

Postbuckling and Failure Analysis of Axially Compressed Composite Panels Using FEM

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In order to improve confidence in composite structure design, a better description of the failure of laminates is necessary. In this paper the buckling and postbuckling behaviour of axially compressed layered composite panels is studied by means of Finite Element Method (FEM). A series of experiments was conducted to verify the FEA-results, but also to address the stability and strength of the composite structure. Combining a geometric nonlinear finite element analysis (FEA) based on the von Karman theory and High Order Shear Deformation Theory (HOST) are used to study the first-ply failure behavior as well as the postbuckling behavior of laminated type composite structures. For this purpose and for the investigation of the failure responses, the improved 4-node layered shell finite elements are used. The finite element formulation is based on the third order shear deformation theory with four-node shell finite elements having eight degrees of freedom per node. A simple method is proposed to predict buckling loads and the post-buckling behaviour. The comparisons between the numerical and the experimental results show quite a good agreement.

Key words: composite materials, panel, fracture mechanics, stability analysis, initial failure analysis, Finite Element Analysis (FEA).

Introduction

DUE to their high specific properties, the use of fiber-reinforced composites has increased drastically during the past few years in a large range of industrial applications. However, in order to meet the aspirations of aeronautical companies for lighter, safer and less polluting planes, the next generations of aircraft will integrate more and more composite components.

Fibre-reinforced composite laminates are materials with high specific as well as absolute stiffness and strength and therefore are promising alternatives to conventional structural materials. Availability of reliable computational techniques for prediction of stiffness and stability would be a major contribution to improved quality assessment of preliminary design, as well as to saving by replacement or at least reduction of the number of tests required for qualification. Nowadays modern computational facilities, with the enormously increased computer capabilities, offer the opportunity to investigate the complex buckling phenomena with robust nonlinear numerical analysis [2-5]. Numerous numerical solutions based in shell finite elements have been continuously developed for the analysis of layered composite structures. Formulation of the shell finite elements based on the classical laminate plate theory (CLPT) is inadequate. Laminated plates/shells made of advanced filamentary composite materials, whose elastic to shear modulus ratios are very large, are susceptible to thickness effects because the effective transverse shear module is significantly smaller than the effective elastic module along fiber directions. These high ratios of elastic to shear modulus render the classical laminated plate theory inadequate for the analysis of thick composite plates. A

higher order shear deformation theory (HOST) with imposed conditions on vanishing of the surfaced shear stresses is needed for laminated anisotropic shells [1, 6]. In the present work a quadrilateral isoparametric shell finite element is developed based on combining the HOST [11] and membrane elements with drilling/rotational degrees of freedom (DOF) [13, 14, 16, 17]. Membrane elements with drilling DOF are of particular importance as they form the building block of shell facets with full rotational DOFs.

This paper covers the experimental verification of the predictive capabilities of such Finite Element Analysis (FEA). For these principal investigations a representative substructure of the axially compressed composite panels is considered. The problem statement, experimental set up, FE-modelling as well as the presentation and discussion of results are outlined. Subsequently, some experimental results concerning the aspects of stability and strength are also presented.

Higher order shear deformation theory

The higher-order shear deformation theory used here [1] takes into account the parabolic distribution of transverse shear stress along the laminate thickness. This requires the use of a displacement field in which the in plane displacements are cubic functions of the thickness coordinate and the transverse deflection is constant along the plate thickness. This definition of the displacement field satisfies the condition that the transverse shear stress be zero on the plate surface and not zero in any other place. So that the displacement field is given by

$$u_1(x, y, z) = u(x, y) + z \psi_x(x, y) + z^2 \xi_x(x, y) + z^3 \zeta_x(x, y)$$

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$$u_2(x, y, z) = v(x, y) + z\psi_y(x, y) + z^2\xi_y(x, y) + z^3\zeta_y(x, y) \quad (1)$$

$$u_3(x, y, z) = w(x, y)$$

where u , v and w are the displacements of a point (x, y) of the midplane and ψ_x , ψ_y are the rotations of the normals to the midplane about the axis x and y , respectively. The functions ξ_x , ξ_y and ζ_x , ζ_y are determined by using the condition that the transverse shear stresses σ_4 and σ_5 are zero on the plate surfaces

$$\tau_{xz} = \sigma_5(x, y, \pm \frac{h}{2}) = 0 \quad (2)$$

$$\tau_{yz} = \sigma_4(x, y, \pm \frac{h}{2}) = 0$$

For orthotropic plates or plates laminated in orthotropic layers, these conditions are equivalent to the requirement that the corresponding strains be zero on these surfaces

$$\begin{aligned} \varepsilon_5 &= \Psi_x(x, y) + 2z\xi_x(x, y) + 3z^2\zeta_x + \frac{\partial w}{\partial x} \\ \varepsilon_4 &= \Psi_y(x, y) + 2z\xi_y(x, y) + 3z^2\zeta_y + \frac{\partial w}{\partial y} \end{aligned} \quad (3)$$

