

Contribution to analysis of stress state at points around elliptic opening in an in-plane stressed orthotropic plate

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The paper discusses modified expressions for the calculation of additional stress tensor components at points around elliptic openings in the case of equal or closes values of complex parameters of the orthotropic plate because the procedure given in Ref. [1] can not be applied for these cases.

For e.g., this is the case of the frequently used quasiisotropic stacking sequence of the multilayer plate.

In that way, the procedure of calculating the stress state around elliptic openings in an in-plane stressed plate is enabled for any orthotropic stacking sequence.

Key words: orthotropic plate, plane stress state, elliptic opening, stress state, plain state

Introduction

AN opening in the stressed plate is a source of the stress concentration and it causes a local stress change at points around the opening, unlike the stresses at the same points of a plate without an opening, which is stressed in the same way.

In case of an in-plane stressed orthotropic plate without an opening, stress tensor components have constant values in all points of the plate. This stress state in the opening – free plate is defined as the basic stress state $\{\sigma^\circ\}$.

If an opening of any shape and dimensions is made in the plate, it causes additional stress tensor components $\{\sigma^\circ\}$ at the points near the opening contour. These components have the highest values at the opening contour, but they decrease rapidly with the increase of distance from the opening contour.

So, the total plane stress state of the loaded plate with the opening can be defined by the stress components

$$\begin{aligned}\sigma_x &= \sigma_x^\circ + \sigma_x^* \\ \sigma_y &= \sigma_y^\circ + \sigma_y^* \\ \tau_{xy} &= \tau_{xy}^\circ + \tau_{xy}^*\end{aligned}\quad (1)$$

The ratio of the maximum stress at the opening contour and the maximum normal stress that occurs at the points far from the opening represents the stress concentration coefficient / factor.

Stress concentration can be considerably more expressed at the points of stressed plates made of orthotropic materials than in case of plates made of isotropic materials, depending on the relation between the elastic constants of the orthotropic material.

For e.g., for a plate with a circular opening, which is subjected to extension, according to Ref. [2], p. 290, the stress concentration factor is 3 for isotropic material, 4 for

unidirectional glass/epoxy, and about 6 for boron/epoxy, while for graphite/epoxy it goes up to 9.

Such high stress concentration factors influence the reduction of the static strength and fatigue resistance of the structure in question, due to the rapid propagation of the initial damage, which occurs in the zone of the stress concentration source in the form of crack initiation and propagation.

Complex variable methods are most frequently used for the analysis of the stress state around the opening. The basic principles of these methods can be found, for e.g., in [3] (p. 424), [4] and [5].

The method of solving the problem of stress distribution in a thin isotropic plate which is weakened by any type of opening, by applying conformal mapping, was developed by N. I. Muskhelishvili [6]. Muskhelishvili's method was adapted by G. N. Savin [7] for anisotropic materials. A great contribution in this field was given by S. G. Lehnicki [8].

The aforementioned method can be applied on multilayer plates that, under certain conditions (see Ref. [1] and [2], pp.162–172), on macroscopic level, can be analyzed as orthotropic.

There are several authors renowned worldwide who study the stress state around elliptic openings in multilayer orthotropic plates. The most important among them, in the author's opinion, are J. M. Whitney, R. J. Nuismer, S. C. Tan and T. K. Tung. Their most significant papers are cited in [9].

Basic stress state for the stressed plate with an opening

An infinite thin orthotropic plate with an elliptic opening, subjected to shear and extension in two mutually perpendicular directions (Fig.1), which satisfies the conditions of plane stress state is being considered,

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assuming that:

1. The plate is a thin material surface and there is no shear between the layers of the plate.
2. The contour of the elliptic opening is free from external stresses, which is the case for almost all constructive, technological and maintenance openings, except those for the fasteners, and
3. External loads do not cause instability of the plate, neither global nor local at the opening, i.e. buckling of the plate and out-of-plane of points of the plate middle surface do not occur.

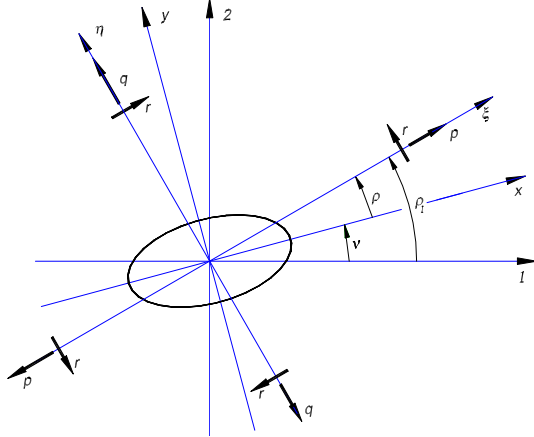


Figure 1. Loading scheme of the plate with elliptic opening

In relation to the principal material direction of orthotropy 1, the larger semi-axis x of the elliptical opening makes an angle ν and the perpendicular directions (ξ and η) of the extension make the angles ρ_1 and $(\rho_1 + 90^\circ)$.

