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A New Integrated GPS/INS Method

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This paper examines the scope of satisfying high accuracy and low cost navigation system requirements by using integrated navigation systems, in which StrapDown Inertial Navigation System is aided by GPS measurements. A new GPS/INS integration scheme was developed and designated as a semi-coupled integration scheme. The coupling effects source from the control vector determined from the INS outputs from all three channels, and are used as input to the Kalman filter. The Kalman filter design for GPS/INS was tested for different flight conditions and inertial sensors accuracy.

Key words: **flight mechanics, navigation, numerical algorithm, Kalman filter, GPS.**

Nomenclature

Convention

$\mathbf{C}_{A_1}^{A_2}$ – direction cosine matrix transforming vector from A_1 to A_2 frame

$\mathbf{b}^A$ – vector $\boldsymbol{b}$ with components in A frame

$\boldsymbol{\omega}_{A_1A_2}^{A_3}$ – angular rate of A_2 frame relative to A_1 frame expressed by components in A_3 frame

Axis systems

i – inercial reference frame

e – Earth-fixed reference frame

n – navigation reference frame

b – body reference frame

Earth quantities

g_n – nominal gravitational acceleration ($\phi = 45^\circ$)

R_0 – radius of equivalent (standard) spherical Earth, $R_0 = 6356766\,\mathrm{m}$

ω_{ie} – Earth turn rate with respect to i frame, $\omega_{ie} = 7.292116\times10^{-5}$ rad/s

$\mathbf{g}_i$ – local gravity column matrix

r – radius of a vehicle from the Earth center $(r=R_0+h)$

Kinematic quantities

t, T – time (sampling time, updating time, filtering time)

$\mathbf{V}_e$ – kinematic velocity (velocity of the vehicle relative to the Earth)

V_N, V_E and V_D – the north, east and down components of kinematic velocity in n frame

$\mathbf{V}_e^n$ – kinematic velocity expressed in n frame

f_x^b, f_y^b and f_z^b – components of body specific force expressed in b frame

$\mathbf{f}^b$ – column matrix of components of body specific force in b frame

$\mathbf{f}^n$ – column matrix of components of body specific force expressed in n frame

x^N, x^E, x^D – components of position resolved in n frame

$\mathbf{R}^n$ – column matrix of components of position resolved in n frame where $\mathbf{R}^n = \begin{bmatrix} x^N & x^E & x^D \end{bmatrix}^{\mathrm{T}}$

$h = -x^D$ – altitude

ϕ – latitude

λ – longitude

Definition of important matrices

$(\boldsymbol{\omega}^A\times)$ – skew symmetric matrix with components of $\boldsymbol{\omega}$ in A frame

$$(\boldsymbol{\omega}^A\times) = \boldsymbol{\Omega}(\boldsymbol{\omega}^A) = \begin{bmatrix} 0 & -\omega_{zA} & \omega_{yA} \\ \omega_{zA} & 0 & -\omega_{xA} \\ -\omega_{yA} & \omega_{xA} & 0 \end{bmatrix}$$

$\mathbf{F}$ – dynamic system matrix

$\mathbf{w}$ – process noise column matrix

$\mathbf{Gu}$ – column matrix of the components of the control vector effort

$\mathbf{P}$ – covariance matrix

$\mathbf{H}$ – measurement matrix

$\mathbf{x}$ – column matrix of the components of the state vector

$\mathbf{z}$ – column matrix of the components of the measurement vector

$\mathbf{Q}$ – continuous process noise matrix

Superscripts

$\sim$ – approximated values

$\wedge$ – estimated values

N, E and D – North, East and Down position components (n frame components)

Subscripts

k – index for Kalman filter discrete equations (filtering index)

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N, E and D – North, East and Down components (n frame components)

Abbreviations

IMU – inertial measurement unit
INS – inertial navigation system
SDINS – strapdown inertial navigation system
GPS – global positioning system
GPS/INS – integrated GPS and INS system (GPS-aided-INS)
AV – average value

Introduction

THE integration of the GPS and an INS has been an important development in modern navigation. The research has found an efficient way to limit INS navigation errors by updating the INS velocity and position with external GPS measurements which have consistent accuracy over time, through the integrated GPS/INS system. In this case, the INS becomes an error bounded navigator when GPS data is available.

The benefits of updating the INS systems using the GPS measurements of position and velocity can be demonstrated by Fig.1.

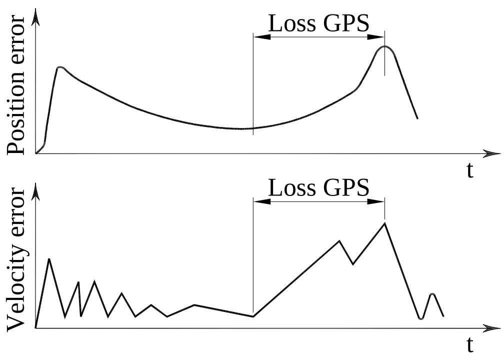

Figure 1. Basic principle of GPS/INS integrated system

The using of the GPS data will reduce the error in the computed values of the position and velocity (improve the accuracy of the system). When the GPS data is lost, it is clear that the error will increase and the accuracy will be reduced. The integration of these two systems into an integrated navigation system requires the development of non-traditional approaches and algorithms [1].

Generally speaking, the integration is possible on two levels: the hardware and software level. For the integration on the hardware level the components of the systems are combined in one box and interface. This approach offers advantages in the reacquisition of satellite signals after loss or lock but, due to hardware integration, it is almost impossible to modify the system for applications different from the ones it was designed for. Therefore, the second approach to turn the hardware units independently and combine the output of both systems on the software level is commonly used [2].

The degree of complexity of the integration approach should reflect the mission requirement; it may also be limited by the investment made to obtain these objectives. Integration strategies and mechanisms can be very simple or relatively complex. Generally speaking, the GPS/INS integration schemes have three categories, namely the uncoupled mode, loosely coupled mode and tightly coupled mode [3]. For each category, variations may exist across applications.

In 2000 Salychev, and Voronov as well as others [1] examined the feasibility of using low cost sensors, specifically the MotionPak, integrated with GPS information, to enable the navigation to bridge GPS outages in tens of seconds. Advanced algorithms are used to integrate the GPS and low cost sensors data. These include INS error damping, calculated platform corrections using GPS output, velocity correction, attitude correction and error model estimation for prediction.

Wolf [3] developed an integrated GPS/INS attitude determination system using low cost sensor unit, i.e. MotionPak unit, and the Trimble Advanced Navigation Sensor (TANS) vector receiver system as GPS component, which is a multi antenna attitude determination and position system. Real time INS navigation software to calculate position, velocity and attitude for the outputs of the IMU was developed. These INS computed values were integrated with velocity, position and attitude information from the GPS component in Kalman filter. The obtained results show that the attitude accuracy of 0.1 degree could be achieved.

Xiang [4] conducted a series of land vehicle tests and high dynamic flight vehicle simulation using GPS/INS integrated system with Kalman filter. The results of land vehicle tests showed that when the GPS signal is available the position accuracy of low cost GPS and low cost SDINS integration is about the same as that of GPS only. The simulation resulted based on typical flight trajectory validate that the developed algorithms are applicable to flight vehicle navigation.

The effect of a constant transmission delay that occurs in delivering measurements form GPS receiver to Kalmen filter for GPS/INS integrated system was analyzed by Hyung and Lee [5].

