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Optimal Division of the Roll Angle Gyroscope Encoder for the Antitank Missile

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A formula for the estimate of the standard deviation of the missile roll rate in the buster phase is derived for an antitank line of sight-guided missiles where the free gyroscope is used to measure the roll angle and roll rate is determined by "two-point differencing". Taking into account process noise (model uncertainty) an expression for the optimal division of the free gyro encoder (optimal number of information on roll angle per revolution) is obtained. By numerical simulation of a complete system of equations of missile motion and guidance and control system, the estimate of the standard deviations of the coordinates from the line of sight is determined by using the Monte-Carlo simulation technique. The estimation of the roll rate and the roll angular acceleration by the Kalman filter is included optionally in the model. The influence of the division of free gyro encoder on maximal values of the estimate of the standard deviation of coordinates is determined by numerical simulation and its optimal value is found in order to obtain minimum of the standard deviation estimate.

Key words: **anti-tank missile, guided missile, roll rate, angular rate, encoder, Kalman filter, free gyro, pulse width modulation, finite difference method, numerical simulation.**

Glossary of Symbols

$\Delta h, \Delta y$	– vertical and horizontal deviation of the missile coordinates from the line of sight [m];
n_g	– number of measured angles of roll position for one missile revolution – number of radial slots on the free gyro encoder, [rad,°];
p	– missile angular velocity around the longitudinal axis – roll rate, [rad/s];
$\dot{p}$	– missile angular acceleration around the longitudinal axis – roll acceleration, [rad/s^2];
T_k, T_0,	– sampling interval of the roll angle at the time t_k and at the time t=0, [s];
t	– time, [s];
δ, $\delta_{\max}$	– deflection of the aerodynamic control surface and its maximum value, $\delta \in [-\delta_{\max}, +\delta_{\max}]$;
η, ς	– command coefficients of the pulse width modulated command in the vertical and horizontal plane respectively, $\eta, \varsigma \in [-1, +1]$;
η_d, ς_d	– demanded command coefficients;
Φ	– missile roll angle, [rad,°];
$\Delta\Phi_g$	– sampling interval of the missile roll angle (gyroscope characteristic), [rad,°];
σ_p	– standard deviation of the roll rate [rad,°];
$\sigma_{p.\Sigma}$	– standard deviation of the total variation of the missile roll rate from the nominal value [rad,°];
σ_Φ	– standard deviation of the variation of the missile coordinates roll angle (gyroscope characteristic), [rad,°];
$\sigma_\Sigma(\Delta h)$, $\sigma_\Sigma(\Delta y)$	– standard deviation of the variation of the missile coordinates in the vertical and horizontal direction under the effect of all disturbances, [m];
$\sigma_\Sigma(\Delta r)$	– resultant (total) standard deviation of the variation of the missile coordinates under the effect of all disturbances, [m];
	Lower indexes
0	– initial state;
k	– values at the instant t_k;
min, max	– minimum and maximum value;
	Upper indexes
$\cdot$	– time derivative;
^	– estimated value based on the Kalman filter;
~	– estimated value based on the mathematical model;
*	– measured quantity, reduced – nondimensional quantity;

Introduction

THE paper analyzes an one-channel pulse width modulation control system, with the aerodynamic fin or thrust vector of the guided antitank missile – simple called impulse control. This sort of control is applied on many tactical projectiles in service (MALJUTKA, HOT, MILAN, FAGOT).

The idea of the one-channel impulse control of the rotated missile is based on the fact, that, when particular conditions are fulfilled, the missile is moving proportionally not to the instant value of the force, but to the mean value of the force in the period of revolution. The missile is rotated around the longitudinal axis under the effect of rolling moment (usually the aerodynamic moment due to the fin cant angle), the roll rate being one order higher than natural frequency of the longitudinal short period motion.

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For one missile revolution the aerodynamic control surface is deflected to a maximum value in one direction and forced to stay at that position for some time, and then it is deflected to the opposite maximal deflection position and forced to stay at that position for some time. The time of the control surface movement from one to the other boundary position is T_δ (Fig.1).

