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New procedure for calculating altitude coefficients

Dragoslav R. Petrović, PhD (Eng)[1)]

New procedure for determination altitude coefficients for certain meteorological elements needed as data in firing tables is presented. Altitude coefficients can be used in artillery practice when meteo-message, the so called "meteo-average" is used for field calculation of meteorological influences and appropriate corrections. Numerical data of altitude coefficients for certain ballistic conditions are given. New procedure eliminates calculation of weighted functions widely used up to now in ballistic practice.

Key words: **external ballistics, ballistic measurements, meteorological measurements, firing tables, artillery shooting.**

Introduction

EFFECTS of actual meteorological elements (wind, temperature, pressure) on trajectory elements that determine projectile position in space is cumulative. It can be calculated for cummulative or individual meteorological elements from results of trajectory calculation elements. In actual ballistic practice, effects of individual meteorological elements (the so called differential elements) are used for calculating ballistic values of meteorological elements. Ballistic values of meteorological elements depend on projectile ballistic characteristics and projectile's initial space position. Ballistic values can be determined by the so called layer weights or their functions. After the application of meteorogical message i.e. meteo-average in artilerry practice [1, 2], a need for calculating altitude coefficients incorporating projectile ballistic characteristics emerged. Clasical procedure for altitude coefficients calculations is based on layer weights or their functions in graphic forms. For this purpose, projectile trajectory must be devided into leyers and unit meteorological element influence on trajectory elements calculated, related to meteorological elements influence in all layers, i.e. along total trajectory height. In this article, a new procedure for altitude coefficients calculation that eliminates the need for layer weights or their function calculation for each of the meteorological elements (longitudinal and directional wind and temperature) is presented.

Altitude coefficients calculation procedures

Numerous procedures for layer weight calculation [3,4,5] that are basis for classical altitude coefficients calculation exist. Layer weights can not be obtained in mathematical form, but their graphs can be formed as functions of node points r_i, Y_i/Y_S, where r_i is discrete value of layer weight function for Y_i/Y_S, where Y_i is layer altitude and Y_S is trajectory height. Graphs can be mechanized [6], but in case of firing tables production, it is not convenient to use them. Instead of graphs, the same data can be used for numerical integration by certain integration methods. Result of numerical integratioin is surface S under virtual graph driwen through node points r_i, Y_i/Y_S (0-2) Fig.1. By mechanized procedure in [6], altitude coefficients $(K)_{Wx,Wz,\Delta\tau}$ for $Wx, Wz, \Delta\tau$, can be calculated where W_x, W_z are meteorological elements, longitudinal and directional wind components and $\Delta\tau$ temperature differences related to nominal values. Formula for altitude coefficients $(K)_{Wx,Wz,\Delta\tau}$ as function of surface S under graph is [6]:

$$(K)_{Wx,Wz,\Delta\tau} = 2(-S) \tag{1}$$

Established dependence eliminates the need for manual graph drawings and estimating surface equality under and above the line drawn from origin O to 1 Fig.1. (In Fig.1, for eq. $(K)_{W_x} = 1.3$), as it is described in [1,2].

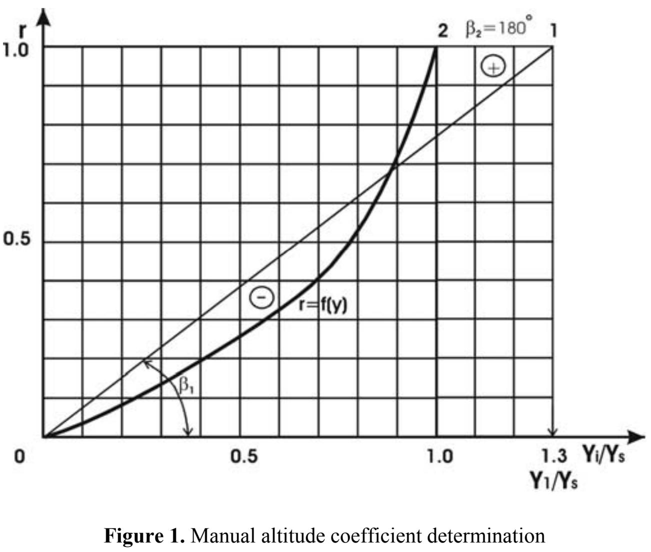

Figure 1. Manual altitude coefficient determination

As follows from (1), what is required is not the graph $r= f(Y_i/Y_S)$, but nodal points r_i, Y_i/Y_S, where Y_i is current