In 1985 Hubert [6] used the distributed Kalman filter in strapdown attitude heading reference system integrated with GPS (SAHRS/GPS). The distributed filter was chosen to eliminate the problem of computer burden. Different integration configuration were examined to ensure that the system would be optimal to stand-alone GPS or SAHRS systems in the event of a failure in one or more sensors. The error states were modeled as a combination of random variable and stochastic processes. The obtained results showed that when the GPS is lost the error in position and velocity increased dramatically.

In 1979 Bose [7] investigated the radar updated SDINS used in the midcourse guidance of ship launched missile with terminal seeker. The integration of radar/INS was performed using Kalman filter. Both high accuracy and low cost sensors were considered.

The GPS/INS integration is commonly performed using Kalman filter. In order to integrate these systems efficiently a proper choice of filtering type should be considered. In 1998 Zrachan, Jesionoweisky and Lawton [8] studied these different types of filters for the problem of tracking target with range and angle measurements.

In this paper the INS outputs from the developed digital navigation algorithms [11] will be updated by the GPS position or both position and velocity measurements. A new integration scheme will be proposed and used in GPS/INS integrated system.

Semi-coupled integration scheme of GPS/INS

One type of the tightly coupled integration scheme with its analysis for land vehicle application was given in [4]. To formulate an extended Kalman filter a linear dynamic model of errors was given. The state vector consists of the errors in the attitude, velocity and position, gyroscopes drift and accelerometer bias. The measurement vector is the difference of the velocity and position calculated by SDINS algorithm and given by GPS receiver as measurement variables.

The main disadvantage of this model in the case of a flying vehicle is due to a linear error model which does not provide an accurate representation of the actual system when the instrument errors become large. A new approach in the design of the integrated GPS/INS is developed in this paper. The Kalman filter is based on the equivalent linear dynamic system in terms of arc-coordinates and velocity in a local geodetic frame. The state vector (position and velocity) from SDINS to determine the difference with GPS information as measurement vector was not used. In this case the vectors of specific forces and angular rate measured by IMU are used for calculation of the control vector effort through already developed numerical algorithm [11] as the input to Kalman filter.

The new developed scheme of GPS/INS integrated system is named semi-coupled integration scheme, since the coupling effects source from the control vector, using information from all three channels (N, E and D). The block diagram for this new integration scheme is shown in Fig.2.

The type of Kalman filter used for GPS/INS integration navigation in this paper is the decoupled (linear) Kalman filter. Decoupled filters imply that the whole six-dimensional state variable is broken up into three two-dimensional states which are updated in filter equations independently, each composed of position and velocity along one of the transformed new Cartesian coordinates of the East, North and Down navigation frame coordinates.

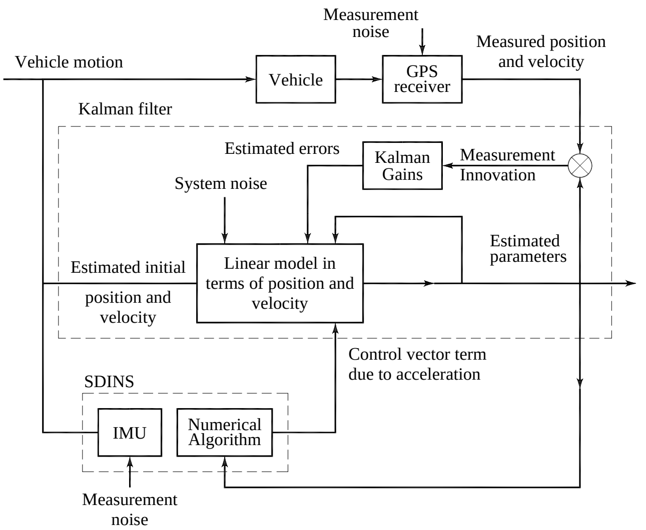

Figure 2. Semi-coupled GPS/INS integration scheme

Coordinate transformation

In order to apply the decoupled filters for each of the three (East, North and Down) channels, the position and its derivative must be transformed from geodetic coordinates into new East, North and Down coordinates (x^E , x^N and x^D). The coordinate transformation can be derived by starting with the rate equations of geodetic latitude, longitude and height given in [12] (Titterton, page 53).

$$\dot{h} = -V_D$$

$$\dot{\lambda} = \frac{1}{(R_0 + h)\cos\phi} V_E \tag{1}$$

$$\dot{\phi} = \frac{1}{(R_0 + h)} V_N$$

From eq.(1) the velocity components in terms of rate of change of geodetic coordinates are

$$V_D = -\dot{h}$$

$$V_E = \dot{\lambda}(R_0 + h)\cos\phi \tag{2}$$

$$V_N = \dot{\phi}\,(R_0 + h)$$

The velocity components given in eq. (2) can be expressed in terms of the rate of change of new coordinate system as following

$$\dot{x}^D = -\dot{h} = V_D$$

$$\dot{x}^E = \dot{\lambda}(R_0 + h)\cos\phi = V_E \tag{3}$$

$$\dot{x}^N = \dot{\phi}\,(R_0 + h) = V_N$$

Equations (3) can be integrated to obtain the incremental position in the new coordinate in the time interval t_{k-1} to t_k . The time interval $[\, t_{k-1},\ t_k \,]$ is the GPS sampling or filtering time interval and it is defined as slower than the position updating cycle in complete INS digital algorithm (n -cycle) [10], [11]. The filtering cycle (k -cycle) is shown in Fig.3.

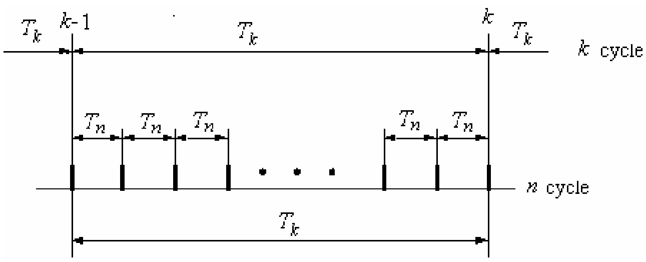

Figure 3. Definition filtering cycle (k-cycle)

The frequency of the k-cycle is defined as

$$f_k = \frac{f_n}{K_n} \tag{4}$$

where K_n is the number of n-cycle inside the k -cycle. The sampling time or filtering time can be obtained as

$$T_k = \frac{1}{f_k} \quad \text{or} \quad T_k = t_k - t_{k-1} \tag{5}$$

The incremental position in terms of transformed coordinates can be obtained by integrating eq. (3) in the following way:

$$\Delta x^D_{k-1,k} = \text{-}(h_k - h_{k-1}) = \Delta x^D_k \tag{6}$$

$$\Delta x^N_{k-1,k} = (R_0 + h)_{\text{AV}}\,\Delta\phi_{k-1,k} = \Delta x^N_k \tag{7}$$

$$\Delta\phi_{k-1,k} = \phi_k - \phi_{k-1} \tag{8}$$

$$\Delta x^E_{k-1,k} = [(R_0 + h)\cos\phi]_{\mathrm{AV}}\, \Delta\lambda_{k-1,k} = \Delta x^E_k \tag{9}$$

$$\Delta\lambda_{k-1,k} = \lambda_k - \lambda_{k-1} \tag{10}$$

Since the altitude h changes slowly during the filtering time interval, the term $(R_0 + h)$ was taken as the average value in evaluating the integration.