Taking into account that ε_4 and ε_5 are zero on the plate surfaces, the following expressions are obtained

$$\xi_x = 0 \quad \xi_y = 0$$

$$\zeta_x = -\frac{4}{3h^2}(w_x + \Psi_x) \quad \zeta_y = -\frac{4}{3h^2}(w_y + \Psi_y) \quad (4)$$

Introducing eq. (4) into eq. (1), the displacement field becomes

$$\begin{aligned} u_1 &= u + z \left[\Psi_x - \frac{4}{3} \left(\frac{z}{h} \right)^2 (w_x + \Psi_x) \right] \\ u_2 &= v + z \left[\Psi_y - \frac{4}{3} \left(\frac{z}{h} \right)^2 (w_y + \Psi_y) \right] \\ u_3 &= w \end{aligned} \quad (5)$$

The displacement field (5) accommodates a quadratic variation of transverse shear strains and vanishing of transverse shear stresses on the top and bottom of a general laminate composed of orthotropic layers. The HOST provides a slight increase in accuracy relative to the first order shear deformation theory solution at expense of a significant increase in computational effort. Thus there is no need to use shear correction factors in a higher order theory. Eq. (5) can be adapted in the next form

$$\begin{aligned} u_1 &= u + z \left[-\alpha \frac{\partial w}{\partial x} + \beta \Psi_x - \lambda \frac{4}{3} \left(\frac{z}{h} \right)^2 (w_x + \Psi_x) \right] \\ u_2 &= v + z \left[-\alpha \frac{\partial w}{\partial y} + \beta \Psi_y - \lambda \frac{4}{3} \left(\frac{z}{h} \right)^2 (w_y + \Psi_y) \right] \\ u_3 &= w \end{aligned} \quad (6)$$

Eq. (6) can be used for all plate theories: classical plate theory ($\alpha=1$, $\beta=0$ and $\lambda=0$), first order shear deformation theory ($\alpha=0$, $\beta=1$ and $\lambda=0$) and HOST ($\alpha=0$, $\beta=1$ and $\lambda=1$).

Finite Element Analysis

Finite element analysis (FEA) is employed to investigate the buckling and postbuckling behaviour of composite

panels under axial compression. The 4-node shell finite elements [8, 9] are used in this investigation. These elements present eight degrees of freedom at each node: three translations, three rotations about the nodal x , y and z axes and two higher order terms.

Based on the higher order shear deformation plate theory in the present analysis, a four-noded quadrilateral element (Q4-RT) with 8 degrees of freedom per node [2, 8] is used. The formulation of a 4-nodes shell finite element that can be good enough also if applied to the thin multilayered plates/shells is by no means an easy matter. The author's experience has shown that a good approach to the formulation of a 4-node shell finite element can be based on the application of the Discrete Kirchhoff's Theory (DKT) [12] for bending behavior. DKT ensures C^1 continuity at discrete points on inter-element boundaries. The improved 4-noded layered shell element is derived combining the HOST and the DKT, Fig.1.

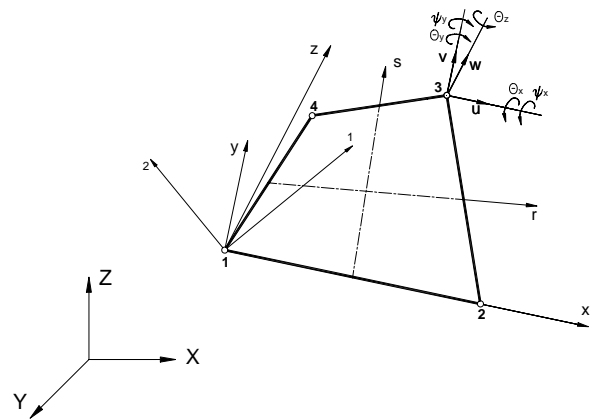


Figure 1. Description of the 4-noded shell finite element

More details about that element can be found in [9] and [11]. In the C^0 finite element theory the continuum displacement vector within the element is defined by

$$a = \sum_{i=1}^M N_i(r, s) a_i \quad (7)$$

where $N_i(r, s)$ is the interpolation function associated with the node i and expressed through the normalized coordinates (r, s) ; M is the number of nodes in the element and a_i is the generalized displacement vector in the mid-surface. In the case of the negligible mid-surface normal stress σ_z the stress-displacement relationships, stress resultants and the constitutive equations associated with the higher-order shear deformation theory are given in [2] and [8]. The total stiffness matrix of the element is obtained by the linear superposition of the following three independent parts:

- (i) Membrane stiffness matrix K_M
- (ii) Bending stiffness matrix K_B , and
- (iii) Rotational stiffness matrix $K_{\theta z}$

The four-node quadrilateral layered shell element, for the geometrical nonlinear analysis, is derived combining a higher order shear deformation theory and membrane elements with drilling/rotational degrees of freedom. This finite element has 8 degrees of freedom (DOF) per node

$$a^T = (u_0, v_0, w_0, \theta_x, \theta_y, \theta_z, \psi_x, \psi_y) \quad (8)$$

where $u_0, v_0, w_0, \theta_x, \theta_y$ represent conventional degrees of freedom, θ_z are in-plane vertex rotations. The terms ψ_x and ψ_y are the corresponding higher order terms in the Taylor's series expansion used in the theory and are also defined at the reference plane. This element is obtained by the superposition of the refined membrane element with rotational degrees of freedom and the discrete Kirchhoff model for bending.

In order to avoid singularity in the assembled matrix using flat elements in a global coordinate system, we used here a membrane element which includes in-plane nodal rotations, θ_z , as a degree of freedom. In this work Allman's approach is used. Allman's approach begins by selecting a quadratic form for the normal component of displacement, U_n , and a linear form for the tangential component of displacement, U_t , along each element edge $i-j$, Fig.2.

$$U_n = \left(1 - \frac{\xi}{l_{ij}}\right) U_{n1} + \left(\frac{\xi}{l_{ij}}\right) U_{n2} + \frac{\xi}{l_{ij}} \left(1 - \frac{\xi}{l_{ij}}\right) (\theta_{z2} - \theta_{z1}) \quad (9)$$

$$U_t = \left(1 - \frac{\xi}{l_{ij}}\right) U_{t1} + \left(\frac{\xi}{l_{ij}}\right) U_{t2} \quad (10)$$

where φ is the running distance from one end and $(U_{n1}, U_{t1}, v_{z1}), (U_{n2}, U_{t2}, v_{z2})$ are the translational and rotational components displacements at each end of the edge.

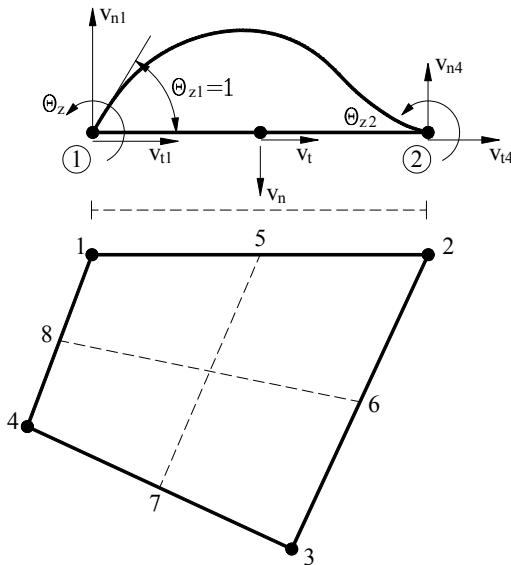


Figure 2. Membrane part of the shell element

As Harder and Mac Neal [14] noted, eqs. (9) and (10) can be used to eliminate the translational displacements at the midpoint of the edge in favor of the degrees of freedom at the adjacent corner points so that, in this way, any eight-noded membrane quadrilateral can be converted into an element with corner translations and rotations as DOFs. Because of geometric complexity and large deformation in the postbuckling state of the laminate, a geometrically non-linear finite-element analysis is performed.