Load components are defined as unit forces through the plate thickness

$$p = \frac{N_x}{T}; \quad q = \frac{N_y}{T}; \quad r = \frac{N_{xy}}{T} = \frac{N_{yx}}{T} \quad (2)$$

where N_x and N_y are extension forces and N_{xy} is shear force per unit length of the outer plate edge.

For a plate without an opening, which is loaded according to Fig.1, stress tensor components have constant values in all points of the plate. This stress state in the opening, free plate, is defined as the basic stress state $\{\sigma^o\}$.

For the coordinate system $x - y$, the basic stress state is obtained by using the transformation equations for expressing stresses in an $x - y$ coordinate system in terms of stresses in a $\xi - \eta$ coordinate system (according to Ref. [10], p.51)

$$\begin{aligned} \sigma_x^o &= p \cos^2 \rho + q \sin^2 \rho - r \sin 2\rho \\ \sigma_y^o &= p \sin^2 \rho + q \cos^2 \rho + r \sin 2\rho \\ \tau_{xy}^o &= \frac{1}{2}(p - q) \sin 2\rho + r \cos 2\rho \end{aligned} \quad (\rho = \rho_1 - \nu) \quad (3)$$

General expressions for the additional stress tensor components for the case when the opening contour is free from external stress

The expressions for the additional components of stress tensor at the points at the contour of the elliptic opening free from external stresses is given by S. C. Tan (see Ref.

[11] and [9], p.29) in the following form

$$\begin{aligned} \sigma_x^* &= \text{Re} \left\{ \frac{1}{\mu_1 - \mu_2} [\mu_1^2 h_1 g_2 - \mu_2^2 h_2 g_1] \right\} \\ \sigma_y^* &= \text{Re} \left\{ \frac{1}{\mu_1 - \mu_2} [h_1 g_2 - h_2 g_1] \right\} \\ \tau_{xy}^* &= -\text{Re} \left\{ \frac{1}{\mu_1 - \mu_2} [\mu_1 h_1 g_2 - \mu_2 h_2 g_1] \right\} \end{aligned} \quad (4)$$

where Re denotes real parts of the expressions in brackets and where the following designations are introduced

$$\left. \begin{aligned} h_j &= \frac{1 - i\mu_j k}{\delta_j \sqrt{\delta_j^2 - 1 - \mu_j^2 k^2} + \delta_j^2 - 1 - \mu_j^2 k^2} \\ \delta_j &= \cos \theta + \mu_j k \sin \theta \\ g_j &= ik\sigma_x^o - \mu_j \sigma_y^o - (1 - i\mu_j k)\tau_{xy}^o \end{aligned} \right\} (j = 1, 2) \quad (5)$$

In expressions (4) and (5), the following designations are used:

- μ_1 and μ_2 are complex parameters that depend on the elastic constants of the plate material (see Ref. [1])
- θ is the angle parameter of the ellipse (see Fig.2)
- $k = \frac{b}{a}$ is the parameter of the opening shape, i.e. axis ratio of the ellipse and
- $i = \sqrt{-1}$ is an imaginary unit.

Based on expression (1), the components of the total tensor of the stress state at the points along the contour of the opening are obtained.

The curvilinear rectangular coordinate system (Fig.2) is often used for the analysis of the stress state. The axes are set in the directions of the tangent and the normal at the points of the opening contour.

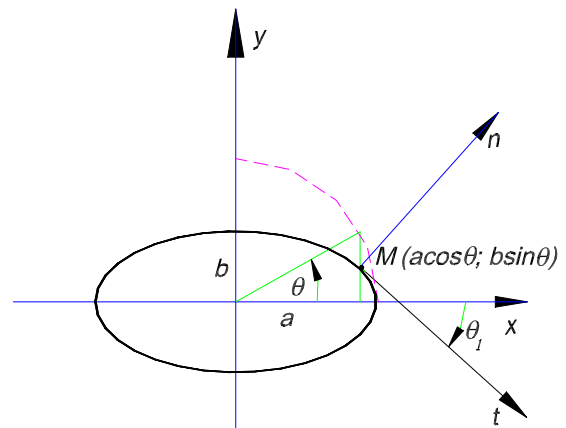


Figure 2. Curvilinear rectangular coordinate system for the analysis of the stress state along the opening contour

