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Elasticity in Humanoid Robotics

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This paper gives a new approach to modeling of the kinematics and dynamics of a complex humanoid robotic system with elastic and stiff elements of joint. Each biped joint is defined with the states of motor (active or locked) and gear type (rigid or elastic). In this way, the new Denavit-Hartenberg parameters, transformation matrix and Jacobi matrix are defined. Mathematical model defines the dynamics of the locomotion robotic system with elasticity elements walking on an immobile platform. Furthermore, it is written in a general, universal, form, describing the motion dynamics of any robotic system that contains elastic elements. Reference trajectory of each joint is defined so as to encompass or not encompass elastic deformations. The synthesized new software FLEXI makes it possible to choose robotic configuration. The user can form very diverse biped configurations and the FLEXI software will define an appropriate mathematical model. The analysis of the simulation results of the humanoid robot motion on an immobile platform gives evidence for all the complexity of this system and shows how much system parameters (choice of trajectory, configuration, geometry, elasticity characteristics, motor, etc.) influence stabilization of its humanoid motion. All research in humanoid robotics aims at creating a robot similar to a human that would be his servant, worker or soldier and that would replace him in all dangerous situations.

Key words: **robotics, humanoid robot, modeling, locomotive systems, joints, joint elasticity, programmed trajectory, software.**

Introduction

MODELING of locomotion mechanisms of rigid anthropomorphic structure is certainly a very interesting problem.

Over the last four decades a strong development of a new class of mechanisms capable of performing diverse artificial motions of locomotion-manipulation type [14, 11, 7, 13] has occurred.

For a long while now, researchers have been thinking about the implementation of elastic joints into the already complex model of the locomotion mechanism.

Let some of the works dealing with robotic mechanisms that involve elasticity at joints be briefly reviewed in the continuation.

The method proposed in [8] provides a general approach to the problem of defining mechanical configuration models and kineto-ealstodynamic effects. In [9], the author discusses the problem of force control of a manipulator with an elastic joint. In [4], the authors expand the approach of singular perturbation to control a manipulator with elastic joint according to the case of known parameters onto the adaptive case. The authors of [6] use the technique of classical analysis for presenting the measured speed of motor shaft in a PD scheme. The paper [16] considers the control of a robot with elastic joint in contact with dynamic environment. This procedure has been used in [3] to achieve a transformed state, completed with the construction of an appropriate manifold sliding in which the second array of the sliding mode is powered. In [10], control of the manipulator with an elastic joint has been studied in detail.

Using the same principles defined by Spong [8] back in 1978, this work introduces elastic joints into the mathematical model. The newly generated FLEXI software defines the mathematical model of the robotic locomotion system. The analysis of simulation results was carried out on a 13-DOFs (degrees of freedom) humanoid robot. In the present research phase the robot's motion on an immobile horizontal platform is analyzed.

Modeling of Humanoid Robotic System

Kinematics and dynamics of a humanoid locomotion robotic system walking on an immobile platform are modeled. Since elasticity elements are introduced, it is necessary to explain in detail, first of all, the kinematics of these systems in order to have dynamic modeling as efficient as possible.

Kinematics

In the presence of elasticity elements, the notion of a joint (DOF) is given a new meaning which is necessary to expand and explain. The configuration of each joint may consist of:

1. motor and/or
2. gearing system.

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The motor may be
* *Active*, if it realizes the motion whereby the motor deflection angle $\overline{\theta}_i \neq const$ or
* *Locked*, if it is fixed at a certain position, and its speed and acceleration are zero, then $\overline{\theta}_i \neq const$.

Behind any motor, a gearing system, which may be:
~ *Elastic*, if the deformation is elastic, then the joint deflection angle $\xi_i \neq 0$.
~ *Rigid*, if elastic deformation is $\xi_i = 0$.
can be implemented.

This means that a joint can be defined in dependence of the working state of the motor and type of gearing. Four types of joints exist.

1. If the motor is active and gear elastic, whereby elastic deformation occurs in the direction of the motor motion, then such type of joint is shortly denoted by AE. The gearing is represented by the stiffness spring $C_{\xi i}\,[\text{Nm/rad}]$ and damping by $B_{\xi i}\,[\text{Nm/(rad/s)}]$. In Fig.1 a spatial sketch of a joint of the AE type is given, whereas in Fig.2 a geometric sketch of the same type of joint is presented. The overall coordinate q_i contains components $\overline{\theta}_i$ and ξ_{ei} .

   All these angles vary in the course of robotic task realization.

$$q_i = \overline{\theta}_i + \xi_{ei} \tag{1}$$

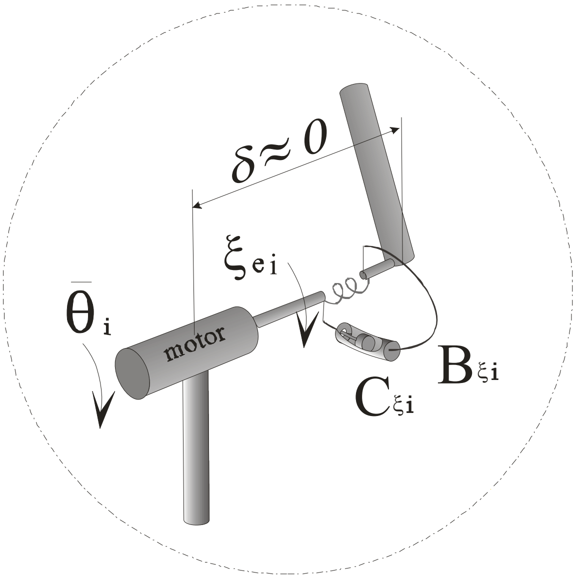

Figure 1. Joint of the AE type

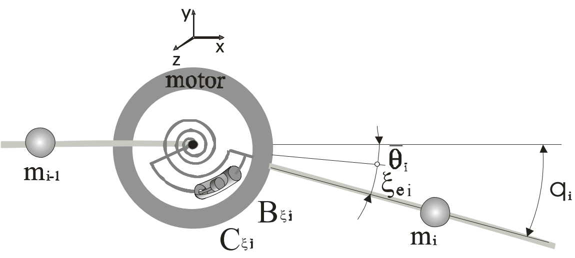

Figure 2. Geometry of the AE joint.

2. If the motor is locked and gear elastic, then such type of joint is denoted by LE.

   A locked motor no longer has the role of an actuator. It is present only as a mass. For this reason the same designation LE is also used in case when there is no motor at all, but only an elastic element (gear).

