7}$$

$$\tau_k = I_k \cdot \ddot{\bar{\theta}}_k + B_k \cdot \dot{\bar{\theta}}_k + S_k \cdot (H_k \cdot \ddot{q} + h_k + J_k^T \cdot F_{uk}) \tag{8}$$

$0_{ne} \in R^2$, $H_e(q) \in R^{2\times 15}$ $h_e(q,\dot{q}) \in R^2$,
$C_n = diag[C_{ni}] \in R^{2\times 2}$, $B_n = diag[B_{ni}] \in R^{2\times 2}$, $q_{el} \in R^2$,
$J_e(q) \in R^{6\times 2}$, $\tau_e \in R^2$, $I_e = diag[I_i] \in R^{2x2}$,
$B_e = diag[B_i] \in R^{2x2}$, $S_e = diag[S_i] \in R^{2x2}$, $\bar{\theta}_e \in R^2$.
$\tau_k \in R^{11}$, $I_k = diag[I_i] \in R^{11x11}$, $B_k = diag[B_i] \in R^{11x11}$,
$S_k = diag[S_i] \in R^{11x11}$, $H_k \in R^{11\times 15}$, $h_k \in R^{11}$, $J_k(q) \in R^{6\times 11}$,
$F_{uk} \in R^6$, $\bar{\theta}_k \in R^{11}$.

It is evident that the environment force F_{uk} participates in generating the overall coordinates q, and the same do the deflection angles ξ. This force is mapped on the direction of the generalized coordinates via the elements of the Jacobi matrix J.

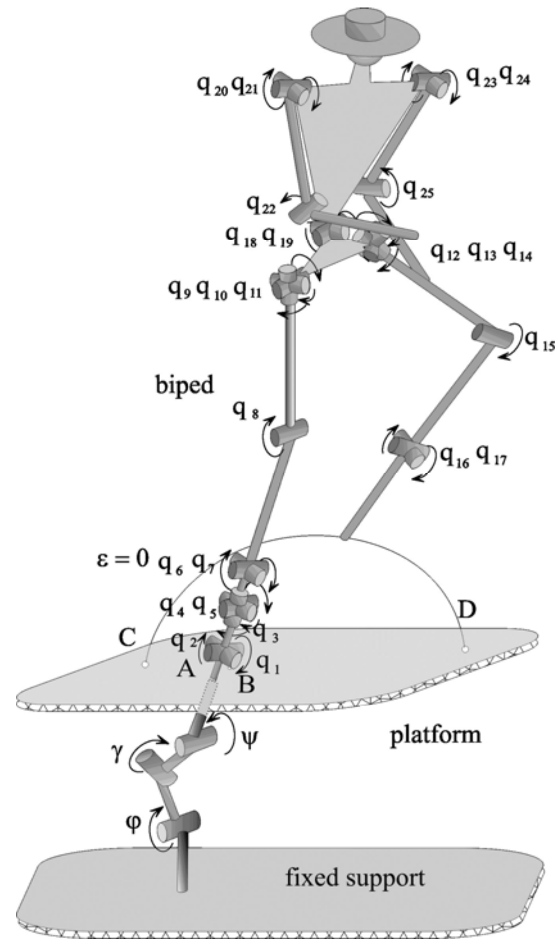

Figure 2. Humanoid robotic mechanism with 25 DOFs on the mobile platform with 3 DOFs.

Reference Trajectory

When considering the motion of a humanoid robotic system one should distinguish two, mutually dependent, aspects of defining the reference trajectory:

- reference trajectory of the motion of each joint,
- reference trajectory of the ZMP^{o} [3], [8]).

These two approaches to defining the references are of equal importance and should act in a coordinated way in order to achieve a harmonious human-like motion.

The problem of creating a reference joint trajectory of elastic robots has been dealt with only in several papers, a procedure being presented first by [1]. More recently in [2]), this was done for one-DOF robotic system with an elastic joint and a rigid link, and separately, for a stiff joint and an elastic link.

There are two aspects of defining the reference trajectory of the motor angle [14], [15] and [16], viz.:

1. Elastic deformation is considered as a quantity which is not encompassed by the reference trajectory. This is the case when the elasticity characteristics in the system are not known and are not included in the reference trajectory definition. We have two approaches to defining the references:

- which does encompass the property of the biped-platform coupling at the reference level
- in real conditions, which does not know the coupling property.

2. Elastic deformation is a quantity which is at least partly encompassed by the reference trajectory. It is assumed that all elasticity characteristics in the system (both of stiffness and damping) are 'known', at least partly, and at that level can be included into the process of defining the reference motion. We have two approaches to defining the references motion. We have two approaches to defining the references:

- which does encompass the property of the biped-platform coupling at the reference level,
- in real conditions, which does not know the coupling property.

Control of the biped walking on a mobile platform

The aim of control here is not to stabilize the humanoid robot motion on the mobile platform to make the ZMP to be as close as possible to zero, or some similar criterion. Here, the aim of the control synthesis is to emphasize the effect of the present elasticity elements and demonstrate the existence of coupling properties.

