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Stochastic Adaptive Control Using the Robust Least Squares Algorithm

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This paper considers properties of the Astrom-Wittenmark self tuning tracker for MIMO systems described with the ARX model. It is supposed that the stochastic noise has the non-Gaussian distribution (condition always present in practice). The consequence of that fact is a nonlinear transformation of the tracking error in the direct adaptive minimum variance controller. The system under consideration is the minimum phase with different dimensions for input and output vectors. Using the concept of the Kronecker product it is possible to represent unknown parameters in the form of a vector. The tensor calculus is thus avoided. Global stability is proved without any modification of the matrix gain in the recursive algorithm. The paper also discusses the relation of the assumption about the absolutely continuous finite-dimensional distributions and different modifications of a high-frequency gain. The paper presents theoretical results but the adaptive control methodology has already been present for many years in military systems (CH-47 helicopter and X-15 aircraft).

Key words: **adaptive control, ARX model, non-Gaussian disturbance, self-tuning tracker, system stability, least squares method.**

Introduction

THE analysis of adaptive controllers is a very important topic in the control area [1]. In this reference it is shown that if the least squares parameters estimates converge to some limit then the adaptive controller must be optimal but, as noted, it is very difficult to prove that the estimates are indeed convergent. After that much attention has been drawn to establishing the global stability and the asymptotic optimality for adaptive controllers. Significant progress in this direction was made in [14] where global convergence has been established for a class of stochastic adaptive control algorithms based on the stochastic approximation method. The next important step is a result presented in [23]. Namely, from the practical point of view, least squares generally have a superior rate of convergence in comparison with the stochastic approximation algorithm. But, in that case, it was necessary to modify the gain matrix for the global convergence of algorithms. In [17] an attempt was made to remove the above restriction. For a minimum phase system where adaptive noise is i.i.d. and Gaussian, using the Bayesian embedding method and the properties of normal equations, a least squares-based adaptive tracker converges outside an exceptional set of the Lebesque measure zero in the parameter space. In this approach the restrictions are: Gaussianity and independency of noise and the exceptional set. Very important results are presented in [13] where the Astom-Wittenmark self-tuning regulator and the ELS-based adaptive tracker are considered. It is shown by a careful analysis of growth rates how to avoid the need to establish parameter convergence. Also, convergence of the original Astrom-Wittenmark self-tuning regulator is proven rigorously. Using the ideas from [17] and [13], reference [21] presents a more comprehensive theory of stochastic adaptive filtering, control and identification. It is also established that the parameters converge to the null space of a certain matrix. The results from [13] are used for some problems in the model reference adaptive control [20]. Weighted estimation and tracking for a multivariable ARMAX model is considered in [2]. This paper introduces a random weighting sequence and shows that the given algorithm has the performance of the ELS for the strong consistency and matches the best result of SG for the adaptive tracking. Some aspects of tuning of self-tuning controllers is discussed in [24]. A further important step is reference [11] where the best convergence rate of self-tuning regulators (logarithmic law of STR) is found, The overview of adaptive methodology is given in [12] and [16]. This paper will consider the Astrom-Wittenmark controller when the disturbance is non-Gaussian. The non-Gaussianity introduces nonlinear transformation of the tracking error in the estimation algorithm. A special case of such situation is the case when there is an priori information about the class of distribution to which the real disturbance belongs. In such situation the theory of min-max estimation can be applied and so the given algorithm is known as a robust algorithm.

Reference [6] considers the robust SG algorithm (nonrobust version of SG algorithm is considered in [4] and [18]) as well as the stability and optimality of the minimum variance controller. The parallel result for the ELS algorithm for SISO systems described by the ARMAX model is presented in [8]. It is shown that for the stability of the adaptive controller no modification of the gain matrix is necessary. A tracking problem when the noise is non-Gaussian and when unmodeled dynamics is also present is considered in [7]. Robust predictor for SISO systems is presented in [9].

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In this paper we will consider the adaptive controller for the system described by the multivariable ARX model. It is supposed that the system is the minimum phase and that the input and the output vector have different dimensions (rectangular systems). The system is established by the concept of stochastic Lyapunov function stability and optimality of feedback. The extension of the results for system control described by multivariable ARMAX models is presented in [10].