In Fig.3 a spatial sketch of a joint of the LE type is given, whereas in Fig.4 a geometric sketch of the same joint is presented. The overall coordinate q_i is:

$$\begin{aligned} q_i &= \xi_{ni} + const \quad &&\text{when the motor exists and it is locked or} \\ q_i &= \xi_{ni} &&\text{when the motor is missing} \end{aligned} \tag{2}$$

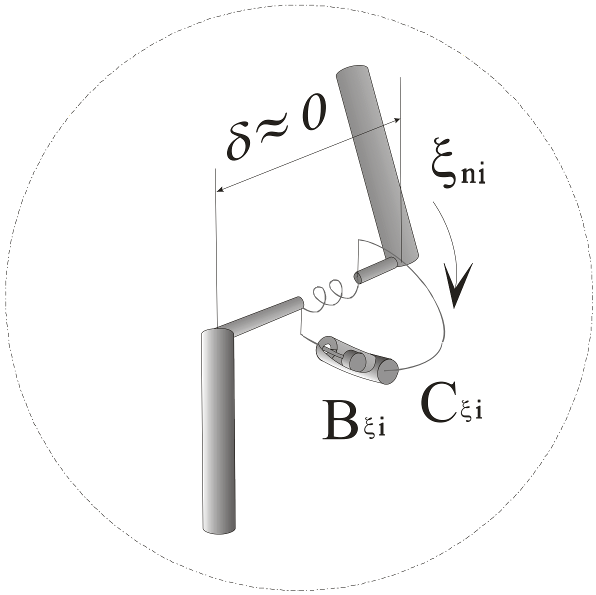

Figure 3. Joint of the LE type

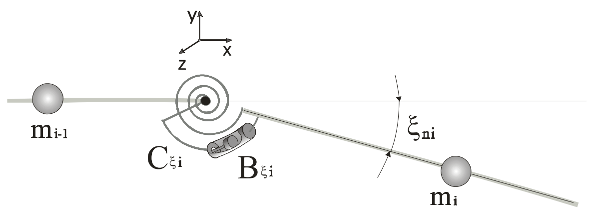

Figure 4. Geometry of the LE joint

Such DOFs belong to the foot sole. The foot sole has no motor but has elastic elements with the deformations about the axes x , y or z of the local coordinate frame, caused by the present load moments.

3. If the motor is active and the gear is rigid, such joint is denoted as AR. In this case, the overall coordinate q_i is equal to the rotation angle of the motor $\overline{\theta}_i$.

$$q_i = \overline{\theta}_i \tag{3}$$

4. If the motor is locked and the gear behind the motor is rigid, such type of joint is denoted by LR.

$$q_i = const \ \text{ or } \ q_i = 0 \,. \tag{4}$$

Therefore, is analyzed in this work the behavior of a humanoid robotic system which may contain joints of the AE, LE, AR and LR type. The rotation matrix which describes the change of the position (Cartesian coordinates) and orientation (Euler angles) of the tip of every link is of the form:

$$T_{e_i}^{i-1}=\begin{bmatrix}\cos q_i & -\sin q_i\cdot\cos\alpha_i & \sin q_i\cdot\sin\alpha_i & l_i\cdot\cos q_i\\ \sin q_i & \cos q_i\cdot\cos\alpha_i & -\cos q_i\cdot\sin\alpha_i & l_i\cdot\sin q_i\\ 0 & \sin\alpha_i & \cos\alpha_i & d_i\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (5)$$

where α_i, l_i and d_i are the Denavit-Hartenberg parameters. The overall transformation matrix is of the form:

$$T_{en}^{0}=T_{e1}^{0}\cdot T_{e2}^{1}\cdot T_{e3}^{2}\cdots T_{ei}^{i-1}\cdots T_{en}^{n-1} \quad (6)$$

The Jacobi matrix for a manipulator with elastic joints maps the velocity vector of the external coordinates $\dot{p}_s$ into the velocity vector of internal coordinates $\dot{q}$:

$$\dot{q}=J_e^{-1}(q)\cdot\dot{p}_s \quad (7)$$

where $\dot{p}_s=\left[\dot{x}\ \dot{y}\ \dot{z}\ \dot{\psi}\ \dot{\wp}\ \dot{\varphi}\right]^T$ defines the velocity of a given point of the robotic system in the Cartesian coordinates, whereas $\dot{q}=\left[\dot{q}_1\ \dot{q}_2\ \dot{q}_3\ \dot{q}_4\ \dots\ \dot{q}_n\right]^T$ defines the velocity vector of internal coordinates of the joints defined by Eqs. (1), (2), (3), (4). Elements of the Jacobian are only functions of the elements of the homogenous transformation matrix T_{en}^{0}. It is clear that each branched chain in the complex mechanism has its finite transformation matrix, as well as its Jacobi matrix.

Dynamics

Mathematical model of the humanoid robotic system with joints of AR type (Fig.5) is of the form:

$$\tau=I\cdot\ddot{q}+B\cdot\dot{q}+S\cdot(H(q)\cdot\ddot{q}+h(q,\dot{q})+J(q)^T\cdot F_{uk}) \quad (8)$$

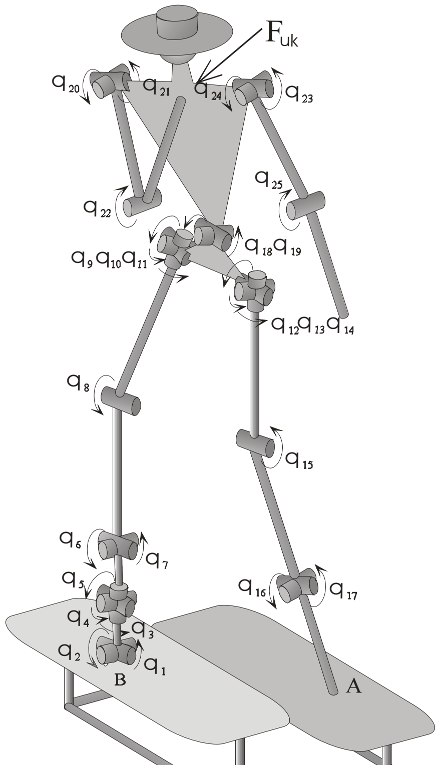

Figure 5. Humanoid robotic mechanism with 25 DOFs.

The robot considered has 25 DOFs. $\tau\in R^{25}$ is the control signal; $I=diag[I_i]\in R^{25x25}$ is the matrix of inertia moment of the motor rotor and reduction gear; $B=diag[B_i]\in R^{25x25}$ is the matrix of viscous friction coefficients; $S=diag[S_i]\in R^{25x25}$ is the matrix of the elements defining the gear geometry; $H\in R^{25\times25}$ is the matrix of mechanism inertia; $h\in R^{25}$ is the vector of centrifugal, Coriolis and gravitational forces; $J\in R^{6\times k}$ is the Jacobian matrix; k characterizes the point of the environment force action; $F_{uk}\in R^{1x6}$ is the dynamic/non-dynamic contact force.

A biped in single-support phase of its motion (Fig.5) is considered. The configuration is adopted with 25 joints. In order to avoid too long a simulation, at the same time preserving the relevant dynamic effects, it is assumed that certain joints 1, 2, 10, 11, 12, 13, 20, 21, 22, 23, 24, 25 are defined as LR. Other joints can be of the AE, LE or AR type. These conditions allow the model of humanoid robots from Fig.5 to be reduced to the model in Fig.6.

