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Geometrical influence on solid rocket propellant ignition

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The solid rocket propellant geometrical properties influence on the ignition process is discussed. The critical review of the recent investigations is presented. The mathematical model of the ignition process which enables determination of the two-dimensional temperature field in the solid rocket propellant heated by the external thermal flux, in the presence of the interior heat source due to exothermal chemical reaction, is developed. Reactant consumption is also involved. Nonsteady and nonlinear equations of the energy and mass conservation are numerically solved using the method of finite differences. The obtained results are presented in the form of tables and diagrams and qualitative agree with the reference data. The simplified model is also developed and the application possibility considered.

Key words: **solid rocket propellant, ignition, geometrical properties.**

Introduction

THE fact that combustion of solid rocket propellants and explosive materials often begins with the ignition of sharp edges, angles and convex parts due to their faster heating and achieving higher temperature during some heating regimes in comparison with flat surface is known to investigators in this domain. The surface decomposition causes faster propellant decomposition and owing to the production of additional energy, accelerates the ignition process and shortens time delay. Keller, Baer and Ryan described the effect of surface roughness on solid propellant ignition based on experimental results, in their early work [6], while Merzhanov and Averson theoretically analyzed the problem in [7].Complex nature of the problem caused development of various models with different approximations [1-5]. In some references the received results are shown in the form of relations between non-dimensional parameters including various quantities important for the ignition process [1, 2], while some other models (for example [4, 5]) consider simplified process so they can be applied only in particular cases. That was the reason for realizing two-dimensional solid rocket propellant ignition mathematical model where surface roughness is approximated with arbitrary wedge angle ($0 < \phi < 180°$). The energy and concentration conservation equations, non steady and nonlinear, are solved numerically employing finite differences method. In such a way propellant temperature history and reactant depletion are determined during the ignition process. The obtained results are compared with usually considered cases of propellants as flat semi-infinite plates. Because the calculation time is long, a simplified one-dimensional model is also developed, but with insufficient precision. This model can be used only for some fast, preliminary analysis.

Geometrical properties effect on solid rocket propellant ignition process

Vorsteveld,L.G. and Hermance,C.E. considered a homogeneous, nontransparent reactive solid with constant physical properties, independent of temperature and decomposition degree, in a form of infinity square corner wedge as a representative of solid rocket propellant in ref. [1]. The temperature distribution in the propellant was defined with two-dimensional heat conduction equation, including exothermal chemical reaction of Arrhenius type, in dimensionless form:

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\xi^2}+\frac{\partial^2\theta}{\partial\eta^2}+A(1-\varepsilon)\cdot\exp(-E'/\theta) \qquad (1)$$

Concentration change of the consumed part of propellant is given by equation:

$$\frac{\partial\varepsilon}{\partial\tau}=\left(\frac{A}{B}\right)\cdot(1-\varepsilon)\cdot\exp(-E'/\theta) \qquad (2)$$

The initial and boundary conditions of eqs. (1) and (2) are:

$$\theta(\xi,\eta,0)=1,\quad \varepsilon(\xi,\eta,0)=0 \qquad (\xi>0,\ \eta>0) \qquad (3)$$

$$\begin{gathered}\frac{\partial\theta}{\partial\xi}(\infty,\eta,\tau)=\frac{\partial\theta}{\partial\eta}(\xi,\infty,\tau)=0;\\ \frac{\partial\theta}{\partial\xi}(0,\eta,\tau)=\frac{\partial\theta}{\partial\eta}(\xi,0,\tau)=-1\end{gathered} \qquad (4)$$

In the above equations the following marks are used:

$A=Q\cdot z\cdot\lambda T_i\cdot\dot{q}^2$	- dimensionless ratio of chemical energy to external heat flux;
$B=Q/\rho c T_i$	- dimensionless parameter of heat release;
$Q\left[\mathrm{J/m^3}\right]$	- heat of chemical reaction;
$z\left[\mathrm{s^{-1}}\right]$	- pre-exponentional factor in Arrhenius law determining exotherm chemical reaction in the solid propellant;
$\lambda\left[W/mk\right]$	- thermal conductivity;
$\rho\left[\mathrm{kg/m^3}\right]$	- propellant density;
$c\left[\mathrm{J/kgK}\right]$	- specific heat of the propellant;

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$E[\text{J/mol}]$ - activation energy;
$R[\text{J/molK}]$ - universal gas constant;
$E' = E/RT$ - nondimensional activation energy;
$\theta = T/T_i$ - dimensionless temperature;
$\xi = \dot{q}x/\lambda T_i$ - dimensionless distance along x axis;
$\eta = \dot{q}y/\lambda T_i$ - dimensionless distance along y axis;
$\dot{q}\left[W/\text{m}^2\right]$ - external heat flux;
$\tau = \dot{q}^2 t/\lambda\rho c T_i^2$ - dimensionless time;
ε - fraction of reactant consumed;
T_i - initial temperature.

The equations (1) to (4) were solved using the same parameter range $\left(A = 1\cdot 10^8 - 1{,}5\cdot 10^{29},\ B = 0{,}9 \text{ and } 4{,}5,\ E' = 33\frac{1}{3};\ 50;\ 66\frac{2}{3};\ 83\frac{1}{3} \text{ and } 100\right)$, as in some former works (for example [8, 9]). Maximum values of θ and ε is always obtained at the tip of the wedge. The ignition temperature θ_c dependence on delay time is shown (for all considered parameter combinations) by the equation:

$$\theta_c = 1 + 2{,}572\sqrt{\tau_c} \tag{5}$$

with 4% accuracy (the results with high values of ε_c are neglected). In the inert heating case there is dependence in the form

$$\theta_i = 1 + 4\sqrt{\tau_c/\pi} \tag{6}$$

and the following relation can be writen

$$\frac{\theta_c - 1}{\theta_i - 1} = 1{,}14 \tag{7}$$

which points that the ignition temperature of the solid rocket propellant, with the described profile, exceeds inert material value for 14%. The heating time ratio of reactive and inert solid, when the same temperature value is achieved, is obtained equal to 0,77.