In this section the method of transformation of the coordinates has been given and both the system and measurement equations can be derived using the new transformed coordinates

$$x^N_k = x^N_{k-1} + \Delta x^N_k \tag{11}$$

$$x^E_k = x^E_{k-1} + \Delta x^E_k \tag{12}$$

System equations

It has been shown that the first derivatives of position components expressed in terms of new coordinates are equal to the components of the kinematic velocity in the navigation frame. Acceleration (the second derivative of position) is given by the navigation equation [12]. To compensate the uncertainties in modeling the acceleration a white noise w will be added to the acceleration expression. Mathematically, these two expressions for three directions are:

$$\begin{bmatrix} \dot{x}^J \\ \ddot{x}^J \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x^J \\ \dot{x}^J \end{bmatrix} + \begin{bmatrix} 0 \\ a_J \end{bmatrix} + \begin{bmatrix} 0 \\ w_J \end{bmatrix} \tag{13}$$

where $J = N, E, D$

The components of the acceleration in the n frame (n frame $\equiv$ local geographic frame) a_N, a_E and a_D are defined by the navigation equation given in [12] as follows

$$\dot{\mathbf{V}}^n_e = \begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} = \begin{bmatrix} a_N \\ a_E \\ a_D \end{bmatrix} = \mathbf{f}^n - [\mathbf{\Omega}(\boldsymbol{\omega}^n_{en}) + 2\mathbf{\Omega}(\boldsymbol{\omega}^n_{ie})]\mathbf{V}^n_e + \mathbf{g}^n_l \tag{14}$$

The kinematic model of a vehicle to estimate its position and velocity by using Kalman filter can be described by the matrix differential equation [15]:

$$\dot{\mathbf{x}} = \mathbf{F}\,\mathbf{x} + \mathbf{G}\mathbf{u} + \mathbf{w} \tag{15}$$

where $\mathbf{x}$ is the column vector with the state of the system, $\mathbf{F}$ is the system dynamic matrix, $\mathbf{Gu}$ is the control vector effort, and $\mathbf{w}$ is the process noise. By comparing eq. (15) with eq. (13), both the system matrix and the control vector can be obtained as

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{Gu} = \begin{bmatrix} 0 \\ a_J \end{bmatrix} \tag{16}$$

There is a process noise matrix $\mathbf{Q}$ relating the process-noise vector according to

$$\mathbf{Q} = \mathrm{E}\left[\mathbf{w}\,\mathbf{w}^{\mathrm{T}}\right] \tag{17}$$

If $\mathbf{w} = 0$ and the state at the time t_{k-1} is known, $\mathbf{x}_{k-1}$, the state vector at t_k is given by [13]

$$\mathbf{x}_k(t_k) = \mathbf{\Phi}(t_k, t_{k-1})\,\mathbf{x}_{k-1} + \int_{t_{k-1}}^{t_k} \mathbf{\Phi}(t,\tau)\mathbf{G}\mathbf{u}\,d\tau \tag{18}$$

where $\mathbf{\Phi}(t_k, t_{k-1}) = \mathbf{\Phi}_k$ is the transition matrix for the linear dynamic system. From the matrix superposition, the effect of the input on the state from t_{k-1} to t_k can be obtained as

$$\mathbf{G}_k\mathbf{u}_{k-1} = \int_{t_{k-1}}^{t_k} \mathbf{\Phi}(t,\tau)\mathbf{G}\mathbf{u}\,d\tau = \mathbf{x}_k - \mathbf{\Phi}_k\,\mathbf{x}_{k-1} \tag{19}$$

The continuous and discrete transition matrices are:

$$\mathbf{\Phi}(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}, \quad \mathbf{\Phi}_k = \begin{bmatrix} 1 & T_k \\ 0 & 1 \end{bmatrix} \tag{20}$$

By using the value of the transition matrix given by eq. (20) and the definition of the state, the discrete form of the control vector action given by eq. (19) is transformed into

$$\mathbf{G}_k\mathbf{u}_{k-1} = \begin{bmatrix} x^J_k \\ \dot{x}^J_k \end{bmatrix} - \begin{bmatrix} 1 & T_k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x^J_{k-1} \\ \dot{x}^J_{k-1} \end{bmatrix} = \begin{bmatrix} x^J_k - x^J_{k-1} - \dot{x}^J_{k-1}T_k \\ \dot{x}^J_k - \dot{x}^J_{k-1} \end{bmatrix} = \begin{bmatrix} \Delta x^J_k - \dot{x}^J_{k-1}T_k \\ \Delta\dot{x}^J_k \end{bmatrix} \tag{21}$$

where Δx^J_k is determined using eqs. (6), (7) and (9), the velocity increments are given by

$$\Delta\dot{x}^J_k = V_{Jk} - V_{Jk-1} \tag{22}$$

The control vector action in fact is calculated by using data from the INS particular solution provided by the output of the position and velocity computed by the developed INS digital algorithms [11]. Since the discrete form of the process noise is simply $\mathbf{w}_k$, the discrete time system can be written as

$$\mathbf{x}_k = \mathbf{\Phi}_k\,\mathbf{x}_{k-1} + \mathbf{G}_k\,\mathbf{u}_{k-1} + \mathbf{w}_k \tag{23}$$

Measurement equation

The Kalman filter formulation requires the measurements from GPS to be linearly related to the states according to

$$\mathbf{z}_{\mathrm{GPS}} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{24}$$

where $\mathbf{z}$ is the measurement vector, $\mathbf{H}$ is the measurement matrix, and $\mathbf{v}$ is the white measurement noise, which is always expressed as a vector. The values of the measurement matrix $\mathbf{H}$ and the measurement noise matrix $\mathbf{R}$, which is related to the measurement noise vector $\mathbf{v}$ according to $\mathbf{R} = \mathrm{E}\left[\mathbf{v}\mathbf{v}^T\right]$, depend on the implementation of the measured data by GPS. In case of using position data for the purpose of updating only, the $\mathbf{H}$ matrix is defined as

$$\mathbf{H} = [1 \quad 0] \tag{25}$$

and the measurement matrix $\mathbf{R}$ is defined as a scalar quantity of

$$\mathbf{R} = \sigma^2_p \tag{26}$$

where σ_p is the general notation for the measurement standard deviation of the GPS position coordinate and may be

$$\sigma_p = \sigma_{xN}, \sigma_{xE} \text{ and } \sigma_{xD}$$

When both velocity and position measurements for updating the $\mathbf{H}$ are used, matrix has the following form

$$\mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{27}$$

and the measurement noise matrix becomes

$$\mathbf{R} = \begin{bmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \tag{28}$$

where σ_V is the general notation for the measurement of standard deviation of the GPS velocity component and may be

$$\sigma_V = \sigma_{V_N}, \sigma_{V_E} \text{ and } \sigma_{V_D}$$

The discrete form of the Kalman filtering measurement equation is given as

$$(\mathbf{z}_k)_{\mathrm{GPS}} = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \tag{29}$$

where $\mathbf{x}_k$ is the state and $(\mathbf{z}_k)_{\mathrm{GPS}}$ is measurement vector obtained by GPS receiver.

The values of GPS measurement parameters (σ_{xJ} and σ_{V_J}, $J = N, E, D$) are given in Appendix A.