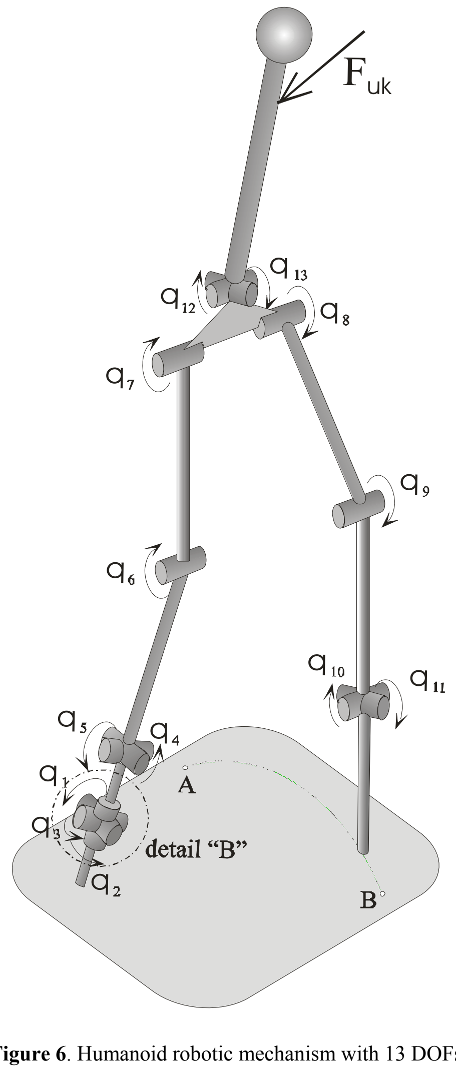

Figure 6. Humanoid robotic mechanism with 13 DOFs.

The selected configuration (Fig.6) has another specific trait i.e. the foot sole elasticity. In general, the sole can experience six deformation motions - three rotational motions about the local coordinate frames and three translational motions in the directions of the x, y and z axes. These joints are of LE type. In view of the fact that the translational DOFs in the foot sole do not influence the biped motion significantly, they have been excluded from the analysis. Also, the same was done with the motion at the sole joint q_1 which is realized about the z-axis of the local coordinate frame, because it was not of interest in example.

In the sole of the foot, only two joints are active, viz., the two rotations q_2 and q_3. The elasticity was assigned to the sole joints q_2 and q_3 which rotates respectively about x and y-axis, of the local coordinate frame. Only these two rotations (deformations) are analyzed because they are actually capable of influencing significantly the dynamic balance of the biped in its motion.

The dynamic model of elastic robotic system is defined on the basis of classical principles of dynamics, addressing the realization of these principles starting from the fact that elastic deformations are the effects that are always present in reality, directly dependent on the instantaneous dynamic load of the mechanism. The dynamic model of finite equations of the system motion is obtained using Lagrangian equations.

Angles q_1, q_2, …, q_{13} ($\dot{q}_1$, $\dot{q}_2$, …, $\dot{q}_{13}$ and $\ddot{q}_1$, $\ddot{q}_2$, …, $\ddot{q}_{13}$) appear in the dynamic model and participate in the forming of dynamic forces (inertial, centrifugal, Coriolis, gravitational, and elasticity).

It is necessary to define the overall coordinate q_i, motor angle $\overline{\theta}_i$ and/or elastic deformation ξ_i for the i-th kinematics pair, and it is these quantities, if they exist in the configuration, that are selected as generalized coordinates. The deflection angle of the i-th joint ξ_{ei}, joint of the AE type from eq. (1), is not select as generalized coordinate because it may be expressed via the already selected generalized coordinates q_i and $\overline{\theta}_i$:

$$\xi_{ei} = q_i - \overline{\theta}_i \tag{9}$$

Potential energy of an elastic element in the general form is:

$$E_{p\xi i} = \frac{1}{2} \cdot C_{\xi i} \cdot \xi_i^2 \tag{10}$$

If expressed as function of the generalized coordinates, then:

$$\begin{aligned} E_{p\xi i} &= \frac{1}{2} \cdot C_{\xi i} \cdot (q_i - \overline{\theta}_i)^2 \quad \text{for the joint of the AE type or} \\ E_{p\xi i} &= \frac{1}{2} \cdot C_{\xi i} \cdot \xi_{ni}^2 \quad \text{for the joint of the LE type.} \end{aligned} \tag{11}$$

Elastic elements also possess dissipative energy:

$$\Phi_{\xi i} = \frac{1}{2} \cdot B_{\xi i} \cdot \dot{\xi}_i^2 \tag{12}$$

The dissipative energy can also be expressed in dependence of the generalized coordinates:

$$\begin{aligned} \Phi_{\xi i} &= \frac{1}{2} \cdot B_{\xi i} \cdot (\dot{q}_i - \dot{\overline{\theta}}_i)^2 \quad \text{for the joint of the AE type or} \\ \Phi_{\xi i} &= \frac{1}{2} \cdot B_{\xi i} \cdot \dot{\xi}_{ni}^2 \quad \text{for the joint of the LE type.} \end{aligned} \tag{13}$$

The equations of the overall model of the biped (Fig.6) change their structure depending on whether a given joint is defined as AE, LE, AR or LR. In order to define the mathematical model of the considered locomotion mechanism of the anthropomorphic structure with 13 DOFs for any possible combination of joint types, it would be necessary to define in total 3^{13} mathematical models, which would be a painstaking job. Therefore, the task of modeling was solved as a programming one, so that the user should only select the type of each joint, along with other system parameters based on which, the FLEXI software generates the model, making it possible to analyze the dynamic behavior of the model.

When defining some of the joints as AE, LE or AR and others as LR, the system structure will change, and so will the structure of the matrices in it. In order to show the structure of the mathematical model of the mechanism with randomly selected joint types, certain joints of the biped should be fixed in the following way (Fig.5):

$eo = 5$ chosen joints of type AE,
$ne = 3$ chosen joints of type LE,
$nr = 5$ chosen joints of type AR,
$nc = 12$ chosen joints of type LR.

In case of such joint types, the mathematical model of the overall system has the following form:

$$0_{ne} = H_n \cdot \ddot{q} + h_n + C_n \cdot \xi_n + B_n \cdot \dot{\xi}_n + J_n^T \cdot F_{uk} \tag{14}$$

$$0_{eo} = H_e \cdot \ddot{q} + h_e + C_e \cdot \xi_e + B_e \cdot \dot{\xi}_e + J_e^T \cdot F_{uk} \tag{15}$$

$$\tau_e = I_e \cdot \ddot{\overline{\theta}}_e + B_{me} \cdot \dot{\overline{\theta}}_e - S_e (C_e \cdot \xi_e + B_e \cdot \dot{\xi}_e) \tag{16}$$

$$\tau_k = I_k \cdot \ddot{\overline{\theta}}_k + B_k \cdot \dot{\overline{\theta}}_k + S_k \cdot (H_k \cdot \ddot{q} + h_k + J_k^T \cdot F_{uk}) \tag{17}$$

