UDK: 621.375:621.396.62
COSATI: 17-01, 09-03

Symbolic analysis and design of current-differencing-amplifier filters

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A circuit model of an ideal current-differencing amplifier and its use in symbolic analysis and design of current-differencing-amplifier filters is presented. The analysis and design of a bandpass filter with one current-differencing amplifier is demonstrated. The advantages of the current-differencing-amplifier filters are highlighted.

Key words: **current-differencing amplifier, symbolic analysis, active filter, current processing.**

Introduction

MANY industrial electronic control systems are designed to operate using only a single-power-supply voltage. The conventional integrated-circuit operational amplifier (IC op amp) is typically designed for split power supplies (±15 V DC) and suffers from a poor output voltage swing and a rather large minimum common-mode input voltage range (approximately +2 V DC) when used in a single power supply applications. In addition, some of performance characteristics of these op amps could be sacrificed – especially in favor of reduced costs.

To meet the needs of the designers of low-cost, single-power-supply control systems, a new internally compensated amplifier has been designed that operates over a power supply range of +4 V DC to 36 V DC with small changes in performance characteristics and provides an output peak-to-peak swing that is only 1 V less than the magnitude of the power supply voltage [1]. The new amplifier, Fig.1, is designated *Current-Differencing Amplifier* (CDA), and represents a departure from a conventional amplifier design. Instead of using a standard transistor differential amplifier at the input, the non-inverting input function has been achieved by making use of a "current-mirror" to "mirror" the non-inverting input current about ground and then to extract this current from that which is entering the inverting input terminal. Whereas the conventional operation amplifier (OPAMP) differentiates input voltage, this amplifier differentiates input currents and therefore the name "Norton Amp" has been used to indicate this new type of operation [1,2].

It is also well known that the frequency range of active filters using operation amplifiers is limited due to the finite OPAMP's bandwidth. Since the CDA as a current processing device has a much wider bandwidth than the classical OPAMP [3], its use in active filter realizations can significantly improve the useful filter bandwidth.

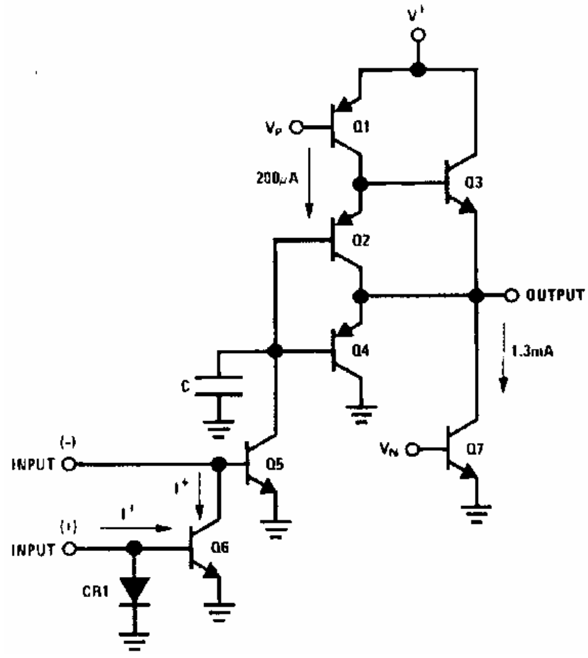

Figure 2. Current-Differencing Amplifier symbol

A new symbol for the "Norton" amplifier is shown in Fig.2.

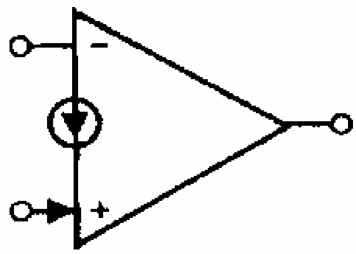

Figure 2. Current-Differencing Amplifier symbol

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The current source symbol between the inputs implies this new current mode of operation. In addition, it signifies that current is removed from the (-) input terminal. Also, the current arrow on the (+) input lead is used to indicate that this terminal functions as a current input.