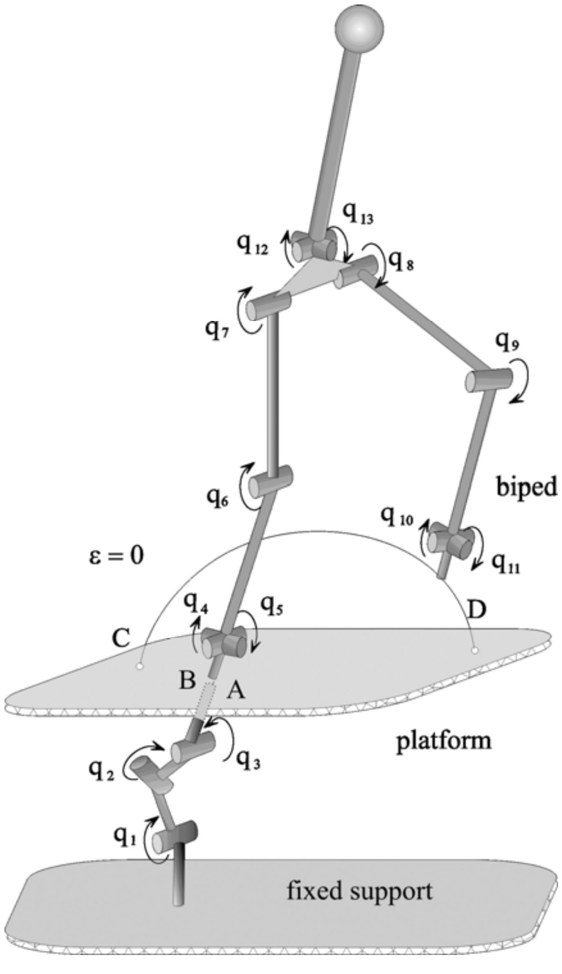

Figure 3. Humanoid robotic mechanism with 10 DOFs on the mobile platform with 3 DOFs.

For the simulation example to analyze the humanoid motion on the mobile platform we choose the control law τ_u, which encompasses CR (Centralized Reference control, calculated from the reference state) τ_{rr}° plus LO (control via LOcal feedbacks of motor motion with respect to position and velocity) τ_l.

$$\tau_u = \tau_{rr}^{\circ} + \tau_l \qquad (9)$$

$$\tau_l = K_{lpi} \cdot (\bar{\theta} - \bar{\theta}_i) + K_{lvi} \cdot (\dot{\bar{\theta}}_i^{o} - \dot{\bar{\theta}}_i) \qquad (10)$$

Since, among the others, the biped-platform coupling is also subject of this work, we will define the CR control for real conditions, namely as a quantity τ_{rr}° which does not encompass

1. the property of the biped-platform coupling at the reference level, and
2. elastic deformation at the reference level.

The overall control $\tau_{rr}^{\circ} = \begin{bmatrix} \tau_p^{\circ} & \tau_b^{\circ} \end{bmatrix}^T$ contains:

- The CR platform control τ_p° defined by eq. (4) under the condition that all biped joints are of the LR type. The biped is in the state of rest on the platform. The platform tracks the reference trajectory without additional disturbances when the biped is placed on it. All the platform joints are of the AR type.
- The CR biped control τ_b° is defined by eq. (2) in the reference regime. All biped joints are defined as being of the AR type. τ_b° is defined under the condition that the humanoid walking on the immobile platform tracks without additional disturbances the reference trajectory. This means that all platform joints are defined as being of the LR type.

Such τ_{rr}° is defined under the special, real conditions of not knowing the coupling property. The conditions of defining the CR control τ_{rr}° are significantly different from the real conditions of the biped motion on the mobile platform.

If we analyze the elastic robotic biped-platform system defined by eqs. (6 - 8), then τ_{rr}° does not encompass the property of

1. coupling, and neither the property of
2. elasticity.

Simulation Examples

General conditions that hold for all the simulation examples of the humanoid walking on the mobile platform are:

Simulations were carried out for the elastic humanoid robotic mechanism that moves without constraints. This means that no environment force is present in the simulation analyses.

The geometry of the humanoid robotic system is defined so to mimic the geometry of a human 1.8 [m] tall, with a mass of 70 [kg], proportionally distributed over the links. The foot size with respect to the desired position of the ZMP in the x-direction is ± 0.15 [m] and in the y-direction ± 0.075 [m], so that the overall foot size is $0.3x0.15$ [m].

The elasticity module of aluminum is: $E_l = 69.3 \cdot 10^9$ [N/m^2].

The initial deviations of the angles of the motor are:

$$\delta\bar{\theta}_i(t_0) = [0;0;....0,0][\text{rad}],\ \delta\dot{\bar{\theta}}_i(t_0) = [0;0;...;0;0][\text{rad/s}].$$