For reasons of comparing results with those obtained using one-dimensional models (propellant described as semi-infinite flat plane) of Bradley [9]

$$A = (E')^{1/2}\left[1 + 2(\tau_c/\pi)^{1/2}\right]^{-1}\exp\left\{E'\Big/\left[1 + 2(\tau_c/\pi)^{1/2}\right]\right\} \tag{8}$$

and Linan and Williams [8]

$$A = 0{,}65(E')^{1/2}(\pi\tau_c)^{-1/4}\cdot \left[1 + 2(\tau_c/\pi)^{1/2}\right]^{-1}\exp\left\{E'\Big/\left[1 + 2(\tau_c/\pi)^{1/2}\right]\right\} \tag{9}$$

the authors presented correlation equation of the calculated data, which couples the ignition delay time τ_c with the previously noted parameters A and E'

$$A = C\cdot(E')^{1/2}\left[1 + 4\left(\frac{\tau_c}{\pi}\right)^{1/2}\right]^{-1}\cdot (\pi\tau_c)^{-1/4}\exp\left\{E'/\left[1 + 4(\tau_c/\pi)^{1/2}\right]\right\} \tag{10}$$

The constant C is equal to 5(±15%) and implies that the process is accelerated in relation to one-dimensional models (analytical in [8] and empirical in [9]), so that the calculated ignition delay times are shortened by factor of 2.5 to 3.4 in comparison with usually studied one-dimensional solid rocket propellant cases (for the same parameter values). The obtained ignition temperatures are slightly higher than in flat plane case.

The same group of authors described in ref. [2] the case of reactive solid with acute angle wedge profile under identical suppositions as in [1].The mathematical model of propellant ignition was presented in cylindrical coordinates in domain $0 < \phi < \phi_0$ and $0 < r < b$ to adjust with the studied geometry:

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial\theta}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2\theta}{\partial\phi^2} + A(1-\varepsilon)\cdot\exp(-E'/\theta) \tag{11}$$

$$\frac{\partial\varepsilon}{\partial\tau} = \frac{A}{B}(1-\varepsilon)\exp(-E'/\theta) \tag{12}$$

with initial and boundary conditions

$$\theta(r,\phi,0) = 0 \qquad \varepsilon(r,\phi,0) = 0 \tag{13}$$

$$\frac{\partial\theta}{\partial r}(b,\phi,\tau) = 0,\ \frac{\partial\theta}{r\partial\phi}(r,0,\tau) = -\frac{\partial\theta}{r\partial\phi}(r,\phi_0,\tau) = -1 \tag{14}$$

In the equations (11-14) the denotations are as follows:
ϕ - angular coordinate
$r = \dot{q}r'/\lambda T_i$ - dimensionless radial coordinate (distance);
$r'[\text{m}]$ - radial distance and
$b[\text{m}]$ - wedge length (the limited value contrary to [1] where semi-limited solid is analyzed).

Due to the problem of symmetry, an additional condition is applied along the wedge centerline:

$$\frac{\partial\theta}{r\partial\phi}(r,\phi_0/2,\tau) = 0 \tag{15}$$

and assumption of constant external heat flux is also used. Numerical method of finite differences was applied aiming at solving the system of equations (11-15). The range of studied parameters was as in [1], except A which maximum value attained 10^{28}. The dimensionless ignition temperature dependencies on parameter τ_c for some definite angles are correlated (within an error range of ±5%):

$$\phi_0 = \pi \quad \theta_c = 1 + 1{,}3\sqrt{\tau_c} \tag{16a}$$

$$\phi_0 = \frac{\pi}{2} \quad \theta_c = 1 + 2{,}57\sqrt{\tau_c} \tag{16b}$$

$$\phi_0 = \frac{\pi}{4} \quad \theta_c = 1 + 5{,}24\sqrt{\tau_c} \tag{16c}$$

$$\phi_0 = \frac{\pi}{8} \quad \theta_c = 1 + 9{,}94\sqrt{\tau_c} \tag{16d}$$

In case when dimensionless activation energy E' is equal to $66\frac{2}{3}$ and parameter $B = 4.5$ ignition time delays are related approximately, for angles $\phi_0 = 22.5^\circ, 45^\circ, 90^\circ$ and 180°:

$$\tau_c(\pi) \approx 3\tau_c\left(\frac{\pi}{2}\right) \approx 9\tau_c\left(\frac{\pi}{4}\right) \approx 27\tau_c\left(\frac{\pi}{8}\right) \tag{17}$$

This means that increasing the angle ϕ_0 for factor 2 causes an increase of ignition time delay approximately for factor 3 in the studied parameter range. For purposes of comparison with previously obtained investigation results (eqs. (8-10)) calculated delay times τ_c and parameters A and E' were correlated in the following form:

$$A = C_1 \left(E'\right)^{1/2} \left(\pi\tau_c\right)^{-1/4} \cdot \left[1 + C_2\left(\tau_c/\pi\right)^{1/2}\right]^{-1} \exp\left\{E'\Big/\left[1 + C_2\left(\tau_c/\pi\right)^{1/2}\right]\right\} \quad (18)$$

Magnitudes of constants C_1 and C_2 in function of the angle ϕ_0 are presented in Table 1.

Table 1.

$\phi_0\left(^\circ\right)$	180	90	45	22,5
C_1	0,65	6	12	24
C_2	2	4	6	8

The difference between constant C_1 for $\phi_0 = 90^\circ$ and data from [1] where $C_1 = 5$ can be seen.

Margolin, Mohin and Krupkin theoretically investigated the ignition of sharpened form (wedge, cone) reactive bodies with homogeneous exothermic chemical reaction in reference [3]. They obtained heat conduction equations including volume thermal energy source for flat wedge (index 1) and cone (index 2):

$$L_1(\Theta) = -\frac{\partial\Theta}{\partial u} + \frac{1}{\eta}\left(\eta\frac{\partial\Theta}{\partial\eta}\right) + \frac{1}{\eta^2}\frac{\partial^2\Theta}{\partial\varphi^2} + {} + \Omega\exp\left[\Theta/(1+\beta\Theta)\right] = 0 \quad (19)$$

$$L_2(\Theta) = -\frac{\partial\Theta}{\partial u} + \frac{1}{\eta}\left(\eta^2\frac{\partial\Theta}{\partial\eta}\right) + \frac{1}{\eta^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial\Theta}{\partial\phi}\right) + {} + \Omega\exp\left[\Theta/(1+\beta\Theta)\right] = 0 \quad (20)$$

where $\Theta = E\left(T - T^*\right)/RT^{*2}$, $\eta = r/r_0$, $u = \lambda t/\left(\rho r_0^2\right)$, $r_0 = \lambda RT^*/qE$, $\beta = RT^*/E$ and $\Omega = r_0 QZ\exp\left(E/RT^*\right)$ are dimensionless variables and T^* is the characteristic ignition temperature.