[1)] Jurija Gagarina 146/50, 11070 Belgrade, SERBIA

trajectory height and Y_S is total trajectory height - summit. Nodal points r_i, Y_i/Y_S tought to be used for numerical integration by certain integration methods for surface S under virtual curve 0-2. Procedures described for layer weight functions calculations $r=f(Y_i/Y_S)$ in discrete form r_i, Y_i/Y_S [5, 6] are not suitable for the purposes of firing tables production. The same references indicate that the value of surface S is dependent on the number of nodal points n. The number of nodal points n is not known in advance, but for the sake of simplicity it must be taken as finite number covering all possible graph forms. Ten nodal points give satisfactory results [6], but at the same those ten trajectories must be calculated. Consequently, the total number of trajectory calculations must be 3n for three meteorological elements for only one distance in the firing tables, which is not satisfactory.

New method for calculating altitude coefficients

The study of graph layer weights functions done in [6] enables detecting the influence of meteorological elements on the projectile flight. Some meteorological elements prevail on initial trajectory path, and some at summit. Therefore, from all the graphs, it can be seen that they are surrounded by two curves, *I* and *II* Fig.2.

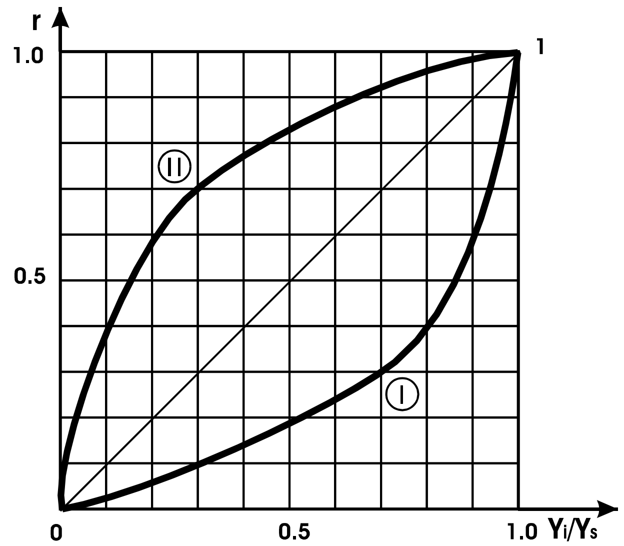

Figure 2. Extreme forms of layer weight functions

Curve *I* indicates prevailing meteorological elements at trajectory summit and *II* prevailing meteorological elements on initial trajectory path. The case when the curve is a straight line that connects points 0-1 and is a diagonal of a square with sides 0-1-1-1 is when the straight line gives the value of surface S = 0.5 and when it is introduced into (1) for the value of altitude coefficient of an actual meteorological element will be equal to one. In other words, all trajectory layers are of equal influence. When considering the influence on the horizontal distance variation when the trajectory is exposed to meteorological elements variation with height similar to that given in Fig.3, under the assumption that meteorological elements' influences are the same on all heights, the horizontal distance variation must be equal zero, i.e. $\Delta X_\mu = X_\mu - X_N = 0$.

It can be said at a glance that when the influence of the meteorological elements is similar to that given in Fig.3, when $\Delta X_\mu > 0$, the form of the layer weight functions must be similar to that given as curve *I* (Fig.2), and when $\Delta X_\mu < 0$, the form of the layer weight functions must be similar to that given as curve *II* (Fig.2). If trajectory is exposed to the influence of meteorological elements in the form given in Fig.4, and when their value equals 10 units, distance variation is the differential coefficient $\Delta D_\mu = 0.1 \Delta X_\mu \times \mu_b$, where μ_b is the ballistic value of the actual meteorological element.