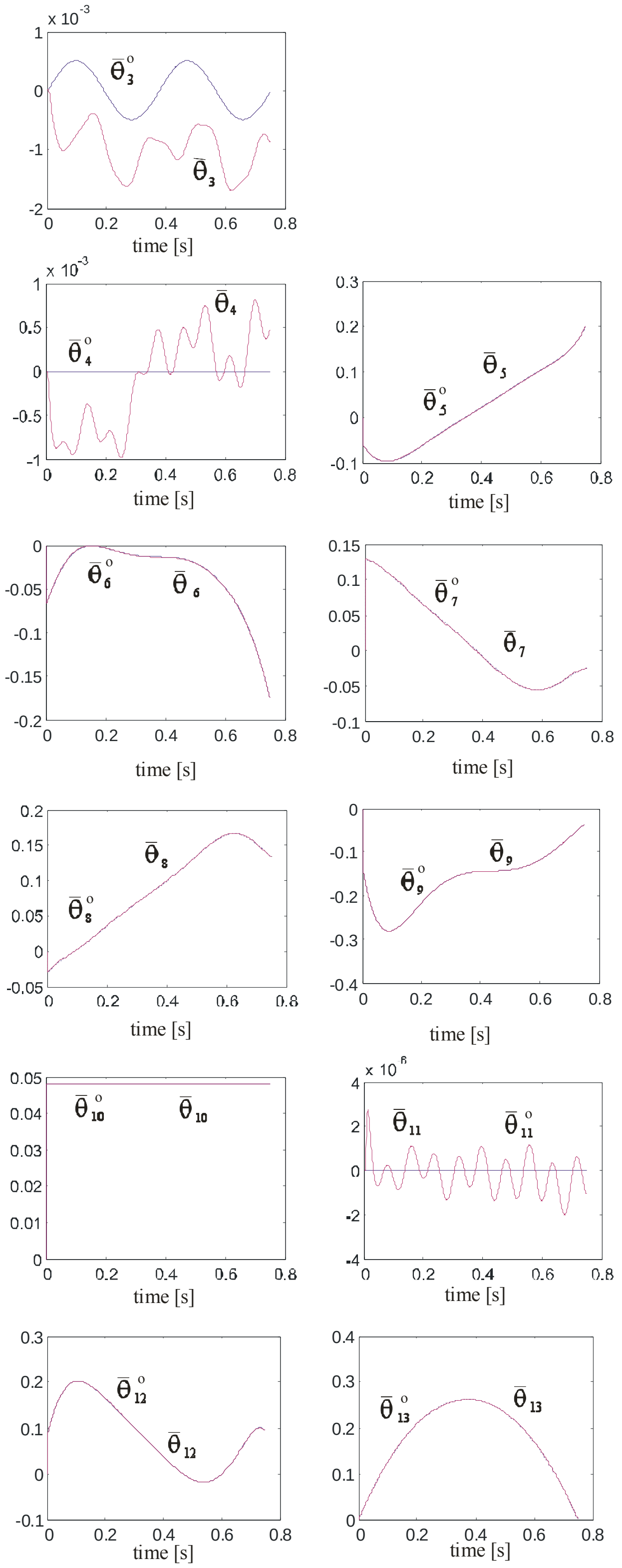

Figure 4. Referenced $\overline{\theta}_i^{\circ}$ and real $\overline{\theta}_i$ motor angle [rad] (*Example 1*).

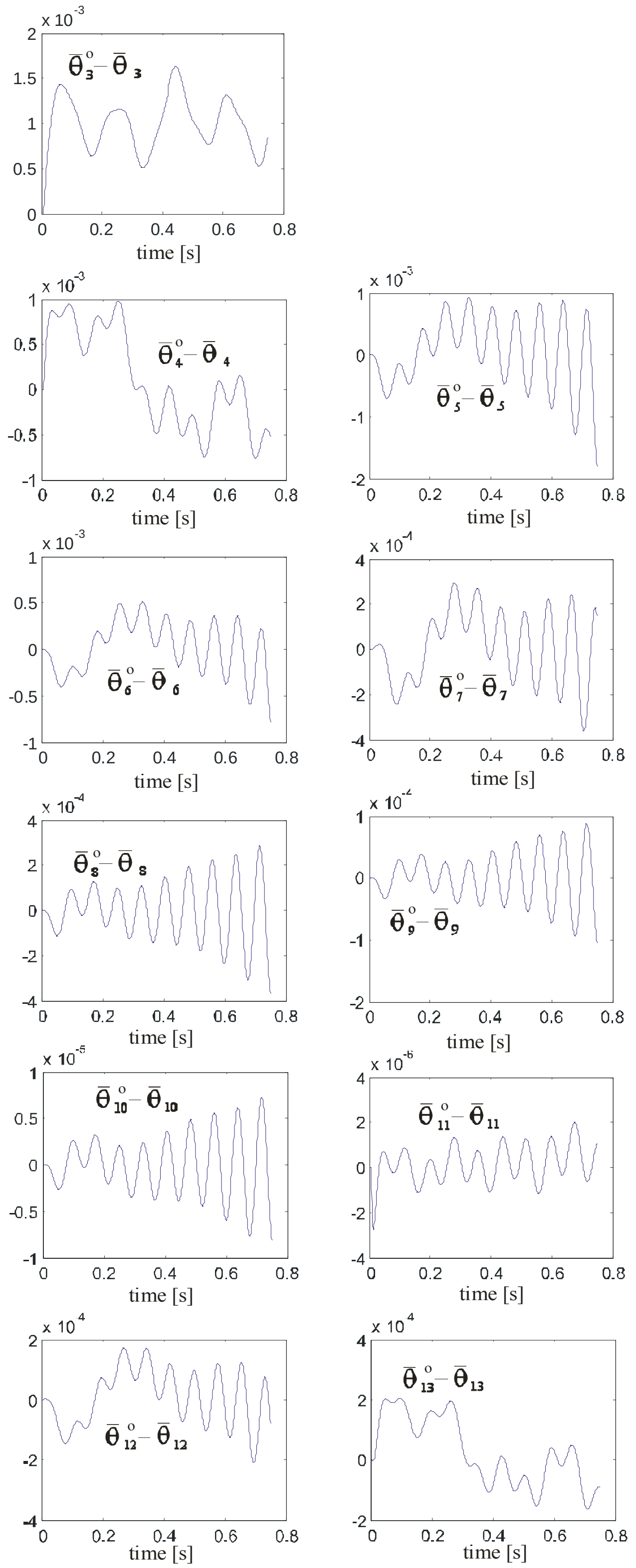

Figure 5. Deviation referenced $\overline{\theta}_i^{\circ}$ from real $\overline{\theta}_i$ [rad] (*Example 1*).