Using analytical solution of the inert heating case equation $(\Omega = 0)$ under assumption that chemical reactions' thermal effect appears immediately before the ignition moment, they obtained the first approximation relations for the ignition time:

$$u_1^* = \Theta_i^2\phi_0^2/\pi\,,\ u_2^* = 0{,}25\cdot\pi\cdot\Theta_i^2\cdot tg^2\left(\phi_0/2\right) \quad (21)$$

For small angle values, middling of equations (19) and (20) by angle ϕ was done. The attained one-dimensional equations were numerically studied in the range $5 < |\Theta_i| < 25$ for $\beta = 0$, and results approximated by formulas (with error up to 15%):

$$\omega_1^* = 0{,}33 + 2{,}44|\Theta_i|^{-0{,}65},\ \omega_2^* = 0{,}65 + 3{,}3|\Theta_i|^{-0{,}5} \quad (22)$$

with $\omega_1 = \Omega\phi_0^2$ and $\omega_2 = 4\Omega tg^2\left(\phi_0/2\right)$.

Comparing with results for $\phi_0 = \pi/2$ the authors established the ignition criterion interpolation expressions in function of the angle (valid for small angles):

$$\Omega_1^* = \left(0{,}13\cos\phi_0 + \left(1{,}3 - 0{,}3\cos\phi_0\right)|\Theta_i|^{-2/3}\right)\cdot\left(\pi/2\phi_0\right)^2 \quad (23a)$$

$$\Omega_2^* = \left(0{,}2\cos\phi_0 + \left(1{,}3 - 0{,}25\cos\phi_0\right)|\Theta_i|^{-2/3}\right)\cdot ctg^2\left(\frac{\phi_0}{2}\right) \quad (23b)$$

Transition to the dimensional variables produced the following ignition delay time formulations:

$$t_1^* = \frac{\pi}{4}\frac{\lambda\rho c\cdot\left[T_1^*(\phi_0) - T_i\right]^2}{\dot{q}^2}\cdot\left(\frac{2\phi_0}{\pi}\right) \quad (24a)$$

and

$$t_2^* = \frac{\pi}{4}\frac{\lambda\rho c\cdot\left[T_2^*(\phi_0) - T_i\right]^2}{\dot{q}^2}\cdot tg^2\left(\frac{\phi_0}{2}\right) \quad (24b)$$

Values $T_1^*(\phi_0)$ and $T_2^*(\phi_0)$ can be received from (23a) and (23b).

Ratios of the propellant ignition delay times for wedge or cone to the case of flat semi-infinite plane were presented as:

$$\frac{t_1^*(\phi_0)}{t^*(\pi/2)} = \left(\frac{T_1^*(\phi_0) - T_i}{T^*(\pi/2) - T_i}\right)^2\left(\frac{2\phi_0}{\pi}\right)^2 \quad (25a)$$

$$\frac{t_2^*(\phi_0)}{t^*(\pi/2)} = \left(\frac{T_2^*(\phi_0) - T_i}{T^*(\pi/2) - T_i}\right)^2 tg^2\left(\frac{\phi_0}{2}\right) \quad (25b)$$

Comparison with other investigators' results (experimental or theoretical) is not shown in the paper.

The same authors, in continued work [4], employed the stationary mathematical model describing the constant thermal flux ignition of the propellant in the shape of a wedge or cone with optional angle and the base hold at constant (initial) temperature. The heat conduction equation middling by the ϕ and the term $\exp(-E/RT)$ expanding produced the expression:

$$\frac{1}{x^k}\frac{d}{dx}\left(x^k\frac{d\Theta}{dx}\right) + \frac{k}{x} + \omega\exp\left(\Theta/(1+\beta\Theta)\right) = 0 \quad (26)$$

with boundary conditions $x^k\Theta'(0) = 0$, $\Theta(x_0) = -x_0$. Here $\Theta = E\left(T - T_s\right)/RT_s^2$, $x = r/r_0$, $x_0 = L/r_0$, $\beta = \frac{RT_s}{E}$, $r_0 = \lambda RT_s^2\phi_0/\dot{q}E$. Parameter $\omega = r_0^2 EQZ\cdot\exp\left(-E/RT_s\right)/\lambda RT_s$ characterizes the relation between the chemical reaction and the external heat flux. The approximate dependencies of dimensionless parameters $\omega^* = \omega^*\left(X_0\right)$ and $\Theta^*\left(X = 0,\ \omega = \omega^*\right)$ (in the interval $0 < X_0 < 50$) were numerically determined for the wedge:

$$\omega^* = 2/x_0^2 + 0.46/x_0 + 0.37/\ln\left(1 + x_0\right) \quad (27a)$$

$$\Theta^*(0) = 1 + 1/\ln\left(15 + x_0^2\right) \quad (27b)$$

and the cone

$$\omega^* = 3.32/x_0^2 + 1.533/x_0 + 0.51 \quad (28a)$$

$$\Theta^*(0) = 1.45 + 0.15/(1 + 0.24x_0) \quad (28b)$$

but without calculating results presentation.