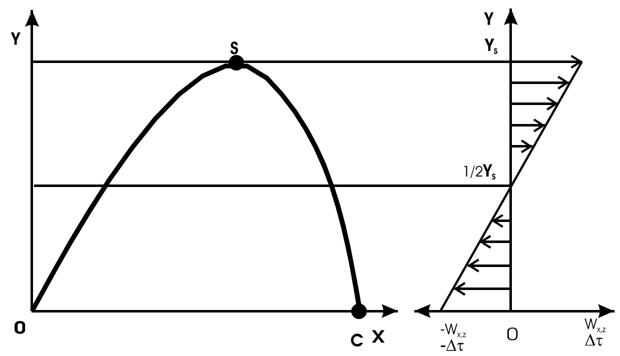

Figure 3. Continual meteorological elements variation with height

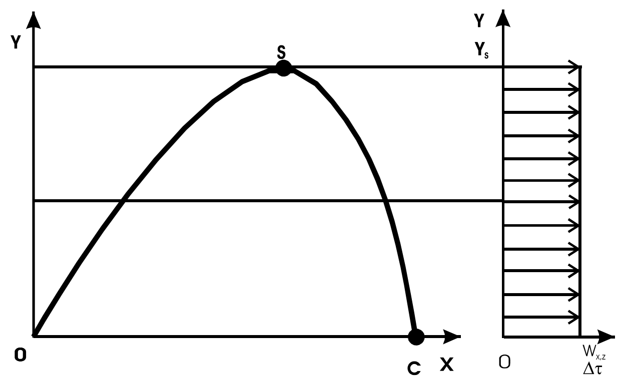

Figure 4. Constant variation of meteorological elements with height

When calculating altitude coefficients $(K)_{Wx,Wz,\Delta\tau}$, the distance variation ΔX_μ is used when meteorological elements' influences are in the form given in Fig.3 and distance variation ΔD_μ when meteorological elements' influences are in the form given in Fig.4. As for time saving, if n trajectory calculation is eliminated, for longitudinal wind $W_x = 10$ (m/s) for e.g., tree trajectory calculations ought to be done. First trajectory calculation ought to be done for determining the summit height, for nominal meteorological elements yielding for values Y_N , ie. Y_S and X_N , second for $\Delta X_{W_x} = X_{W_x} - X_N$, third for ΔD_{W_z} , and fourth for ΔX_τ . Calculation of Y_N , i.e. Y_S and X_N are already done when doing the firing tables. Therefore only one trajectory calculation must be done for determining the value of ΔX_μ when meteorological elements' influences are in the form given in Fig.3. i.e. three components of meteorological elements: W_x , W_z , of $\Delta\tau$ for determining distance variations ΔX_{W_x} , ΔZ_{W_z} , ΔX_τ . Can a relationship between ΔX_μ and altitude coefficients $(K)_{Wx,Wz,\Delta\tau}$ be established? From equation (1)

$$K_{Y_1} = 2(1 - S)$$

it is evident than when $\Delta X_\mu = 0$, then S should be 0.5 when introduced into (1)

$$\left(K_1\right)_{Wx,Wz,\Delta\tau} = 1$$

In case when $\Delta X_\mu \# 0$, extreme coefficient values are:

a) $$\Delta X_\mu \cong \Delta D_\mu \ (S \cong 0);\ K_{Wx,Wz,\Delta\tau} \cong 2 \quad (2)$$

b) $$-\Delta X_\mu \cong \Delta D_\mu \ (S \cong 1);\ K_{Wx,Wz,\Delta\tau} \cong 0$$

From the above speculation it can be concluded that the values of the altitude coefficients $(K)_{Wx,Wz,\Delta\tau}$ given with (1), using (2) can be obtained from intuitive equality

$$K_\mu = 1 + \frac{\Delta X_\mu}{\Delta D_\mu} \quad (3)$$

Necessary values of distance variations ΔX_μ for (3) can be obtained from trajectory calculations when the influences of meteorological elements $\mu(W_x, W_z, \Delta\tau)$ are in the form given in Figures 3, 5 or 6.

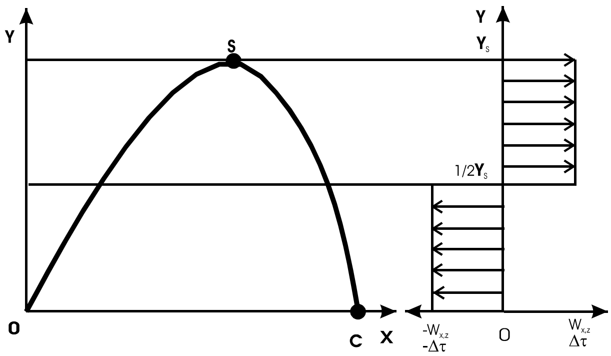

Figure 5. Discontinual meteorological elements' variation with height

The values of all altitude coefficients $(K)_{Wx,Wz,\Delta\tau}$ for certain projectile velocity, ballistic coefficient and set of projectile angles - gun elevations can be calculated in this way. The obtained values can be compared with those obtained by other methods. Usually, individual values are not suitable in practice, and they must be summed and divided by tree. From the above speculation, it follows that only tree projectile trajectories should be calculated, but not $3 \times n$, where ***n*** is the number of trajectory layers, and ***n*** is usually 10. Further time saving can be achieved if only one trajectory is calculated, as it is done in [5].