For the new coordinate system, the components of the stress state are obtained by transforming the coordinates

$$\begin{aligned} \sigma_t &= \sigma_x \cos^2 \theta_1 + \sigma_y \sin^2 \theta_1 + \tau_{xy} \sin 2\theta_1 \\ \sigma_n &= \sigma_x \sin^2 \theta_1 + \sigma_y \cos^2 \theta_1 - \tau_{xy} \sin 2\theta_1 \\ \tau_{tn} &= \frac{1}{2}(\sigma_y - \sigma_x) \sin 2\theta_1 + \tau_{xy} \cos 2\theta_1 \end{aligned} \quad (6)$$

where θ_1 is the angle between the tangent of the ellipse, at the given point and the positive direction of $x -$ axis. From

the analytic geometry, the following relation between the angle θ_1 and the angle parameter of ellipse θ is obtained

$$\operatorname{tg} \theta_1 = -k \cdot \operatorname{ctg} \theta \quad (7)$$

For the opening boundary free from external stresses, the stress state on the opening edge is reduced to the state in which the component σ_t differs from zero, while the components σ_n and τ_{tn} equal zero.

Basic limiting forms for the case of equal or close values of complex parameters

Expressions (4) become undefined for the equal value of complex parameters, so they can not be applied in this case. In practice, however, machine calculation based on expressions (4) can result in enormous errors even in cases approaching infinity, i.e. when $\mu_1 \approx \mu_2$. These situations occur for isotropic materials ($\mu_1 = \mu_2 = i$) as well as for some orthotropic materials. For e.g., in case of quasiisotropic stacking sequence of the multilayer plate $\mu_1 \approx \mu_2 \approx i$. In such cases, the limiting forms are used for modification of the expressions (4), so accurate results (with usual machine precision) are obtained.

Consider complex parameters $\mu_2 = \mu$ and $\mu_1 = \mu_2 + \Delta$, with $\Delta \rightarrow 0$. In this case, the following limiting forms (see Ref. [12]) exist in the expressions (4)

$$\begin{aligned} \lim_{\substack{\Delta \rightarrow 0 \\ (\mu_1 \rightarrow \mu_2)}} \frac{f(\mu_1) - f(\mu_2)}{\mu_1 - \mu_2} &= \frac{df(\mu)}{d\mu} = f'(\mu) \\ \lim_{\substack{\Delta \rightarrow 0 \\ (\mu_1 \rightarrow \mu_2)}} \frac{\mu_1 f(\mu_1) - \mu_2 f(\mu_2)}{\mu_1 - \mu_2} &= (\mu f)' = \mu f'(\mu) + f(\mu) \quad (8) \\ \lim_{\substack{\Delta \rightarrow 0 \\ (\mu_1 \rightarrow \mu_2)}} \frac{\mu_2 f(\mu_1) - \mu_1 f(\mu_2)}{\mu_1 - \mu_2} &= \mu f'(\mu) - f(\mu) \end{aligned}$$

Calculation by using expressions (8) gives accurate results for equal values of complex parameters and parameters that have close values should be brought to their average value, i.e.

$$\mu = \frac{\mu_1 + \mu_2}{2} \quad (9)$$

Complex parameters are, in general case, obtained in the form

$$\mu_1 = \alpha_1 + \beta_1 i; \quad \mu_2 = \alpha_2 + \beta_2 i \quad (10)$$

where $\alpha_1, \alpha_2, \beta_1$ and β_2 are real numbers.

Author of this paper defines criterion for the closeness of complex parameters by applying the difference of their real and imaginary parts. On the basis of this criterion, it is assumed that complex parameters (10) are close if the following conditions are satisfied

$$|\alpha_1 - \alpha_2| \leq 0.001 \quad \wedge \quad |\beta_1 - \beta_2| \leq 0.001 \quad (11)$$

Modification of general expressions by using limiting forms

In all expressions (4), the following terms exist

$$A(\mu_1, \mu_2) = h_1 g_2 \quad B(\mu_1, \mu_2) = h_2 g_1 \quad (12)$$

By introducing the designations (5), these terms can be

written in the following form

$$\begin{aligned} A(\mu_1, \mu_2) &= \varphi_1(\mu_1) - \mu_2 \varphi_2(\mu_1) \\ B(\mu_1, \mu_2) &= \varphi_1(\mu_2) - \mu_1 \varphi_2(\mu_2) \end{aligned} \quad (13)$$

When expressions (13) are put in (4) and limiting forms (8) are used, the expressions for calculation of additional stress tensor components, for equal or close values of complex parameters, can be written in the following form

$$\begin{aligned} \sigma_x^* &= \operatorname{Re} \left[\varphi_1'(\mu) - \mu \varphi_2'(\mu) + \varphi_2(\mu) \right] \\ \sigma_y^* &= \operatorname{Re} \left[\varphi_1'(\mu) - \mu \varphi_2'(\mu) + \varphi_2(\mu) \right] \\ \tau_{xy}^* &= -\operatorname{Re} \left[\varphi_1'(\mu) - \mu \varphi_2'(\mu) + \varphi_2(\mu) \right] \end{aligned} \quad (14)$$

where the mean value of complex parameters is given by (9) and derivatives of corresponding functions with respect to μ are denoted by '.