Kalman filtering equation and algorithm

The Kalman filtering or update equation can be written using the proceeding matrices in discrete time form [15]

$$\begin{aligned} \hat{\mathbf{x}}_k &= \mathbf{\Phi}_k \hat{\mathbf{x}}_{k-1} + \mathbf{G}_k \mathbf{u}_{k-1} + \\ &+ \mathbf{K}_k \left[(\mathbf{z}_k)_{\mathrm{GPS}} - \mathbf{H}\mathbf{\Phi}_k \hat{\mathbf{x}}_{k-1} - \mathbf{H}\mathbf{G}_k \mathbf{u}_{k-1} \right] \end{aligned} \tag{30}$$

Eq.(30) can be shortened by defining the filter's estimate of the state at time t_k before the measurement

$$\tilde{\mathbf{x}}_k = \mathbf{\Phi}_k \hat{\mathbf{x}}_{k-1} + \mathbf{G}_k \mathbf{u}_{k-1} = \mathbf{\Phi}_k \hat{\mathbf{x}}_{k-1} + \begin{bmatrix} \Delta x_k^J - \dot{x}_{k-1}^J T_k \\ \Delta \dot{x}_k^J \end{bmatrix} \tag{31}$$

which is identical to the prior estimate expression with the control vector term. Eq.(29) will be used to estimate the state after the measurements.

It is clear from eq.(30) that in order to estimate the state, the only quantity necessary to be determined is the Kalman filter gain column vector $\mathbf{K}_k$, since all other matrices have been previously defined. The Kalman filter recursive algorithm given in [15] or [14] is applied to determine the gain vector of each filter.

The discrete time process noise for the decoupled Kalman filter will be assumed to have a piecewise-constant linear model in order to reduce the computation load and storage requirements for evaluating the optimal gain in case of on-line computations. The assumption of the piecewise constant acceleration in this model means that the acceleration of the object of interest (effect of the error produced by the numerical algorithm of SDINS) is assumed to be constant during the sampling (or filtering) period T_k and these accelerations are uncorrelated from one period to another. For the state of position and velocity, and provided that the w_k is constant acceleration during the filtering period, then the increment of velocity during the period is $w_k T_k$, while the effect of this acceleration on the position is $w_k T_k^2/2$. According to this assumption the discrete time process noise term in eq.(23) can be written as

$$\mathbf{w}_k = \begin{bmatrix} T_k^2/2 \\ T_k \end{bmatrix} w_k = \mathbf{\Gamma}_1 w_k \tag{33}$$

The discrete process noise matrix can be obtained from the covariance of the noise process given by eq.(17)

$$\begin{aligned} \mathbf{Q}_k &= \mathbf{E}\left[\mathbf{\Gamma}_1 w_k w_k \mathbf{\Gamma}_1^{\mathrm{T}} \right] = \begin{bmatrix} T_k^2/2 \\ T_k \end{bmatrix} \begin{bmatrix} T_k^2/2 & T_k \end{bmatrix} \sigma_w^2 \\ &= \begin{bmatrix} \dfrac{T_k^4}{4} & \dfrac{T_k^3}{2} \\ \dfrac{T_k^3}{2} & T_k^2 \end{bmatrix} \sigma_w^2 \end{aligned} \tag{34}$$

After the estimation of states is obtained, the new coordinates should be transformed again into the geodetic coordinates to be compatible with the INS algorithms. After estimation these coordinates are basically determined using the filter estimate of the altitude $\hat{h}$ to calculate the estimated geodetic radius $\hat{r}$ for the previous and current altitude estimations, $\hat{h}_{k-1}$ and $\hat{h}_k$ respectively, as follows

$$\hat{r}_{k-1} = (R_0 + \hat{h}_{k-1}) \tag{35}$$

$$\hat{r}_k = (R_0 + \hat{h}_k) \tag{36}$$

Average of the estimated geodetic radius $\hat{r}_{AV}$ is

$$\hat{r}_{AV} = \frac{1}{2}\left(\hat{r}_k + \hat{r}_{k-1} \right) \tag{37}$$

The incremental estimated position in the North direction $\Delta \hat{x}_k^N$ is simply the difference between the current and the previous estimates

$$\Delta \hat{x}_k^N = \hat{x}_k^N - \hat{x}_{k-1}^N \tag{38}$$

The incremental estimated latitude $\Delta \hat{\phi}_k$ can be calculated using eq.(7) in the following way

$$\Delta \hat{\phi}_k = \frac{\Delta \hat{x}_k^N}{\hat{r}_{\mathrm{AV}}} \tag{39}$$

Using eq.(39) the current estimated latitude $\hat{\phi}_k$ can be updated by

$$\hat{\phi}_k = \hat{\phi}_{k-1} + \Delta \hat{\phi}_k \tag{40}$$

Since the estimated value of latitude is obtained, the estimated horizontal component of the geodetic radius $\hat{r}_H$ using both the previous and current estimations of geodetic radius $\hat{r}$ and the latitude $\hat{\phi}_k$ respectively can be obtained as follows

$$\hat{r}_{H_{k-1}} = \hat{r}_{k-1} \cos \hat{\phi}_{k-1} \tag{41}$$

$$\hat{r}_{H_k} = \hat{r}_k \cos\hat{\phi}_k \qquad (42)$$

The average value of this component then is calculated by

$$\left(\hat{r}_H\right)_{\mathrm{AV}} = \frac{1}{2}\left(\hat{r}_{H_k} + \hat{r}_{\,k-1}\right) \qquad (43)$$

The incremental estimated longitude $\Delta\hat{\lambda}_k$ can be calculated in terms of the component geodetic radius average value $\left(\hat{r}_H\right)_{\mathrm{AV}}$ and the incremental estimated East position $\Delta\hat{x}_k^E$, eq.(38)

$$\Delta\hat{\lambda}_k = \frac{\Delta\hat{x}_k^E}{\left(\hat{r}_H\right)_{\mathrm{AV}}} \qquad (44)$$

Using eq. (44) the current estimated latitude $\hat{\lambda}_k$ becomes

$$\hat{\lambda}_k = \hat{\lambda}_{k-1} + \Delta\hat{\lambda}_k\,. \qquad (45)$$

GPS/INS integration computer program flow chart

The flow chart of the computer program is shown in Fig.4. For the case of no GPS-aided-INS navigation, the computer program will work for the INS stand-alone navigation system and the performance will depend on the accuracy of the inertial sensors. For the case of GPS-aided-INS application, the measurements of geodetic position and altitude from the GPS receiver along with the velocity component measurements will be taken. In this program the GPS measurements picking up were simulated by reading the data file with the exact values of position and velocity and by adding noise measurements. The decoupled Kalman filter flow chart is shown in Fig.5.

Numerical simulation results

Numerical simulations were conducted on the pitch programmed trajectory in order to test the proposed GPS/INS integration scheme using the developed algorithm shown in Figures 4 - 5. The pitch programmed flight is divided into two parts: maneuvering phase with variable velocity due to the thrust ($t \le 30$s) and pure ballistic flight ($t > 30$s). The generated angular rate and specific force are shown in Fig.6. The GPS measured data have been generated by integrating the navigation equations using the 4th Runge-Kutta method with integration time step of 0.001s. Two grades of inertial sensors, the high accuracy/high cost and low accuracy/low cost, were used in these numerical experiments.

The INS solutions were updated by GPS measurements by using either the position and velocity measurements or position only measurements. The effect of the process noise used for filter tuning will be examined.

Recognizing that the simulation outputs based on random inputs can vary from one simulation to another (from run to run) the Monte Carlo method will be used in these analyses to obtain system performance and the influence of the input parameters on the accuracy of the simulation. The Monte Carlo method is simply repeated using simulation trails and post-processing of the resultant data in order to make an assembly averaging to get the mean and the standard deviation.

Monte-Carlo method results

In this method, each source of error in both the inertial navigation system (sensor errors and alignment errors) and the GPS system (velocity and position measurement errors) was modeled as a random variable or random process. Data for the Monte-Carlo numerical simulation are given in Appendix A for both high accuracy and low cost sensors. The mathematical model includes 12 independent random process and 14 independent random variables with normal distribution. In order to obtain the proper statistics on GPS/INS system by Monte-Carlo method the number of simulations (number of runs) should be large enough to generate estimates of the variances of the position and velocity estimates provided by the integrated GPS/INS system. The number of simulations used in this study was 70 simulations or runs for a period of 70 s.