$0_{ne} \in R^{ne}$, $H_n(q) \in R^{ne\times 13}$, $q \in R^{13}$, $h_n(q,\dot{q}) \in R^{ne}$,
$C_n = diag[C_\xi] \in R^{ne\times ne}$, $B_n = diag[B_\xi] \in R^{ne\times ne}$, $\xi_n \in R^{ne}$,
$J_n(q) \in R^{6\times ne}$.
$0_{eo} \in R^{eo}$, $H_e(q) \in R^{eo\times 13}$ $h_e(q,\dot{q}) \in R^{eo}$
$C_e = diag[C_\xi] \in R^{eo\times eo}$, $B_e = diag[B_\xi] \in R^{eo\times eo}$, $\xi_e \in R^{eo}$,
$J_e(q) \in R^{6xeo}$, $\tau_e \in R^{eo}$, $I_e = diag[I_i] \in R^{eoxeo}$,
$B_{me} = diag[B_i] \in R^{eoxeo}$, $S_e = diag[S_i] \in R^{eoxeo}$, $\overline{\theta}_e \in R^{eo}$.
$\tau_k \in R^{nr}$, $I_k = diag[I_i] \in R^{nrxnr}$, $B_k = diag[B_i] \in R^{nrxnr}$,
$S_k = diag[S_i] \in R^{nrxnr}$, $H_k \in R^{nr\times 13}$, $h_k \in R^{nr}$,
$J_k(q) \in R^{6\times nr}$ $\in$, $F_{uk} \in R^6$, $\overline{\theta}_k \in R^{nr}$.

By analyzing Eqs. (14), (15), (16), (17) it can be concluded that the overall model can also be written in a compact form, but the equations are presented in this form for the sake of clarity.

It is evident that the environment force F_{uk} participates in the generation of the overall coordinates q, as well as the deflection angles ξ. The environment force is mapped onto the direction of the generalized coordinates via the elements of the Jacobian matrix J.

Model (14 - 17) does not define dynamics of the locomotion robotic system with elasticity elements walking on an immobile platform, but it is written in a general, universal form and describes the motion dynamics of any robotic system that contains elastic elements at its joints of the AE, LE, AR and/or LR type.

Referenced Trajectory

When considering the motion of humanoid robotic system two mutually dependent, aspects of defining the referenced trajectory a should be distinguished:
* referenced trajectory of the motion of each joint,
* referenced trajectory of the motion of Zero-Moment Point (ZMP^o, see [15, 5 and 12]).

These two approaches to defining the references are of equal importance and should act in a coordinated way in order to achieve a harmonious human-like motion.

The problem of creating a referenced joint trajectory of elastic robots has been dealt with only in several papers, a procedure being presented first in [1]. In [2], one-DOF robotic system with an elastic joint and rigid link, and separately, a stiff joint and elastic link, was presented.

There are two aspects in defining the referenced trajectory of the motor angle, viz.:

1. Elastic deformation is considered as a quantity which is not encompassed by the referenced trajectory. This is the case when the elasticity characteristics in the system are not known and are not included in the referenced trajectory definition. The reference is defined as for a rigid system. The motor angle $\overline{\theta}_i^\circ$ is given as a desired value. However, in the real motion of the mechanism, elastic deformations ξ_i are always involved and represent "disturbance" to the system that is the deviation of the real from the referenced motion. The working ZMP trajectory, in the absence of other disturbances, deviates from the referenced ZMP^o trajectory by the extent of the influence of all elastic deformations. The referenced trajectory is defined as a referenced trajectory of the rigid system of the same configuration. Elastic deformations do not participate in control synthesis. This has for a long time, been, the only way of formulating the references. A consequence of defining the references in this way must not be the requirement that the elastic deformation should vanish under the action of the control law. Such an assumption is unrealistic because elastic deformation is the consequence of the present forces and hence an actually present characteristic in the course of a robotic task realization. The referenced trajectory, and hence the control law, does not encompass the elasticity characteristic, and it represents a permanent disturbance to the system. In this case it is necessary to devise some new control laws that will ensure a synchronized and humanoid motion of the robot, regardless of the present "disturbances" due to unknown elasticity characteristics.
2. Elastic deformation is a quantity which is at least partly encompassed by the reference trajectory. It is assumed that all elasticity characteristics in the system (both of stiffness and damping) are "known", at least partly and at that level can be included into the process of defining the reference motion. To begin with, it is necessary to identify the robotic configuration, i.e. to define mathematically the robotic model which includes all elasticity elements and estimate unknown parameters: stiffness and damping of the elastic elements. If all is done well, then it is possible to synthesize the reference trajectory of the new system (with elasticity elements) so that, at least, to the accuracy level of estimated parameters, it encompasses elastic deformation in the task realization. The overall coordinate q_i° is defined as the desired one. In this case, the reference coordinate of the motor $\overline{\theta}_i^\circ$ and of elastic deformation deflection angle of the gear ξ_i° are obtained from the overall reference dynamics of the mechanism in the course of motion in the absence of disturbances. Various control laws can be applied. The only drawback is that this is an idealized case, whereby it is assumed that the elasticity characteristics can be estimated with sufficient accuracy and included in the process of defining the reference trajectory. For such a choice of reference trajectory it is assumed that elastic characteristics of each joint at the reference level are known sufficiently, so that the ZMP^o of the humanoid is close enough to zero. The magnitude of the ZMP deviation from the reference is sufficiently small during the biped robotic task realization. If this is not satisfied, it is necessary to define new control laws that will compensate for the insufficient information on the elasticity characteristics.

These two aspects of trajectory forming are interesting; although they are essentially different as the aspect 1) is only a special, boundary case of the aspect 2).

Control

As far as the control strategy is concerned, the user can choose from the known control laws:
* control by Inverse Dynamics Method, whereby actuators are not implemented in the biped model. This idealized control, which does not encompass the actuators' dynamics in the overall system dynamics, is not of interest here and therefore will not be analyzed.
* additive control, CR (Centralized Referenced control calculated from the referenced states) τ°, plus LO (control via local feedbacks of the motor angle with respect to the position and velocity) τ_l. Only this law was analyzed, as it encompasses the actuators' dynamics. The selected control is of the form:

$$\tau_i = \tau^\circ + \tau_l \,. \tag{18}$$

$$\tau_l = K_{lpi} \cdot (\overline{\theta}_i^\circ - \overline{\theta}_i) + K_{lvi} \cdot (\dot{\overline{\theta}}_i^\circ - \dot{\overline{\theta}}_i) \,. \tag{19}$$

The τ° is a quantity that can be defined in two ways, namely as:
* an idealized quantity defined in the case when elastic deformation exists at the referenced level, τ_i° or as
* a real quantity defined for the case when elastic deformation is not defined at the referenced level τ_r°.

Simulation Examples

Simulations were carried out for elastic humanoid robotic mechanisms with 13 DOFs, moving without constraints. This means that the simulation analyses are carried out with no environment force involved.
The left-leg tip of the humanoid robot moves in a half-step from the point "A" to the point "B" in a prescribed time of $T = 0.75[s]$ (see Fig.6).

