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Robust Control of Systems over Communication Network

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In this paper the problem of robust control of constrained linear dynamic systems, in the presence of communication networks with queues, is considered. The communication network is between the process and the controller. We assume that the queue is in the sensor. The closed-loop system may face the problem of induced random delays caused by the communication network and that delay would deteriorate the system performance as well as its stability. Digital control systems with random but bounded delay in the feedback loop can be modeled as finite dimensional discrete-time jump linear systems with transition jumps being modeled as finite state Markov chains. The queue is modeled as a nonlinear systems to which feedback linearisation methodology is apllied. Then the complete system (process, communication network, queue) can be presented as a discrete-time jump system. For such a system, without unmodeled dynamics, the constrained quadratic control is proposed. The analysis of the system is then performed in the presence of unmodeled dynamics. This kind of systems has wide applications in military systems and process control systems.

Key words: **communication system, communication network, network control, robust control, dynamic system, linear system, random delay.**

Introduction

IN many industrial systems, especially those with remote sensors, actuators and controllers, a communication network is used to gather sensor data and send control signals. A communication network is a cost-effective and reliable way to coordinate different modules of control systems. Utilization of a multi-user network with random demands, affecting the network traffic, could result in random delays in the feedback-loop. These delays will deteriorate the system performance as well as stability. The problem becomes more complicated when, queue formation is also, considered for closed-loop data transmission.

Time-delays are important components of many dynamical systems that describe interconnection between dynamics, propagation or transport phenomena and heredity and competition in population dynamics. In monograph [14] the stability and stabilization of such systems are considered using a unified eigenvalue-based approach. Application of methodology is demonstrated on the congestion anlysis in a high performance communication network.

The paper considers a situation when a communication network is incorporated between the process and the controllers (up link case). The control systems, involve a queue as well. Such kind of systems is considered in [3]. In this paper the queue has a FIFO (first-in, first-out) structure. With the known maximum bufter size and the upper bound of random delay in the communication link, the recursive relation for controller input is described in the above paper. The static controller is then considered for jump systems.

In [17] the problem of control systems with random communication delays is considered. It is shown that the control systems with random and bounded delays in the feedback loop can be modeled as finite dimensional discrete-time jump linear systems with the transition jumps being modeled as finite-state Markov chains. The important conclusion of this the paper is that control of the augmented state-space model is an output feedback problem even if a state feedback law is intended for the original system.

Many physical systems are subject to frequent unpredictable structure changes (random failures, sudden environment disturbances, abrupt variation of operating points). Such systems can be described with Markovian jump systems. The system is a hybrid system with the state vector which has two components x_k and $r(k)$. The first one is generally referred to as the state, and the second one is regarded as the mode. Stability of stochastic systems with Markovian switching is considered in [1], [13] and [20]. Stochastic nonlinear hybrid systems are considered in [7].

In this paper we will consider the robust control of systems over a communication network and a queue. The results in this paper differ in the following from the known results in the literature

(i) For a network buffer (queue) in this paper we use a nonlinear mathematical model. The feedback linearization is applied to that model.
(ii) The process model has unmodeled dynamics
(iii) The controller is robust and designed in the presence of a set of constraints
(iv) The transition probability of Markov chains is not exactly known but belongs to the convex hull.
(v) Using the LMI tool, the robust stability, in the sense of mean square stability, is proved.

The problem under consideration belongs to the field of discrete-time Markovian jump linear systems. That kind of systems is considered in [5].

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Problem formulation

The system under consideration is presented in Fig.1.

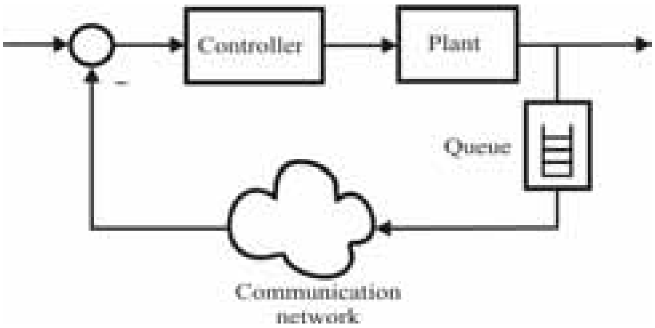

Figure 1. Control systems involving the queue and the communication link

It is supposed that the queue is in the sensor. The discrete-time linear time-invariant (LTI) process model with unmodeled dynamics is

$$x_{k+1}^1 = \left(A^1 + A_\Delta^1\right) x_k^1 + B^1 u_k \tag{1}$$

where $x_k^1 \in R^n$, $u \in R^m$. The uncertainty A_Δ^1 has a finite matrix norm, i.e.