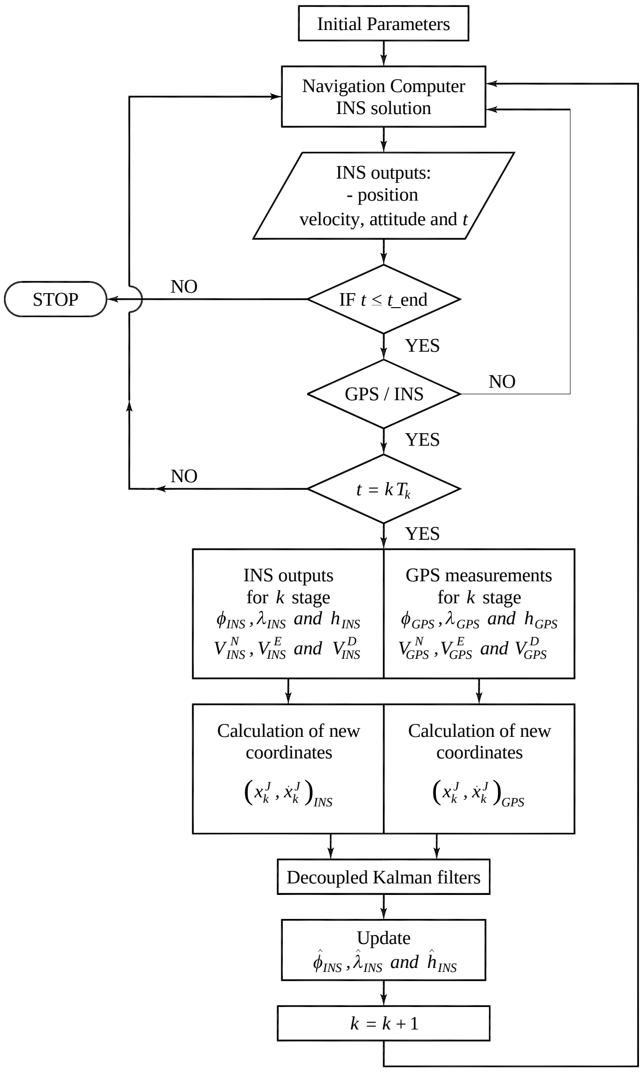

Figure 4. GPS/INS integration flow chart

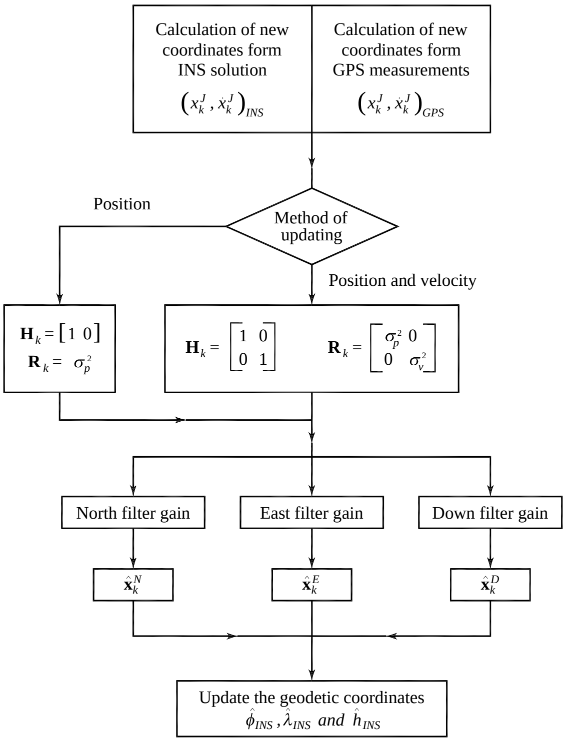

Figure 5. Decoupled Kalman filter flow chart

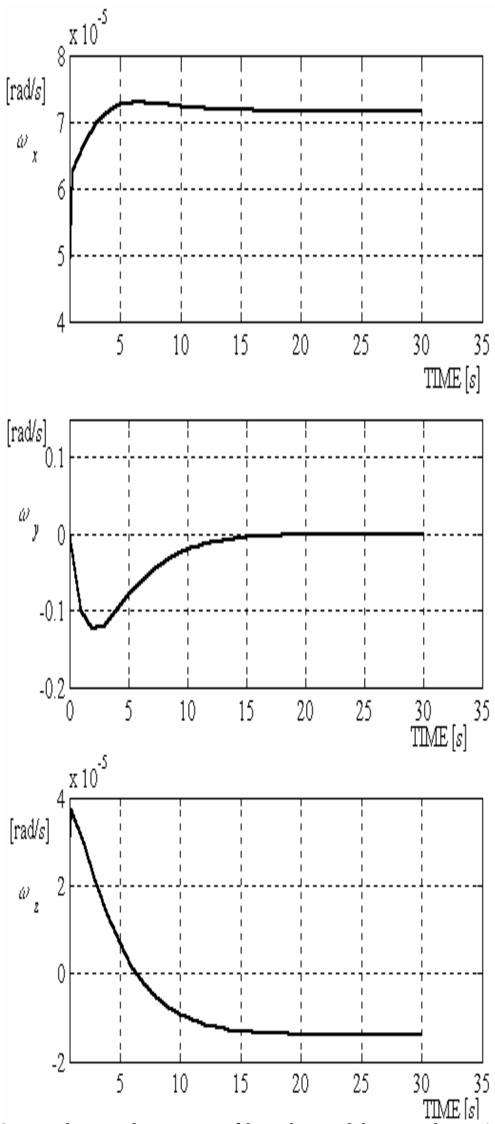

Figure 6-1 Body angular rate profiles obtained by simulator (trajectory with maneuver and variable velocity)

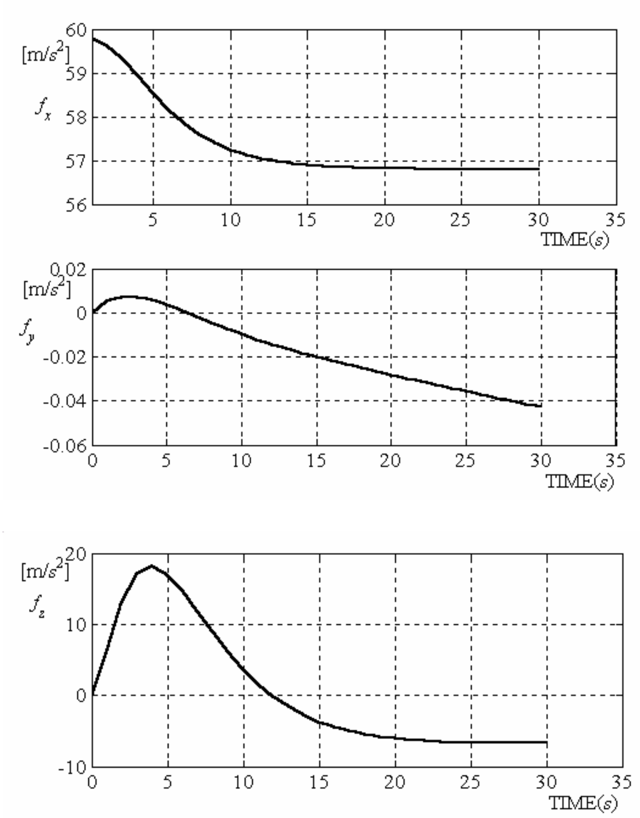

Figure 6-2 Specific force profiles obtained by simulator (trajectory with maneuver and variable velocity)

Influence of process noise

In order to obtain accurate results from the integrated GPS/INS system via Kalman filter, the proper values of filter parameters should be selected. The main parameter is the system or process noise which compensates the uncertainty in the mathematical model of the real system including inertial sensors errors in GPS/INS integrated system. The proper values of the process noise were numerically adjusted in such a way to achieve the stability and minimum standard deviation of the estimated parameters.