Problem formulation

Let the system under consideration be described by a linear multiple-imput/multiple-output ARX model with m- and l-dimensional output and input respectively.

$$B(z) = B_1 + B_2 z + \cdot\cdot + B_q z^{q-1}, \quad q \ge 1$$

$$A(z)\, y_{n+1} = B(z) u_n + w_{n+1}\,,\ n \ge 0 \tag{1}$$

$$y_n = w_n = 0\,,\ u_n = 0\,,\ n < 0$$

where $A\ (z)$ and $B\ (z)$ are the matrix polynomials in the shift-back operator $z\ y_n\ =\ y_{n-1}$ with the order p and q respectively , i.e.

$$A(z) = I + A_{1z} + \ldots + A_p z^p\,,\ \ p \ge 0 \tag{2}$$

$$B(z) = B_1 + B_{2z} + \ldots + B_q z^q\,,\ \ q \ge 0 \tag{3}$$

The noise $\{w_n\}$ is assumed to be a martingale - difference sequence with respect to a nondecreasing family of σ- algebras $\{F_n\}$.

The unknown matrix coefficients are

$$\theta^M = \left[-A_1 \ldots - A_p B_1 \ldots B_q\right]^T \tag{4}$$

Model (1) can be rewritten in the next form

$$y_{n+1} = (\theta)^T\ \varphi_n + w_{n+1} \tag{5}$$

where

$$\varphi_n^T\ =\ \left[\ y_n^T \cdot\cdot\ y_{n-p}^T \quad u_n^T \cdot\cdot\ u_{n-q+1}^T \right] \tag{6}$$

Let us introduce the matrix x_n^0 in the form

$$X_n = \begin{bmatrix} \varphi_n^T & & 0 \\ & \ddots & \\ 0 & & \varphi_n^T \end{bmatrix} = \mathrm{I} \otimes \varphi_n^T \tag{7}$$

where $\otimes$ is the symbol for the Kronecker product. Also, a new vector θ is constructed by stocking the columns of the θ^M matrix one on top of the other. Relation (5) now has the form

$$y_{n-1} =\ X_n\ \theta\ +\ w_{n+1} \tag{8}$$

In this paper we will consider the direct adaptive minimum variance controller. The algorithm for the estimation of unknown parameters can be given by minimizing the next functional

$$J(\theta) = E\{\Phi(\varepsilon_{n+1})\}, \quad \Phi : R^m \to R^1 \tag{9}$$

whereby ε_{n+1} is the prediction error, i.e. $\varepsilon_{n+1} = y_{n+1} - \hat{y}_{n+1}$ where $\hat{y}_{n+1}$ is the prediction of y_n.

The functional $J(\theta)$ depends on the probability distribution of observations which is, generally, non-Gaussian. From the identification theory it is known that

$$\Phi(x) = -\log p\,(x),\ \ x \in R^m \tag{10}$$

where $p(\cdot)$ is the probability density. Using the methodology from [5], from (8) and (9) one can get

$$\theta_{n+1} = \theta_n + P_{n+1} X_n^T \Psi\ \left(y_{n+1} - X_n \theta_n\right) \tag{11}$$

$$P_{n+1} = P_n - P_n X_n^T\ \left[X_n P_n X_n^T + M^{-1}\right]^{-1} X_n P_n \tag{12}$$

$$\Psi(x) = -\nabla_x \log p(x), \quad \dim \Psi(x) = mx1 \tag{13}$$

$$\varphi_n^T = \left[y_n^T \cdot\cdot y_{n-p+1}^T u_{n-p}^T \cdots u_{n-q+1}^T\right]\,,\ X_n = I \otimes \varphi_n^T \tag{14}$$

$$M = E\{\nabla_x \Psi(x)\}, \quad x \in R^m \tag{15}$$

For the minimum variance controller, the control u_n is chosen as [22]

$$X_n \theta_n = y_{n+1}^* \tag{16}$$

where $\{y_{n+1}^*\}$ is a sequence of bounded deterministic signals.

Remark 1. Using the concept of the Kronecker product (relation (7)) one can represent unknown parameters in the vector form. The tensor calculus is thus avoided.

Remark 2. If we can use an a priori assumption that the distribution of real noise lies in a specified class of the distribution F which is convex and vaguely compact ([15] and [19]) it is possible to construct a robust real-time procedure in min-max sense. The members of F are symmetric and contain the standard normal distribution N. Two important classes are

a) the gross error model

$F_1\varepsilon = \{\ F : F = (1\text{-}\varepsilon)\ N + \varepsilon G,\ G$ is symmetric$\}$

b) the Kolmogorov model

$F_{2,\varepsilon} = \{\ F : F$ is symmetric and $\sup_x \left|F(x) - N(x)\right| \le \varepsilon\ \}$

Analysis of adaptive algorithms

In this part of the paper the global stability of the control system and self-optimizing property of the adaptive controller is established. What is more important, the above mentioned facts are proved for algorithms (11)-(16) without any modification of the gain matrix.