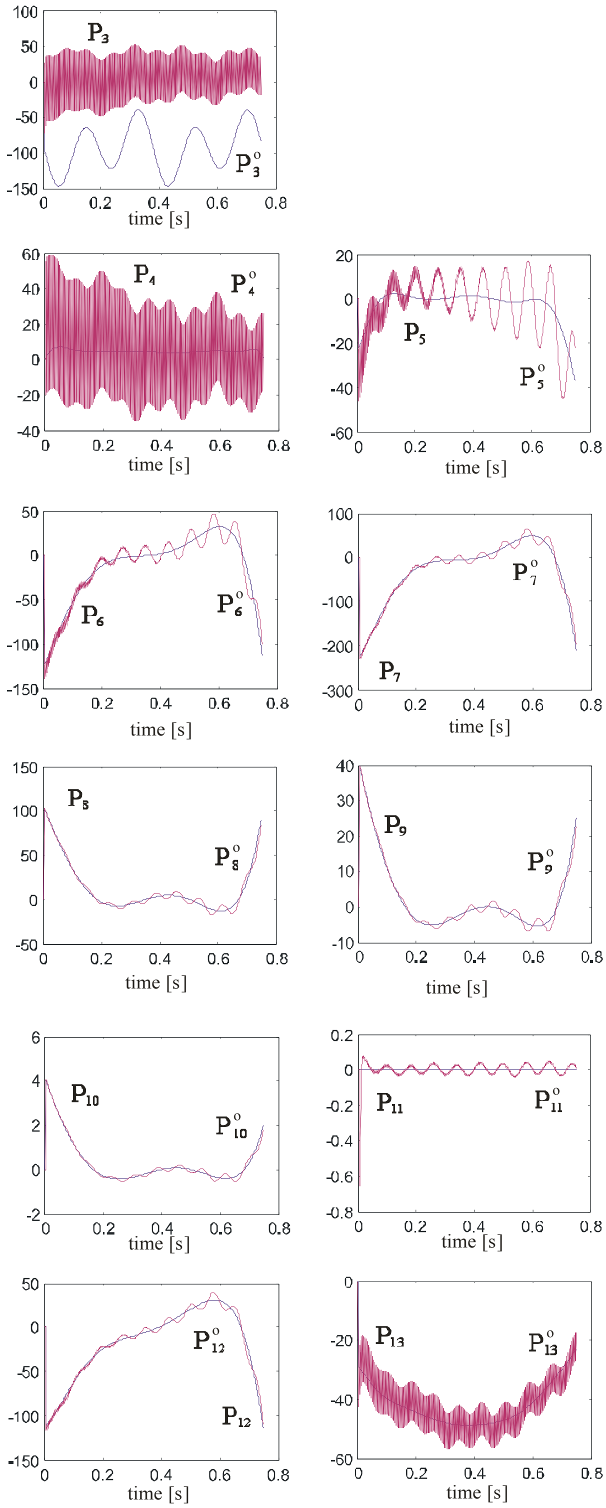

Figure 6. Referenced P_i° and real P_i moment load [Nm] (*Example 1*).

The initial deviations were deliberately chosen as zeros in order to emphasize the importance of the other effects of motion dynamics. The sampling period is: $dt = 0.001[\mathrm{s}]$. In the simulation examples, the reference trajectory is defined as for a rigid system, so that no elastic deformations are present at the reference level, $\xi_i^{\circ} = 0$.

Because of that, only the real values of elastic deformations ξ_i are designated in the figures. The reference trajectory is defined for real conditions, which does not encompass the property of biped-platform coupling.

Let us now deal with the simulations.

The tip of the left leg of the 10-DOFs humanoid robot moves in a half-step from the point "C" to the point "D" on a mobile platform in a predicted time $T = 0.75[\mathrm{s}]$ (Fig.3).

We select the following parameters of the motor and the gear: I=0.3534 $[\mathrm{kg\,m\Omega A/N}]$, B = 7.3534$[\mathrm{V/(rad/s)}]$, S=0.2297 $[\Omega\mathrm{A/Nm}]$.

Simulations were carried out for the following biped-platform system: the platform joints, 1, 2 are of the type LE (they can move about the x, y axes of the local coordinate frame, respectively) and 3 are of the type AR (can move about the z axes of the local coordinate frame).

The biped joints, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13, are also of the AR type. The control is defined by $\tau_{rr}^{\circ} + \tau_l$.

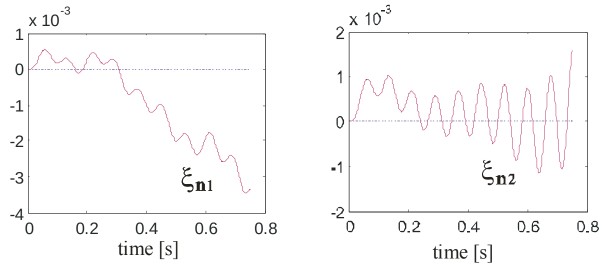

Figure 7. Real elastic deformations ξ_{ni} ($\xi_{ni}^{\circ} 0$) [rad] (*Example 1*).

The control gains for stabilizing drives of the platform are: $K_{lp} = diag[22500]$ and $K_{lv} = diag[300]$, and of the biped $K_{lp} = diag[2025]$ and $K_{lv} = diag[90]$.

Example 1:

If in the preceding example we define the characteristics of stiffness and damping of the platform joints 1 and 2 of the type LE to $C_{\xi 1} = 100$ [Nm/rad], $C_{\xi 2} = 10$ [Nm/rad], $B_{\xi 1} = 100$ [Nm/(rad/s)], $B_{\xi 2} = 1000$ [Nm/(rad/s)], the ZMP position is stabilized, although oscillations are present.

Fig.4 shows the referenced $\bar{\theta}_i^{\circ}$ and the real trajectories of motor angles $\bar{\theta}_i$ for all 11 DOFs. There was the disposal the trajectory where the referenced ZMP° is very close to zero during the half-step realization.