The sensitivity of the obtained results to the value of the process noise is shown in Fig. 7 – 8 for the case of INS with low accuracy sensors aided by GPS position measurements. The applied values of σ_w are: 0.1, 1 and 10 m/s2. The results of the error in the computed Down velocity and altitude are presented. The best values of the process noise standard deviation σ_w are: σ_w =10 m/s^2 for $t \le 30\,\text{s}$ and σ_w =1 m/s^2 for $t > 30\,\text{s}$.

The filter instability in the errors of altitude and down velocity is obtained during the maneuvering flight if $\sigma_w \le$ 1 m/s^2.

In the case of high accuracy sensors the best value of the process noise standard deviation is equal to σ_w =0.1 m/s^2

High accuracy sensor results

The mean and standard deviation values for δh and δV_D are given in Fig. 9 for position updating. The system noise used for compensating the uncertainty in modeling the real system is σ_W= 0.1 m/s^2. Fig. 9 shows that the value of standard deviation of δh at t = 30 s is reduced to

5.67 m (0.3 times the GPS error) while its value at t = 70 s is 4.21 m. For the error δV_D the standard deviation value at t = 30 s is 0.54 m/s and at t = 70 s is 0.19 m/s.

Using the GPS velocity measurements along with the position measurements in the GPS/INS system leads to an increase of accuracy in the computed parameters especially in the computed velocity components (Fig.10). The standard deviation of δh at t = 30 s is reduced to 2.29 m (~0.4 times the GPS/INS with position measurements only) while its value at t = 70 s is 1.57 m (~0.4 times the GPS/INS with position measurements only). The value of δV_D at t = 30 s is 0.1 m/s and at t = 70 s becomes 0.07 m/s.

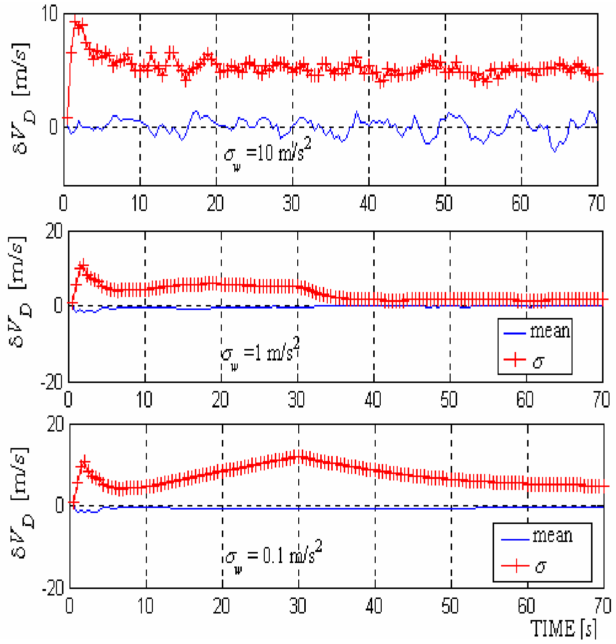

Figure 7. The effect of system noise on Down velocity of GPS/INS system (σ_w = 0.1, 1 and 10 m/s^2, low accuracy sensor, GPS position data only)

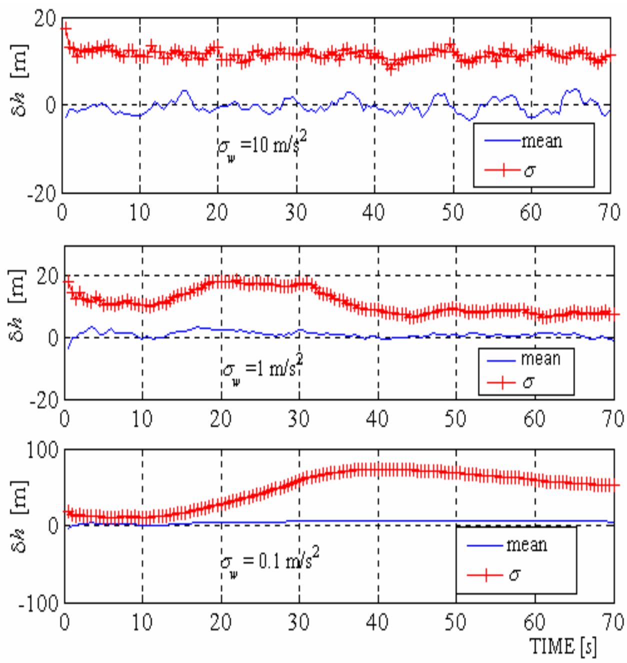

Figure 8. The effect of system noise on the altitude of GPS/INS system (σ_w = 0.1, 1 and 10 m/s^2, low accuracy sensor, GPS position data only)

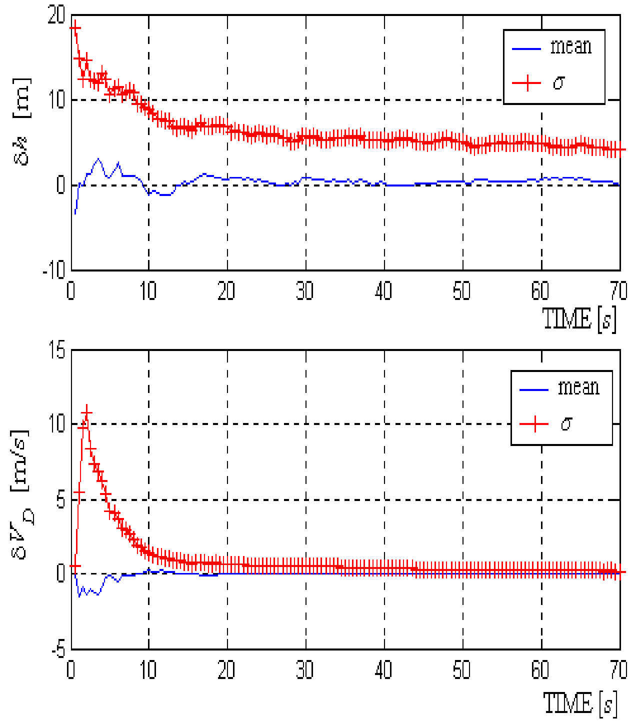

Figure 9. Mean and standard deviation values of δh and δV_D (High accuracy sensors GPS/INS with position updating, σ_w = 0.1 m/s^2)