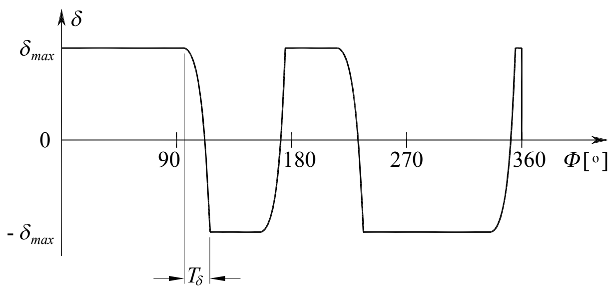

Figure 1. Pulse width modulated command for one missile revolution

Discrete aerodynamic one-channel control of fast rotating missiles has been used for many years. However, there are no published papers from that field, particularly not those concerning the estimation of the missile roll rate. One of the basic factor which influences precession of guided anti-tank missiles with one-channel pulse width modulation control systems, with the aerodynamic fin or thrust vector is the accuracy of the prediction of time of actuation of the control device from the equilibrium position. For this reason the accurate estimate and prediction of the roll angle is necessary. In order to estimate the roll angle, it is necessary to estimate the roll angular rate and the roll angular acceleration.[1].

Namely, when, based on the guidance law, the roll angles of control device deflection (there are four angles in the general case, $\Phi_j,\ j=1,2,3,4$) are determined, the time of control activation is determined by

$$\Delta t_j=\frac{\Phi_j}{\hat{p}_k}\left[1-\frac{1}{2}\frac{\dot{\hat{p}}_k\Phi_j}{\left(\hat{p}_k\right)^2}\right]=\frac{\Phi_j}{\hat{p}_k}-\frac{1}{2}\frac{\dot{\hat{p}}_k\left(\Phi_j\right)^2}{\left(\hat{p}_k\right)^3},\ \begin{matrix} j=1,2,3,4. \\ k=1,2,... \end{matrix} \quad (1)$$

where: $\hat{p}_k$ - estimated value of the roll rate at the time instant t_k, $\dot{\hat{p}}_k$ - estimated value of the roll angular acceleration at the time instant t_k. Any error in estimation of these quantities causes the mean control vector in one missile revolution in magnitude and direction to deviate from a desired one. As a consequence, the movement of the missile will not correspond to the demanded one.

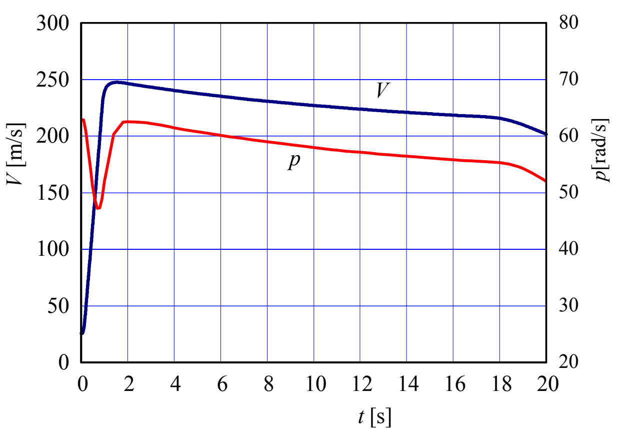

Figure 2. Velocity, roll rate and path on the reference trajectory for the hypothetical missile [1].

The angular rate p (Fig.2) is changed during the flight for two main reasons. First, the equilibrium angular rate is changed due to the change of missile speed, and second, due to the missile movement around the center of mass, the flight parameters are changed.

In this report the missiles with a clearly distinguished buster phase will be considered (Fig.2). In this type of missiles the first – buster phase is the most complex for analysis and synthesis and therefore, emphasize will be placed on this phase.

In order to measure the roll angle, the free gyroscope is built in the missile. The measurement is done discretely with some measurement error.

The roll angle sensor is an encoder consisting of a disk fixed to the outer gyroscope frame. On the disk perimeter there are n_g radial slots, and on the missile rolling body there is a photo diode and a light source. The light, while passing through the slots, produces a current impulse based on which the time of passing the slot on the disk through the referent point is determined. Accordingly, after the each angle $k\times\Delta\Phi_g$, the time t_k is measured, where $\Delta\Phi_g=2\pi/n_g=\text{const}$.

In this work the methods for estimating the optimal division of the free gyroscope encoder are considered in order to obtain minimum of the estimate of standard deviations of the missile deviation of coordinates from the line of sight.

Estimation of the Missile Roll Rate by the Method of Finite Difference of the First Order

The roll rate, necessary for calculating the roll angles and the times of the actuation of controls can be determined based on the measured values of the roll angle using the finite difference method of the first order ("two-point differencing "):

$$p_k=\frac{\Phi_k-\Phi_{k-1}}{T_k}=\frac{\Delta\Phi_k}{T_k} \quad (2)$$

where: Φ_k and Φ_{k-1} – the measured values of the angles at the moment t_k and t_{k-1}, and $T_k=t_k-t_{k-1}$ - the time interval between two successive measurements of the angles Φ_k. The roll angle Φ is measured by the free gyroscope with a measurement error. When the angular rate is constant, the error of determining the angular rate rises only due to the measurement error of the roll angle. But, in this article the missile model which is using the angular rate is variable (Figures 2 and 3), so the time interval T_k sampling is a function of the time of flight. For the purpose of this work all necessary data are calculated for a hypothetical missile using the program SimDVPTR, [7].