Failure Criterion

Initial failures of a layer within the laminate of a composite structure can be predicted by applying an appropriate failure criterion or the first-ply failure theory. Failure modes in laminated composite panels are strongly

dependent on ply orientation, loading direction and panel geometry. There are four basic modes of failure that occur in laminate composite structures. These failure modes are: matrix cracking, fiber-matrix shear failure, fiber failure and delamination. Various first-ply failure theories are incorporated in the pre-, buckling and post-buckling failure analysis of the laminated fibrous composite structures. Failure criteria used with finite element results from the nonlinear postbuckling solutions qualitatively predict the load level and the location of local failures in the laminates that correspond to experimental results. A finite element computational procedure is incorporated for the first-ply failure analysis of laminated composite shells. The procedure is based on the higher-order shear deformation theory and the tensor polynomial failure criterion

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j + F_{ijk} \sigma_i \sigma_j \sigma_k + \dots \geq 1 \quad (11)$$

where σ_i are the stress tensor components in material coordinates and F_i, F_j and F_{ijk} are the components of the strength tensors. Most failure criteria are based on the stress state in a lamina. The Tsai-Wu criterion [15] was considered in this work in order to determine the first-ply failure. The failure of an axial compressed panel was initiated near the region with severe local bending gradients. The computation and experimental results indicate that local failures occurred in regions of large radial displacements. These local failures are associated with the brittle failure characteristics of the graphite-epoxy material system. The procedure of calculating the first-ply failure load of laminated composite panels refers to calculating stress and strains at all the nodes for each layer of laminate and then the maximum values of stress and strain are picked up. The failure loads for the weakest ply in the plate/shell are then calculated using various failure criteria using the iteration procedure. The increment in the load level can be made suitable for predicting the failure load.

Tsai- Wu criterion

The coefficients F_i and F_{ij} in eqn (11) are functions of the unidirectional lamina strengths and are presented below for the Tsai-Wu criterion:

$$F_1 \sigma_1 + F_{11} \sigma_1^2 + F_2 \sigma_2 + F_{22} \sigma_2^2 + F_{12} \sigma_1 \sigma_2 + F_{66} \sigma_6^2 \leq 1 \quad (12)$$

where

$$F_1 = X_t^{-1} - X_c^{-1}, \quad F_2 = (Y_t^{-1} - Y_c^{-1})$$

$$F_{11} = (X_t X_c)^{-1} \quad F_{22} = (Y_t Y_c)^{-1}$$

$$F_{12} = -(X_t X_c Y_t Y_c)^{\frac{1}{2}} \quad F_{66} = S^{-2}$$

Here: X_t, X_c are the longitudinal tensile and compressive strengths, Y_t, Y_c are the transverse tensile and compressive strengths, S is the rail shear strength.

Numerical and experimental results

A numerical analysis of critical and post critical behaviour of axially compressed composite panels was done applying 4-node shell finite elements (Q4-RT) and it was compared to the experimental results. The geometry, loads and boundary conditions are shown in Fig.3. During

the experiments the panel was clamped (c) along the lateral edges where the loads were introduced and it was simply supported (ss) along its longitudinal edges (dashed lines). The mesh of finite elements was formed for the whole panel and the boundary conditions were defined in the same way as it was done during the experiments (Fig.4).

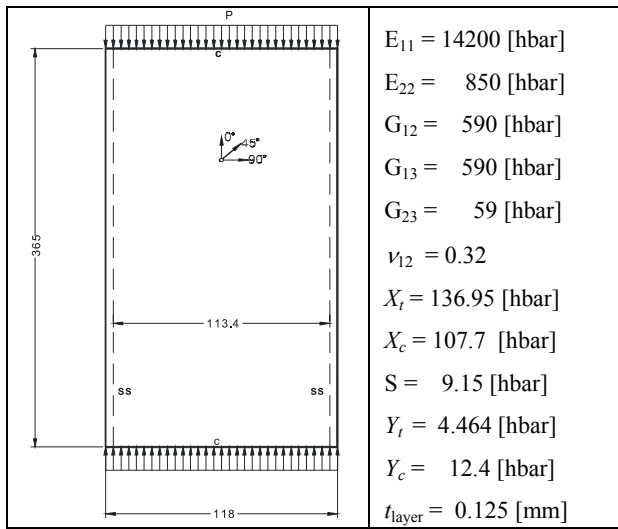


Figure 3. Axially compressed composite panel – Geometry and boundary conditions

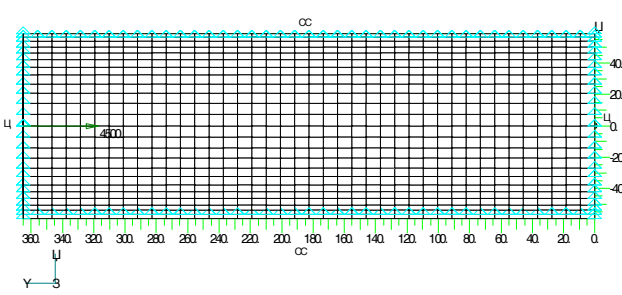


Figure 4. Axially compressed panel – mesh of finite elements

The analysis of critical and post critical behaviour was done for three stacking sequences:

A-[0°/90°/±45°]_{2s},

B-[0°/±45°/90°]_{2s},

C-[45°/0°/-45°/90°]_{2s}.