$$\left\| A_\Delta^1 \right\| \le \gamma \tag{2}$$

Remark 1. The uncertainty can also be presented, as:

$$A_\Delta^1 = \sum_{i=1}^{l} q_i A_i \ , \ |q_i| < \gamma_1 \ , \ i = 1,...,l \tag{3}$$

The paper supposes that the communication network has a scheduler for queues. One kind of scheduling is presented in [16]. For the queue and the communication network the following assumptions were made:

The queue flow strategy is FIFO (first-in, first-out).

The communication network is shared with other control loops. The scheduling can be in the form of stochastic process [19].

The important parameters for the queue are queue capacity (maximum number of customers that can be accomodated in the actual queuing space) and queuing discipline (refers to the rule according to which the next customer to be served is selected from the queue [2].

Mathematical model of the jump systems

In this paper we will consider a dynamic model of the buffer. Different kinds of models are considered in literature [9], [15] and [16]. We will first consider the fluid flow model for the scalar continuous case. Using the flow conservation principle for a single queue and assuming no packet losses, the following differential equations are obtained:

$$\dot{x}_i^b(t) = -f_{out,i}(t) + f_{in,i}(t) \tag{4}$$

where $x(t)$ is the state of the queue and $f_{in}(t)$ and $f_{out}(t)$ represent respectively the number of incoming and outgoing packets. According to Fig.2 we obtain from (4)

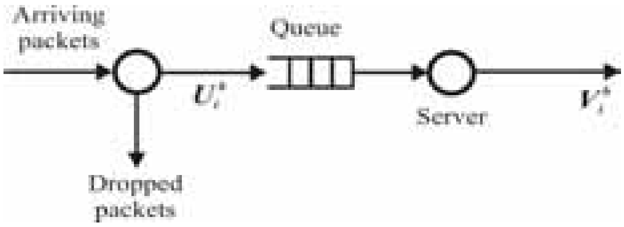

Figure 2. Network buffer

$$x_i^b(t) = u_i^b(t) - v_i^b(t) \tag{5}$$

Let us introduce the processing rate $r\left(x_i^b(t)\right)$. In the literature there are different forms of the function $r(\cdot)$. In this paper we will use $r(\cdot)$ as in [9]

$$r\left(x_i^b(t)\right) = \frac{x_i^b(t)}{\theta\left(x_i^b(t)\right)} \tag{6}$$

where $\theta(\cdot)$ is the average packet residence time. For the linear relationship between θ and the buffer level x_i^b there is

$$r\left(x_i^b(t)\right) = \frac{\mu x_i^b(t)}{a_i + x_i^b(t)} \tag{7}$$

where $a_i > 0$ is the parameter and μ is the service rate of the network server. From (5) and (7) the fluid model of the buffer is finally obtained

$$x_i^b(t) = u_i^b(t) - \frac{\mu x_i^b(t)}{a_i + x_i^b(t)} \tag{8}$$

Remark 2. For $a_i = 1$ in relation (8) there is a classic formula of the queuing theory for the M/M/1 system [11].