It will be assumed that the measurement error is uncorrelated quantity, and that its probable density error follows Gauss' law with null mathematical expectations and the standard deviation σ_Φ. Further, it will be assumed that the measurement error of the roll angle is a small value compared to the measured angle itself, so from formulae (2) the error of determining the angular rate and its dispersion is

$$\sigma_p^2=\sigma^2\left(p_k\right)=\frac{1}{\left(T_k^2\right)^2}\left[\sigma^2\left(\Phi_k\right)+\sigma^2\left(\Phi_{k-1}\right)\right]=2\frac{\sigma_\Phi^2}{T_k^2} \quad (3)$$

where σ_Φ^2 - dispersion of the measured roll angle by the free gyro.

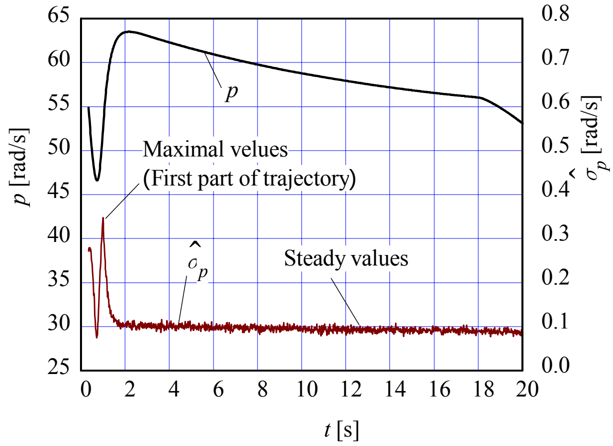

Figure 3. Estimate of the root mean square deviation of the roll angular rate $\hat{\sigma}_p$ obtained by the "two-point differencing" in the function of time $\sigma_\Phi = 0.87\,\text{mrad}$.

In the preceding analysis it was assumed that the measured angle dispersion is the same during time.

From relation (3) it can be seen that the dispersion of the missile roll rate is inversely proportional to the square of the sampling time T_k

When the angular rate is variable, which is the case in the buster phase of flight, one can write

$$\Phi_k \approx \Phi_{k-1} + p_k T_k + \dot{p}_k \frac{T_k^2}{2} \tag{4}$$

wher $\dot{p}_k$ - the missile angular acceleration. From (4) it is

$$p_k = \frac{\Phi_k - \Phi_{k-1}}{T_k} - \dot{p}_k \frac{T_k}{2} \tag{5}$$

Upon comparison of (2) and (5) it can be concluded that, when the angular rate is variable, and when the first order of differencing is used for the estimation of the angular rate, the process noise (model noise) is:

$$-\dot{p}_k \frac{T_k}{2} \tag{6}$$

It is proportional to the sampling time. So, the decrease in the sampling time results in the process noise decrease and the component of the roll rate estimation error increases due to the measurement noise. This is the main disadvantage of the finite difference method.

The total dispersion of the roll rate is equal to the sum of the dispersion of the component due to measurement noise and the dispersion of the component due to process noise.

$$\sigma_{p.\Sigma}^2 = 2\frac{\sigma_\Phi^2}{T_k^2} + \left(-\dot{p}_k \frac{T_k}{2}\right)^2 \tag{7}$$

The minimum of the dispersion (7) is obtained when the first derivative of (7) with respect to T_k is equal to zero:

$$\frac{\partial\left(\sigma_{p.\Sigma}^2\right)}{\partial\left(T_k\right)} = \dot{p}_k^2 \frac{T_k}{2} - 4\sigma_\Phi^2 \left(T_k\right)^{-3} = 0 \tag{8}$$

From the equation above the value of T_k which corresponds to the minimum is obtained:

$$\left(T_k\right)_{\min} = \sqrt[4]{8}\sqrt{\frac{\sigma_\Phi}{|\dot{p}_k|}} \tag{9}$$

It can be seen that the sampling time, which corresponds to the minimum of the dispersion of the roll rate estimation is proportional to the square root of the standard deviation of the roll angle measurement noise and inversely proportional to the square root of the roll angular acceleration.

