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Mechanized procedure for the calculation of altitude coefficients

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Mechanized procedure for determining altitude coefficients is presented. These coefficients are used in artillery shooting tables in cases when the so called "meteo-average" meteo-message is applied. Numerical experimental results are given.

Key words: **external ballistics, ballistic measurements, meteorological measurements, firing tables artillery shooting, artillery projectiles, projectile flight, numerical results.**

Introduction

IT has been proven that during ground ballistic firing, the influence of actual meteorological elements (wind with two components, temperature and pressure as most significant) on the values that determine projectile position in space is not partial but cumulative. When other ballistic influences are eliminated in the process of ballistic data processing, the rest of unusual metrological influences can not be prescribed to actual individual meteorological elements. These partial influences can be determined on the bases of trajectory calculations for determining the differential coefficients. In the ballistic practice, individual meteorological elements influences are determined by differential meteorological elements coefficients and ballistic values of meteorological elements. Ballistic values of meteorological elements are constants whose influence is the same that of the actual meteorological elements varying with altitude. Ballistic values of meteorological elements depend on projectile ballistic characteristic and flight conditions. Traditionally, ballistic values of meteorological elements are calculated by the so called *layer weight*. In order to calculate the *layer weight*, projectile trajectory summit is divided into layers and performed calculations: the unit of meteorological element deviation is calculated and divided with total meteorological element deviation, resulting in *layer weight* of the actual layer. Curves of the *layer weight* can not be obtained as analytical functions, but as discreet values that serve as node points for graphical presentation of the obtained set of values. The same discreet values serve for ballistic values calculations. It is understandable that accuracy depends on the number of discreet values, i.e. node points. This method in not suitable for practical application and in [1] an approximation an imagined curve of *layer weights* by a straight line is suggested. The consequence of this approximation is that the ballistic value of an individual meteorological element can be obtained as an average value of all node points of the meteorological elements from ground a certain altitude, not equal to the summit of the projectile trajectory. In references, the actual value of such altitude is called *conditional altitude*. For the sake of simplicity, the calculation for obtaining *conditional altitude* relation with the *projectile trajectory summit* is made and that relation is called *altitude coefficient*. *Conditional altitudes* are usually given in *Firing tables* as trajectory data that depend on ballistic projectile characteristic. Ballistic values of individual meteorological elements obtained as an average value of all node points of each meteorological element from ground to a certain altitude are coded into meteorological message, called "meteo-average". Up to the moment of introduction of the "meteo-average" meteorological message into the field artillery practice, *layer weights* are not calculated in a ballistic institute but at a meteorological station that has made meteorological processing for obtaining ballistic values of the measured meteorological elements and forming an appropriate meteorological message. Therefore, *layer weights* are calculated on the bases of trajectory elements calculated by parabolic theory which is valid for vacuum space, i.e. space without air resistance. Having introduced "meteo-average" meteorological message into the field artillery practice, *layer weights* must be calculated in a ballistic institute for obtaining *altitude coefficients* and by it *conditional altitudes* that are given in *Firing tables* as trajectory data. In reference [2] the results of the performed analysis, more possibilities for the calculation of *layer weights* and obtained numerical results are given. This paper presents a mechanized procedure for determining *altitude coefficients*. Numerical experimental results are given.

Layer weights concept

In order to facilitate the consideration meteorological elements influences, let projectile trajectory is divided into two layers equal in thickness – "1" and "2", Fig.1. If meteorological elements during firing time are nominal – standard, then projectile trajectory is of I form and it coincides with the calculated trajectory for nominal meteorological elements, and projectile will be placed in the point 1 (*C*) in the horizon at the end of its flight. If meteorological elements remain nominal, and one of them, say longitudinal wind, first changes for 10 units in the layer "1" – II trajectory, and then in the layer "2" - III trajectory,

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the projectile will be placed in points 2 and 3.