The functions presented in expressions (14) can be written in the form

$$\begin{aligned} \varphi_1(\mu) &= \frac{u(\mu)}{v(\mu)} & \varphi_2(\mu) &= \frac{w(\mu)}{v(\mu)} \\ \bar{\varphi}_1(\mu) &= \mu^2 \varphi_1(\mu) = \frac{\bar{u}(\mu)}{v(\mu)} & \bar{\varphi}_2(\mu) &= \mu^2 \varphi_2(\mu) = \frac{w(\mu)}{v(\mu)} \\ \varphi_1(\mu) &= \mu \varphi_1(\mu) = \frac{\hat{u}(\mu)}{v(\mu)} & \varphi_2(\mu) &= \mu \varphi_2(\mu) = \frac{w(\mu)}{v(\mu)} \end{aligned} \quad (15)$$

By introducing the above defined functions, as well as their derivatives with respect to μ , in (14), the following expressions are obtained

$$\begin{aligned} \sigma_x^* &= \operatorname{Re} \left\{ \frac{1}{v^2} \left[\bar{u}' \cdot v - v' \cdot \bar{u} - \mu (\bar{w}' \cdot v - v' \cdot \bar{w}) + v \cdot \bar{w} \right] \right\} \\ \sigma_y^* &= \operatorname{Re} \left\{ \frac{1}{v^2} \left[u' \cdot v - v' \cdot u - \mu (w' \cdot v - v' \cdot w) + v \cdot w \right] \right\} \\ \tau_{xy}^* &= -\operatorname{Re} \left\{ \frac{1}{v^2} \left[\hat{u}' \cdot v - v' \cdot \hat{u} - \mu (w' \cdot v - v' \cdot w) + v \cdot w \right] \right\} \end{aligned} \quad (16)$$

where all the variables in brackets are, generally speaking, complex. The derivatives of the corresponding variables with respect to μ are denoted by '.

The variables μ, v and v' exist in all expressions (16). The mean value of complex parameters can be written in the form

$$\mu = \alpha + \beta i \quad (17)$$

where α and β are its real and imaginary part, respectively.

The value v is defined by the expression

$$v = \delta \sqrt{\delta^2 - 1 - \mu^2 k^2} + \delta^2 - 1 - \mu^2 k^2 \quad (18)$$

and its derivative with respect to parameter μ is then defined by the expression

$$\begin{aligned} v' &= \frac{k \delta (\delta \sin \theta - \mu)}{\sqrt{\delta^2 - 1 - \mu^2 k^2}} + \\ &+ k \left(\sqrt{\delta^2 - 1 - \mu^2 k^2} \sin \theta + 2 \delta \sin \theta - 2 \mu k \right) \end{aligned} \quad (19)$$

From expressions (18) and (19), it can be seen that values v and v' depend on the calculation point, i.e. that they are functions of the angle parameter of the ellipse θ .

The remaining values in the expressions (16) can be presented in the following form

$$\begin{aligned}
u_{\text{Re}} &= \alpha k^2 \sigma_x^\circ - z_2 \tau_{xy}^\circ; & u_{\text{Im}} &= k(z_2 \sigma_x^\circ + \alpha \tau_{xy}^\circ) \\
u'_{\text{Re}} &= k^2 \sigma_x^\circ; & u'_{\text{Im}} &= k \tau_{xy}^\circ \\
\bar{u}_{\text{Re}} &= z_6 u_{\text{Re}} - 2\alpha \beta u_{\text{Im}}; & \bar{u}_{\text{Im}} &= z_6 u_{\text{Im}} + 2\alpha \beta u_{\text{Re}} \\
\bar{u}'_{\text{Re}} &= kz_7 \sigma_x^\circ - z_8 \tau_{xy}^\circ; & \bar{u}'_{\text{Im}} &= kz_8 \sigma_x^\circ + z_7 \tau_{xy}^\circ \\
\hat{u}_{\text{Re}} &= \alpha u_{\text{Re}} - \beta u_{\text{Im}}; & \hat{u}_{\text{Im}} &= \alpha u_{\text{Im}} + \beta u_{\text{Re}} \\
\hat{u}'_{\text{Re}} &= 2\alpha k^2 \sigma_x^\circ - z_5 \tau_{xy}^\circ; & \hat{u}'_{\text{Im}} &= k(z_5 \sigma_x^\circ + 2\alpha \tau_{xy}^\circ)
\end{aligned} \quad (20)$$