In our case the buffer coresponds to the *n*-dimensional vector. Therefore, eq. (8) gives

$$x^b(t) = \begin{bmatrix} x_1^b(t) \\ \vdots \\ \vdots \\ \vdots \\ x_n^b(t) \end{bmatrix} \ , \ u^b(t) = \begin{bmatrix} u_1^b(t) \\ \vdots \\ \vdots \\ \vdots \\ u_n^b(t) \end{bmatrix} \ ,$$

$$f\left(x^b(t)\right) = \begin{bmatrix} \dfrac{\mu x_1^b(t)}{a_1 + x_1^b(t)} \\ \vdots \\ \vdots \\ \vdots \\ \dfrac{\mu x_1^b(t)}{a_n + x_1^b(t)} \end{bmatrix} \tag{9}$$

From (8) and (9), the fluid model in the vector form is obtained

$$\dot{x}^b(t) = u^b(t) - f\left(x^b(t)\right) \tag{10}$$

For the nonlinear system in (10), feedback linaerization [10] is applied. The model is simple and it can be heuristicaly read

$$u^b(t) = f\left(x^b(t)\right) + h^b(t) \tag{11}$$

Eqs. (10) and (11) give the linear continuous system

$$\dot{x}^b(t) = h^b(t) \tag{12}$$

$$x_{k+1}^b = x_k^b + h_k^b \tag{13}$$

From Fig.1 it can be concluded that

$$h_k^b = x_k^1 \tag{14}$$

The last two relations give

$$x_{k+1}^b = x_k^b + x_k^1 \tag{15}$$

Models (1) and (15) can be presented as a common model.

$$x_{k+1}^2 = \left(A^2 + A_\Delta^2\right) x_k^2 + B^2 u_k \tag{16}$$

$$x_k^2 = \begin{bmatrix} x_k^1 \\ x_k^b \end{bmatrix},\ A^2 = \begin{bmatrix} A^1 & 0 \\ I & I \end{bmatrix},\ A_\Delta^2 = \begin{bmatrix} A_\Delta^1 & 0 \\ I & I \end{bmatrix}$$

$$B^2 = \begin{bmatrix} B^1 \\ 0 \end{bmatrix}$$

Between model (16) and the controller, there is the communication network. Now we will find the common model for model (16) and the communication network.

It is assumed that there are random but bounded delays from the sensor to the controller. Let us denote the finite delay bound with d. Using methodology from [6] we increase the state variable

$$x_k = \left[\left(x_k^2\right)^T \left(x_{k-1}^2\right)^T \ldots \left(x_{k-d}^2\right)^T\right]^T \tag{17}$$

where $x_k \in R^{2(d+1)n}$. As in [17] we can obtain a model which includes the process, the queue and the communication network (in the sense of stochastic time delay).

$$x_{k+1} = \left(A + A_\Delta\right) x_k + B u_k \tag{18}$$

$$y_k = C_{r(k)} x_k \tag{19}$$

$$A = \begin{bmatrix} A^2 & 0 & \cdots & 0 & 0 \\ I & 0 & & 0 & 0 \\ 0 & I & & \vdots & \\ \vdots & & \ddots & & \\ 0 & & & I & 0 \end{bmatrix},$$

$$A_\Delta = \begin{bmatrix} A_\Delta^2 & 0 & \cdots & 0 & 0 \\ I & 0 & & 0 & 0 \\ 0 & I & & \vdots & \\ \vdots & & \ddots & & \\ 0 & & & I & 0 \end{bmatrix},\quad B = \begin{bmatrix} B^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C_{r(k)} = \left[0 \cdots 0\ I\ 0 \cdots 0\right]$$

where $\{r(k)\}$ is a bounded random integer sequence with $0 \le r(k) \le d < \infty$ and $C_{r(k)}$ has all elements being zero except for $r(k)$-th block being an identity matrix. The system of (18) and (19) is the discrete-time jump linear system. These equations are in the form of an output feedback control problem.

The important problem is how to model the $r(k)$ sequence. One method is proposed in [3] where the $r(k)$ is modeled as a finite state Markov process with the transition probability

$$P = \left\{r(k+1) = j \mid r(k) = i\right\} = p_{ij} \tag{20}$$

where $0 \le i$, $j \le d$. The model is general and can include packet losses. The structured transition probability matrix is

$$P_s = \begin{bmatrix} p_{00} & p_{01} & 0 & 0 & \cdots & 0 \\ p_{10} & p_{11} & p_{12} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \\ & & & & & p_{d-1,d} \\ p_{d0} & p_{d1} & p_{d2} & p_{d1} & & p_{d,d} \end{bmatrix} \tag{21}$$

where

$$0 \le p_{ij} \le 1 \ \text{ and } \sum_{j>0}^{d} p_{ij} = 1 \tag{22}$$

Each row represented the transition probabilites from a fixed state to the whole state.