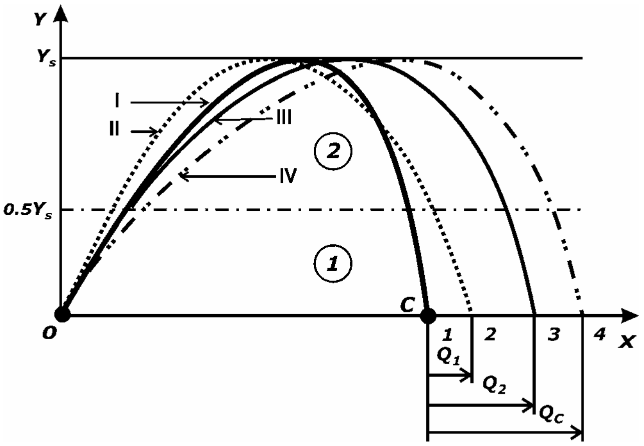

Figure 1. Layer influence on projectile flight

It can be seen from Fig.1 that distances Q_1 and Q_2 from point 1 (C) are not the same for meteorological elements in the layers "1" and "2" i.e. their influences are different in different layers. In case of longitudinal wind of 10 units, the acts simultaneously in layers "1" and "2"- IV trajectory; the projectile will be in point 4 and the distance from point 1 will be Q_C. If the distance variation Q_C is equal in magnitude to differential coefficient for longitudinal wind or distance correction for longitudinal wind, then $Q_c = Q_1 + Q_2 = \Delta X_{\mu 1} + \Delta X_{\mu 2}$ where variables denote the following: Q_C – distance variation of projectile in the horizon from point 1 (C) due to the constant wind magnitude that acts in both layers, i.e. on the whole trajectory; Q_1, Q_2 - distance variation of projectile in the horizon from point 1 (*C*) due to the constant wind magnitude that first acts in layer "1", and later only in layer "2", $\Delta X_{\mu 1}$, $\Delta X_{\mu 2}$ - distance variation of the projectile due to the longitudinal wind when it acts as in case of Q_1, Q_2.

Generally speaking, the summit projectile altitude can be divided into "n" layers and consideration made for wind influences in the same order as in case of two layers. Then distance variation of the projectile in the horizon from point 1 (*C*) are $Q_1, Q_2, ...Q_n$ and at same time are $Q_1 \neq Q_2, ..., \neq Q_n$ and the sum of individual distance variations given in case of linear influence (by default it is nearly linear)

$$Q_1 + Q_2 + ... + Q_n = Q_C \qquad (1)$$

Dividing (1) with Q_C, it is obtained

$$\frac{Q_1}{Q_C} + \frac{Q_2}{Q_C} + ... + \frac{Q_i}{Q_C} + ... + \frac{Q_n}{Q_c} = 1 \qquad (2)$$

Relations $\frac{Q_i}{Q_C}$ $(i = 1, 2, ..., n)$ are called *layer weights* and are denominated by q_1. The sum of *layer weights* q_1 is equal to one:

$$q_1 + q_2 + ... + q_i + ... + q_n = 1 \qquad (3)$$

The same conclusion is valid for *layer weights* for other meteorological elements, i.e. for temperature and pressure as most significant among these.

Weight function

Let *layer weighs* q_i from horizon to a certain altitude r_1 be summed. The following relations are:

For the first layer

$$r_1 = q_1$$

For the second layer

$$r_2 = q_1 + q_2 = r_1 + q_2 \qquad (4)$$

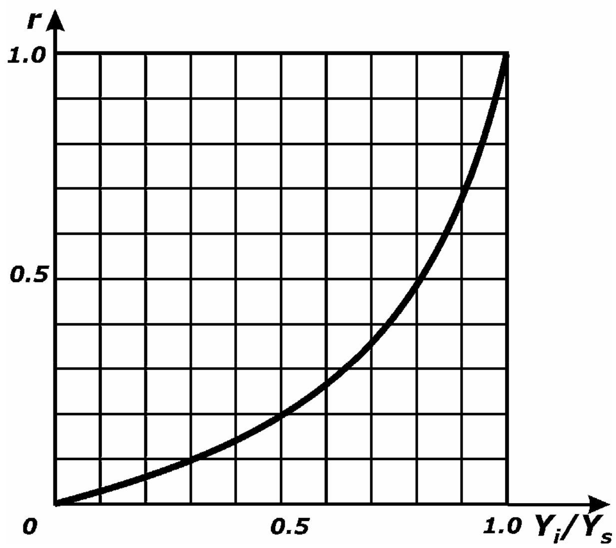

Figure 2. Weight function

For i -th and n -th layer $r_1 = r_{i-1} + q_i$; $r = r_n = r_{n-1} + q_n$, and sum of all r_i equals:

$$r_n = \sum_{i=1}^{n} q_i = 1$$

If the obtained values for r_i are calculated in function of altitude relations Y_i/Y_S where Y_S is trajectory summit altitude and Y_i is altitude from the horizon to the upper layer limit, the obtained curve is called *weight function* – Fig.2. Having obtained the curve of weight function, this curve can be used for obtaining *layer weights* of certain layer as

$$q_i = r_{Y_i} - r_{Y_{i-}} \qquad (5)$$

where are: q_i - *layer weight* in scope of altitudes Y_i and Y_{i-1} and r_{Y_i} and $r_{Y_{i-1}}$ numerical values of *weight functions* for the relations $0 - Y_i/Y_S$ and $0 - Y_{i-1}/Y_S$.