Strakovskiy,L.G. investigated the ignition of explosive materials with semi-infinite wedge geometry [5] using the equations:

$$\frac{\partial\Theta}{\partial\tau} = \frac{\partial^2\Theta}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\Theta}{\partial\xi} + \frac{1}{\xi^2}\frac{\partial^2\Theta}{\partial\phi^2} + \exp\left(\frac{\Theta}{1+\beta\Theta}\right),$$
$$\tau \geq 0,\; 0 \leq \xi < \infty,\; 0 < \phi \leq \alpha/2 \quad (29)$$

$$\Theta(\xi,\phi,0) = \Theta_i,\; \frac{\partial\Theta}{\partial\xi}(0,\phi,\tau) = -\sigma \quad (30)$$

$$\frac{1}{\xi}\frac{\partial\Theta}{\partial\phi}(\xi,\alpha/2,\tau) = \sigma,\quad \frac{\partial\Theta}{\partial\phi}(\xi,0,\tau) = 0 \quad (31)$$

where: $\tau = t/t_{ad}$, $t_{ad} = \frac{c}{QZ}\frac{RT_*^2}{E}\exp(E/RT_*)$, $\xi = x/x_*$, $x_* = \sqrt{at_{ad}}$, $\Theta = E(T - T_*)/RT_*^2$, $\beta = RT_*/E$, $\sigma = \frac{\dot{q}x_*}{\lambda}\frac{E}{RT_*^2}$ and α - the wedge top angle. Above equations were treated numerically. The obtained parameters were compared to the corresponding values for the case of explosive material in the flat semi-infinite plane form. It was deduced that the ratio of the ignition delay time $t_{ig,k}$ and the time delay of the same reactive material under the influence of the identical thermal flux for the level plane $t_{ig,\infty}$, in the studied range of parameters $\Theta_0 \in (-24,-13)$, $\beta \in (0.02,\ 0.042)$, and $\sigma \in (1.16,\ 6.9)$ is practically independent of $t_{ig,\infty}$ $(\omega_k = t_{pr,k}/t_{pr,\infty} \approx const)$, so the following expression (with error $\leq$ 10%) can be used:

$$t_{ig,k} = \omega_k A/\dot{q}^m \quad (32)$$

if the relation $t_{ig,\infty} = A/\dot{q}^m$ is known. For the hexogen $\omega_k = 0.29$. In the case of hexogen ignition with heat flux $\dot{q} = 5{,}5\ \mathrm{MW/m^2}$ the calculated value of $t_{ig,k}$ is about $100 \cdot 10^{-6}\,\mathrm{s}$. Experimentally measured delay times were in the interval $(60 \div 80) \cdot 10^{-6}\,\mathrm{s}$.

The decay of the angle ϕ effect on the material temperature field in regions near the wedge tip, for the examined process parameters was indicated. Introducing the medium dimensionless temperatures $\bar{\Theta}$ for angles ϕ the following expressions were obtained:

$$\frac{\partial\Theta}{\partial\tau} = \frac{\partial^2\Theta}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\Theta}{\partial\xi} + \frac{2\sigma}{\alpha\xi} + \exp\left(\frac{\Theta}{1+\beta\Theta}\right) \quad (33)$$

$$\Theta(\xi,0) = \Theta_i,\; \frac{\partial\Theta}{\partial\xi}(0,\tau) = -\sigma,\; \frac{\partial\Theta}{\partial\xi}(\infty,\tau) = 0 \quad (34)$$

This is practically one-dimensional wedge ignition model and computing time is considerably reduced (more than one order of magnitude). In Table 2, the calculated ignition delay time ratios for two-dimensional $(t_{ig,2D}$, eqs. $(29-31))$ and one-dimensional $(t_{ig,1D}$, eqs. $(33-34))$ model are presented in function of α.

Table 2.

$\alpha(^\circ)$	30	45	60	90	120
$\frac{t_{ig,2D}}{t_{ig,1D}}$	0,995	0,99	0,97	0,94	0,86

The satisfactory agreement of simplified and complete process model is achieved for $\varphi \leq 90^\circ$, but calculating conditions were not indicated.

The mathematical model of geometrical properties influence on solid rocket propellant ignition process

The significance of this problem, insufficient clarity in some results and incomplete modeling in others [1-5], created the need for one comprehensive solid rocket propellant ignition development for the shape different from the usually considered flat semi-infinite plane. Propellant, in the form of definite length wedge ($0 < r \leq L$) with arbitrary angle ϕ $(-\phi_0 \leq \phi \leq \phi_0,\quad 0 < \phi_0 \leq \pi/2)$ exposed to the external heat flux $\dot{q}$ (Fig.1) with thermal source due to exothermal chemical bulk reaction of Arrhenius type is considered.

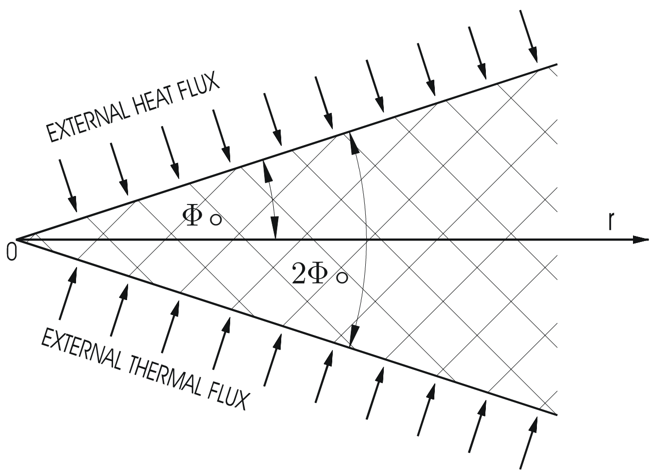

Figure 1. Schematic illustration of solid rocket propellant in the wedge form ignited by external thermal flux $\dot{q}$.

For the propellant temperature profiles determination, non-steady two-dimensional heat conduction equation in cylindrical coordinates is applied:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{a}{r^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{QZ(1-\varepsilon)}{\rho c}\exp(-E/RT) \quad (35)$$

The initial and boundary conditions are:

$$T(r,\phi,0) = T_i \quad \text{for } 0 < r \leq L,\; -\phi_0 \leq \phi \leq \phi_0 \quad (36)$$

$$\frac{\lambda}{r}\frac{\partial T}{\partial\phi}(r,\phi_0,t) = \dot{q},\; \frac{\partial T}{\partial\phi}(r,0,t) = 0,\; \frac{\partial T}{\partial\phi}(L,\phi_0,t) = 0 \quad (37)$$

$$\frac{\partial T}{\partial r}(0,\phi,t) = 0,\; \frac{\partial T}{\partial r}(L,\phi_0,t) = 0 \quad (38)$$

The concentration change of the reacted propellant part during ignition process is governed by equation:

$$\frac{\partial \varepsilon}{\partial t} = (1-\varepsilon) Z \cdot \exp(-E/RT) \tag{39}$$

with initial condition $\varepsilon(r,\phi,0)=0$.