The minimum of the dispersion of the roll rate estimate is obtained when the expression for $\left(T_k\right)_{\min}$ determined by (9) is substituted into expression (7)

$$\sigma_{p.\Sigma}^2 = \frac{6}{\sqrt{8}}|\dot{p}_k|\sigma_\Phi \tag{10}$$

Based on expressions (2) and (9) the angle $\Delta\Phi_k$, which corresponds to the minimum of the dispersion of the roll rate estimation can be put into the following expression

$$\Delta\Phi_g = \Delta\Phi_k = p_k\left(T_k\right)_{\min} = p_k\sqrt[4]{8}\sqrt{\frac{\sigma_\Phi}{|\dot{p}_k|}} \approx 1.7\,p_k\sqrt{\frac{\sigma_\Phi}{|\dot{p}_k|}} \tag{11}$$

Expression (11) can be used for estimating the increment of the measurement of the free gyro roll angle under the condition that the minimum of the dispersion of the estimate of the missile roll rate is achieved, when the roll rate, roll angular acceleration and standard deviation of the measurement of the missile roll angle are known. For example, for $p_k = 10\,\text{Hz} = 62.8\,\text{rad/s}$, $\sigma_\Phi = 0.1° = 1.7\,\text{mrad}$, $\dot{p}_k = 20\,\text{rad/s}^2$ it is $\Delta\Phi_g = 1\,\text{rad}$, and $n_g = 2\pi/\Delta\Phi_g = 2\pi/1 \approx 6$ and $\sigma_{p.\Sigma} = 0.27\,\text{rad/s}$. By decreasing σ_Φ twice the number of division becomes $n_g = 9$ and $\sigma_{p.\Sigma} = 0.19\,\text{rad/s}$. The roll rate and the roll angular acceleration are changed several times on the trajectory, so $\Delta\Phi_k$ is variable on the trajectory, which is one of the disadvantages of the derived algorithm. But the main disadvantage of the algorithm is a considerable error of the roll rate estimate defined by (7), especially at the highly variable roll angle.

It should be mentioned that the dispersion of the error defined by expression (3) comprises only the component of additive measurement noise. It is well known that the error due to differentiation depends on the applied method and the increment of the independent variable.

The error of the roll rate estimate can be decreased by increasing the differentiation algorithm order. That analysis will not be considered in this paper, but only some basic conclusion will be given. They are: with the increase of the order of the algorithm the process noise (error due to the method) is decreased, but, the component of the error due to the measurement noise is increased.

In the whole analysis given above it is assumed that the standard deviation of the roll angle measurement is constant in time.

Estimation of the States by the Kalman Filter

It is known that the optimal estimate of the system states can be obtained by applying the Kalman filter, assuming that a particular necessary condition is fulfilled [5]. The detailed analysis of the estimation of the states of the hypothetical missile by using the Kalman filter is given in papers [1, 2, 3]. These works examine the estimates of the roll rate with the Kalman filter with a true model, the Kalman filter with a polynomial kinematical model and the Kalman filter with a kinematical model with constant coefficients (alpha-beta-gamma filter). The results of this research are as follows:

The efficiency of the proposed method is analyzed by the numerical simulation. The numerical simulation is also used to obtain the estimate of the standard deviation of the roll rate in respect to the reduced value of the process noise. The process noise optimal value for achieving the minimal dispersion of the estimate of the roll angle and the roll rate is determined. It has been shown that there exists the minimum of the standard deviation estimate for a particular value of the process noise for all three applied methods. The conditions for determining the method of estimate of the roll rate and the process noise are determined. The Kalman filter application results in three times lower estimate of the standard deviation of the roll rate in the transition process (maximum values) and about ten times lower steady value compared to the values obtained by the numerical differentiation using the first order finite difference method ("two-point differencing").

The parallel analysis of the applied method shows that influence of the process noise on the standard deviation of the roll angle and the roll rate is different on the first part of the trajectory (maximum values) and in the second part (steady values). At the beginning of flight, where the changes of the angular rate are considerably greater, the application of the Kalman filter with the exact mathematical method gives better results when compared to those obtained by the kinematical polynomial method at lower values of the process noise covariance. The exact mathematical model better describes the rolling motion dynamics at low values of the process noise, while at higher values of the process noise the influence of the mathematical model is of less importance, so both, exact and kinematical polynomial model, give the same results.