Now we will quote two lemmas which will be useful for future reference.

Lemma 1. Let the next assumptions hold

A1. the function $\Psi(\cdot)$ is uniformly bounded

A2. $\lambda_{\min}\{\,M\,\} > 0$

Then

$$\left|\sum_{i=0}^{n} \Psi^T(\varepsilon_{i+1}) X_i P_i X_i^T \Psi(\varepsilon_{i+1})\right| = O(\log r_n)$$

where $r_{n+1} = r_n + tr X_n^T M X_n$

Proof: Can be found in [7].

The next lemma has the form

Lemma 2. Let $\{x_n, F_n\}$ be a martingale difference sequence and assume that the following assumption is satisfied

A1: $\sup_n E\left\{\|x\|^2 | F_{i-1}\right\} < \infty$ *a.s.* where $x \in R^{p1}$

A2: $\|a_n\| \le k_a$, $k_a \in (0,\infty)$, $a_n \in R^{p1}$

Then for $\exists\, \delta > 0$

$$\lim_{n\to\infty} \frac{1}{n^{(1+\delta)/2}} \sum_{i=1}^{n} x_i^T a_i = 0 \qquad \bullet$$

Proof: The proof is given in [8].

Now we will quote the key lemma.

Lemma 3. Consider model (5) and algorithms (11) - (16) subject to the following assumptions

A1: $\{w_n\}$ is a martingale-difference with symmetric distribution $P(\cdot)$ and

$$\sup_n E\left\{\|w_{n+1}\|^2 | F_n\right\} < \infty \quad a.s.$$

A2: The function $\Psi(\cdot)$ is odd and continuous almost everywhere

A3: The function Ψ is uniformly bounded

A4: There exists the passive operator H such that

$$HZ_1 = \Psi(Z_1) - \frac{1+k_0}{2} Z_1, \quad k_0 > 0$$

$$Z_1 = -X_n \tilde{\theta}_n + w_{n+1}, \quad \tilde{\theta}_n = \theta_n - \theta$$

Then

$$\sum_{i=0}^{n} \left\|X_i \tilde{\theta}_i\right\|^2 \le 0(1) + 0(\log r_n)$$
$$-2\sum_{i=0}^{n} w_i^T \Psi\left(X_i\tilde{\theta}_i - w_{i+1}\right)$$

where $c > 1\bullet$

Proof: Introducing the stochastic Lyapunov function

$$V_{n+1} = \tilde{\theta}_{n+1}^T P_{n+1}^{-1}\, \tilde{\theta}_{n+1}$$

where

$$\tilde{\theta}_n = \theta_n - \theta,$$

one can get using (11)

$$\begin{aligned} V_{n+1} &= \left[\tilde{\theta}_n + P_{n+1} X_n^T \Psi(\varepsilon_{n+1})\right]^T P_{n+1}^{-1} \cdot \left[\tilde{\theta}_n + P_{n+1} X_n^T \Psi(\varepsilon_{n+1})\right] \\ &= \tilde{\theta}_n^T + P_{n+1}^{-1}\tilde{\theta}_n + 2\tilde{\theta}_n^T X_n^T \Psi \cdot \\ &\cdot(\varepsilon_{n+1}) + \Psi^T(\varepsilon_{n+1}) X_n P_{n+1} X \Psi(\varepsilon_{n+1}) \end{aligned} \tag{17}$$

Using the matrix inversion lemma (12) can be rewritten in the next form

$$P_{n+1}^{-1} = P_n^{-1} + X_n^T\, M X_n \tag{18}$$

From (17) and (18) follows

$$\begin{aligned} V_{n+1} = V_n + \left(X_n\tilde{\theta}_n\right)^T M X_n \tilde{\theta}_n + 2\tilde{\theta}_n^T X_n^T \Psi \\ \cdot(\varepsilon_{n+1}) + \Psi^T(\varepsilon_{n+1}) X_n P_{n+1} X_n^T \Psi(\varepsilon_{n+1}) \end{aligned} \tag{19}$$