$$\begin{aligned}
w_{\text{Re}} &= z_2 \sigma_y^\circ - \alpha k^2 \tau_{xy}^\circ; & w_{\text{Im}} &= -k(z_2 \tau_{xy}^\circ + \alpha \sigma_y^\circ) \\
w'_{\text{Re}} &= -k^2 \tau_{xy}^\circ; & w'_{\text{Im}} &= -k \sigma_y^\circ \\
\bar{w}_{\text{Re}} &= z_6 w_{\text{Re}} - 2\alpha \beta w_{\text{Im}}; & \bar{w}_{\text{Im}} &= z_6 w_{\text{Im}} + 2\alpha \beta w_{\text{Re}} \\
\bar{w}'_{\text{Re}} &= z_8 \sigma_y^\circ - kz_7 \tau_{xy}^\circ; & \bar{w}'_{\text{Im}} &= -kz_8 \tau_{xy}^\circ - z_7 \sigma_y^\circ \\
w_{\text{Re}} &= \alpha w_{\text{Re}} - \beta w_{\text{Im}}; & w_{\text{Im}} &= \alpha w_{\text{Im}} + \beta w_{\text{Re}} \\
w'_{\text{Re}} &= z_5 \sigma_y^\circ - 2\alpha k^2 \tau_{xy}^\circ; & w'_{\text{Im}} &= -k(z_5 \tau_{xy}^\circ + 2\alpha \sigma_y^\circ)
\end{aligned} \quad (21)$$

where real and imaginary part of the corresponding value are denoted by the subscripts Re and Im, respectively.

The following designations are introduced into expressions (20) and (21)

$$\begin{aligned}
z_1 &= k\beta; & z_2 &= 1 + z_1; & z_3 &= 2 + 3z_1; & z_4 &= z_3 - 1 \\
z_5 &= z_1 + z_2; & z_6 &= \alpha^2 - \beta^2; & z_7 &= 3k\alpha^2 - z_3\beta; & z_8 &= 2\alpha z_4
\end{aligned} \quad (22)$$

From expressions (20) – (22), it can be seen that the defined values are constant for a certain problem, i.e. for the given material, shape and position of the opening and basic stress state, so these values do not depend on the calculation point at the contour of the opening, i.e. on the angle parameter θ .

Expressions (16) – (22) enable calculating additional stress tensor components at points at the contour of the opening for equal or close values of complex parameters of the orthotropic plate. This means that they can also be used for the isotropic plate. Therefore, expressions (32) in Ref. [1] have not been used further.

Total stress state at the contour of the elliptic opening is defined by the stress components (1) and this stress state is critical for the plate strength estimation.

Numerical examples

The practical application of the model of the stress analysis of the stressed plate with the elliptic opening was illustrated in Ref. [1] by four practical examples of comparative analysis of the stress state at the discrete points along the contour of the elliptic opening for orthotropic plates and isotropic material.

The examples' results were obtained by using the author's own FORTRAN PC program which is intended for the stress analysis of the stressed plate with the elliptic opening and in which all the necessary equations and assumptions were incorporated.

But, at this moment it was not possible to calculate the stacking sequences which have close or equal values of the complex parameters as it is the case with quasiisotropic stacking sequence.

Then, expressions (13)–(22) are incorporated in the aforementioned FORTRAN PC program so it is applicable for any stacking sequence of the multilayer orthotropic plate as well as isotropic one.

For the given shape (parameter $k = \frac{b}{a}$) and the position

of the opening (angle ν), the calculation is made for the general case of plate loading, which is given by the ρ_1 angle and ratio between loading components $p : q : r = 1 : a_1 : a_2$, or, in particular cases, for pure shear loading ($p = q = 0 \wedge r \neq 0$).

According to the described model, the reduced value of the normal stress, $\bar{\sigma}_t$, for the relative stress state at the points along the opening contour in the direction of the tangent line to the opening contour in each calculation point, is obtained as the result of the calculation, where

$$\bar{\sigma}_t = \frac{\sigma_t}{p} = f_1(\theta) \vee \bar{\sigma}_t = \frac{\sigma_t}{r} = f_2(\theta) \quad (23)$$

depending on whether it is a general case of load or a special case of pure shear.

In the last version of the FORTRAN PC program, the calculation step is not fixed $\Delta\theta = 1^\circ$ (see Ref. [1]), but it can be chosen depending on the required calculation accuracy, for $0^\circ \leq \theta \leq 180^\circ$.

Based on the results obtained for each stacking sequence, extreme values of undimensional coefficients are determined, which are maximum and minimum values of the reduced value of the normal stress for the relative stress state.

$$\begin{aligned}
K_1 &= (\bar{\sigma}_t)_{\text{max}} \quad \text{for } \bar{\sigma}_t \geq 0 \\
K_2 &= (\bar{\sigma}_t)_{\text{min}} \quad \text{for } \bar{\sigma}_t < 0
\end{aligned} \quad (24)$$

In a certain way, these coefficients are a measure of sensitivity of the material to the presence of an opening of the given shape and position and for the given external load.