The application of the Kalman filter with the kinematical polynomial model provides 50% less estimate of standard deviation of the roll rate and the roll angle when compared to the Kalman filter with the exact model for steady values. Besides, by using the polynomial model it is achieved that the estimation process doesn't depend on dynamic coefficients of equation of missile motion and the real trajectory, which is not known due to the stochastic nature of disturbing forces and moments.

By applying the Kalman filter with the polynomial kinematical model the first and the second derivative of the roll angle are obtained directly:

$$\begin{aligned}\hat{\Phi}_k &= \tilde{\Phi}_k + K_\Phi\left(\Phi_k^* - \tilde{\Phi}_k\right)\\ \hat{p}_k &= \tilde{p}_k + K_p\left(p_k^* - \tilde{p}_k\right)\\ \dot{\hat{p}}_k &= \dot{\tilde{p}}_k + K_{\dot{p}}\left(\dot{p}_k^* - \dot{\tilde{p}}_k\right)\end{aligned} \tag{12}$$

Simulation of the Guidance and Control System and Measured Values of Roll Angle

In order to achieve the stated objective the very complex digital program SimDVPTR, [7], is developed. The program is based on a full mathematical model of a guided missile as a nonlinear unsteady object with a digital control system and the Monte-Carlo simulation technique which comprises simultaneous acting of all significant disturbances in flight.

The model includes all processes in the system from the estimation of missile states up to control and guidance which should be realized in the microcontroller: estimates of coordinates in the locater, filtering and estimation of the coordinates and their derivatives, estimation of the roll angle and its derivatives, calculation of the command coefficient for compensating the missile weights and command coefficients for the compensation of the influence of the Magnus force and moments, calculation of the controls in the compensator, calculation of the angles of the equivalent lags in commands, calculation of the roll angles of control surface deviation, and calculation of times of activation of the control surfaces.

By testing gyroscopes in laboratory and by measurements, in-flight characteristics of gyroscopes can be obtained. For the purpose of this work, the standard deviation of measurement of the roll angle by the free gyroscope will be taken $\sigma_\Phi = 0.05^\circ = 0.87\,\text{mrad}$, while the mathematical expectation is assumed to be zero. Based on the taken assumption, the measured noise is simulated by the pseudo normal distribution of random numbers $n_k \sim \mathcal{N}\left(0, \sigma_\Phi^2\right)$.

Calculations and measurements in flight for the missile under consideration give the mean value of the initial angular rate and the estimate of its standard deviation: $p_0 = 62.8$ rad/s, $\sigma_{p0} = 3$ rad/s, [1]. Note that these values correspond to the normal temperature of rocket motor propellant. The initial value of the roll angle is taken to be $\Phi_0 = 0$. With these values, the measured values of the roll angles during flight are simulated by using the program SimDVPTR [7]. The roll rate is also simulated as the estimate of the root mean deviation of the roll rate $\hat{\sigma}_p$ obtained by the "two-point differencing" in the function of time. It is shown in Fig.3 together with the simulated roll rate. For the simulation by the Monte-Carlo method [4] 200 realizations of the trajectory were used. From Fig.3 it can be seen that the values of $\hat{\sigma}_p$ are several times higher in the vicinity of significant changes of the roll rate compared to the intervals where the roll rate is changed monotonically.

Analysis of the Realization of the Demanded Command

In the beginning of this report it has been shown that there is the minimum of the estimate of the dispersion of the roll rate $\sigma_{p.\Sigma}^2$ in the function of the standard deviation of the measurement of the missile roll angle σ_Φ and the sampling time T_k, or in the function of the increment of the free gyro encoder $\Delta\Phi_g = 2\pi/n_g$. The minimum is a function of the missile roll rate p_k and the roll angular acceleration $\dot{p}_k$. It is further concluded that, for the chosen parameters of the hypothetical missile, the minimum of the dispersion of the roll rate estimate is achieved for $\Delta\Phi_g \approx 1\,\text{rad}$ for the chosen value of the standard deviation of the roll angle $\sigma_\Phi = 1\,\text{mrad}$. The question is how the division of the roll free gyro encoder and the estimation of the roll angle and the roll rate affect the realization of the given command in the pitch and yaw plane and how they affect deviation of the perturbed trajectories with respect to the undisturbed – reference trajectory. The answer to that question is obtained by the numerical investigation of that influence by using the program SimDVPTR.

In Figures 4 and 5 the demanded and realized command coefficient in the vertical and horizontal plane are shown for one trajectory for the first two seconds of flight when the roll angle is changed most on the trajectory (Figures 2 and 3).