It is well known that in [22] the predicted error has the form

$$\varepsilon_{n+1} = -X_n \tilde{\theta}_n + w_{n+1} \tag{20}$$

Using (19) and (20) we have

$$\begin{aligned} V_{n+1} &= V_n + \left(X_n\tilde{\theta}_n\right)^T M X_n\tilde{\theta}_n - 2\left[\tilde{\theta}_n^T X_n^T - w_{n+1}^T\right] \\ &\cdot\left\{\Psi\left(X_n\tilde{\theta}_n - w_{n+1}\right) - \frac{1+k_0}{2}\left[X_n\tilde{\theta}_n - w_{n+1}\right]\right\} \\ &-(1+k_0)\left\|X_n\tilde{\theta}_n - w_{n+1}\right\|^2 - 2w_{n+1}^T \Psi\left(X_n\tilde{\theta}_n - w_{n+1}\right) \\ &+\Psi^T(\varepsilon_{n+1}) X_n P_{n+1} X_n^T \Psi(\varepsilon_{n+1}) \end{aligned} \tag{21}$$

Summing both sides of (21) from 0 to n we obtain

$$\begin{aligned} V_{n+1} &= V_0 + \sum_{i=0}^{n} \left(X_i\tilde{\theta}_i\right)^T M X_i\tilde{\theta}_i - 2\, S_n + 2k_1 - \\ &(1+k_0)\sum_{i=0}^{n}\left\|X_i\tilde{\theta} - w_{i+1}\right\|^2 - 2\sum_{i=0}^{n} w_{i+1}^T \Psi\left(X_i\tilde{\theta} - w_{i+1}\right) \\ &+\sum_{i=0}^{n} \Psi^T(\varepsilon_{i+1}) X_i P_{i+1} X_i^T \Psi(\varepsilon_{i+1}) \end{aligned} \tag{22}$$

where

$$\begin{aligned} S_n &= \sum_{i=0}^{n}\left[\tilde{\theta}_i^T X_i^T - w_{i+1}^T\right] \\ &\cdot\left\{\Psi\left(X_i\tilde{\theta}_i - w_{i+1}\right) - \frac{1+k_0}{2}\left[X_i\tilde{\theta}_i - w_{i+1}\right]\right\} \\ &+k_1 \ge 0 \end{aligned} \tag{23}$$

for $k_0 > 0$, $k_1 \ge 0$ and $\forall n \ge 0 \quad \bullet$

Using assumption A3 of the lemma, from (22) and (23) it follows

$$\begin{aligned} V_{n+1} &\le 0(1) + \|M\|\sum_{i=0}^{n}\left\|X_i\tilde{\theta}_i\right\|^2 \\ &-(1+k_0)\sum_{i=0}^{n}\left\|X_i\tilde{\theta}_i - w_{i+1}\right\|^2 - 2\sum_{i=0}^{n} w_{i+1}^T\Psi\left(X_i\tilde{\theta}_i - w_{i+1}\right) \\ &+\left|\sum_{i=0}^{n} \Psi^T(\varepsilon_{i+1}) X_i P_{i+1} X_i^T\ \Psi\ (\varepsilon_{i+1})\right| \end{aligned} \tag{24}$$

Having in mind a simple inequality

$$\|y\| - \|x\| \le \|\ y - x\ \| \tag{25}$$

relation (24) can be rewritten in the next form

$$\begin{aligned} V_{n+1} &\le 0(1) + \|M\|\sum_{i=0}^{n}\left\|X_i\tilde{\theta}_i\right\|^2 - (1+k_0)\sum_{i=0}^{n}\left\|X_i\tilde{\theta}_i\right\|^2 \\ &+2(1+k_0)\sum_{i=0}^{n} w_{i+1}^T X_i\tilde{\theta}_i - 2\sum_{i=0}^{n} w_{i+1}^T\Psi\left(X_i\tilde{\theta}_i - w_{i+1}\right) \\ &+\sum_{i=0}^{n}\Psi^T(\varepsilon_{i+1}) X_i P_{i+1} X_i^T \Psi(\varepsilon_{i+1}) = 0(1) \end{aligned}$$

$$-\left(1+k_0-\|M\|\right)\sum_{i=0}^{n}\left\|X_i\tilde{\theta}_i\right\|^2+2\left(1+k_0\right)\sum_{i=0}^{n}w_{i+1}^T X_i\tilde{\theta}_i$$
$$-2\sum_{i=0}^{n}w_{i+1}^T\Psi\ \left(X_i\tilde{\theta}_i-w_{i+1}\right) \quad (26)$$
$$+\left|\sum_{i=0}^{n}\Psi^T\left(\varepsilon_{i+1}\right)X_iP_{i+1}X_i^T\Psi\left|\left(\varepsilon_{i+1}\right)\right.\right|$$

In [3] the following result is proved

$$\left|\sum_{i=0}^{n}w_{i+1}^T X_i\tilde{\theta}_i=0\ \right|\ \left[\left[\left(\ \sum_{i=0}^{n}\left\|X_i\tilde{\theta}_i\right\|\ \right)^{\beta}\right]\right]+0\ \left(\log r_{n+1}\right) \quad (27)$$

for $\beta \in$ (1/2, 1) and $\forall c > 1$.