As an illustration of this sensitivity, three symmetrical stacking sequences of the composite plate, each with 8 layers satisfying the conditions of orthotropy, are analyzed:

– stacking sequence A $[+45^\circ / -45^\circ / +45^\circ / -45^\circ]_S$

– stacking sequence B $[0^\circ / 90^\circ / 0^\circ / 90^\circ]_S$ and

– quasiisotropic stacking sequence $[+45^\circ / -45^\circ / 90^\circ / 0^\circ]_S$

First two were analyzed in Ref. [1], but the third one could not be, for aforementioned reasons.

The material is graphite/epoxy T300/5208 [12] with the following elastic constants

$$E_1 = 181000 \frac{N}{mm^2} \quad E_2 = 10300 \frac{N}{mm^2}$$

$$G_{12} = 7170 \frac{N}{mm^2} \quad \nu_{12} = 0.28$$

and layer thickness 0.125 mm.

Example 1. Load is defined by the ratio between components $p : q : r = 1 : 1.2 : 0.5$ and by the angle $\rho_1 = 20^\circ$. The position of the opening is defined by the angle $\nu = 10^\circ$. The shape parameter of the elliptical opening is $k = 0.5$. The calculation step is $\Delta\theta = 0.5^\circ$.

Changes in the reduced values of the normal stress $\bar{\sigma}_t$ along the contour of the opening for all stacking sequences are shown by diagrams in Fig.3.

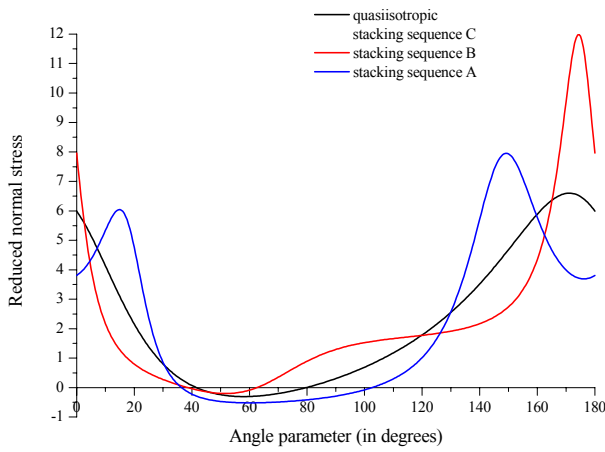


Figure 3. Changes in the values of $\bar{\sigma}_i$ at the points on the opening contour (for Example 1)

Calculation based on (24) shows that $K_1=11.99476$ (for $\theta=174.5^\circ$) and $K_2=-0.20277$ (for $\theta=52^\circ$) for the stacking sequence B and $K_1=7.95156$ (for $\theta=149^\circ$) and $K_2=-0.5185$ (for $\theta=60^\circ$) for the stacking sequence A, while $K_1=6.60019$ (for $\theta=171^\circ$) and $K_2=-0.30541$ (for $\theta=58^\circ$) for the quasiisotropic stacking sequence C.

It can be seen that stacking sequences A and B are more sensitive to this source of the stress concentration than quasiisotropic stacking sequence for the given loading conditions: stacking sequence A about 20% and stacking sequence B as high as 82%. The stacking sequence B is about 51% more sensitive than the stacking sequence A.

Example 2. Load is pure shear. Direction of the load is defined by the angle $\rho_1=30^\circ$. The position of the opening is defined by the angle $\nu=40^\circ$. The shape parameter of the elliptical opening is $k=0.5$. The calculation step is $\Delta\theta=0.5^\circ$.

Changes in the reduced values of the normal stress $\bar{\sigma}_i$ along the contour of the opening for all stacking sequences are shown by diagrams in Fig.4.

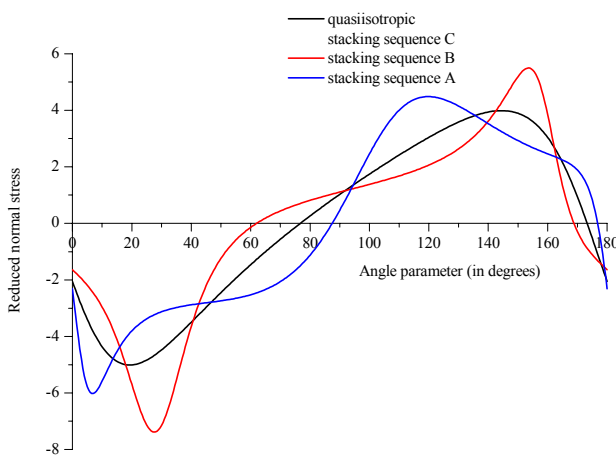


Figure 4. Changes in the values of $\bar{\sigma}_i$ at the points on the opening contour (for Example 2)

Calculation based on (24) shows that $K_1=4.48354$ (for $\theta=120^\circ$) and $K_2=-6.01965$ (for $\theta=7^\circ$) for the stacking sequence A and $K_1=5.49655$ (for $\theta=153.5^\circ$) and $K_2=-7.38643$ (for $\theta=27.5^\circ$) for the stacking sequence B, while $K_1=3.98697$ (for $\theta=144.5^\circ$) and $K_2=-5.01277$ (for $\theta=19.5^\circ$) for the

quasiisotropic stacking sequence C.