R*emark* 3. In [17] it is showed that the delay $r(k)$ can increase at most by 1 step each and can decrease as many steps as possible.

Remark 4. The diagonal elements of the probability matrix P_S are the probabilites of data coming in sequence with equals delays. The elements above diagonal are the probabilites of encoutering longer delays and the elements below the diagonal include packet losses. In [18], the stability conditions of networked control systems with both arbitrary and Markovian packet losses are established via a packet-loss dependent Lyapunov approach.

In this paper it is supposed that the transition probability matrix P_S is not precisely known, but belongs to the convex hull. For any set $\{H_1, ..., H_r\}$ the convex hull is defined as

$$conv\{H_1, ..., H_r\} = \\ = \left\{H : \sum_{l=1}^{r} \alpha_l H_l, \alpha_l > 0, \sum_{l=1}^{r} \alpha_l = 1\right\} \tag{23}$$

Constrained quadratic control

In this part of the paper we consider the system of (18) and (19) without unmodeled dynamics, i.e.

$$x_{k+1} = A x_k + B u_k \tag{24}$$

$$y_k = C_{r(k)} x_k \tag{25}$$

Also, we will consider a situation when the system is subject to constraints on the state (finite queue capacity) and control input (saturation). The mathematical description of constraints is

$$\|F_i x_k + G_i u_k\| \le \rho_i \quad \text{w.p.1} \tag{26}$$

$$k = 0,1,\ldots; \quad i = 1,\ldots,t$$

Remark 5. The theory of constrained systems is very important from the practical point of view [8]. It is usually true that higher performance levels are associated with pushing the limits. Therefore, there is astrong incentive for the system to operate on constraint boundaries. To be more specific, the target is to maximise performance whilst ensuring that the relevant constraints on both inputs (manipulated variables) and states (process variables) are not violated.

It is supposed that the transition probability of the Markov chain P_S is not exactly known, but belongs to $conv\{P_{S1}, P_{S2} ..., P_{Sk}\}$ The goal of the optimization problem is to find the upper bound for the next function

$$J = \sum_{k>0}^{\infty} E\left\{x_k^T C_{r(k)}^T C_{r(k)} x(k) + u_k^T D^T D u_k\right\} \tag{27}$$

where $E\{\cdot\}$ is mathematical expectation. Using the methodology from [4] the controller can be found

$$K_j = Y_j Q_j^{-1} \quad , \; j = 1,2,...,d \tag{28}$$

which is the solution of four LMI. Let us define

$$K = (K_1, K_2, ..., K_d) \text{ and } u_k = K_{r(k)} x_k \tag{29}$$

where $r(k) \in \{1,2,...,d\}$. Then

$$J(K) \le \delta \tag{30}$$

where δ is a bounded real number.

Robust stability of constrained quadratic control

In this part of the paper we will consider the stability of the following control system

$$x_{k+1} = (A + A_\Delta) x_k + B u_k \tag{31}$$

$$u_k = K_{r(k)} x_k \tag{32}$$

where $r(k) \in \{1,2,...,d\}$ and $K = (K_1, K_2, ..., K_d)$ as in the previous section.

The closed loop system is

$$x_{k+1} = \left(A + BK_{r(k)}\right) x_k + A_\Delta x_k = R_{r(k)} x_k + A_\Delta x_k \tag{33}$$

Let us introduce the indicator function

$$1_A(\omega) = \begin{cases} 1 & if \quad \omega \in A \\ 0 & other\ wise \end{cases} \tag{34}$$

for $\forall i \in d \in N$, $1_{\{r(k)=i\}}(\omega) = 1$ for $r(k) = i$ and 0 otherwise. Now we will formulate the formal results.