For the clarity of analysis the calculation is done for constant demanded values of command coefficients with values $\eta_d = 0.5$ in the vertical plane and $\varsigma_d = 0$ in the horizontal plane. By definition, the command coefficient is constant during one revolution of the missile. But the period of revolution is variable along the trajectory due to the variable roll rate (Figures 4 and 5). Due to the variable roll rate, the impulses of the control surface are not realized at the demanded (calculated) roll angles, but at the estimated roll angles which deviate from the demanded values, so the realized commands are not equal to the demanded ones. These deviations depend on the methods of estimation of the roll rate and the roll angle. When the roll rate is estimated by the classical first order differencing, and the roll angle is measured once per revolution (curves "n_g =1, no filtering" in Figures 4 and 5) the largest deviation of the realized command coefficients with respect to the demanded is achieved. The command coefficients deviate in both vertical and horizontal planes. Although the demanded command coefficient in the horizontal plane is zero, there is a parasite realized command coefficient of the absolute value, approximately 0.3.

When the angular rate is estimated by the Kalman filter as explained in [1, 2, 3], and the roll angle is measured once per revolution (curves "n_g =1, with filtering" in Figures 4 and 5) the deviations of the realized command coefficients from the demanded ones are 30 to 50% lower from the realized command coefficients in the case "n_g =1, no filtering", which considerably simplifies the synthesis of the guidance law and decreases the dispersion of the missile in the first phase of flight.

The lowest deviations of the realized command coefficient from the demanded one are achieved when the roll rate is estimated by the Kalman filter as it is given in [1, 2, 3], and the roll angle is measured eight times per revolution (curves "$n_g = 8$, with filtering" in Figures 4 and 5). In this case, the deviations of the realized command coefficients when compared to the demanded ones, are negligible, so that the estimation of the roll rate and the roll angle can be considered as optimal.

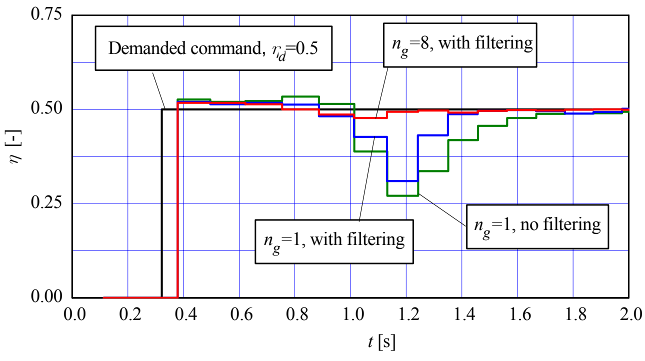

Figure 4. Demanded and realized command coefficients in the vertical plane in the function of the method used for the estimation of the roll rate.

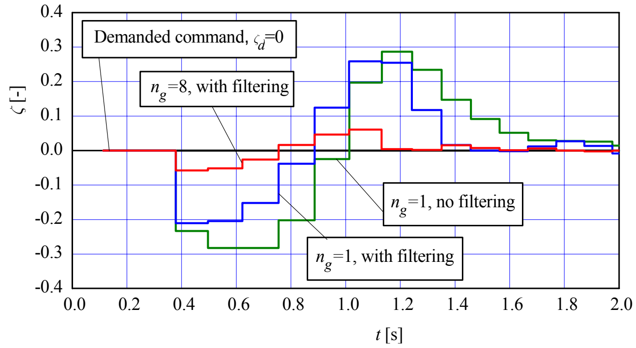

Figure 5. Demanded and realized command coefficients in the horizontal plane in the function of the method used for the estimation of the roll rate.

Optimal Division of the Free Gyro Encoder - Stohastic Analysis

By using the mentioned SimDVPTR program the estimates of the standard deviations of the missile coordinates in the vertical $\hat{\sigma}_\Sigma(\Delta h)$ and the horizontal $\hat{\sigma}_\Sigma(\Delta y)$ plane are calculated, as well as their resultant value $\hat{\sigma}_\Sigma(\Delta r)$ in the function of the time of flight, [1], (Fig.6). It can be seen that the maximum values of the estimates of the standard deviations of the missile coordinates occur nearly at the end of the first – buster phase of flight $(t \approx 1\,\mathrm{s})$. This is a well known feature of that kind of missile which has a large axial force developed by the rocket motor in the buster phase [1].