So, stacking sequences A and B are more sensitive to this source of the stress concentration than quasiisotropic stacking sequence for the given loading conditions: stacking sequence A about 12% (K_1) and 20% (K_2), while stacking sequence B about 38% (K_1) and 47% (K_2). The stacking sequence B is about 23% more sensitive than the stacking sequence A.

As it was explained in Ref. [1], the described procedure can also be applied to determine the best, as well as the worst, orientation of the opening for the given loading conditions.

With the defined value of the calculation step $\Delta\theta$, the aforementioned FORTRAN PC program makes the calculation of the coefficients K_1 and K_2 for each value from 0° to 180° of the angle ν with the chosen step $\Delta\nu$. The following relations are obtained

$$K_1 = f_1(\nu) \wedge K_2 = f_2(\nu) \Rightarrow K(\nu) = \max\{|K_1|, |K_2|\} \quad (25)$$

The best orientation of the opening is defined by the angle ν for which the minimum value of coefficient $K(\nu)$ is obtained. The worst orientation of the opening is defined by the angle ν for which the maximum values of $K(\nu)$ is obtained.

Relation (25) is illustrated on the aforementioned stacking sequences for two loading cases.

Example 3. Load is defined by the ratio between components $p:q:r=1:1.20:0.5$ and by the angle $\rho_1=20^\circ$. The shape parameter of the opening is $k=0.5$. The calculation steps are $\Delta\theta=0.5^\circ$ and $\Delta\nu=1^\circ$.

The dependence of $K(\nu)$ on the angular orientation of the opening regarding the principal directions of orthotropy for all stacking sequences is given in Fig.5.

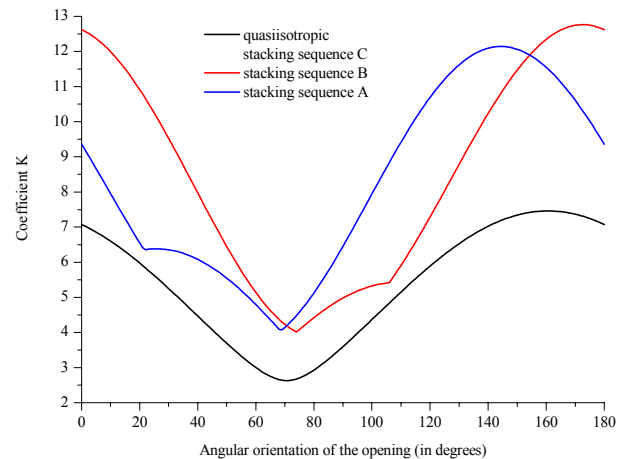


Figure 5. Dependence of the coefficient $K(\nu)$ on the angle of orientation of the opening (for Example 3)

The calculation shows that, for the stacking sequence A, the best orientation of the opening is defined by the angle $\nu=69^\circ$ ($K_{\min}=4.07049$) and the worst by the angle $\nu=144^\circ$ ($K_{\max}=12.14243$).

For the stacking sequence B, the best orientation of the opening is defined by the angle $\nu=74^\circ$ ($K_{\min}=4.01612$) and the worst by the angle $\nu=173^\circ$ ($K_{\max}=12.76451$).

For the quasiisotropic stacking sequence C, the best orientation of the opening is defined by the angle $\nu=71^\circ$ ($K_{\min}=2.63019$) and the worst by the angle $\nu=161^\circ$ ($K_{\max}=7.49919$).

Example 4. Load is pure shear. Direction of the load is defined by the angle $\rho_1 = 30^\circ$. The shape parameter of the opening is $k = 0.5$. The calculation steps are $\Delta\theta = 0.5^\circ$ and $\Delta\nu = 1^\circ$.

The dependence of $K(\nu)$ for all stacking sequences on the angular orientation of the opening regarding the principal directions of orthotropy is given in Fig.6.

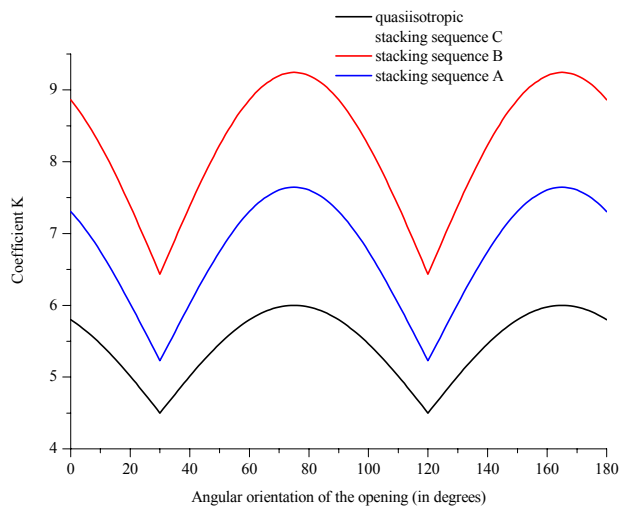


Figure 6. Dependence of the coefficient $K(\nu)$ on the angle of orientation of the opening (for Example 4)