Using Lemma 1 and relations (26) and (27), one can get the result of lemma•

Now we will formulate the main result of the paper.

Theorem 1. Suppose that for model (5) and algorithms (25) - (26) the following conditions are satisfied

C1: B_1 is of full rank and B_1^+ $B(z)$ is an asymptotically stable matrix polynomial where B_1^+ denotes the pseudo inverse of B_1

C2: All finite-dimensional distributions of $\{ x_0 ,\ w \}$ are absolutely continuous with respect to the Lebesque measure and

$$x = \{y_0 ,..,y_{1-n},\ u_0,...,\ u_{1-n}\ ;\ w_0...w_{1-n}\ \}$$

C3: Reference signal $\left\{y_n^*\right\}$ is uniformly bounded

C4: $\{w_n\ ,\ F_n\}$ is a martingale sequence with

$$\sup_{n\geq 1}\quad E\ \left\{\left\|w_n\right\|^2\ \mid\ F_{n-1}\right\}<\infty\qquad a.s.$$
$$\lim_{n\to\infty}\ \frac{1}{n}\sum_{i=0}^{n}w_{i+1}w_{i+1}^T=\ R>0\qquad a.s.$$

C5: Conditions A1-A4 of Lemma 2 hold

Then the self-tuning tracker is stable and optimal in the following sense

$$\lim_{n\to\infty}\ \frac{1}{n}\sum_{i=0}^{n}\left(\left\|y_{i+1}\right\|^2+\left\|u_{i+1}\right\|^2\right)<\infty\quad a.s.$$
$$\lim_{n\to\infty}\ \frac{1}{n}\sum_{i=0}^{n}\left(y_{i+1}-y_{i+1}^*\right)\ \left(y_{i+1}-y_{i+1}^*\right)=R\quad a.s.$$

Proof: We will first prove the global stability of the self - tuning tracker. It is a well known fact [22] that

$$\lim_{n\to\infty}\frac{r_n}{n}<\infty\ \ a.s. \quad (28)$$

Using the matrix inversion lemma from relation (12) it follows

$$P_{n+1}^{-1}=P_n^{-1}+X_n^T M X_n \quad (29)$$

and from (3.11) we can define the recursive quantity r_n , i,e.

$$r_{n+1}=r_n\ +tr\left(X_n^T M X_n\right),\ \ r_n=trP_n^{-1} \quad (30)$$

Using relations (23) and (25), the definition of the matrix X_n and C1, C2 and C4 conditions of the Theorem, one can get a relation for the global stability of the self-tuning tracker.

In the second part of the proof we will prove optimality of the tracker. From relation (8), one can write

$$y_{i+1}-y_{i+1}^*=-X_i\tilde{\theta}_i+w_{i+1}\ \ ,\ \ \tilde{\theta}_i=\theta_i-\theta \quad (31)$$

Now we have

$$\sum_{i=0}^{n}\left(y_{i+1}-y_{i+1}^*\right)\left(y_{i+1}-y_{i+1}^*\right)^T$$
$$=\sum_{i=0}^{n}w_{i+1}-w_{i+1}^*+O\left(\left|\sum_{i=0}^{n}w_{i+1}^T X_i\tilde{\theta}_i\right|\right)+O(\sum_{i=0}^{n}\left\|X_i\tilde{\theta}_i\right\|^2) \quad (32)$$

From Lemma 2 it follows

$$\frac{1}{n}\sum_{i=0}^{n}.w_{i+1}^T\Psi\left(\varepsilon_{i+1}\right)=0\quad a.s. \quad (33)$$

Using (32), (33) and Lemma 3 one can get the second statement of the theorem •

Remark 3. In the Theorem1 condition C2 is restrictive. To avoid such type of conditions it is possible to make some modifications.

In the original scheme of Astrom and Wittenmark [1] it is supposed that the matrix coefficient B_1 is a priori known.