Lemma 1. Let us suppose that for the system of (31) and (32) the following is valid:

1. System without unmodeled dynamics is

$$x_{k+1} = R_{r(k)} x_k \;, \; R_{r(k)} = A + BK_{r(k)}$$

$$k \in \{1,...,d\}$$

2. Transition probabilites belong to the

$$conv\{P_{S1}, ..., P_{Sp}\}$$

3. $r(k)$ represents a Markov chain taking values in N and having the initial distribution.

$$v = \{v_i : i \in N\}$$

4. x_0 is a random variable
5. Bounded linear operator in a Banach space is given in the following form

$$T_j(U) = \sum_{i=1}^{d} p_{ij} R_i U_i R_i^T$$

where $U_i(k) = E\left\{x_k x_k^T 1_{\{r(k)=i\}}\right\}$

If the system without unmodeled dynamics is mean square stable, then

$$E\left\{\|x_k\|^2\right\} \le \beta \xi^k E\left\{\|x_0\|^2\right\}$$

where $\beta \ge 1$ and $0 < \xi < 1$

Lemma is, in essence, a collection of different results from [12] and [5].

Lemma 2. For the system with unmodeled dynamics, the following assumption is valid:

1. (31)-(32) is the system with unmodeled dynamic
2. x_0 is a random variable
3. $U_i(k) = E\left\{x_k x_k^T 1_{\{r(k)=i\}}\right\}$
4. $R_{r(k)} = A + BK_{r(k)}$ is stable for

$$r(k) \in \{1,2,...,d\}$$

5. The unmodeled dynamics has the bound $\|A_\Delta\| \le \gamma < \infty$

Then

$$\sum_{i=1}^{d} tr\left\{A_\Delta E\left\{x_k x_k^T\right\} A_\Delta^T 1_{\{r(k)=i\}}\right\} +$$
$$+2\sum_{i=1}^{d} tr\left\{R_i E\left\{x_k x_k^T\right\} A_\Delta\right\} \left\{1_{\{r(k)=i\}}\right\} \le$$
$$\le c\gamma d(\gamma + 2\alpha) E\left\{\|x_k\|^2\right\}$$

where $(c, d, \alpha, \gamma) \in (0, \infty)$

Proof: We have

$$G=\sum_{i=1}^{d} tr\left\{A_\Delta E\left\{x_k x_k^T\right\} A_\Delta^{+} 1_{\{r(k)=i\}}\right\}+$$
$$+2\sum_{i=1}^{d} tr\left\{R_i E\left\{x_k x_k^T\right\} A_\Delta\right\} A_\Delta \left\{1_{\{r(k)=i\}}\right\}=$$
$$=d\sum_{i=1}^{d}\left\|A_\Delta U_i(k) A_\Delta^T\right\|+2d\sum_{i=1}^{d}\left\|R_i U_i(k) A_\Delta\right\| \le$$
$$\le d\sum_{i=1}^{d}\left\|A_\Delta\right\| \left\|U_i(k)\right\| \left\|A_\Delta^T\right\|+2d\sum_{i=1}^{d}\left\|R_i\right\| \left\|U_i(k)\right\| \left\|A_\Delta\right\| \quad (35)$$

The matrix $R_i = A + BK_i$ is stable by the design of the optimal controller and

$$\left\|R_i\right\| \le \alpha < \infty \quad (36)$$

From (35), (36) and assumption 5° of **Lemma 2** it follows

$$G \le d\gamma^2 \sum_{i=1}^{d}\left\|U_i(k)\right\|+2d\gamma\sum_{i=1}^{d}\left\|U_i(k)\right\|=$$
$$=d\gamma(\gamma+2\alpha)\sum_{i=1}^{d}\left\|U_i(k)\right\|=\gamma d(\gamma+2\alpha)\left\|U(k)\right\|_1 \quad (37)$$

Owing to the fact that the system (31) and (32) is mean square stable, the constant $c\in(u,\infty)$ can be always found, such that

$$E\left\{\ \left\|U(k)\right\|_1\right\} \le cE\left\{\ \left\|U(0)\right\|_1\right\} \quad (38)$$

then

$$G \le c\gamma d(\gamma+2\alpha)E\left\{\ \left\|U(0)\right\|_1\right\} \quad (39)$$

On the other hand

$$E\left\{\ \left\|U(0)\right\|_1\right\}=E\left\{\sum_{i=1}^{d}\left\|U_i(u)\right\|\right\}=$$
$$E\left\{\sum_{i=1}^{d}\left\|Ex(0)x^T(0)1_{\{x(0)=i\}}\right\|\right\}$$
$$\le E\left\{\sum_{i=1}^{d}E\left\|x_0\right\|1_{\{r(0)=i\}}\right\}=E\left\{\left\{\ \left\|x_0\right\|^2\right\}\right\}=E\left\{\ \left\|x_0\right\|^2\right\} \quad (40)$$

Finally it is

$$G \le c\gamma d(\gamma+2\alpha)E\left\{\ \left\|x_0\right\|^2\right\} \quad (41)$$

The proof is completed

Now we will, in the form of a theorem, formulate the main result of the paper.