Calculations of ballistic values

Procedure for *layer weights* calculations can be used for *weight function* curve creation with slight modification in the layer definition and its lower and upper limits. The procedure in calculations are carried out makes no difference, since are used for ballistic values determinations constant values of meteorological elements. However, as a result of atmosphere sondage constant values of meteorological elements are obtained certain f altitudes and they generally vary with altitude. For projectile trajectory division into layers that do not have to be of the same thickness, meteorological elements equal to average values in every layers can be taken as constant values. If dependence among differential coefficients and variations of meteorological elements are linear, distance variation ΔD_μ can be written as

$$\Delta D_\mu = 0.1\left(\Delta X\mu_1 \times \mu_1 + ... + \Delta X\mu_i \times \mu_i + ... + \Delta X\mu_n \times \mu_n\right) \quad (6)$$

where - $\mu_i\,(i = 1,2,...,n)$ -average values of variations of meteorological elements related to nominal values in the same layer; - $\Delta X\mu_i\,(i = 1,2,...,n)$ - distance variations (differential coefficients) for 10 units of meteorological elements in i-th layer. If ballistic values of meteorological elements μ_b are known, equation (6) can be written as

$$\Delta D_\mu = 0.1 \times \Delta X\mu \times \mu_b \quad (7)$$

Equalizing (6) and (7) the following is obtained

$$\mu_b = \frac{\Delta X\mu_1}{\Delta X\mu}\mu_1 + ... + \frac{\Delta X\mu_i}{\Delta X\mu}\mu_i + ... + \frac{\Delta X\mu_n}{\Delta X\mu}\mu_n$$

if

$$\frac{\Delta X\mu_i}{\Delta X\mu} = \frac{Q_i}{Q_c} = q_i (i = 1,2,...,n)$$

then

$$\mu_b = \sum_{i=1}^{n} \mu_i \times q_i \quad (8)$$

If n has a tendency to words infinity, limes of sum (6) can be taken as

$$\lim_{n \to \infty} \sum_{i=1}^{n} \mu_i \times q_i = \lim \sum_{i=1}^{n} \mu_i \times \Delta r_i =$$
$$= \lim \sum_{i=1}^{n} \mu_i \times \frac{\Delta r_i}{\Delta Y_i} \times \Delta Y_i$$

Substituting Σ with integral, the following equation is obtained

$$\mu_b = \int_0^{Y_S} \mu(y) \times r'(y) \times dy \quad (9)$$

According to equation (9) ballistic values of all meteorological elements can be calculated. It can be seen from the same equation that the ballistic value of a specific meteorological element μ_b is it constant which gives the same distance variation $\Delta X\mu$ as the specific meteorological element $\mu(y)$ whose values vary with altitude. For that kind of calculation have to be previously formed a curve of *weight function* $r'(y)$ and function of values $\mu(y)$. There are more methods for calculating *weight functions* - [1–4]. Calculation for *difference method* is done in steps. In first step the distance from gun to target for nominal meteorological elements should be calculated. Let this distance be X_N, and summit altitude for that trajectory Y_S - Fig.3.

In the second step variation distance $\Delta X\mu_i$ is calculated when one of the meteorological elements in every layer has unit value different from its nominal value. The variation distance is