The Current-Differencing Amplifier has a wide range of applications [1,2] such as:

– AC amplifiers (non-inverting amplifier, inverting amplifier, high input impedance high gain amplifier, amplifier with DC gain control, line-receiver amplifier)
– DC amplifiers (DC coupled power amplifier, ground referencing a differential voltage, unity gain buffer amplifier)
– Voltage regulators
– *RC* active filters (single-amplifier lowpass, single-amplifier highpass, single-amplifier bandpass, two-amplifier bandpass, three-amplifier state-variable universal biquad low/high/band/pass)
– Waveform generators (sinewave, squarewave, pulse, triangle, sawtooth, staircase)
– Phase-locked loops (PLL) and voltage controlled oscillators (VCO)
– Digital and switching circuits (OR gate, AND gate, bistable multivibrator, trigger flip-flops, monostable multivibrator, comparators, Schmitt trigger)
– Current sources and sinks
– Tachometers
– Squaring amplifiers
– Differentiators and integrators
– Sample and hold (S/H) circuits

For an automated computer-aided analysis of linear time-invariant circuits with CDAs, a CDA device model must be established. In this paper, we formulate an ideal model of CDA that is suitable for an automated computer-aided symbolic analysis [4-13]. Next, a filter circuit is analyzed in full detail to effectively show the use of the CDA model in conjunction with the Modified Nodal Analysis (MNA) equations. Finally, the filter design equations are derived from the symbolic analysis and an illustrative design is presented.

Symbolic analysis of a CDA circuit model

Fig.1 shows that the (+) and (-) inputs are both clamped by (equivalent) diodes to force them to be one-diode drop above ground. The diode shown across the (+) input actually exists as a diode in the circuit, Fig.1. The diode across the (-) input is the base-emitter junction that acts as an equivalent diode. The current entering the (+) input is "mirrored" or reflected about ground and then extracted from the (-) input [1,2].

The Current-Differencing Amplifier, Fig.2, is modeled as a four-terminal three-port linear time-invariant lumped element. The four terminals are

– non-inverting terminal, or equivalently (+) terminal
– inverting terminal, or equivalently (-) terminal
– output terminal
– ground, normally not drawn in Fig.2.

The three ports are constituted as follows:

– non-inverting port, (+) terminal and ground
– inverting port, (-) terminal and ground
– output port, output terminal and ground.

The ground terminal is the common terminal for the three ports; hence, the ideal CDA is a grounded circuit element.

The definition equations of the ideal CDA are based on the fact that the input currents are equal, and the input voltages are clamped to zero. The "associated" reference directions for port currents and voltages are assumed to be standard. The currents at (+), (-) and output terminals are assumed to be directed towards the element.

The ideal CDA definition equations are

- $v_+ = 0$, plus-terminal-to-ground voltage is zero,
- $v_- = 0$, minus-terminal-to-gorund voltage is zero,
- $i_+ - i_- = 0$, input-terminal currents are equal.

Note that the above equations are quite opposite from the ideal OPAMP definition equations which state that the input voltages are equal, and that the input currents are clamped to zero.

If the bias conditions or the circuit topology force one of the CDA input currents to zero, then the ideal CDA equations become equal to the ideal OPAMP equations.

The CDA currents are not known, and cannot be expressed in terms of the CDA voltages. Therefore, every CDA increases the order of Modified Nodal Analysis (MNA) system of equations by three. Also, it introduces three additional non-voltage variables – the CDA port currents.

It should be pointed out that the CDA is neither the Differential Current Amplifier nor the Current-Feedback Operational Amplifier. Advances in integrated circuit technologies have meant that state-of-the-art analog IC design is now able to exploit the potential of current-mode analog signal processing, providing attractive and elegant solutions for many circuit and system problems [14,15]. A further consequence of the current-mode processing has been the emergence of new analog building-blocks. It is important not to confuse the *Current-Differencing Amplifier* (CDA) with the *Differential Current Amplifier* (DCA), or with the *Current-Feedback Operational Amplifier* (CFOA). We summarize below the definition equations for the three elements (CDA, DCA, CFOA), along with the definition equations of the ideal operational amplifier.