The geometry of the humanoid robotic system is defined so as to mimic the geometry of a human 1.8 [m] tall, with a mass of 70 [kg], ideally distributed over all links. The foot size with respect to the desired ZMP^o position in the x-direction $\pm 0.15[\mathrm{m}]$ and in the y-direction is $\pm 0.075[\mathrm{m}]$, so that the overall foot size is $0.3\times 0.15[\mathrm{m}]$.

The elasticity module of aluminium is: $E_l = G = 69.3\cdot 10^9\left[\mathrm{N/m^2}\right]$. The initial deviations of the motor angle are: $\delta\bar{\theta}_i(t_0)=[0;\ 0;....0,0][\mathrm{rad}]$, $\delta\dot{\bar{\theta}}_i(t_0)=[0;0;...;0;0][\mathrm{rad/s}]$. The initial deviations are deliberately selected as zeros, to emphasize the other dynamic motion effects. The selected parameters of the motor and gear are: I=0.0229 $[\mathrm{kg\,m\Omega A/N}]$, B=0.0328 $[\mathrm{V/(rad/s)}]$, S=0.0022 $[\mathrm{\Omega A/Nm}]$.

Sampling period is $dt = 0.001[\mathrm{s}]$.

Control gains for the motion stabilization are: $K_{lp} = diag[300,...,300]$ and $K_{lv} = diag[35,...,35]$.
Characteristics of stiffness and damping of the joints are:

$$C_{\xi} = 2\cdot diag\left[10^3;10^4;10^4;10^3;10^3;10^4;10^4;\right.$$
$$\left.10^4;10^4;110^3;10^3;0^4;10^4\right][\mathrm{Nm/rad}]$$

and

$$B_{\xi} = diag[10;100;100;10;10;30;30;$$
$$30;30;10;10;30;30;]\ [\mathrm{Nm/rad/s}]$$

In the simulation examples, the reference trajectory is defined as for a rigid system, so that no elastic deformations are present at the reference level, $\xi_i^\circ = 0$. Because of that, only the real values of elastic deformations ξ_i are designated in the figures. It is assumed that all other characteristics in the system are known and included into the process of defining the reference motion.

The simulation demonstrates how the lack information on the elasticity properties at the reference level influences the stability of the humanoid motion.

Example:

In the simulation example, the joints 1, 2, 3 (in the sole of the foot) were of LR type (see Fig.6). The joints 4, 5, 10 and 11 are AR types, and the joints 6, 7, 8, 9, 12 and 13 are AE types.

The references trajectory was defined as for a rigid robotic system. This means that elastic deformations do not exist on the referenced level. The control is defined by $\tau_r^\circ + \tau_l$.

Simulation results are given only for the previously selected set of parameters. The values of these parameters and selection of the control gains had a significant effect on the simulation results. In Fig.7 the referenced $\bar{\theta}_i^\circ$ and real trajectories of motor angles $\bar{\theta}_i$ for all 10 DOFs are shown. The trajectory where the referenced ZMP° is very close to zero during the half-step realization was used. If the control is defined well, it is expected that the ZMP does not come out of the foot support zone.

In Fig.8 the deviation of $\bar{\theta}_i^\circ$ from $\bar{\theta}_i$ is given.

It is interesting to have a look at Fig.9, showing the reference reduced torque P° caused by the reference system dynamics during robotic task realization, as well as real reduced torque P caused by the real system dynamics. Namely, the moment P encompasses the elastic effects occurring during the motion of the elastic humanoid robotic mechanism.

In Fig.10 are shown the elastic deformations ξ_{ei} of the joints 6, 7, 8, 9, 12 and 13. It is evident that the level of elastic deformation in the real states is directly dependent of the load moment P_i for the joint considered. On the referenced level $\xi_{ei}^\circ = 0$.

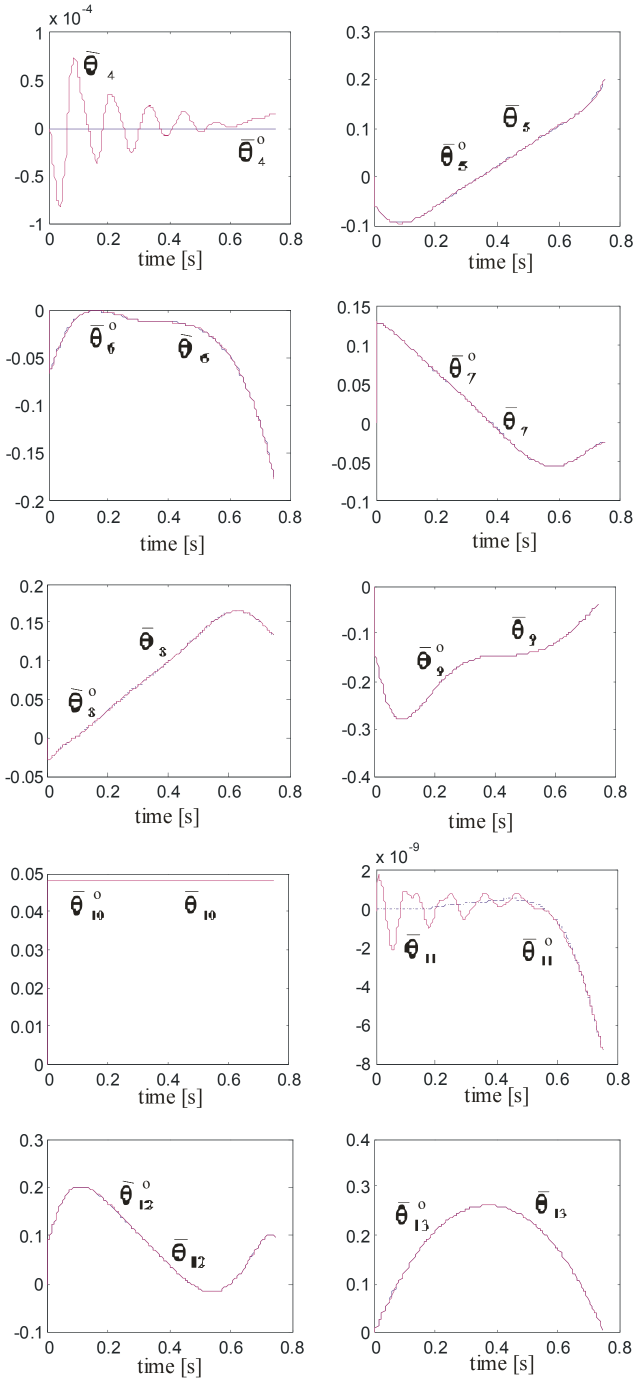

Figure 7. Referenced $\bar{\theta}_i^\circ$ and real $\bar{\theta}_i$ motor angle [rad].