Introducing the variables $\theta = T/T_i$, $r_1 = r/L$, $\phi_1 = \phi/\phi_0$ and $E_1 = E/RT_i$ in order to facilitate the calculation process the system of equations is transformed into a nondimensional mode:

$$\frac{\partial \theta}{\partial t} = \frac{a}{L^2}\left(\frac{\partial^2 \theta}{\partial r_1^2} + \frac{1}{r_1}\frac{\partial \theta}{\partial r_1}\right) + \frac{a}{L^2\phi_0^2}\frac{1}{r_1^2}\frac{\partial^2 \theta}{\partial \phi_1^2} + \frac{QZ(1-\varepsilon)}{\rho c T_i}\exp(-E_1/\theta) \tag{40}$$

$$\frac{\partial \varepsilon}{\partial t} = (1-\varepsilon) Z \cdot \exp(-E_1/\theta) \tag{41}$$

$$\frac{\partial \theta}{\partial \phi_1}(r_1,1,t) = \frac{L r_1 \phi_0}{T_i}\frac{\dot{q}}{\lambda},\ \frac{\partial \theta}{\partial \phi_1}(r_1,0,t)=0,\ \frac{\partial \theta}{\partial \phi_1}(1,1,t)=0 \tag{42}$$

$$\frac{\partial \theta}{\partial r_1}(0,\phi_1,t)=0,\ \frac{\partial \theta}{\partial r_1}(1,1,t)=0 \tag{43}$$

The partial differential equations system (40-43) is nonlinear and non steady and solved by the numerical finite-difference method [13, 14]. The implicit scheme is used for the spatial derivatives decomposition and explicit one for the temporal derivative. For the radial variable derivative, the following approximation is applied [12]:

$$\left[\frac{\partial^2 \theta}{\partial r_1^2} + \frac{1}{r_1}\cdot\frac{\partial \theta}{\partial r_1}\right]_{i,j}^{n} \approx \approx \frac{1}{(\Delta r_1)^2}\left[\left(1+\frac{1}{2i}\right)\theta_{1+1,j}^{n} - 2\theta_{i,j}^{n} + \left(1-\frac{1}{2i}\right)\cdot\theta_{1+1,j}^{n}\right] \tag{44}$$

where $r_1 = i\cdot\Delta r_1$. The numerical calculation procedure is presented in full in the reference [16].

The ignition temperature of the semi-infinite level plane solid rocket propellant form, defined as in ref. [11] and [15] is employed for the ignition criterion:

$$T_{s,ig} = \frac{E}{R\ell n\left\{\dfrac{\frac{\pi}{2}\rho Q Z \lambda (T_{s,ig}-T_p)}{0{,}15\ell n10(\dot{q}_{ig})^2}\right\}} \tag{45}$$

Because the temperature gradients in time moments immediately before the ignition (for $\dot{q}$ intensities of practical interest in the solid rocket propellant ignition) have very high values, $T_{s,ig}$ for $\phi_0 = \pi/2$ $(2\phi_0 = 180^\circ)$ are chosen for all angles $\phi_0 < \pi/2$ in the case of the same propellant and thermal flux.

Fig.2 comparatively presents curves of the maximum temperatures change in function of time for the well-known Russian propellant N + 1% C [10] with thermo-physical properties $E = 1{,}4654\cdot10^5$ [J/mol], $Q = 1{,}13\cdot10^6$ J/kg, $Z = 9.08\cdot10^{13}$ $[s^{-1}]$, $\rho = 1600$ $[\mathrm{kg/m^3}]$, $c = 1465$[J/kg], $\lambda = 0{,}2345$[W/mk] for three wedge tip angle values $2\phi_0 = 45^\circ$, 60° and 90°. The propellant is heated by external thermal flux $\dot{q} = 0{,}20934\,\mathrm{MW/m^2}$ (5 cal/cm²s). The angle decreasing under the same conditions causes diminution of the ignition delay times.

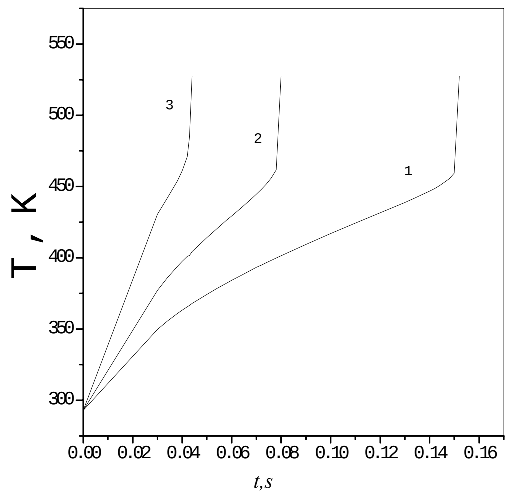

Figure 2. Comparison of maximum temperatures rise for the propellant [10] heated by $\dot{q}$=0,20934 MW/m^2 : 1- $2\phi_0$=90° ; 2- $2\phi_0 = 60^\circ$ and 3- $2\phi_0 = 45^\circ$.

The temperature variation along the surface of the propellant [10] with the angle $2\phi_0 = 22{,}5^\circ$ and heated by $\dot{q} = 0{,}41868\,\mathrm{MW/m^2}$ for different time periods as shown in Fig.3. It can be seen that the greatest rate of temperature growth appears near the wedge top.