CDA	**DCA**	**CFOA**	**OPAMP**
$v_+ = 0$	$v_+ = 0$	$v_+ = v_-$	$v_+ = v_-$
$v_- = 0$	$v_- = 0$	$i_+ = 0$	$i_+ = 0$
$i_+ = i_-$	$i = a(i_+ - i_-)$	$v = ri_-$	$i_- = 0$

All equation parameters are real.

The assumed reference directions for voltages and currents are shown in Fig.3.

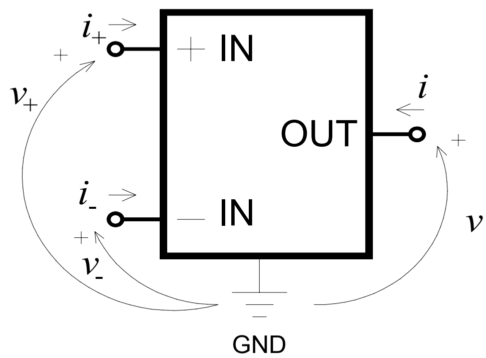

Figure 3. Reference directions for a grounded four-terminal three-port network that represents CDA, DCA, or CFOA.

Notice that CDA, DCA, CFOA, and OPAMP are four-terminal, three-port, lumped, linear, time-invariant, grounded elements.

Symbolic analysis of the single-CDA bandpass filter

A typical CDA filter application is a single-amplifier bandpass *RC* active filter, Fig.4, suitable for low frequency, low gain, and low quality-factor requirements.

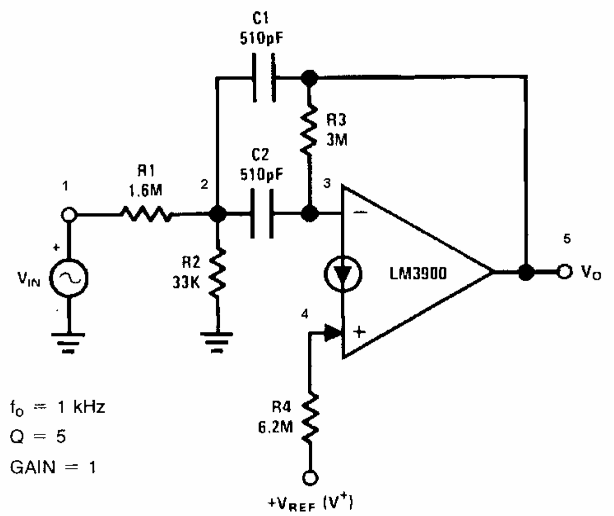

Figure 4. Single-CDA bandpass filter.

The automated symbolic analysis formulates the MNA system of equations as follows [4,5]:

Node equations

$$\frac{\underline{V}_1-\underline{V}_2}{R_1}+\underline{I}_{GEN}=0$$

$$\frac{\underline{V}_1-\underline{V}_2}{R_1}+\frac{\underline{V}_2}{R_2}+C_2 s\left(\underline{V}_2-\underline{V}_3\right)+C_1 s\left(\underline{V}_2-\underline{V}_5\right)=0$$

$$C_2 s\left(\underline{V}_3-\underline{V}_2\right)+\frac{\underline{V}_3-\underline{V}_5}{R_3}+\underline{I}_-=0$$

$$\frac{\underline{V}_4}{R_4}=\underline{I}_+=0$$

$$\frac{\underline{V}_5-\underline{V}_3}{R_3}+C_1 s\left(\underline{V}_5-\underline{V}_2\right)+\underline{I}_{OUT}=0$$

Independent voltage generator definition equation

$$\underline{V}_1=\underline{V}_{IN}$$

CDA definition equation

$$\underline{V}_+=0$$

$$\underline{V}_-=0$$

$$\underline{I}_+-\underline{I}_-=0$$

The variables $\underline{V}_1$ to $\underline{V}_5$ are the node voltages, $\underline{I}_+$ and $\underline{I}_-$ are the CDA input currents, $\underline{I}_{\mathrm{OUT}}$ stands for the CDA output current, and $\underline{I}_{\mathrm{GEN}}$ designates the generator current.