In Fig.11 the referenced forces F_i° (and moments M_i°) and real F_i (M_i) by which the humanoid robot acts on the immobile platform in all 6 directions of Cartesian coordinates are presented. Fig.12 illustrates the ZMP° and ZMP motion during the robotic task realization. The shaded area belongs to the biped foot. Immediately after the simulation has begun the ZMP comes out of the biped foot contour. Theoretically, the biped is falling. Irrespective of this, the simulation is continued, as if the foot were of much bigger dimensions, in order to show the dynamic possibility of model. In Fig.13 the ZMP movement related to the foot is presented.

The ZMP badly tracks the "desired trajectory". Not knowing the elasticity at the reference level represents a significant disturbance to the system and causes biped's instability during the motion. For this reason, it is necessary to devise new control laws.

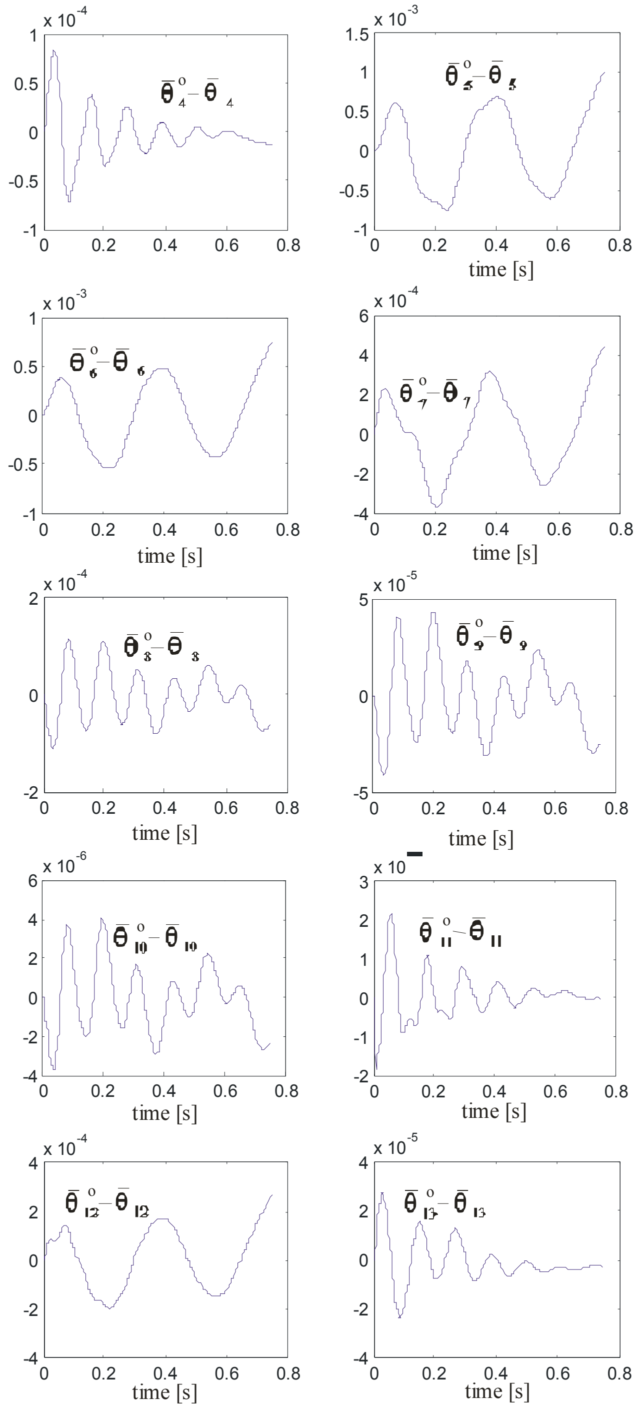

Figure 8. Deviation referenced $\bar{\theta}_i^\circ$ from real $\bar{\theta}_i$ [rad].

Conclusion

The model kinematics and dynamics of a complex humanoid robotic system with elastic and stiff elements of joint are presented in this paper. A joint is defined in a new way, as dependence on the motor state (active or locked) and type of elastic or rigid gear behind the motor. With the types of joints that may appear in a robotic construction defined in this way, it is possible to use the known equations to calculate the matrices of transformation and Jacobi matrix. An analysis made of the choice of reference trajectory, which depends on the level of the known elasticity characteristics, was made. The estimated elasticity characteristics may be included into the reference trajectory, and thus into the control law.

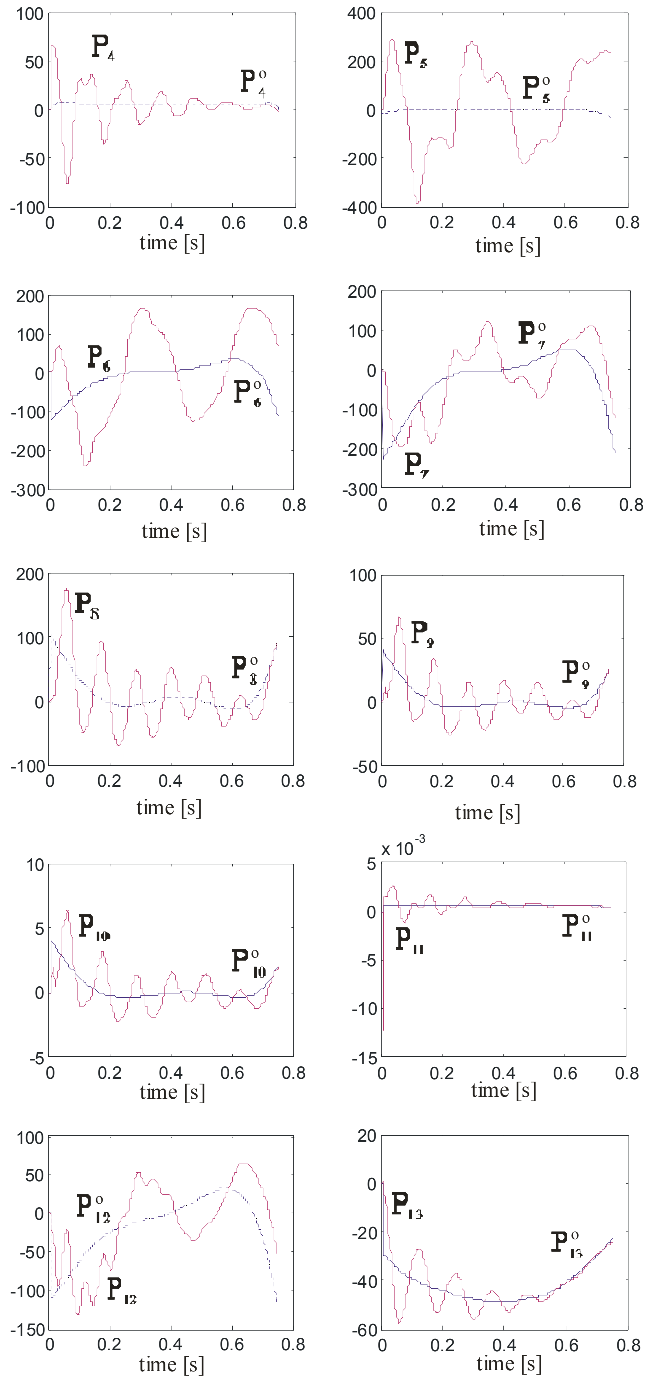

Figure 9. Referenced P_i° and real P_i moment load [Nm].