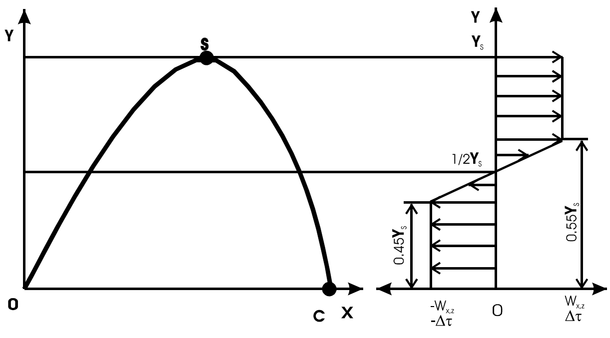

Figure 6. Mixed meteorological elements variation with height

All types of meteorological elements' variations with height are not suitable for various mathematical models of projectile trajectories. Depending on the appropriate mathematical model, the type of meteorological elements variation with height can be chosen.

Numerical test data

Results of numerical test of altitude coefficients $(K)_{Wx,Wz,\Delta\tau}$ obtained by formulae (1) are given in Table 1. Projectile ballistic characteristics are: standard law of resistance cx"1943", and ballistic coefficient $c = 0.5$. Muzzle velocities are in the range 100-900m/s with increment of 100 meter, and elevation angels are 10,25,45,60 and 80 degrees. Data are given for Vencel standard atmosphere (Vencelj) [7]. In the program for numerical calculation of altitude coefficients, 3D (three degree of freedom) mathematical model for calculating projectile trajectory is taken. Numerical test data are given in Table 1. For determining the method accuracy by comparison, altitude coefficients are calculated according to the existing procedure using the definition for calculating layer weights and node points r_i, Y_i / Y_S. Surface S is used in equation (1) and it is numerically determined by Simpson's rule for node points $n = 20$ in all cases for avoiding numerical errors. Numerical altitude coefficients and their mean values for W_x, W_z and $\Delta\tau$ are given in Table 1. Altitude coefficients W_x, W_z and $\Delta\tau$ are calculated according to the method existing in practice and equation (1) signed with subscript s. Under these values of altitude coefficients calculated by equation (3) are given and signed with n in subscript for meteorological elements variation with height similar to those given in Fig.2.

Table 1 Altitude coefficients

$V_0\,(\text{m/s}) / W_{x,z,\tau}$	$\theta = 10^\circ$	$\theta = 25^\circ$	$\theta = 45^\circ$	$\theta = 60^\circ$	$\theta = 80^\circ$
100. W_x	1.264	1.260	1.263	1.242	1.075
W_z	1.328	1.312	1.263	1.195	1.060
$\Delta\tau$	1.244	1.170	1.130	1.061	0.931
$(S_r)s$	1.279	1.247	1.219	1.166	1.022
$(S_r)n$	1.316	1.288	1.239	1.165	0.968
200. W_x	1.257	1.264	1.262	1.236	1.075
W_z	1.328	1.308	1.250	1.175	1.033
$\Delta\tau$	1.192	1.175	1.118	1.048	0.921
$(S_r)s$	1.259	1.249	1.210	1.153	1.010
$(S_r)n$	1.303	1.276	1.224	1.176	0.996
300. W_x	1.131	1.134	1.165	1.160	1.032
W_z	1.131	1.286	1.218	1.136	0.991
$\Delta\tau$	1.077	1.027	0.987	0.926	0.813
$(S_r)s$	1.113	1.149	1.123	1.074	0.945
$(S_r)n$	1.071	1.076	1.077	1.028	0.874
400. W_x	1.478	1.255	1.001	0.939	0.825
W_z	1.259	1.106	1.010	0.925	0.789
$\Delta\tau$	1.359	1.161	0.871	0.752	0.641
$(S_r)s$	1.365	1.174	0.961	0.872	0.752
$(S_r)n$	1.478	1.240	0.765	0.681	0.586
500. W_x	1.411	1.521	1.220	1.041	0.806
W_z	1.333	1.179	1.006	0.893	0.755
$\Delta\tau$	1.282	1.357	1.087	0.927	0.794
$(S_r)s$	1.342	1.352	1.104	0.954	0.785