Theorem. The System of (31) and (32) is mean square stable if and only if

$$E\left\{\ \left\|x_0\right\|^2\right\} \le \beta\xi^k E\left\{\ \left\|x_0\right\|^2\right\}+c\gamma d(\gamma+2\alpha)E\left\{\ \left\|x_0\right\|^2\right\}$$

for $\beta \ge 1$, $0<\xi<1$ and $(c,\gamma,d,\alpha)\in(0,\infty)$

Proof: ***(If)*** Suppose that the system of (31) and (32) is robustly stable in the mean square sense. Relations (31) and (32) give

$$x_{k+1}x_{k+1}^T=\left(R_{r(k)}x_k+A_\Delta x_k\right)\left(R_{r(k)}x_k+A_\Delta x_k\right)^T=$$

$$\left(R_{r(k)}x_k+A_\Delta x_k\right)\left(x_k^T R_{r(k)}^T+x_k^T A_\Delta^T\right)=$$
$$R_{r(k)}x_k x_k^T R_{r(k)}^T+A_\Delta x_k x_k^T A_\Delta^T+2R_{r(k)}x_k x_k^T A_\Delta^T \quad (42)$$

Further it can be written

$$E\left\{\left\|x_{k+1}\right\|^2\right\}=E\left\{tr\left(x_{k+1}x_{k+1}^T\right)\right\}=$$
$$=\sum_{i=1}^{d} tr\left\{R_i E\left\{x_k x_k^T\right\} R_i^T 1_{\{r(k)=i\}}\right\}+$$
$$+\sum_{i=1}^{d} tr\left\{A_\Delta E\left\{x_k x_k^T\right\} A_\Delta^T 1_{\{r(k)=i\}}\right\}+$$
$$+2\sum_{i=1}^{d} tr\left\{E\left\{x_k x_k^T\right\} A_\Delta^T 1_{\{r(k)=i\}}\right\} \quad (43)$$

Using Lemma 1 and Lemma 2 we obtain

$$E\left\{\ \left\|x_0\right\|^2\right\} \le \beta\xi^k E\left\{\ \left\|x_0\right\|^2\right\}+c\gamma d(\gamma+2\alpha)E\left\{\ \left\|x_0\right\|^2\right\} \quad (44)$$

(Only if) If (44) is satisfied then

$$E\left\{\ \left\|x_0\right\|^2\right\} \to c\gamma d(\gamma+2\alpha)E\left\{\ \left\|x_0\right\|^2\right\} \text{ as } k\to\infty \quad (45)$$

Then the system under consideration is robustly mean square stable

Remark 6. When the model of system (31) is exact (absence of unmodeled dynamics, i.e. $\gamma=0$) from relation (45) it follows

$$E\left\{\ \left\|x_0\right\|^2\right\} \to 0 \text{ as } k\to\infty$$

and the system is mean square stable.

Conclusion

In this paper the stabilizing controllers for specially structured discrete-time jump linear systems with random but bounded delays in the feedback loop and buffers, as consequence of the scheduling, are considered. The main task of the paper is the investigation of robust stability for Markov jump linear systems. The buffer (queue) is modeled in the form of a fluid model. For exact linearization, the feedback linearization is used. It means that in the control loop there are two controllers. The first one performs feedback linearization of the buffer and the second one is the main feedback controller. The next interesting and more difficult problem is the investigation of a system which also has the communication network between the controller and the actuators (down link).