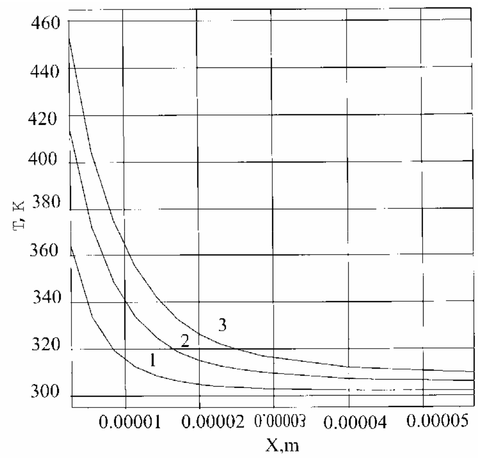

Figure 3. Temperature profile of the wedge surface with angle $2\phi_0 = 22{,}5^\circ$ for propellant [10] exposed to heat flux $\dot{q} = 0{,}41868$ MW/m^2 in the moments: 1 – 0.001s, 2 – 0.002s and 3 – 0.003s.

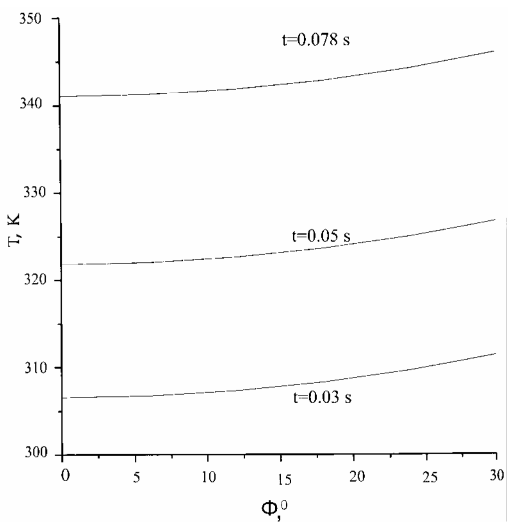

Figure 4. The propellant [10] temperature distribution in function of the angle ϕ, at the radial distance $r = 20\cdot10^{-6}$ [m] from the top of the wedge with $2\phi = 60^\circ$, under the effect of $\dot{q} = 0{,}20934\,\mathrm{MW/m^2}$ for time: 1–0.03s, 2–0.05s and 3–0.078s.

Fig.4 shows temperature profile along the angular coordinate ϕ (0<ϕ<30°) at radial distance from the propellant [10] top $r\!=\!20\cdot10^{-6}$ [m], when $2\phi_0\!=\!60^\circ$ and $\dot q\!=\!0,20934\,\mathrm{MW/m^2}$, for the three time values: 0.03s, 0.05s and 0.078s.

The temperature magnitudes are close in all cases at the noticed radial distance. The temperature alternation in function of the angle ϕ is much greater at the $r\!=\!40\cdot10^{-6}$ [m] while the temperature level is lower (Fig.5).

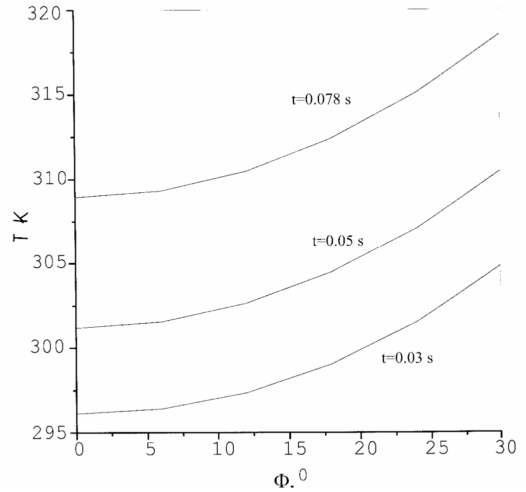

Figure 5. Temperature of the propellant [10] in the form of a wedge with $2\phi\!=\!60^\circ$ along the angle ϕ, for $r=40\cdot10^{-6}\,[\mathrm{m}]$, and $\dot q=0,20934\,\mathrm{MW/m^2}$ at three times: 1 – 0.03s, 2 – 0.05s and 3 – 0.078s.

The ignition delay times of the propellant [10] heated with thermal flux $\dot q\!=\!0,20934\,\mathrm{MW/m^2}$ and their relations to the flat plane shaped propellant cases $\left(t_{ig,180^\circ}\right)$ are presented in Table 3.

Table 3.

Angle $2\phi_0\,(^\circ)$	$\dot q = 0,20934\,\mathrm{MW/m^2}$	
	$t_{ig}\,[\mathrm{s}]$	$t_{ig}/t_{ig,180^\circ}$
180	0,476	1
90	0,152	0,3193
60	0.080	0,16807
45	0,044	0,0924
30	0,0144	0,03025
22,5	0,0088	0,0185
20	0,0072	0,0151

The propellant [10] temperature dependence of the angle ϕ at the radial coordinate $r = 20\cdot10^{-6}$ [m] with incident heat flux $\dot q = 0,41868\,\mathrm{MW/m^2}$ and $2\phi_0 = 20^\circ$ is shown at Fig.6 .The mild temperature increase is visible.

For analysis of the geometrical characteristics in the ignition process French propellant [11] with thermophysical properties $E\!=1.6747\cdot10^5\,[\mathrm{J/mol}]$, $Q\!=2.512\cdot10^5\,[\mathrm{J/kg}]$, $Z\!=1\cdot10^{17}\,[\mathrm{s^{-1}}]$, $\rho=1600\,[\mathrm{kg/m^3}]$, $c=1674,2\,[\mathrm{J/kgK}]$, $\lambda=0.2135\cdot[\mathrm{W/mK}]$ was also used. The calculation was accomplished for the same propellant geometry (the wedge with confined length) and the same angle values. For the heat flux intensity $\dot q = 0.8918\cdot\mathrm{MW/m^2}$ experimentally measured delay time is $t_{ig} = 0.034\mathrm{s}$ and the ignition temperature, according to eq. (45), is $T_{s,ig} = 543\mathrm{K}$.

The maximum temperature values of the propellant [11], for the angles ≤ 30°, are shown in Figures 7 and 8. The obtained results affirmed the fact of the faster ignition of the thinner (sharp) angles.