Fig.5 gives the deviation of $\bar{\theta}_i^{\circ}$ from $\bar{\theta}_i$.

It is interesting to have a look at Fig.6, showing the reference reduced torque P° caused by the reference system dynamics during robotic task realization, as well as the real reduced torque P caused by the real system dynamics.

Namely, the moment P encompasses the influence of elastic effects and also the property of the biped-platform coupling occurring during the motion of the humanoid robotic mechanism on the elastic platform.

It is evident from Fig.7 that the elastic deformations ξ_{n1} and ξ_{n2} were of the order of 10^{-3}. Fig.8 presents the referenced forces F_i° and moments M_i° and the real F_i, M_i by which the humanoid robot acts on the mobile platform in all 6 directions of the Cartesian coordinates.

Fig.9 illustrates the ZMP and ZMP motion during the robotic task realization. The shaded area belongs to the biped foot.

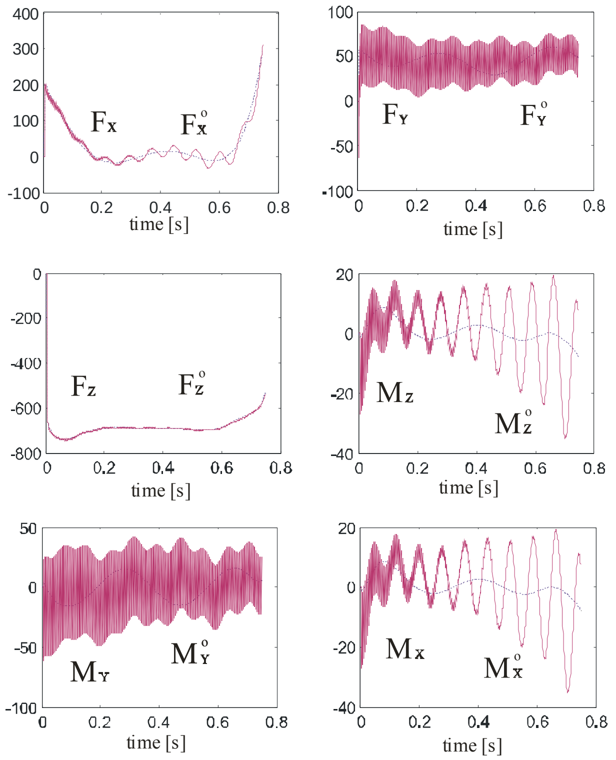

Figure 8. Referenced F_i°, M_i° and real F_i, M_i force [N] ([Nm]) (*Example 1*).

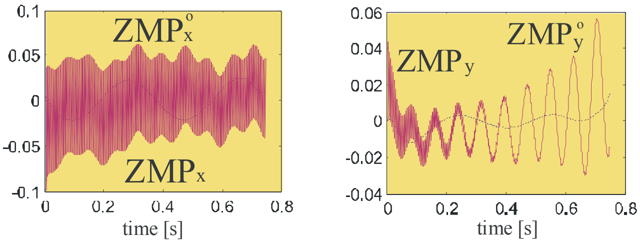

Figure 9. Referenced ZMP° and real ZMP in the x- and y-directions [m] (*Example 1*).

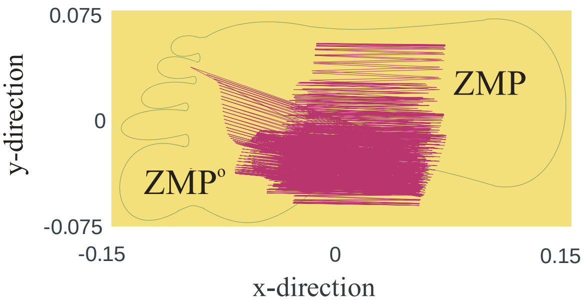

Figure 10. Referenced ZMP° and real ZMP in the x - y plane [m] (*Example 1*).

The ZMP position, despite significant oscillations, does not show the tendency of instability. The ZMP remained inside the footprint area (see Fig.9 and 10).

Example 2:

If in the preceding example we change the characteristics of stiffness and damping of the platform joints 1 and 2 of the type LE to $C_{\xi 1}=2*1000$ [Nm/rad], $C_{\xi 2}=2*1000$ [Nm/rad], $B_{\xi 1}=1000$ [Nm/(rad/s)], $B_{\xi 2}=1000$ [Nm/(rad/s)].

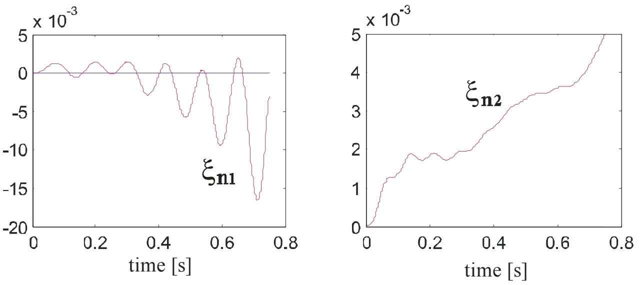

Figure 11. Real elastic deformations ξ_{n1} and ξ_{n2} $\left(\xi_{n1}^{\circ}=\xi_{n2}^{\circ}=0\right)$ [rad] (*Example 2*).

All other characteristics are same as in Example 1.

By analyzing Fig.11 we can see that the elastic deformations ξ_{n1} and ξ_{n2} of the joints of the LE type increased in an oscillatory manner during the robotic task realization, especially for ξ_{n1}, which stands for the motion about the x-axis of the local coordinate frame.

If we analyze the behavior of the ZMP (Figures 12 and 13) we can see its motion in deviating away from the zero position. In the critical moment of $T_h=0.745$[s], when the ZMP comes out of the foot area (and this is the instant when it exceeds $+0.15$[m] in the x-direction), the humanoid is falling down.