The circuit parameters, R_1, R_2, R_3, R_4, C_1, C_2, are kept as symbols, and the filter transfer function is symbolically computed as

$$\underline{H}(\underline{s})=\frac{\underline{V}_5}{\underline{V}_{\mathrm{IN}}}=\frac{b_1\underline{s}}{a_2\underline{s}^2+a_1\underline{s}+a_0}$$

where

$$b_1=-C_2R_2R_3$$

$$a_2=C_1C_2R_1R_2R_3$$

$$a_1=\left(C_1+C_2\right)R_1R_2$$

$$a_0=R_1+R_2$$

A similar symbolic analysis can be carried out for a complex circuit shown in Fig.5.

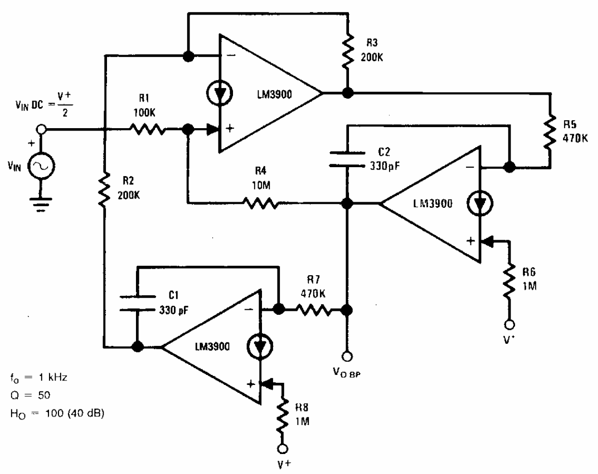

Figure 5. Three-CDA bandpass filter.

It has been reported [16, 17] that the passive sensitivities of CDA filters are lower than the corresponding sensitivities of classical OPAMP filters [18].

Symbolic design equations for the single-CDA bandpass filter

From the transfer function derived in the previous section, the filter center frequency, f_0, quality factor, Q, and gain, H_0, are computed symbolically.

$$f_0=\frac{\omega_0}{2\pi}$$

$$\omega_0=\sqrt{\frac{a_0}{a_2}}=\sqrt{\frac{R_1+R_2}{C_1C_2R_1R_2R_3}}$$

$$Q=\omega_0\frac{a_2}{a_1}=\frac{C_1C_2R_3}{C_1+C_2}\sqrt{\frac{R_1+R_2}{C_1C_2R_1R_2R_3}}$$

$$H_0=\underline{H}(\mathrm{j}\omega_0)=\frac{-C_2R_3}{(C_1+C_2)R_1}$$

The design procedure is as follows:

- given H_0, Q, and $\omega_0 = 2\pi f_0$, find the five element values (capacitances and resistances)
- usually, $C_1 = C_2$ is preferred and the capacitances are chosen according to the available capacitors.
- find resistances R_1, R_2, R_3

The symbolic design expressions are of the general form

$$R_1 = R_1(H_0, f_0, Q; C_1, C_2)$$

$$R_2 = R_2(H_0, f_0, Q; C_1, C_2)$$

$$R_3 = R_3(H_0, f_0, Q; C_1, C_2)$$

and are found in the form

$$R_1 = \frac{-Q}{C_1 \omega_0 H_0}$$

$$R_2 = \frac{Q}{\left(C_1 H_0 + (C_1 + C_2) Q^2\right) \omega_0}$$

$$R_3 = \frac{C_1 + C_2}{C_1 C_2 \omega_0} Q$$

For example, require

$$H_0 = -1$$

$$Q = 5$$

$$f_0 = 1000\,\text{Hz}$$

it follows

$$R_1 = \frac{1}{400 C_1 \pi}$$

$$R_2 = \frac{1}{400(24C_1 + 25C_2)\pi}$$

$$R_3 = \frac{C_1 + C_2}{400 C_1 C_2 \pi}$$

Typically, equal capacitances are preferred: $C_1 = C_2$. The capacitance values are selected from the available standard values. For $C_1 = C_2 = 510\,\text{pF}$ the resistances are found as