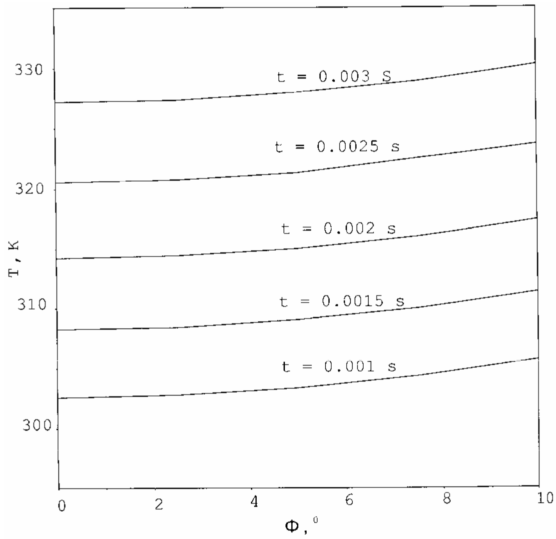

Figure 6. The temperature profiles in the propellant [10] at the radial distance $r=20\cdot10^{-6}\,[\mathrm{m}]$ along the angle ϕ for $\dot q=0,41868\,\mathrm{MW/m^2}$ and $2\phi_0=20^\circ$.

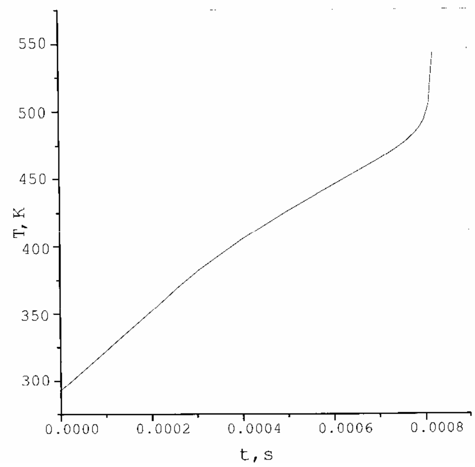

Figure 7. The maximum temperature time history of the propellant [11] for the angle $2\phi_0 = 22.5^\circ$ and the incident heat flux $\dot q$ = 0.8918 MW/m².

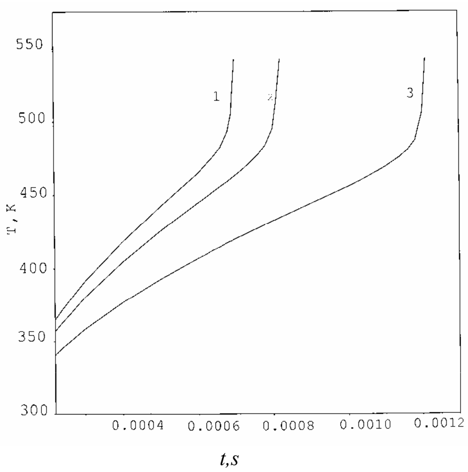

Figure 8. The comparison of maximum temperature grow of the propellant [11] heated by thermal flux $\dot q = 0.8918\,\mathrm{MW/m^2}$ for the wedge top angles: 1- $2\phi_0=20^\circ$, 2 - $2\phi_0=22.5^\circ$, 3 - $2\phi_0=30^\circ$.

Because the calculation takes a long time (small time increment and 1400 points in spatial grid) the possibility of applying the simplified one-dimensional model is analyzed. Temperature distribution in the propellant is then defined by the equation:

$$\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial x^2}+\frac{a}{x}\frac{\partial T}{\partial x}+\frac{a}{\alpha x}\frac{2\dot{q}}{\lambda}+\frac{QZ}{c}\exp\left(-E/RT\right) \qquad (46)$$

with initial and boundary conditions :

$$T\left(x,0\right)=T_i\,,\ \frac{\partial T}{\partial x}\left(\infty,t\right)=0\,,\ \frac{\partial T}{\partial x}\left(0,t\right)=-\frac{\dot{q}}{\lambda} \qquad (47)$$

The comparative values of the ignition delay times for propellant [10] heated by the external thermal flux $\dot{q}=0.20934\ \mathrm{MW/m^2}$ in function of the angle $2\phi_0$ are presented in Table 4.

Table 4.

$2\phi_0$ (°)	20	22.5	30	45	60
$t_{ig,1D}$	0.0074	0.0093	0.0158	0.036	0.065
$t_{ig,2D}$	0.0072	0.0088	0.0144	0.044	0.080

The comparison between ignition delay times calculated by one dimensional and two dimensional models for the same propellant and the doubled heat flux intensity $\dot{q}=0.41868\ \mathrm{MW/m^2}$ is shown in Table 5.

Table 5.

$2\phi_0$ (°)	20	22.5	30	45	60
$t_{ig,1D}$	0.0024	0.003	0.0051	0.0109	0.019
$t_{ig,2D}$	0.0031	0.0037	0.0073	0.0105	0.017

Temperature profiles of the propellant [10] calculated by one and two-dimensional models are presented in Fig.9. Considerable deviation between the computed temperatures is obvious.

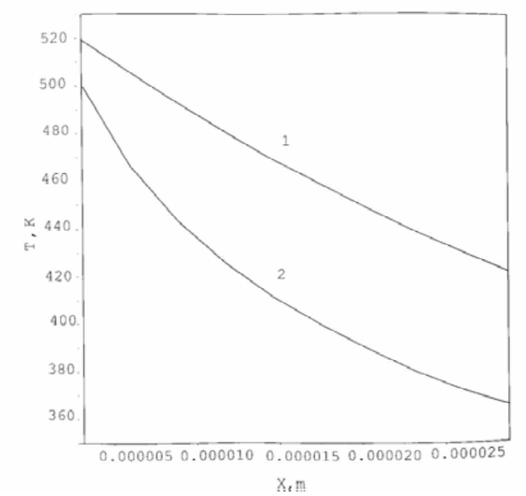

Figure 9. Temperature variation along the propellant [10] with $2\phi_0=20°$ under the effect of heat flux $\dot{q}=0.20834\,\mathrm{MW/m^2}$: 1 – one-dimensional model , 2 – two-dimensional model .

Table 6.

$2\phi_0$ (°)	20	22.5	30	45	60
$t_{ig,1D}$	0.00072	0.00086	0.00142	0.00294	0.00504
$t_{ig,2D}$	0.0007	0.00082	0.00122	0.00264	0.0046

Computed time delay values for the propellant [11] ignited with $\dot{q}$ = 0,8918 MW/m², obtained by complete and simplified models are shown in Table 6.