Table 1. Buckling behaviour of axially compressed composite panels

Panel	BUCKLING LOAD P_{cr} [kN]					
	I	II	III	IV	V	
A1	13.3		15.1	C	13.52	13.78
				E	12.37	
A2	13.4		15.1	C	13.52	
				E	12.37	
A3	13.4	13.3	15.1	C	13.52	
				E	12.37	
A4	13.4	13.8	15.1	C	13.52	
				E	12.37	
A5	12.5	12.9	15.1	C	13.52	
				E	12.37	
A7A	13.1	12.7	15.1	C	13.52	14.90
				E	12.37	
B2	13.0	14.0	16.5	C	14.73	16.038
				E	13.51	
C1	13.6	14.3	16.5	C	14.78	16.038
				E	13.51	
C2	14.0	14.3	16.5	C	14.78	
				E	13.58	

Besides the analysis of the critical and post critical behaviour of composite panels that was done using the nonlinear structural analysis, the critical loads were determined applying the linear eigen-value problem. Both analyses were done for two types of material characteristics: in the cases of compression and extension of the used CFC material. The experimental and numerical results and the values of buckling loads P_{cr} for all stacking sequences are given in Table 1.

- a) Experimental results: Buckling load determined from the Force-Axial displacement curve
 - b) Experimental results: Buckling loads determined from the membrane deformations
 - c) FINEL (FEM Software developed at Imperial College)
 - d) Present numerical results: Buckling loads – FEM solutions obtained solving eigen-values
 - e) Present numerical results: Buckling loads – FEM solutions obtained from the nonlinear analysis (from the Load – Deflection curve)
- E** – Material characteristics - Tension
C - Material characteristics – Compression

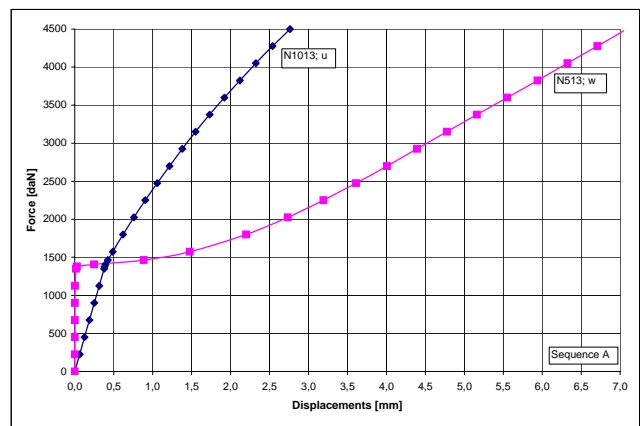


Figure 5. Nonlinear stability analysis: P-u and P-w curves

u, v, w – Displacements in the x, y, z directions

It is obvious from Table 1 that the FEM results agree particularly well with the experiments in both cases: in linear and nonlinear analysis. It is even more interesting to underline the fact that the results obtained using the 4-node shell finite element Q4-RT are closer to the experimental results than the results obtained applying 8-node shell finite elements used by the FINEL software.

Besides the determination of the critical force P_{cr} the analysis of the total post critical behaviour of the panels as well as the analysis of the initial failure - the levels of the load P_f that correspond to the initial failure. Some of these results will be shown for the panel with the stacking sequence A and they are shown in Fig.3. It is necessary to underline that the critical loads for the compressed panels can be determined from the Force – Deflection w curves given in Fig.5: As it can be seen from that curve and Table 1, a good agreement between numerical and experimental results was obtained.

The results of the initial failure are presented in Fig.6 in the form of the distribution of initial failure coefficients F.I.

The value of the load P_f corresponding to the initial failure is around 36 kN and that is sufficiently above the critical load level ($P_f / P_{cr} \approx 2.6$). Similarly high rates were obtained also for other panels and these rates are in accordance with the experiments.