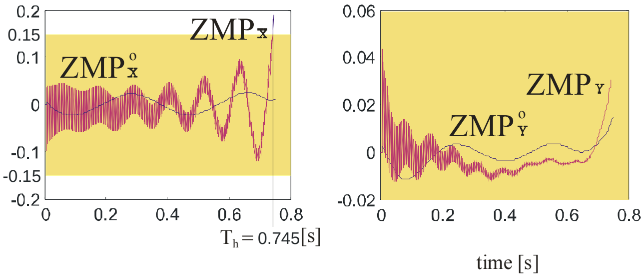

Figure 12. Reference ZMP° and real ZMP in the x- and y-directions [m] (*Example 2*).

To that moment, the ZMP trajectory is shown as a full red line and the rest of the simulation from the instant T_h to the end of motion by a dashed blue line.

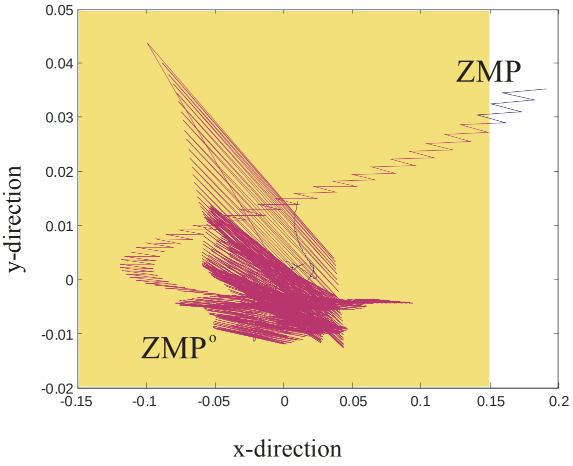

Figure 13. Reference ZMP° and real ZMP in the x - y plane [m] (*Example 2*).

A simulation after the critical moment T_h makes no sense because it was carried out under the assumption that the foot was $>|\pm 0.15[\mathrm{m}]|$ in the x-direction, but it is interesting from a visual point of view, as it shows a ZMP departure from zero. In Fig.14 it is seen how much ZMP really goes out from the foot contour. In this example, tracking of the desired ZMP trajectory is poor.

The lack of the knowledge of the elasticity properties and the coupling characteristic (CR control is τ°_{rr}) at the reference level represents a significant disturbance to the system.

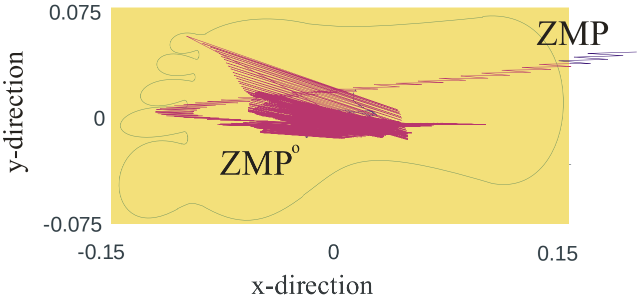

Figure 14. Reference ZMP and real ZMP in the x - y plane with respect to the foot [m] (*Example 2*).

The simulation results show the real problems of the complex biped-platform system control and the necessity of synthesizing new control laws.

Conclusion

The software package developed in this study can be used for the synthesis and analysis of complex elastic robotic systems of a biped-platform type, and can be expanded from various aspects, according to the user's needs.

The algorithm defined generates a mathematical model of a complex humanoid robotic system that walks on a mobile platform. It holds for an arbitrary number of DOFs of the chosen biped and an arbitrary number of DOFs of the platform. In this work, it was applied on a 25-DOFs robot walking on a 3-DOFs platform. The model equations change their structure depending on the nature of particular joints, defined as being of AE, LE, AR or LR type. The task of modeling is solved as a program, so that the user should only select the type of joints and, normally, all the system parameters, and the software package will generate a model which offers the possibility of analyzing its dynamic behavior.

The dynamic model of robotic mechanism encompasses all coupling elements, as well as all dynamic effects of the present forces. Elastic deformations of the elastic elements are defined as a logical consequence of dynamic load moments.

Elasticity dynamics is analyzed in dependence of the dynamics of the overall robotic system. It is shown that there exist strong coupling between the DOFs during the motion of the complex robotic system. The humanoid robotic system which moves on mobile platform is described by a system of differential equations of damped oscillations. For such dynamic model of robotic system, a combined control law, CR plus LO control, is selected.

The simulations were carried out for the case when the reference trajectory did not encompass the magnitude of elastic deformation. The software package offers comfort in the analysis of complex elastic robotic systems. The characteristic of the configuration complexity of the robotic system and the choice of the elasticity characteristic in the system are not any more limiting factors in the analysis of kinematics and dynamics of humanoid robotic systems.

Thus, an essentially new approach is elaborated for considering elastic robotic systems, which opens up some new possibilities in the analysis and modeling of these systems, as well as in the implementation of new control laws.

The paper emphasizes the existence of dynamic coupling between the biped and the platform in the course of robotic task realization. In the moment of robot's stepping on the platform, these two complex contacting systems form a more complex mechanical system. The mathematical model of the overall system must encompass all the elements of the coupling between the DOFs of the biped and the platform. Coupling is an elementary property of robotic systems that has to be taken into consideration when analyzing the motion of the biped and the platform in contact. Simulations and analyses encompassed the dynamics of the biped walk on a mobile platform involving only rotational DOFs. Inclusion of the elementary property of coupling between the biped and the platform suggests the need for synthesizing new control laws to track the ZMP trajectory. In the case when elastic characteristics are included, the coupling becomes stronger and the departure of the real ZMP trajectory from its reference value becomes larger.