The dispersions in the vertical plane and the horizontal plane are not equal and they form an ellipse of dispersion. The resultant standard deviation – square root of the sum of dispersion of the deviation in the vertical plane $\hat{\sigma}^2_\Sigma(\Delta h)$ and the horizontal plane $\hat{\sigma}^2_\Sigma(\Delta y)$ can be taken as a deviation measure , which will be denoted as $\hat{\sigma}_\Sigma(\Delta r)$

$$\hat{\sigma}_\Sigma(\Delta r) = \sqrt{\hat{\sigma}_\Sigma(\Delta h) \times \hat{\sigma}_\Sigma(\Delta y)} \tag{13}$$

where Δh , Δy - deviations of the missile coordinates with respect to the mathematical expectation of the coordinates in the vertical and the horizontal plane respectively. This quantity is nearly equal to the estimate of the mathematical expectation of the quantity $(\Delta r) = \sqrt{(\Delta h)^2 + (\Delta y)^2}$ and it is practically a measure of the circular probable error (CEP), $CEP = 1.18\ \hat{\sigma}_\Sigma(\Delta r)$.

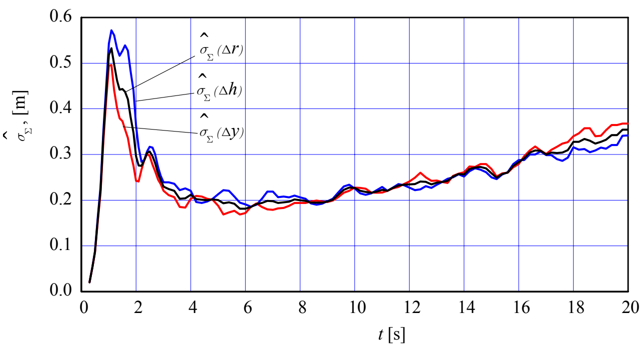

Figure 6. Standard deviation of the missile coordinates in the vertical and the horizontal plane and their resultant value in the function of time of flight while hitting the target at a distance of $\approx 4500\,\mathrm{m}$ upon the action of all relevant disturbances

The results of the statistical analysis are shown on the diagram in Fig.7 where the maximum values of the estimate of the standard deviation of the resultant deviation of the missile coordinates from the line of sight $\hat{\sigma}_{\Sigma}(\Delta r)$ against the number of slots of the free gyro encoder. Two curves are presented. One denoted by "no filtering", for the case when the estimate of the roll rate and its derivative is obtained by the "two-point differencing", and the second one denoted by "with filtering" for the case when the estimate of the roll rate and its derivative is obtained by the Kalman polynomial filter as explained [1, 2, 3]. It can be concluded that for the case when the estimate of the roll rate and its derivative is obtained by the "two-point differencing" there exists the minimum of the estimate of the standard deviation of the coordinates. It is achieved at $n_g = 4 \div 8$, or $\Delta\Phi_g = (\pi/4 \div \pi/2)\,\mathrm{rad}$, which closely corresponds to the value obtained by the analytical method (expression (11). With increasing n_g, the estimate of the standard deviation $\hat{\sigma}_{\Sigma}(\Delta r)$ is increasing inversely proportionally to n_g as it follows from expression (11).

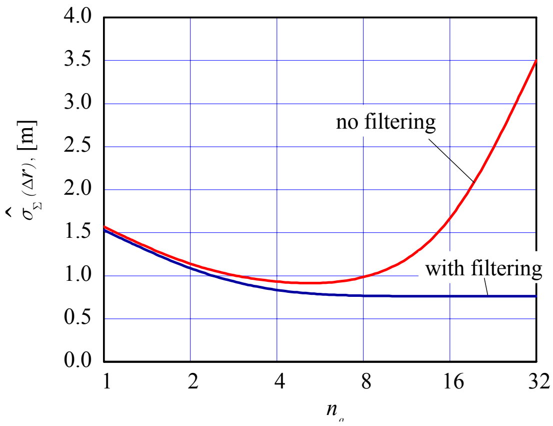

Figure 7. Estimate of the standard deviation of the resultant deviation of the missile coordinates from the line of sight (maximum values) against the number of slots of the free gyro encoder ($\sigma_{\Phi} = 1\,\mathrm{mrad}$)

The curve "no filtering" - $\hat{\sigma}_{\Sigma}(\Delta r) = f(n_g)$ is always above the curve $\hat{\sigma}_{\Sigma}(\Delta r) = f(n_g)$ "with filtering" by the Kalman filter. The difference is small at low values of n_g, and large at high values of n_g

For the case when the estimate of the roll rate and its derivative is obtained by the Kalman polynomial filter (curve denoted by "with filtering") with increasing of the number of information per revolution from the free gyro, the estimate of the standard deviation is decreasing monotonically. Up to $n_g = 4$, for exemple, the influence of the number of slots is considerable, but above $n_g = 4$, with increasing n_g, the value of $\hat{\sigma}_{\Sigma}(\Delta r)$ decreases insignificantly. With the introduction of the Kalman filter, the estimate of the standard deviation is reduced for about 15% in the vicinity of the minimum value of the "no filtering" curve.