$$R_1 = 1.56034\,\text{M}\Omega$$

$$R_2 = 31.8437\,\text{K}\Omega$$

$$R_3 = 3.12069\,\text{M}\Omega$$

Various capacitance values can be tried to obtain the most suitable resistances.

We found out that Application Note 72 [1] on page 18, in equations (8), (9), (10), defines the filter gain as a positive number, i.e., as $|\underline{H}(\mathrm{j}\omega_0)|$. The minus sign is taken into account explicitly in these formulas to provide positive values for the resistances. In addition, these design equations assume a particular case when the capacitances are equal, $C_1 = C_2$.

The formulas presented in this paper can be used for the most general fully symbolic analysis and design case.

Since the output impedance of the CDA is low, higher-order filters can be easily obtained by cascading second-order sections.

Conclusion

Current-mode analog signal processing can be used to extend the bandwidth of integrated-circuit amplifiers. In this paper, a current-differencing amplifier has been investigated and a circuit model for the idealized device has been established. The model has been used in fully symbolic analysis and design of active filters realized with current-differencing amplifiers. The closed-form expressions (keeping all parameters as symbols) have been developed for filter analysis equations, transfer functions, and filter design equations. The definition equations of several current-mode devices have been reviewed in order to clarify the equations behind the underlining model.

References

[1] The LM3900 A New Current-Differencing Quad of ± Input Amplifiers, Application Note 72, National Semiconductor, Sept. 1972

[2] The LM2900/LM3900/LM3301 Quad Amplifiers, Handbook, National Semiconductor Corporation, 1995

[3] DAILEY,D.Y.: Operational amplifiers and linear integrated circuits. McGraw-Hill Book Company, New York, 1989

[4] TOŠIĆ,D.V.: SALECAS - a package for symbolic analysis of linear circuits and systems, in Proc. 4th International Workshop on Symbolic Methods and Applications to Circuit Design, Leuven, Belgium, Oct. 10-11, 1996, pp.227-230. (Editor: Georges Gielen)

[5] TOŠIĆ,D.V.: A contribution to algorithms in computer-aided symbolic analysis of linear electric circuits and systems, Doctoral Dissertation, University of Belgrade, School of Electrical Engineering, Belgrade, 1996

[6] TOŠIĆ,D.V., LUTOVAC,M.D., EVANS,B.L. and MARKOSKI,I.M.: A tool for symbolic analysis and design of analog active filters, in Proc. 5th International Workshop on symbolic Methods and Applications to Circuit Design, SMACD'98, Kaiserslautern, Germany, Oct.8-9, 1998, pp.71-74

[7] HRIBŠEK,M.F., TOŠIĆ,D.V.: Symbolic analysis of current-differencing amplifier circuits, in Proc. XLIV Conf. ETRAN, Soko Banja, Serbia, 2000, pp.CD-ROM.

[8] HRIBŠEK,M.F., TOŠIĆ,D.V.: Ideal current-differencing amplifier as a building block for basic filter structures, in Proc. IEEE Balkan Conference on Signal Processing, Communications, Circuits, and Systems BCSP 2000, Maslak, Istanbul, Turkey, June 2-3, 2000, pp.CD-ROM.