The dynamic model of the robotic mechanism encompasses all coupling elements, as well as all dynamic effects of the forces involved. Elastic deformations of joints are defined as a logical consequence of the dynamic moments and loads of each of the joints.

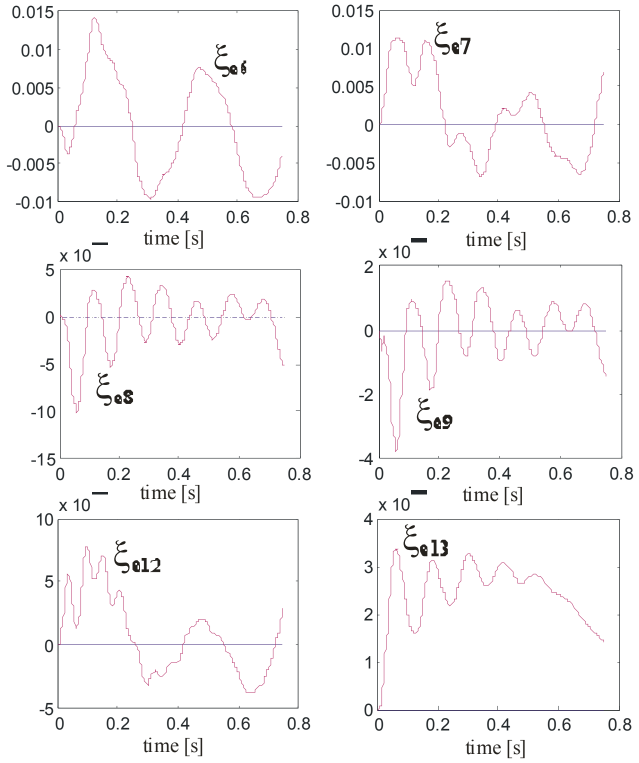

Figure 10. Real elastic deformations ξ_{ei} $\left(\xi_{ei}^{\circ}=0\right)$ [rad].

The elasticity dynamics cannot be analyzed independently of the overall robotic system dynamics. The prominent coupling of joints, appearing in the complex robotic system motion, was demonstrated. The humanoid robotic system is described by a system of differential equations of damped oscillations. For the dynamic model of robotic system thus defined, a combined control law consisting of CR plus the control LO was selected.

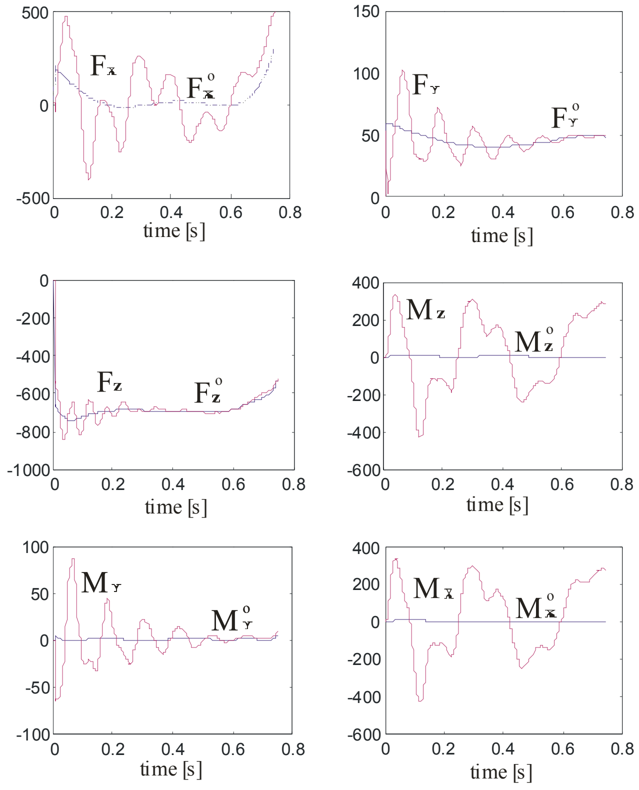

Figure 11. Referenced F_i° $\left(M_i^{\circ}\right)$ and real F_i $\left(M_i\right)$ force [N] ([Nm]).

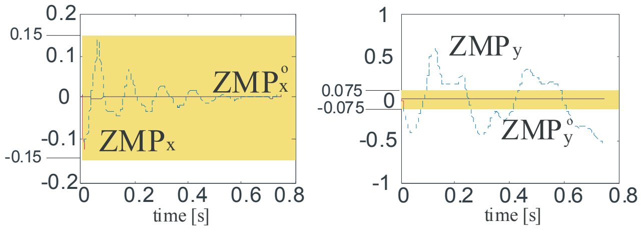

Figure 12. Referenced ZMP° and real ZMP in the x - and y - directions [m].

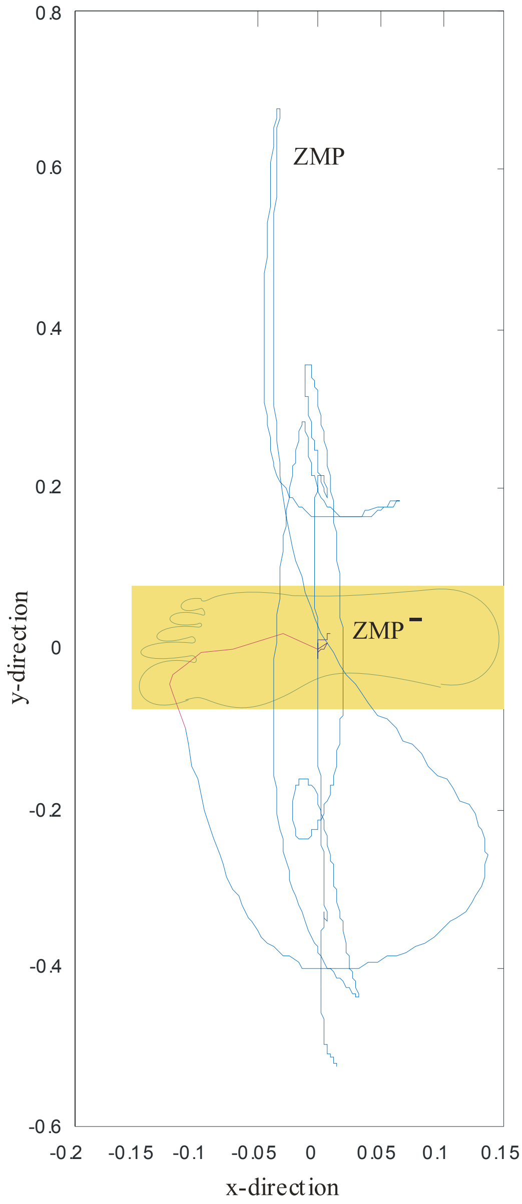

Figure 13. Referenced ZMP° and real ZMP in the x - y plane [m].