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Robusno upravljanje sistema preko komunikacionih mreža:
Pregled rezultata

U ovom radu je razmatran problem robusnog upravljanja linearnih dinemičkih sistema sa ograničenjima u prisustvu komunikacione računarske mreže sa redovima čekanja. Komunikaciona mreža je između procesa i regulatora. U radu se pretpostavlja da je red čekanja smešten u senzor. U ovakvim slučajevima se u sistemu upravljanja pojavljuje slučajno kašnjenje izazvano komunikacionom računarskom mrežom. Prisustvo kašnjenja degradira performanse sistema i dovodi u pitanje njegovu stabilnost. Digitalni sistem sa slučajnim ali ograničenim kašnjenjem može biti modelovan kao konačno dimenzionalan diskretni linearni sistem sa skokovima, pri čemu su tranzicioni skokovi modelovani kao Markovljevi lanci sa konačnim stanjima. Red čekanja se modeluje kao nelinearan sistem na koji se primenjuje metodologija linearizacije sa povratnom spregom. Onda se kompletan sistem (proces, računarska komunikaciona mreža, red čekanja) može predstaviti kao diskretan sistem sa skokovima. Za takav sistem, bez nemodelovane dinamike, predložen je kvadratni regulator sa ograničenjima. Posle toga je izvršena analiza takvog sistema u prisustvu nemodelovane dinamike. Takva vrsta sistema ima široku primenu u upravljanju vojnih i industrijskih sistema.

Ključne reči: **komunikacioni sistem, komunikaciona mreža, upravljanje mrežom, robustno upravljanje, dinamički sistem, linearni sistem, slučajno kašnjenje.**

Живучее управление систем через коммуникационные системы: Обзор и анализ результатов

В настоящей работе рассматривана проблема живучего управления линейных динамических систем с ограничением в наличии коммуникационной вычислительной сети с очерёдностями ожидания. Коммуникационная сеть осуществляется между процессом и контроллером. В работе предполагается, что очерёдность ожидания находится в чувствительном элементе. В таких случаях в системе управления появляется случайная временная задержка, вызвана коммуникационной вычислительной сетью. Наличие временной задержки приводит к устареванию характеристик системы и ставит под угрозу его устойчивость. Цифровая система со случайной но ограниченной временной задержкой может быть моделирована в виде дискретной линейной системы с прыжками конечной размерности, при чём переходные прыжки моделированы как цепи Маркова с конечными состояниями. Очерёдность ожидания моделируется как нелинейная система, на которую применяется методология линеаризации с обратной связью. Тогда полную систему (процесс, вычислительная коммуникационная сеть, очерёдность ожидания) возможно представить как дискретную систему с прыжками. Для такой системы, без немоделированной динамики, предложен квадратический чувствительный элемент с ограничениями. После этого сделан анализ такой системы при наличии немоделированной динамики. Такой вид системы широко применяется в управлении военных и промышленных систем.

Ключевые слова: коммуникационная система, коммуникационная сеть, управление сетью, живучее управление, динамическая система, линейная система, случайная временная задержка.

Commandes robustes des systèmes par les systèmes des communications: tableau des résultats

Ce papier considère le problème des commandes robustes des systèmes linéaires dynamiques avec limitations et dans la présence du réseau de communications aux filtres d'attente. Le réseau de communications est entre le procès et le régulateur. Dans ce travail on suppose que la file d'attente est située dans le sensor. Dans des cas pareils chez le système des commandes apparaît un délai aléatoire causé par le réseau de communications informa- tique. La présence du délai dégrade les performances du système et met en question sa stabilité. Le système digital avec délai aléatoire mais limité peut être modelé comme le système discret linéaire fini et dimensionel avec sauts, mes sauts de transitions étant modelé comme les chaînes de Markov avec les états finis. La file d'attente est modelée comme le système non-linéaire où l'on applique la métodologie de linéarisation avec réactions. Le système complet (procès, réseau de communication informatique, file d'attente) peut alors être présenté comme le système discret avec sauts. Pour ce système, sans dynamique non-modelée, on a proposé un régulateur carré avec contraintes. Ensuite, on a analysé ce système en présence de la dynamique non-modelée. Ce type de système trouve une vaste application chez les commandes des systèmes militaires et industriels.

Mots clés: **système de communications, réseau de communications, commande de réseau, commande robuste, système dynamique, système linéaire, délai aléatoire.**