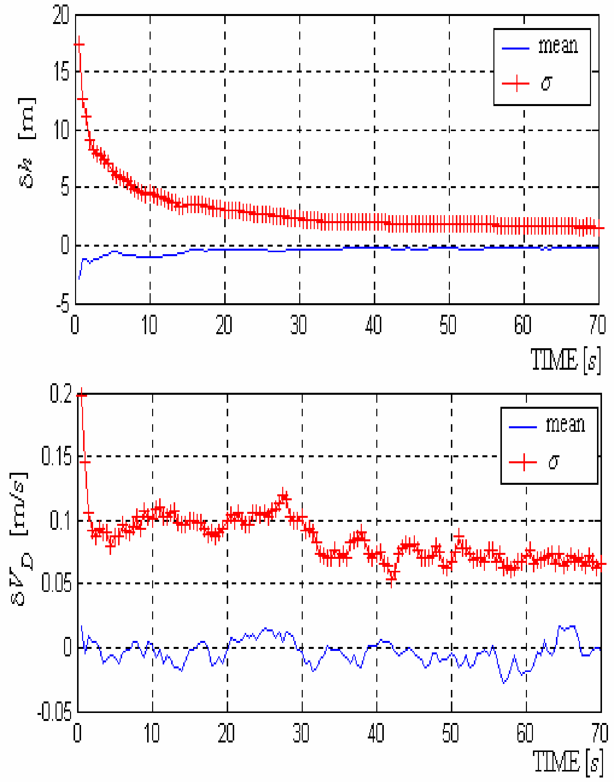

Figure 10. Mean and standard deviation values of δh and δV_D (High accuracy sensors GPS/INS with position and velocity updating, σ_w = 0.1 m/s^2)

Low cost sensor results

The values of process noise used for GPS/INS integrated system with low cost inertial sensors are σ_w = 10 m/s^2 for the maneuvering phase ($t \leq 30$ s) and σ_w =1 m/s^2 for the ballistic phase (t > 30 s).

The results of the mean and standard deviation of δh and δV_D obtained by GPS/INS using position measurements only for updating the INS are shown in Fig.11, which illustrates the benefits of using two values of σ_W for the estimation of h and V_D. The standard deviation values were damped in the ballistic phase (at $t > 30$ sec) and have values less than the maneuvering phase values by 1.4 times and 2.9 times for δh and δV_D, respectively.

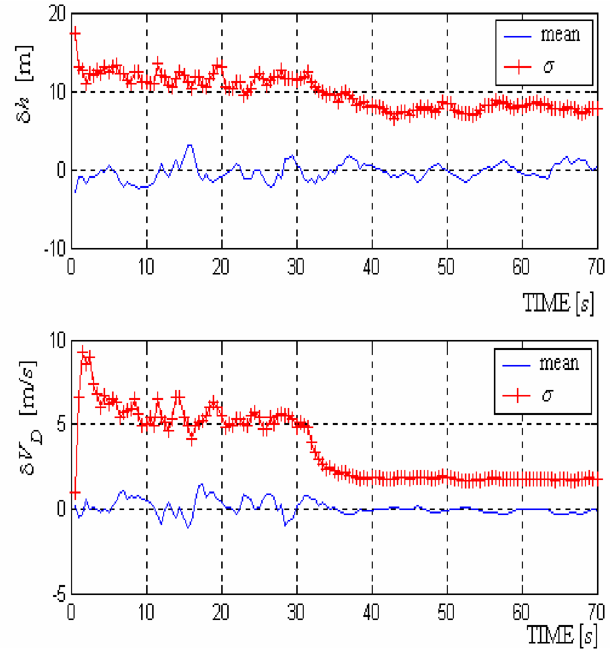

Figure 11. Mean and standard deviation values of δh and δV_D (Low cost sensors GPS/INS with position updating)

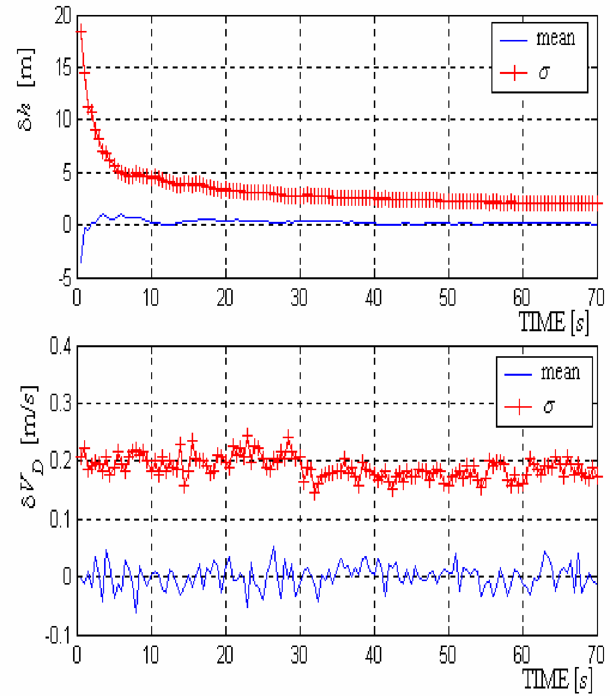

Figure 12. Mean and standard deviation values of δh and δV_D (Low cost sensors GPS/INS with position and velocity updating)

The effect of introducing the GPS velocity measurements in the GPS/INS integrated system on the accuracy of the estimated values of h and V_D is shown in Fig.12. The accuracy in h is increased by ~ 4 times compared with the case of position only updating at $t = 30$ s and by ~ 3.7 times at $t = 70$ s. The accuracy in the down velocity component V_D is greatly increased in the maneuvering phase by ~ 22 times compared with the position only case at $t = 30$ s and by 9 times in the ballistic phase $t > 30$ s.

Conclusion

A new integration scheme was developed and designated as semi-coupled integration scheme because the coupling effects arise only from the control vector effort provided by using the INS outputs from all three channels (N, E and D). The Kalman filter used in the integration scheme was based on the equivalent linear dynamics in terms of arc-coordinate and velocity in local geodetic frame.

The obtained Monte-Carlo simulation results of the maneuvering vehicle using the new developed integration scheme of GPS/INS integrated system showed that:

- The accuracy of the estimation of the vehicle position for the complete trajectory by using low cost INS/GPS is somewhat increased compared to GPS if only GPS position (P) measurements were used; much better results for the same parameter are achieved if both position and velocity (P+V) measurements were used.
- The errors of the low cost INS/GPS/P+V and high accuracy INS/GPS/P+V are approximately the same in the estimation of position. This recommends the application of integrated GPS with low cost INS.
- The application of the high accuracy INS compared to the low cost INS in the integrated GPS/INS system is not effective having in mind its cost.

Appendix A

Data for the Monte-Carlo numerical simulation

The output of the rate gyroscopes ($\boldsymbol{\omega}_{\mathrm{r.g}}$) may be expressed in a simplified form

$$\boldsymbol{\omega}_{\mathrm{r.g}} = (\mathbf{I} + \mathbf{S}_{\mathrm{r.g}})\boldsymbol{\omega}_{ib}^{b} + \mathbf{B}_{\mathrm{r.g}} + \mathbf{n}_{\mathrm{r.g}} \tag{A.1}$$

where

$\boldsymbol{\omega}_{\mathrm{r.g}}$ the outputs of rate gyro [rad/s]

$\mathbf{S}_{\mathrm{r.g}}$ the matrix of rate gyro scale factor errors [non-dimensional]

$\mathbf{B}_{\mathrm{r.g}}$ the matrix of the rate gyro fixed bias [rad/s]

$\mathbf{n}_{\mathrm{r.g}}$ the matrix of rate gyro white noise.