The calculation shows that, for all stacking sequences, the best orientation of the opening is defined by the angles $\nu = 30^\circ$ and 120° ($K_{\min} = 5.22984$ for the stacking sequence A, $K_{\min} = 6.43361$ for the stacking sequence B and $K_{\min} = 4.49998$ for the quasiisotropic stacking sequence C).

For all stacking sequences, the worst orientation of the opening is defined by the angles $\nu = 75^\circ$ and 165° ($K_{\max} = 7.64511$ for the stacking sequence A, $K_{\max} = 9.24428$ for the stacking sequence B and $K_{\max} = 6$ for the quasiisotropic stacking sequence C).

Conclusion

The procedure (presented in Ref. [1]) of the calculation of additional stress tensor components around an elliptical opening in orthotropic plate can not be applied in the case of equal or close values of complex parameters of the orthotropic plate.

In case of equal values of complex parameters, expressions (4) become undefined and cannot be applied. In

practice, however, machine calculation based on expressions (4) can result in enormous errors even for cases approaching infinity, i.e. when $\mu_1 \approx \mu_2$.

That is, for e.g., the situation for quasiisotropic stacking sequence of multilayer plate $\left([45^\circ / -45^\circ / 90^\circ / 0^\circ]_S\right)$ for which $\mu_1 \approx \mu_2 \approx i$. This stacking sequence is used very often in practice and it is used as a reference stacking sequence for the choice of the optimum stacking sequence for the defined load conditions [9].

For these special cases, the author of this paper has used the procedure proposed in [12] and derived expressions (13) – (22) in the presented form. These expressions are incorporated in FORTRAN PC program [1, 9] and, in this way, calculation of the stress state on the contour of the elliptical opening for all orthotropic stacking sequences of the multilayer plate is enabled.

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Prilog proračunu stanja napona u tačkama oko eliptičnog otvora ravanski napregnute ortotropne ploče

U radu su dati modifikovani izrazi za proračun komponenti priraštaja tenzora stanja napona u tačkama na konturi eliptičnog otvora za slučaj jednakih ili bliskih vrednosti kompleksnih parametara ortotropne ploče. To je, na primer, slučaj kod kvaziizotropne šeme slaganja koja se vrlo često koristi u praktičnim konstrukcijama. Time bi bio upotpunjen postupak proračuna naponskog stanja oko eliptičnog otvora ravanski napregnute ortotropne ploče, izložen u [1], koji se ne može primeniti za slučaj jednakih ili bliskih vrednosti kompleksnih parametara.

Ključne reči: ortotropna ploča, višeslojna ploča, eliptični otvor, ravansko stanje napona.

Состояние напряжения в пунктах около эллиптического отверстия плоскоотно напряжённой плиты с рабочей арматурой в обоих направлениях

В настоящей работе приведены модифицированные выражения для расчёта составляющих сложения тензора напряжения в пунктах около эллиптического отверстия в случае одинаковых или близких значений комплексных параметров плиты с рабочей арматурой в обоих направлениях.

Это, на пример, случай у квазиизотропической схемы последовательности в многослойной плите, часто употребляющейся в практике.

Таким способом была бы пополнена процедура расчёта напряжённого состояния около эллиптического отверстия плоскоотно напряжённой плиты с рабочей арматурой в обоих направлениях. Эта процедура, дана в (1), не может быть применена в случае одинаковых или близких значений комплексных параметров.

Ключевые слова: плита с рабочей арматурой в обоих направлениях, многослойная плита, эллиптическое отверстие, напряжённое состояние, плоскоотно состояние.

Etat de tension des points autour de l'ouverture elliptique d'une plaque orthotropique tendu

Ce papier donne les expressions modifiées pour le calcul des composantes additionnelles du tenseur de tension dans les points autour de l'ouverture elliptique, pour le cas où les valeurs des paramètres complexes de la plaque sont égales ou proches. C'est, par exemple, le cas chez le schéma de l'accord quasitropique qui est souvent utilisé pour les constructions pratiques. De cette façon le procédé du calcul de l'état de tension autour de l'ouverture elliptique de la plaque orthotropique plane, exposé dans (1), serait complète. Ce procédé ne peut pas s'appliquer dans le cas où les valeurs sont égales ou proches chez les paramètres complexes.

Mots clés: plaque orthotropique, plaque multicouches, ouverture elliptique, état de tension, état de plan.