It appeared that the choice of parameters (motors, configuration, geometry, elasticity characteristics, trajectory selection, etc.) of such a complex biped-platform system may play an essential role in the process of system stabilization. This is only one aspect of the analysis of dynamic behavior of robotic systems which opens the door to other research aspects. Our final goal is to achieve a humanoid walking on the platform, and this will be possible if much more knowledge is gained about this complex system.

The software package, synthesized specifically for the purpose of allowing more comfortable analysis of such a complex system, gave interesting simulation results. By using it, the user can choose the configuration of the system to be analyzed as well as the types of particular DOFs. Besides, there exists the possibility of expanding the software package from various aspects, to meet the user's diverse needs.

The further development in this area should proceed in the direction of implementing new control laws. All research in this area has a tendency to create a humanoid robot that would be helpful to people in everyday work. War industry especially in highly developed countries invests enormous funds in the development of „robot-solder” that would completely replace people in war circumstances.

References

[1] BAYO,E.: *A Finite-Element Approach to Control the End-Point Motion of a Single-Link Flexible Robot*, J. of Robotic Systems, 1987, Vol.4, No.1, pp.63-75.

[2] LUCA,A.De.: *Feedforward/Feedback Laws for the Control of Flexible Robots*, IEEE Int. Conf. on Robotics and Automation, April 2000, pp.233-241.

[3] JURIČIĆ,D.,VUKOBRATOVIĆ,M.: *Mathematical modeling of a bipedal walking system*, ASME Publ. 72-WA/BHF-13 (1972).

[4] PAUL,R.P.: *Robot Manipulators: Mathematics, Programming, and Control, The Computer Control of Robot Manipulators*, The MIT Press, Cambridge, Massachusetts and London, England, 1981.

[5] SPONG,M.W.: *Modeling and control of elastic joint robots*, ASME J.of Dynamic Systems, Measurement and Control, 1987, 109, pp.310-319.

[6] SPONG,M.W.: *On the Force Control Problem for Flexible Joint Manipulators*, IEEE Transactions on Automatic Control, January 1989, Vol.34, No.1.

[7] VUKOBRATOVIĆ,M., BOROVAC,B., SURLA,D., STOKIĆ,D.: *Scientific fundamentals of robotics,* Vol. 7, Biped Locomotion: Dynamics, Stability, Control and Application, Springer-Verlag, 1989.

[8] VUKOBRATOVIĆ,M., BOROVAC,B.: *Zero-Moment Point – Thirty Five Years of its Life*, International Journal on Humanoid Robots, World Scientific Publishing (Singapore) & Imperial College Press (London), 2004, Vol.1, pp.157-173.

[9] VUKOBRATOVIĆ,M., KIRĆANSKI,M.: *Kinematics and Trajectory Synthesis of Manipulation Robots*, Springer-Verlag, Berlin, Heidelberg 1986.

[10] VUKOBRATOVIĆ,M., HRISTIĆ,D., STOILJKOVIĆ,Z.: *Development of active anthropomorphic exoskeleton*, Medical and Biological Engineering, 1974, 12, No.1, pp.20-28

[11] VUKOBRATOVIĆ,M., JURIČIĆ,D.: *Contribution to the synthesis of biped gait*, IEEE Trans. On Biomedical Engineering, 1969, 16, No.2, pp.1-6.

[12] VUKOBRATOVIĆ,M., POTKONJAK,V., MATIJEVIĆ,V.: *Control of Robots with Elastic Joints Interacting with Dynamic Environment*, Journal of Intelligent and Robotic Systems, 1998, No.23, pp.87-100.

[13] POTKONJAK,V., VUKOBRATOVIĆ,M.: *Dynamics in Contact Tasks in Robotics, Part I General Model of Robot Interacting with Environment*, Mechanism and Machine Theory, 1999. Vol.33.

[14] FILIPOVIĆ,M., VUKOBRATOVIĆ,M.: *Modeling of Flexible Robotic Systems*, Computer as a Tool, EUROCON 2005, The International Conference, Belgrade, Serbia and Montenegro, 21-24 Nov. 2005, Vol.2, pp.1196-1199.

[15] FILIPOVIĆ,M., VUKOBRATOVIĆ,M.: *Contribution to modeling of elastic robotic systems*, Engineering & Automation Problems, International Journal, September 23. 2006, Vol.5, No.1, pp.22-35.

[16] FILIPOVIĆ,M., POTKONJAK,V., VUKOBRATOVIĆ,M.: *Elasticity in Humanoid Robotics*, Scientific Technical Review, Military Technical Institute, Belgrade, 2007, Vol.LVII, No.1, pp.24-33.

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Dinamika kretanja bipeda na pokretnoj platformi u prisustvu elastičnih elemenata