References [3] and [13] suggest the next kind of modification of estimate B_{1n}. Namely, the estimate B_{1n} is replaced with any F_n - measurable $\hat{B}_{1n}^{-1}$ that satisfies the conditions

$$\hat{B}_{1n}^T\hat{B}_{1n}\geq\frac{1}{\log r_{n-1}}I\ \ ,\quad n\geq 1$$
$$\left\|\hat{B}_{1n}\hat{B}_{1n}\right\|\leq\frac{1}{\left(\log r_{n-1}\right)^{1/2}}$$

The weighted ELS recursive algorithm is considered in [2]. The modification to B_{1n} is

$$\hat{B}_{1n}=\begin{cases}\hat{B}_{1n} & if\quad \lambda_{\min}\ \left\{\hat{B}_{1n}^T\hat{B}_{1n}\right\}>0\\ \hat{B}_{1n}+\sqrt{\upsilon_n}P_nQ_n^T & otherwise\end{cases}$$

where P_n and Q_n are the orthogonal matrices associated with the singular value decomposition of $\hat{B}_{1n}$.

The results from the adaptive control of SISO systems [11] about the modification of the high-frequency gain are applicable as well.

Conclusion

This paper considers the problem of global stability and optimality of the Astrom-Wittenmark self-tuning tracker when the noise is, generally, non-Gaussian. The system is modeled as a multivariable ARX model with the rectangular structure. As a special case, the approach includes the robust procedure with respect to the change of disturbance distribution. The paper also discusses relaxation of some assumptions by the high-frequency gain modification. Further investigation will be directed to give the logarithm law of the self-tuning regulator for the algorithms described in this paper.

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Stohastičko adaptivno upravljanje zasnovano na robusnom metodu najmanjih kvadrata: Pregled rezultata

Rad razmatra osobine Astrom-Wittenmark-ovog regulatora u problemu praćenja referentne vrednosti za slučaj multivarijabilnih sistema opisanih sa ARX modelom. Pretpostavlja se da stohastički poremećaj nema Gausovu raspodelu (uslov uvek prisutan u praksi). Posledica toga je nelinearna transformacija greške praćenja u direktnom adaptivnom regulatoru minimalne varijanse. Sistem je minimalno fazni i ima različite dimenzije ulaznog i izlaznog vektora. Korišćenjem Kronekerovog proizvoda nepoznati parametri se predstavljaju u formi vektora. Time se izbegava tenzorski račun. Dokazana je globalna stabilnost bez ikakve modifikacije matričnog pojačanja u rekurzivnom algoritmu. Dokazana je optimalnost regulatora u slučaju praćenja referentne trajektorije U radu su, takođe, razmatrani odnos pretpostavke o apsolutnoj neprekidnosti konačno dimenzionalnih raspodela verovatnoće i modifikacije visokofrekventnog pojačanja.
U radu su predstavljeni teoretski rezultati, ali metodologija adaptivnog upravljanja već postpoji mnogo godina u vojnim sistemima (CH-47 helikopter i X-15 avion).

Ključne reči: **adaptivno upravljanje, ARX model, Negausov poremećaj, sampodešavajući regulator, stabilnost sistema, metoda najmanjih kvadrata.**

Commande adaptive stochastique basée sur la méthode des moindres carrés: tableau des résultats

Ce papier considère les propriétés du régulateur Astrom-Wittenmark pendant la poursuite des valeurs de référence chez les systèmes multivariables décrits à l'aide de modèle ARX. On suppose que la déviation stochastique n'a pas la distribution de Gauss (condition toujours présente en pratique).La conséquence de cela est la transformation non-linéaire de l'erreur de poursuite dans le régulateur direct adaptif de la variance minimale.Le système est de phase minimale et aux différentes dimensions de vecteurs d'entrée et de sortie.Les paramètres inconnus sont présentés en forme de vecteurs au moyen du concept du produit de Kronecker. De cette façon on a évité le calcul de tenseur . La stabilité totale a été prouvée sans aucune modification du profit de la matrice dans l'algorithme récursif. L'optimalité du régulateur est confirmée dans le cas de la poursuite de la trajectoire de référence. Dans ce papier on a considéré aussi le rapport entre la supposition de la continuité absolue des distributions de dimension de la probabilité et la modification de haute fréquence. On a présenté les résultats théoriques mais la méthodologie de commande adaptive existe depuis des années dans les systèmes militaires (hélicoptère CH 47 et avion X-15)/

Mots clés: **commande adaptive, modèle ARX,déviation non-Gauss, régulateur automatique, stabilité du système, méthode des moindres carrés.**