The simulation results were obtained for the case when the reference does not include the magnitude of the elastic deformation (the reference was defined as for a rigid system).

The new FLEXI software as an algorithm for generating the mathematical model of a complex humanoid robotic system with elastic joints is synthesized. The algorithm defined holds for an arbitrary number of DOFs of the selected robot, and was applied for a robotic mechanism with 25 DOFs. Equations of the model change their structures depending on whether particular joints are defined as AE, LE AR or LR. Therefore, the modeling task was solved in the form of a program, so that the user should only select joint type and, subsequently, all other parameters of the system (FLEXI software generates the model), which gives the possibility of analyzing the system's dynamic behavior.

The FLEXI software also allowed modeling, simulation and analysis of the example involving elasticity joints of the LE type in the foot sole. Only two rotations (deformations) were analyzed (the sole elasticity q_2 and q_3 appearing close to the x and y -axis, respectively, of the local coordinate frame) because they influence the dynamic balance of the biped in its motion.

The FLEXI software synthesized in this study can be used for the analysis of elastic robotic systems of anthropomorphic structure and can also be expanded and supplemented from different aspects, in accordance with the user's requirements.

The characteristics of configuration complexity of the robotic system and choice of the presence of elasticity characteristic in the system are no longer limiting factors in the analysis of kinematics and dynamics of humanoid robotic systems. This essentially novel approach in considering elastic robotic systems opens up new possibilities in the analysis and modeling of these systems.

Further development in this area should proceed in the direction of implementing new control laws. It would also be interesting to analyze the motion of the humanoid robotic system on a mobile platform (rigid or elastic). All research in this area has a tendency to create a humanoid robot that would be helpful to people in everyday work. Military industry, especially in highly developed countries, invests enormous funds in developing “robot-solders” that would completely replace people in combat.

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Elastičnost u humanoidnoj robotici

Ovaj rad daje novi pristup u modeliranju kinematike i dinamike složenog humanoidnog robotskog sistema sa elastičnim i krutim elementima zgloba. Svaki zglob bipeda je definisan stanjem motora (aktivan ili zaključan) i tipom prenosnika (krut ili elastičan). Na taj način su definisani novi Denavit-Hartenberg parametri, matrica transformacije i Jakobijeva matrica. Matematički model definiše dinamiku kretanja humanoidnog robotskog sistema sa elementima elastičnosti koji hoda po nepomičnoj platformi, a takođe je napisan u opštoj, univerzalnoj formi i opisuje dinamiku kretanja bilo kog robotskog sistema, koji sadrži elemente elastičnosti. Referentna trajektorija svakog zgloba je definisana tako da obuhvata ili ne obuhvata veličinu elastične defo0rmacije. Sintetizovan je novi softver FLEXI, koji pruža mogućnost izbora konfiguracije robota. U zavisnosti od zelje korisnika može se formirati vrlo različita konfiguracija bipeda. Programski paket formira matematički model. Analiza rezultata simulacija kretanja humanoidnog robota po nepokretnoj platformi ukazuje na složenost ovog sistema i pokazuje koliko parametri sistema (izbor trajektorije, konfiguracije, geometrije, karakteristika elastičnosti, motora ...) utiču na stabilizaciju njegovog humanoidnog kretanja. Sva istraživanja u humanoidnoj robotici imaju za cilj da stvore robota što sličnijeg čoveku, koji bi mu bio sluga, radnik, vojnik i koji bi ga zamenio u svim opasnim situacijama.

Ključne reci: **robotika, humanoidni robot, modeliranje, lokomotorni sistem, zglob, elastičnost zgloba, programirana trajektorija, softver.**

Эластичность в интеллектуальной робототехнике

В настоящей работе показан новый подход в моделировании кинематики и динамики сложных интеллектуальных робототехнических систем с эластичными и жёскими зглобами (шарнирами). Каждый зглоб бипеда определён состоянием двигателя (активный или запертый), и типом передатчика (жёсткий или эластичный). Таким способом определены новые параметры Денавит-Гартенберга, матрица преобразования и матрица Якобы. Математическая модель определяет динамику движения интеллектуальной робототехнической системы с эластичными элементами, ходящей по неподвижной платформе, а тоже написана в общей, универсальной форме и описывает динамику движения любой робототехнической системы с элементами эластичности. Главная траектория каждого зглоба (шарнира) определена так, что охватывает или не охватывает размер эластичной деформации. Синтетизованно новое пеограммное обеспечение ФЛЕЬИ, обеспечивающее выбор конфигурации интеллектуального робота. В зависимости от желания пользователя, возможно формирование различных конфигураций бипеда. Программное обеспечение ФЛЕЬИ формирует математическую модель. Анализ результатов имитации движения интеллектуального робота по неподвижной платформе указывает на сложность этой системы и показывает в какой мере параметры системы (выбор траектории, конфигурации, геометрии, характеристик эластичности, движенияЩ) влияют на устойчивость его интеллектуального движения. У всех этих исследований в интеллектуальной роботике есть одна единственная цель - формировать и сделать робота более подобного человеку, который в будущем был бы его слугой, служащим, работником и веонным и который бы заместил его во всех опасных обстоятельствах.

Ключевые слова: робототехника, интеллектуальный робот, моделирование, локодвигательная система, зглоб (шарнир), эластичность зглоба (шарнира), проектированная траектория, программное обеспечение.

Elasticité dans la robotique humanoide

Ce papier expose une nouvelle approche à la modélisation de cinématique et dynamique d'un système robotique humanoide et complexe, aux éléménts de l'articulation élastique et rigide.Chaque articulation du bipède est définie par l'état du moteur (activé ou fermé à clé) et par le type du translateur (rigide ou élastique.On a défini ainsi les nouveaux paramètres Denavit-Hartemberg, la matrice de transformation et celle de Jacobi. Le modèle mathématique définit la dynamique du mouvement du système robotique humanoide, aux éléments d'élasticité et qui se déplace sur une plate-forme immobile.Ce modèle est présenté en forme générale et universelle et décrit la dynamique du mouvement de n'importe quel système robotique contenant les éléments d'élasticité.La trajectoire référentielle de chaque articulation est défini de façon à inclure ou ne pas inclure les déformation d'élasticité. Le nouveau logiciel FLEXI, qui permet de choisir la configuration du robot, a été synthétisé. En fonction des exigences d'utilisateurs, il est possible de former des configurations très variées du bipède et le progiciel définit le modèle mathématique. L'analyse des résultats de la simulation du mouvement du robot himanoide sur la plate-forme immobile démontre la complexité de ce système et à quel point les paramètres du système (choix de la trajectoire, configuration, géométrie, caractéristiques de l'élasticité, moteur…) influent à la stabilisation de son mouvement humanoide. Dans le domaine de la robotique humanoide, toutes les recherches ont pour but de créer un robot qui ressemblerait à l'homme et dont il serait servant, travailleur, soldat et le remplacerait dans chaque situation dangereuse.

***Mots clés*: robotique, robot humanoide, modélisation, système locomoteur, articulation, élasticité de l'articulation, trajectoire programmée, logiciel.**