U ovom radu je modeliran i analiziran složeni humanoidni robotski sistem koji hoda po pomičnoj platformi. U tu svrhu sintetizovan je programski paket, koji pruža mogućnost izbora konfiguracije humanoida kao i konfiguracije platforme. Svaki zglob bipeda ili platforme korisnik može da definiše stanjem motora (aktivan ili zaključan) i tipom prenosnika (krut ili elastičan). U momentu kada biped stupi na platformu, ona svojim karakteristikama deluje na dinamiku kretanja bipeda, a takodje i biped svojim karakteristikama deluje na dinamiku kretanja platforme. Ova dva složena sistema u kontaktu postaju jedan još složeniji sistem, čiji matematički model mora obuhvatiti sve elemente sprezanja izmedju zglobova platforme i zglobova humanoida. Pokazano je da je fenomen sprezanja još izraženiji kada se u konfiguraciju uključe i elementi elastičnosti. Referentna trajektorija svakog zgloba može biti definisana tako da obuhvata elastičnu deformaciju ili da je ne obuhvata a takođe da poznaje ili ne poznaje karakteristiku kuplovanja između humanoida i platforme. Upravljačka struktura bipeda koji se kreće po platformi treba da bude definisana tako da u svakom trenutku odabiranja bude zadovoljen zahtev da ZMP (*Zero-Moment Point*) bude u zadatim granicama, čime je stabilnost lokomocionog mehanizama u realnom režimu zagarantovana. Analiza rezultata simulacija kretanja humanoidnog robota po pokretnoj platformi ukazuje na složenost ovog sistema i pokazuje koliko parametri sistema (izbor trajektorije, konfiguracije, geometrije, karakteristika elastičnosti, motora ...) utiču na stabilizaciju njegovog humanoidnog kretanja. Sva istraživanja u humanoidnoj robotici imaju za cilj da stvore robota što sličnijeg čoveku, koji bi mu bio sluga, radnik, vojnik i koji bi ga zamenio u svim opasnim situacijama.

Ključne reči: **robotika, humanoidni robot, dinamika kretanja, pokretna platforma, modeliranje, kuplovanje, elastičnost zgloba, programirana trajektorija, programski paket.**

Динамика движения бипеда на подвижной платформе при наличии упругих элементов

В настоящей работе моделирована и анализирована сложная интеллектуальная робототехническая система, шагающая по подвижной платформе. В ту цель синтезирована упаковка программного обеспечения, позволяющая сделать выбор конфигурации интеллектуалјьного робота, а в том роде и конфигурации платформы. Каждый зглоб (шарнир) бипеда или платформы пользователь может определить состоянием двигателяя (активный или запертый) и типом передатчика (жесткий или упругий). В моменте когда бипед шагнет на платформу, она своими характеристиками действует на динамику движения бипеда, а также и бипед своими характеристиками действует на динамику движения платформы. Эти две сложные системы в контакте соединяются в одну еще сложнее систему, чья матеметическая модель обязательно охватывает все элементы соединения между шарнирами платформы и зглобами интеллектуального робота. Доказано, что феномен соединения более виразительным когда в конфигурацию включени и элементы упругости. Начальная траектория каждого зглоба (шарнира) может быть определена так, что охватывает упругую деформацию или не охвативает ее, а также что узнает или не узнает характеристику соединения между интеллектуальным роботом и платформой. Управляющая структура бипеда, шагающего по платформе, должна быть определена так, чтобы в каждом моменте выбирания были выполнены требования , что ZPM (Zero-Moment Point) будет в заданных пределах, чем и устойчивость локодвигательного механизма в номинальном режиме будет гарантирована. Анализ результатов имитации движения интеллектуального робота по подвижной платформе указывает на сложность этој системы и показывает, в какой мере параметры системы (выбор траектории, конфигурации, геометрии, характеристик упругости, двигателя...) влияют на устойчивость его интеллектуального движения. У всех этих исследований в интеллектуальной робототехнике одна единственная цель – создать и формировать интеллектуального робота более похожего на

человека в большей мере, который бы в будущем человеку был слугой, рабочим, солдатом и который бы обменял его во всех опасных случаях и ситуациях.

Ключевые слова: **робототехника, интеллектуальный робот, динамика движения, подвижная платформа, моделирование, соединение, упругость зглоба, планированная траектория, упаковка программного обеспечения.**

Dynamique du mouvement du bipède sur la plate-forme mobile en présence des éléments élastiques

Dans ce travail on a modelé et analysé un système robotique humanoïde et complexe qui marche sur la plate-forme mobile. Un progiciel qui permet le choix de la configuration de l'humanoïde ainsi que la configuration de la plate-forme a été synthétisé dans ce but. L'utilisateur peut définir chaque articulation du bipède ou de la plate-forme par l'état du moteur (actif ou fermé à clé) et par le type du translateur (rigide ou élastique). Au moment ou le bipède marche sur la plate-forme, elle agit par ses caractéristiques sur la dynamique du mouvement du bipède ; le bipède agit aussi, par ses caractéristiques, sur la dynamique du mouvement de la plate-forme. En contact, ces deux systèmes complexes forment un système encore plus complexe, dont le modèle mathématique doit comprendre tous les éléments d'attelage entre les articulations de la plate-forme et celles de l'humanoïde. On a démontré que le phénomène d'attelage est encore plus manifeste lorsque les éléments d'élasticité sont incorporés à la configuration. La trajectoire référentielle de chaque articulation peut être définie de façon à comprendre ou non la déformation élastique et aussi de connaître ou non la caractéristique d'attelage entre l'humanoïde et la plate-forme. La structure des commandes du bipède doit être définie pour satisfaire l'exigence de 2MP (Zero-Moment Point) à chaque moment de choix, ce qui garantit la stabilité du mécanisme locomoteur dans le régime réel. L'analyse des résultats des simulations du mouvement du robot humanoïde sur la plate-forme mobile démontre toute la complexité de ce système et l'influence des paramètres du système (choix de la trajectoire, configuration, géométrie, caractéristiques de l'élasticité, moteur) à la stabilisation de son mouvement humanoïde. Toutes les recherches dans la robotique humanoïde ont pour but de créer un robot similaire à l'homme, qui serait son servant, travailleur, soldat et qui le remplacerait dans chaque situation dangereuse.

Mots clés: **robotique, robot humanoïde, dynamique du mouvement plate-forme mobile, modélisation, attelage, élasticité de l'articulation, trajectoire programmée, progiciel.**