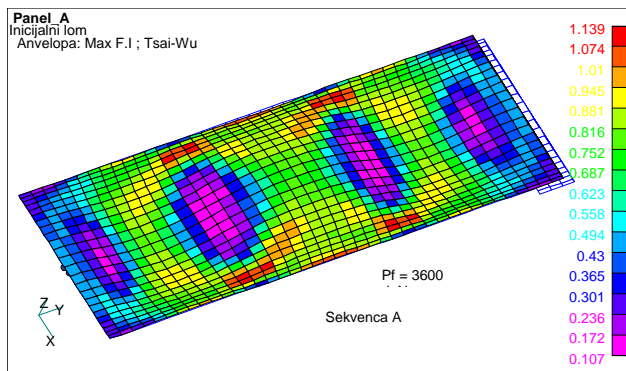


Figure 6. Distribution of initial failure coefficients (F.I.) for the force P_f

Conclusions

The paper has applied a FEM model to analyse the nonlinear response of generally laminated flat composite panels subjected to several geometrical and mechanical boundary conditions. The buckling and postbuckling behavior of axially compressed composite panels is investigated. The results of a numerical and experimental study to evaluate the initiation of damage in nonlinearity deformed flat layered composite panels subjected to axial compression are presented. A good agreement between numerical and experimental results is obtained. The failure criteria used with numerical results from the nonlinear postbuckling solutions qualitatively predict the load level and the location of first-ply failures in the panels that correspond to the experimental results. The comparison of numerically obtained results with the experimental data shows that the improved 4-node shell finite element can be successfully applied for the buckling and postbuckling analysis as well as for the initial failure analysis and for the prediction of the location of initial failures of compressed layered flat composite panels.

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Analiza postkritičnog ponašanja i loma aksijalno pritisnutih kompozitnih panela primenom MKE

Da bi se obezbedila veća pouzdanost za primenu kompozitnih materijala u procesu projektovanja neophodno je dobro poznavanje za modeliranje inicijalnih otkaza. U radu je razmatran problem kritičnog i postkritičnog ponašanja primenom metode konačnih elemenata (MKE). Niz eksperimenata bio je realizovan u cilju poredjenja i verifikacije numeričke rezultate na bazi MKE u domenu stabilnosti i čvrstoće kompozitne strukture. Kombinuju geometrijski nelinearne analize MKE zasnovane na Karnanovoj teoriji i teoriju smicanja višeg reda (TSVR) za formulaciju konačnog elementa su korišćene za analizu inicijalnog loma kao i postkritičnog ponašanja višeslojnih kompozitnih struktura. Za tu svrhu je korišćen poboljšani 4-čvorni konačni element višeslojne ljuske. Formulacija konačnog elementa je zasnovana na teoriji smicanja višeg reda gde svaki čvor ima po osam stepeni slobode. Eksplicitan metod je predložen za analizu kritičnog i postkritičnog ponašanja. Poredjenja između prezentovanih numeričkih i eksperimentalnih rezultata daju dobra slaganja.

Ključne reči: kompozitni materijali, panel, mehanika loma, analiza stabilnosti, analiza inicijalnih otkaza, metoda konačnih elemenata.

Analyse du comportement postcritique et de défaillance des panneaux composites comprimés axialement

Dans le but d'assurer plus grande fiabilité dans l'emploi des matériaux composites lors du projet, il est nécessaire de connaître bien la modélisation des défaillances initiales. Ce papier traite le problème du comportement critique et postcritique par la méthode des éléments finis (MKE). Une série d'essais a été réalisée afin de comparer et vérifier les résultats numériques, basés sur MKE, dans le domaine de la stabilité et de la force chez la structure composite. La combinaison de l'analyse géométrique non-linéaire de l'analyse MKE, basée sur la théorie de Karnan et la théorie de décalage de l'ordre supérieur (TSVR) pour la formulation de l'élément fini, est utilisée pour l'analyse de la défaillance initiale et pour le comportement postcritique des structures composites des couches à plusieurs. A cet effet on a appliqué l'élément fini amélioré de coque à couches à plusieurs et à quatre nœuds. La formulation de l'élément fini se base sur le théorie de décalage de l'ordre supérieur où chaque nœud possède 8 degrés de liberté. La méthode explicite est proposée pour l'analyse du comportement critique et postcritique. La comparaison montre qu'il y a bon accord entre les résultats numériques et expérimentaux.

Mots clés: matériaux composites, panneau, mécanique de fracture, analyse de la stabilité, analyse des défaillances initiales, méthode des éléments finis