The matrices of rate gyro. scale factor, fixed bias and the standard deviation of the white noise are defined as

$$\mathbf{S}_{\mathrm{r.g}} = 0.01\begin{bmatrix} S_{x_{\mathrm{r.g}}} & 0 & 0 \\ 0 & S_{y_{\mathrm{r.g}}} & 0 \\ 0 & 0 & S_{z_{\mathrm{r.g}}} \end{bmatrix}$$

$$\mathbf{B}_{\mathrm{r.g}} = 4.84814\mathrm{x}10^{-6}\begin{bmatrix} B_{x_{\mathrm{r.g}}} \\ B_{y_{\mathrm{r.g}}} \\ B_{z_{\mathrm{r.g}}} \end{bmatrix}$$

$$\boldsymbol{\sigma}_{\mathrm{r.g}} = 4.84814\mathrm{x}10^{-6}\begin{bmatrix}\sigma_{x_{\mathrm{r.g}}}\\ \sigma_{y_{\mathrm{r.g}}}\\ \sigma_{z_{\mathrm{r.g}}}\end{bmatrix}$$

For High Accuracy rate gyro, (1σ) of the parameters are

$$S_{x_{\mathrm{r.g}}} = S_{y_{\mathrm{r.g}}} = S_{z_{\mathrm{r.g}}} = S_{rg} = 0.05\%$$

$$B_{x_{\mathrm{r.g}}} = B_{y_{\mathrm{r.g}}} = B_{z_{\mathrm{r.g}}} = B_{rg} = 5 \text{ degree/hour}$$

$$\sigma_{x_{\mathrm{r.g}}} = \sigma_{y_{\mathrm{r.g}}} = \sigma_{z_{\mathrm{r.g}}} = \sigma_{rg} = 180 \text{ degree/hour}$$

For Low Cost rate gyro, (1σ) of the parameters are

$$S_{\mathrm{r.g}} = 0.5\%$$

$$B_{\mathrm{r.g}} = 50 \text{ degree/hour}$$

$$\sigma_{\mathrm{r.g}} = 180 \text{ degree/hour}$$

The outputs of the accelerometers can be expressed in a simplified form:

$$\mathbf{f}_{\mathrm{acc}} = (\mathbf{I} + \mathbf{S}_{\mathrm{acc}})\mathbf{f}^b + \mathbf{B}_{\mathrm{acc}} + \mathbf{n}_{\mathrm{acc}} \tag{A.2}$$

where

$\mathbf{f}_{\mathrm{acc}}$ the outputs of accelerometers [m/s^2]

$\mathbf{S}_{\mathrm{acc}}$ the matrix of accelerometers scale factor errors [non-dimensional]

$\mathbf{B}_{\mathrm{acc}}$ the matrix of accelerometers fixed bias [m/s^2]

$\mathbf{n}_{\mathrm{acc}}$ the matrix of accelerometers white noise.

The matrices of accelerometer's scale factor, fixed bias and standard deviation of white noise are defined as

$$\mathbf{S}_{\mathrm{acc}} = 0.01\begin{bmatrix}S_{x_{\mathrm{acc}}} & 0 & 0\\ 0 & S_{y_{\mathrm{acc}}} & 0\\ 0 & 0 & S_{z_{\mathrm{acc}}}\end{bmatrix},\ \mathbf{B}_{\mathrm{r.g}} = 0.01\begin{bmatrix}B_{x_{\mathrm{acc}}}\\ B_{y_{\mathrm{acc}}}\\ B_{z_{\mathrm{acc}}}\end{bmatrix} \text{ and}$$

$$\boldsymbol{\sigma}_{\mathrm{acc}} = 0.01\begin{bmatrix}\sigma_{x_{\mathrm{acc}}}\\ \sigma_{y_{\mathrm{acc}}}\\ \sigma_{z_{\mathrm{acc}}}\end{bmatrix}$$

For High Accuracy accelerometers, (1σ) of the parameters are

$$S_{x_{\mathrm{acc}}} = S_{y_{\mathrm{acc}}} = S_{z_{\mathrm{acc}}} = S_{acc} = 0.05\%$$

$$B_{x_{\mathrm{acc}}} = B_{y_{\mathrm{acc}}} = B_{z_{\mathrm{acc}}} = B_{acc} = 0.5\, mg$$

$$\sigma_{x_{\mathrm{acc}}} = \sigma_{y_{\mathrm{acc}}} = \sigma_{z_{\mathrm{acc}}} = \sigma_{acc} = 50\, mg$$

For Low Cost accelerometers, (1σ) of the parameters are

$$S_{\mathrm{acc}} = 2\%$$

$$B_{\mathrm{acc}} = 25\ mg$$

$$\sigma_{\mathrm{acc}} = 50mg$$

The standard deviation (1σ) for the vehicle's position and velocity in GPS are - in North direction ($\sigma_{xN} = 10\,\mathrm{m}$ and $\sigma_{V_N} = 0.2\,\mathrm{m/s}$)

– in East direction ($\sigma_{xE} = 10\,\mathrm{m}$ and $\sigma_{E_N} = 0.2\,\mathrm{m/s}$)

– in Down direction ($\sigma_{xD} = 20\,\mathrm{m}$ and $\sigma_{V_E} = 0.2\,\mathrm{m/s}$)

The alignment uncertainties are given by (1σ) for the initial elevation, azimuth and roll angle:

$$\sigma_{\theta_0} = 0.03 \text{ degree},\ \sigma_{\Psi_0} = 0.03 \text{ degree and } \sigma_{\Phi_0} = 0\,.$$

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Nova integrisana GPS/INS metoda za navigaciju objekta

U radu se ispituje oblast zahteva koje treba da zadovolje senzori (velike i male tačnosti) integrisanih navigacionih sistema kod kojih se besplatformni inercijalni navigacioni sistemi koriguju pomoću GPS mernih podataka. Razvijena je nova GPS/INS šema integracije koja je nazvana «polu-kuplovana» šema estimacije. Efekti među-uticaja potiču od upravljačkog vektora koji se određuje na osnovu izlaza iz sva tri kanala inercijalnog navigacionog sistema i predstavlja ulaznu veličinu za Kalmanov filter. Kalmanov filter, koji je projektovan za GPS/INS, testiran je za različite uslove kretanja i tačnosti inercijalnih senzora.

Ključne reči: **mahanika leta, navigacija, numerički algoritam, kalmanov filter, GPS.**

Новый проинтегрированный GPS/INS метод

В настоящей работе исследуются требования, которым нужны удовлетворить чувствительные элементы (большой и маленькой точности) проинтегрированных навигационных систем, у которых бесплощадные инерциальные навигационные системы корректируются при помощи GPS измеримых данных. Здесь развита новая GPS/INS схема интеграции с названием "полу-соединённая" схема оценивания. Эффекты между-влияния вытекают от управляющего вектора, который определяется на основе выходов из всех трёх каналов инерциальной навигационной системы и представляет входную величину для фильтра Калмана. Над фильтром Калмана, проектированным для GPS/INS, проведен тест для различных условий движения и точности инерциальных чувствительных элементов.

Ключевые слова: механика полёта, навигация, численный алгорифм, фильтр Калмана, GPS.

Nouvelle méthode GPS/INS intégrée

Dans ce papier on examine les exigences que les sensors (de grande et petite précision) des systèmes de navigation intégrés doivent satisfaire et où les systèmes inertiels de navigation sans platformes sont corrigés au moyen des données de mesurement GPS. On a développé un nouveau schéma d'intégration appelé schéma d'estimaton demi-couplé. Les effets d'inter-action proviennent du vecteur du guidage qui est déterminé à l'aide de la sortie de tous les trois canaux du système inertiel de navigation et représente le paramètre d'entrée pour le filtre de Kalman. Le filtre de Kalman, projeté pour GPS/INS, était testé pour le différentes conditions du mouvement et de précision chez les sensors inetiels.

Mots clés: **mécanique du vol, navigation, algorithme numérique, filtre de Kalman, GPS.**