In Fig.10 temperature variations along the surface of the propellant [11] are compared where $2\phi_0=30°$ heated by $\dot{q}$ = 0.8918 MW/m², received using previously described models.

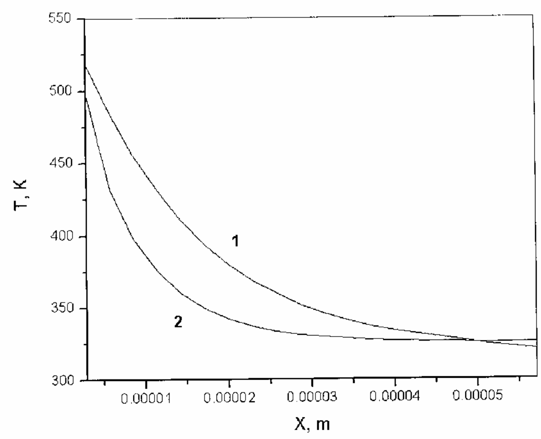

Figure 10. Temperature distibution along the propellant [11] surface, when $2\phi_0=30°$ and $\dot{q}$ = 0,8918 MW/m²: 1 - one-dimensional model and 2 - two-dimensional model.

It is obvious that one dimensional model does not enable sufficiently precise calculations, in the investigated interval of parameters and can be used for preliminary analysis.

Conclusion

Effect of the solid rocket propellant geometrical properties on the ignition process is analyzed in this work. The previous research activities are analyzed first. Some of the former studies were done for the purpose of increasing safety during manufacture, transport and use of the explosive materials. Since results of some models are in the implicit form and some models are much too simplified, the own mathematical model of the process is developed. Two-dimensional temperature field of the solid rocket propellant in the form of wedge with limited length and arbitrary angle is defined numerically by solving the nonsteady heat conduction equation with interior thermal source due to the existence of the exothermal chemical reaction. The propellant consumption determination is included in the model. The numerical calculations are executed for the two types of well-known double-base rocket propellants, with optional angles (from 180°-plane case, to 20°-thin wedge) ignited by different flux intensities. The received results (which are in qualitative agreement with the reference data), imply that wedge angle decreasing (which approximates the surface irregularities) induces the ignition delay time shortening. The maximum temperatures originate from the propellant surface near the wedge top. The process is dynamical and depends on the thermal flux intensity and wedge top angle. Due to the long calculation time, a one-dimensional model is also developed, and compared with the complete one (Tables 4 - 6, Figures 9 and 10). There are considerable differences between results obtained using these two models, so the simplified one-dimensional model can be used for the preliminary analysis only.

Generally, the described propellant geometry properties cause significant ignition process acceleration compared with the case when the propellant is treated as a flat plane. The developed two-dimensional model can be applied for various propellants and whole values of the external heat flux (constant or time dependent) and the propellant top wedge angle.

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Pripaljivanje čvrstih raketnih goriva određenih geometrijskih karakteristika

Analiziran je uticaj geometrijskih parametara na proces pripaljivanja čvrstih raketnih goriva.Dat je kritički prikaz i uticaj geometrijskih parametara na njega. Dat je kritički prikaz do sada objavljenih radova iz ove oblasti. Razvijen je matematički model procesa koji omogućava definisanje dvodimenzionalnog temperaturnog polja unutar raketnog goriva koje se zagreva usled spoljašnjeg toplotnog fluksa,pri postojanju unutrašnjeg izvora toplote zbog egzotermne hemijske reakcije u gorivu. Model omogućava izračunavanje potrošnje goriva za vreme procesa pripaljivanja. Jednačine održanja toplote i mase, nestacionarne i nelinearne, rešene su numerički, metodom konačnih razlika. Dobijeni rezultati prikazani su tabelarno i u vidu dijagrama i kvalitativno se slažu sa literaturnim podacima. Razvijen je i pojednostavljeni model procesa i analizirana mogućnost njegove primene.

Ključne reč: **čvrsto raketno gorivo, pripaljivanje, geometrijski parametri.**

Зажигание твёрдых ракетных топлив определённых геометрических характеристик

В настоящей работе анализировано влияние геометрических характеристик на процесс зажигания твёрдых ракетных топлив и дан критический обзор и влияние геометрических характеристик на процесс, с критическим обзором до сих пор опубликованных работ из этой области. Здесь разработана математическая модель процесса, обеспечивающего определение двухмерного температурного поля внутри ракетного топлива, обогревающего внешним температурным потоком при наличии внутреннего источника температуры из-за экзотермических химических реакций в топливе. Модель обеспечивает вычисление расхода топлива в течении процесса зажигания. Уравнения сохранения температуры и массы, нестационарные и нелинейные, решены цифровым способом, методом конечных разностей. А полученые результаты показаны в таблицах и в виду диаграмм и по качеству согласовываются с данными из литературы. Здесь разработана и упрощена модель процесса и анализированна возможность её применения.

Ключевые слова: твёрдое ракетное топливо, зажигание, геометрические характеристики.

Allumage chez les propergols solides aux caractéristiques géométriques

Ce travail contient l'analyse de l'influence des paramètres géométriques sur le procès d'allumage chez les propergols solides. On a donné un aperçu critique et expliqué l'influence des paramètres géométriques sur ce procès. On a présenté aussi, d'une manière critique, les travaux publiés jusqu'à présent qui se rapportent au domaine étudié. Le modèle mathématique du procès permettant la définition du champ de température à deux dimension à l'intérieur du propergol qui se rechauffe à cause du flux thermique extérieur quand existe une source de chaleur intérieure due à la réaction thérmique exotherme dans le propergol a été développé. Ce modèle permet de calculer la consommation du propergol pendant le procès d'allumage. Les équations de la conservation de chaleur et de masse, non-stationaires et non-linéaires, sont résolues numériqement par la méhode des différences finies. Les résultats obtenus sont présentés en forme de tableaux et diagrammes et ils sont en accord qualitative avec les données existantes dans la littérature technique.On a développé aussi un modèle simplifié de ce procès et analysé les possibilités de son application.

Mots clés: **propergol solide, allumage, caractéristiques géométriques.**