$(S_r)n$	1.438	1.455	1.177	0.818	0.536
600. W_x	1.379	1.553	1.336	1.107	0.820
W_z	1.352	1.244	1.045	0.910	0.767
$\Delta\tau$	1.242	1.348	1.174	1.008	0.866
$(S_r)s$	1.324	1.382	1.185	1.008	0.818
$(S_r)n$	1.391	1.440	1.256	1.045	0.702
700. W_x	1.365	1.520	1.397	1.142	0.832
W_z	1.360	1.284	1.080	0.929	0.780
$\Delta\tau$	1.218	1.293	1.197	1.029	0.877
$(S_r)s$	1.308	1.366	1.225	1.033	0.830
$(S_r)n$	1.367	1.435	1.268	1.055	0.736
800. W_x	1.354	1.471	1.420	1.158	0.882
W_z	1.362	1.306	1.108	0.941	0.779
$\Delta\tau$	1.199	1.231	1.185	1.012	0.831
$(S_r)s$	1.305	1.336	1.238	1.037	0.831
$(S_r)n$	1.382	1.368	1.263	1.039	0.715
900. W_x	1.352	1.424	1.414	1.143	0.787
W_z	1.365	1.317	1.128	0.936	0.751
$\Delta\tau$	1.190	1.179	1.153	0.945	0.744
$(S_r)s$	1.302	1.307	1.232	1.008	0.761
$(S_r)n$	1.355	1.358	1.255	0.972	0.640

Conclusion

Classical procedure for altitude coefficients calculations is based on layer weights or their functions in graphic forms. For this purpose, projectile trajectory must be divided into layers and the influence per unit meteorological element on the trajectory elements calculated, related to meteorological elements influence in all layers, i.e. along total trajectory height. This article presents an original procedure for altitude coefficients calculations that eliminates the need for layer weights or their function calculations for each meteorological element.

The existing meteo-measurements data processing given in [1, 2], apart from not being up to date, is also not justified because they neglect projectile ballistic characteristics. Approximation curve of the weight function enables determining ballistic values of meteorological elements as if they were mean values from the gun position up to the height that is not equal to projectile summit height Y_S . That height is called altitude height, signed as Y_1. Relation Y_1/Y_S is called altitude coefficient. Numerical calculation of the altitude coefficient is a mechanized procedure [5,6], but it is based on existing definitions and methods proposed in references that are not suitable at this time [7]. Introduction of the new method for calculating values of altitude coefficients $(K)_{Wx,Wz,\Delta\tau}$ the number of projectile trajectory calculations is reduced to three. It follows that only three projectile trajectories ought to be calculated, but not $3\times n$, where $\boldsymbol{n}$ is the number of trajectory layers, and $\boldsymbol{n}$ is usually 10 or more. Further time saving can be achieved if only one trajectory is calculated, as it is done in [5].

It is difficult to confirm the accuracy of the method, since the mean value of three individual altitude coefficients for three meteorological elements are given. Results $(S_r)s$ obtained according to "accurate" methods for calculating layer weight functions (signed with subscript s) compared with the new calculations $(S_r)n$ (signed with subscript n) coincide for elevations of 45° on the first decimal position for all muzzle velocities.

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Nov postupak nalaženja koeficijenata uslovnih visina

Dat je nov postupak proračuna koeficijenata uslovnih visina. Oni služe za korišćenje uvedenog u naoružanje biltena meteo-srednji u artiljerijskim jedinicama. Određene su uslovne visine odnosno koeficijenti uslovnih visina za jedan broj uslova gađanja. Izložen postupak ne zahteva nalaženje težina slojeva koji se određuju prema definicijama datim u vreme početka balističke prakse za koje se nepotrebno troši dosta vremena.

Ključne reči: **spoljna balistika, balistička merenja, meteorološka merenja, tablice gađanja, artiljerijsko gađanje.**

Новая процедура вычисления коэффициентов условных высот

В настоящей работе приведён новый поступок расчёта коэффициентов влияния баллистических значений метеорологических элементов и обнаружено, что они могут использоватся для вычисления коэффициентов условных высот нужных артиллерийским частям, которые пользуются средним метеорологическим бюллетенем. Здесь определены условные высоты для определённого числа условий стрельбы, т.е. определены коэффициенты условных высот. Растолкованный поступок не требует ни познания вычисления, ни веса слоев на которые тратится много времени.

Ключевые слова: внешняя баллистика, баллистические измерения, метеорологические измерения, таблицы стрельбы, артиллерийские стрельбы.

Nouvelle procédure pour calculer les coefficients d’altitude

Dans le cadre de ce travail on présente une nouvelle procédure pour déterminer l’influence des valeurs balistiques des éléments météorologiques. On a constaté qu’ils peuvent servir pour calculer les coefficients des altitudes conditionnelles necessaires aux unités d’artillerie qui utilisent le bulletin météo-moyen. Pour un nombre de conditions de tir on a déterminé les altitudes conditionnelles, c’est-à-dire les coefficients des altitudes conditionnelles. Cette nouvelle procédure n’exige pas la connaissance et le calcul du poids des couches.

Mots clés: **balistique extérieure, mesurement balistique, mesurement météorologique, tableaux de tir, tirs d’artillerie.**