On the basis of the previous analysis it can be said that both methods, the simple analytical method and the numerical simulation of the whole system of missile guidance and control, yield practically the same optimal division of the free gyro encoder (from four to six slots on the encoder in the example) and that the introduction of the Kalman filter reduces the estimate of the standard deviation (maximum values) (for about 15% in the example).

Conclusion

The optimal division of the free gyro encoder, used to measure the roll angle of the line of sight of anti-tank guided missiles, is determined analytically and by numerical simulation. In the case of the analytical approach the expression for the optimal number of information per revolution (optimal number of slots on the encoder) is derived from the expression for calculating the estimate of dispersion of the roll rate when the roll rate is determined by the two-point differencing method. Meanwhile, in the numerical approach, the optimal encoder division is obtained from the estimate of the standard deviation of the coordinates from the line of sight which is obtained by the Monte-Carlo simulation of the complete set of equations describing the missile guidance and control system. In the system of equations the Kalman filter is included optionally.

Both methods show that there is an optimal division of the encoder n_g if in the numerical method "no filter" is included. The values of n_g obtained by both methods are close. When the Kalman filter is included in the numerical method for the estimation of the missile roll rate, the curve representing the estimate of the standard deviation of the resultant deviation of the missile coordinates from the line of sight (maximum values) against the number of slots of the free gyro encoder has no minimum, but monotonically decreases with increasing of the number of information per revolution from the free gyro. The influence of the number of slots is considerable up to a value of n_g, but above that value, with increasing n_g, the value of $\hat{\sigma}_{\Sigma}(\Delta r)$ decreases insignificantly. A designer can thus determine a scope of division under the interest.

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Optimalna podela enkodera žiroskopa protivoklopnih raketa

Izveden je izraz za ocenu standardne devijacije ugaone brzine valjanja protivoklopne vođene rakete kad se ona određuje klasičnom metodom numeričkog diferenciranja izmerenih vrednosti ugla valjanja metodom konačnih razlika prvog reda, pri čemu se meri ugao valjanja rakete pomoću žiroskopa sa enkoderom. Određeno je vreme uzorkovanja pri kome se postiže optimalna ocena ugaone brzine pri postojanju greške merenja i procesne greške. Numeričkom simulacijom su određene ocene standardnih devijacija odstupanja koordinata od linije viziranja i od srednje putanje rakete pri istovremenom dejstvu svih definisanih poremećaja za raketu koja se upravlja širinsko modulisanim impulsnim otklonima aerodinamičkog krilca. Ugaona brzina valjanja i ugao valjanja su ocenjeni Kalmanovim filtrom. Na bazi kriterijuma minimuma rezultujuće standardne devijacije koordinata određena je optimalna vrednost podele enkodera žiroskopa.

Ključne reči: **protivoklopna raketa, vođena raketa, brzina valjanja, ugaona brzina, enkoder, Klamanov filtar, slobodni žiroskop, impulsna modulacija, metoda konačnih razlika, numerička simulacija.**

Division optimale de l’encodeur du gyroscope pour les missiles antichars

Dans ce papier on présente la formule dérivée pour estimer la déviation ordinaire de la vitesse d’angle de roulement chez le missile antichar guidé quand elle est déterminée par la méthode classique de la différenciation numérique des valeurs mesurées de l’angle de roulement par la méthode des différences finies du premier ordre et quand l’angle de roulement est mesuré au moyen du gyroscope avec encodeur. On a déterminé le temps d’échantillonnage où l’on obtient l’estimation optimale de la vitesse d’angle quand il y a une erreur de mesurement ou de procès. Par la simulation numérique on a évalué les déviations ordinaires des coordonnées de la ligne de vue et la trajectoire moyenne du missile, pendant l’action simultanée de toutes les déviations définies pour le missile guidé par les déviations modulées d’impulsion des ailettes aérodynamiques. La vitesse de l’angle de roulement et l’angle de roulement sont évalués par le filtre de Kalman. La valeur optimale de la division de l’encodeur du gyroscope est déterminée par le critère de la déviation minimale obtenues des cordonnées.

Mots clés: **missile antichar, missile guidé, vitesse du roulement, vitesse d’angle, encodeur, filtre de Kalman, gyroscope libre, modulation d’impulsions.**