[9] LUTOVAC,M.D., TOŠIĆ,D.V.: Symbolic analysis and design of control systems using Mathematica, International Journal of Control, Special Issue on the Use of Computer Algebra Systems for Computer Aided Control System Design, Nov. 2006, Vol.79, No.11, pp.1368-1381

[10] TOŠIĆ,D.V., LUTOVAC,M.D.: Advances in Symbolic Simulation of Systems, The IPSI BgD Transactions on Advanced Research, Jan. 2007, Vol.3, No.1, pp.9-14

[11] LUTOVAC,M.D., TOŠIĆ,D.V.: Symbolic Signal Processing and System Analysis, Facta Universitatis (Nis), Series: Electronics and Energetics, 2003, Vol.16, No.3, pp.423-431

[12] TOŠIĆ,D.V., HRIBŠEK,M.F., RELJIN,B.D.: Generation and design of new continuous-time second order gain equalizers using program SALEC, International Journal of Electronics and Communications (AEÜ - Archiv für Elektronik und Übertragungstechnik), May 1996, Vol.50, No.3, pp.226-229

[13] LUTOVAC,M.D., TOŠIĆ,D.V., EVANS,B.L.: Filter Design for Signal Processing using MATLAB® and Mathematica®. Upper Saddle River, NJ: Prentice Hall, 785 pages, 1st edition, ISBN 0-201-36130-2, ©2001.

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[14] TOUMAZOU,C., LIDGEY,F.J., HAIGH,D.G.: Analog IC design: the current-mode approach. London, UK: Peter Peregrinus, 1990.

[15] PAYNE,A., TOUMAZOU,C.: Analog Amplifiers: Classification and Generalization, IEEE Trans. CAS-I, Jan. 1996, Vol.43, No.1

[16] HRIBŠEK,M., EL HAMRUNI,E.A.: Single current differential amplifier lowpass filter, in Proc. TELFOR'99, Beograd, 1999, pp.453-454

[17] HRIBŠEK,M., EL AZZABI,I.A.: Self-oscillating DC-DC convertor using current differential amplifier, in Proc. TELFOR'99, Beograd, 1999, pp.455-457

[18] HRIBŠEK,M.F.: Simple allpass sections with complex poles and zeroes, IEE Proc. Circuits Devices Syst., Oct.1995, Vol.142, No.5, pp.273-276

Received: 26.05.2007.

Simbolička analiza i projektovanje filtara sa strujnim diferencijalnim pojačavačima

Predstavlja se ekvivalentna šema idealnog strujnog diferencijalnog pojačavača i njena primena u simboličkoj analizi i projektovanju filtara sa strujnim diferencijalnim pojačavačima. Prikazuje se analiza i projektovanje filtra propusnika opsega sa jednim strujnim diferencijalnim pojačavačem. Naglašavaju se prednosti filtara sa strujnim diferencijalnim pojačavačima.

Ključne reči: **strujni diferencijalni pojačavač, simbolička analiza, aktivni filtar, strujno procesiranje.**

Символический анализ и проектирование фильтров со токовыми дифференциальными усилителями

В настоящей работе представлена эквивалентная схема идеального токового дифференциального усилителя и его применение в символическом анализе и в проектировании фильтров со токовыми дифференциальными усилителями. Здесь приведён анализ и проектирование фильтра полосы пропускания с одним токовым дифференциальным усилителем. Здесь выделяются преимущества фильтров со токовыми дифференциальными усилителями.

Ключевые слова: токовой дифференциальный усилитель, символический анализ, активный фильтр, токовая обработка.

Analyse symbolique et projet des filtres aux amplificateurs différentiels de courant

Le schéma équivalent de l'amplificateur différentiel idéal et son emploi dans l'analyse symbolique ainsi que le projet des filtres aux amplificateurs différentiels de courant sont présentés dans ce papier. On a exposé aussi l'analyse et le projet du filtre passe-bande à l'amplificateur différentiel de courant. Pour finir, on a souligné les avantages des filtres à l'amplificateur différentiel de courant.

Mots clés: **amplificateur différentiel de courant, analyse symbolique, filtre actif, traitement de courant.**