$$\Delta X\mu = X\mu - X_N \quad (10)$$

where $X\mu$ is the distance when a meteorological element acts in layer thickness Y_S, e.g. wind W_x has nominal values and other two are $(W_z, \Delta\tau)$. In the third step the summit altitude Y_S is divided in n partitions, n layers. After that in layer "1" of the trajectory when an actual meteorological element in acts it and it has nominal value in the rest of the layers. If longitudinal wind on 10 units intensity acts on the part of the trajectory from 0 to 1, 10 (m/s), then on part from 1 to 2 its value is nominal, i.e. zero and on the part from 2 to C has the value of 10 (m/s) again. In that case, the variation distance is obtained as $\Delta X\mu_1 = X\mu_1 - X_N$. The same is done with layer "2", from 0 to 1 the wind has 0 value; from 1 to 3it has the value of 10 (m/s); from 3 to 4 it has the value of 0; from 4 to 2 it has the value of 0; from 2 to C it has the value of 0, and it goes in the some order for the rest of the layers. *Layer weights* are obtained as

$$q_1 = \frac{\Delta X\mu_i}{\Delta X\mu};...;q_n = \frac{\Delta X\mu_n}{\Delta X\mu} \quad (11)$$

Weight function can be calculated according to (4), and for the case of its linear combination of q_i, r_2 it can be obtained for the wind of 10 (m/s) from 0 to 3; on part of the trajectory from 3 to 4 the value ought to be 0; and then again 10 (m/s) on the part from 4 to C. In that way *weight functions* can be obtained without calculating q_1 that can easily be calculated as $q_i = r_i - r_{i-1}$. These steps ought to be repeated for all meteorological elements. For e.g., ballistic wind value W_b graphical construction is given on Fig.4 using wind in layers ΔY_i, V_i and the corresponding wind directions.

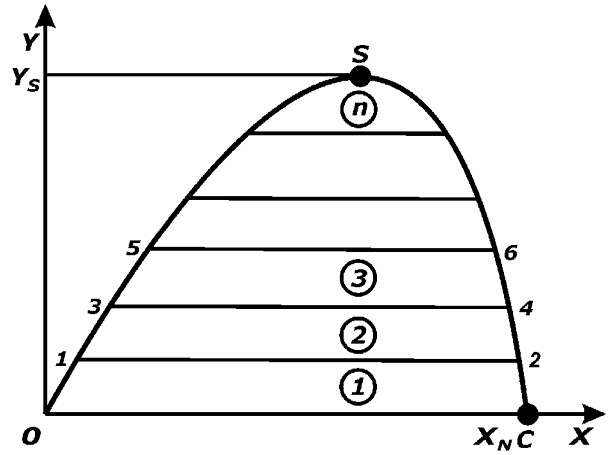

Figure 3. Trajectory layer partitions for the difference method

Drawing wind vector V and its direction α_v, vector intensity ought to be multiplied with *layer weights* q_i with successive addition, final result giving *ballistic wind value*. Let the influence of the *weight function* approximation of the form given on Fig.2 be considered in order to determination the *ballistic value* of a meteorological element μ_b. Let the real *weight function* for any meteorological element be calculated using *difference method.* Its substitution with two lines, line 0-1, and line 1-2 – Fig.5, with give equation (9) the following form

$$\mu_b = r_1 \mu_1 + (1 - r_1)\mu_2 \qquad (12)$$

where: r_1 - *layer weight* from gun horizon to altitude Y_1 which corresponds point 1, $(1 - r_1)$ - *layer weight* from altitude Y_1 to the summit altitude Y_S, μ_1, μ_2 - real value (variation) of the meteorological element in the first layer (from 0-Y_1) and in the second layer (from $Y_1 - Y_S$).

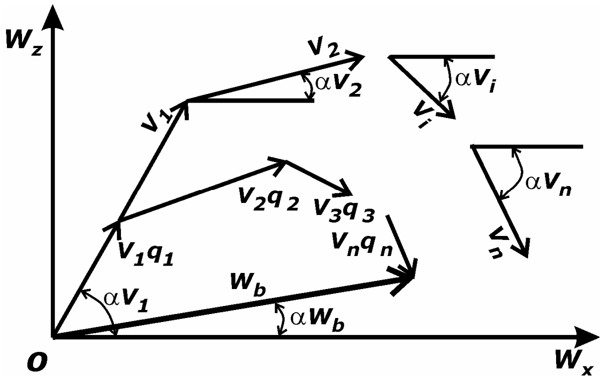

Figure 4. Ballistic wind value graphical construction

Equation (12) is simpler compared with (9) for it has only two elements for performing numerical calculations, and according to results in [5] provides enough accuracy in practice. Further simplification will cause equation (12) to one element only, provided test the weight function is substituted with one line, Fig.6. Line 0-1 can be drown such that the surface between the *weight function* and line upper and lower equal. Then, the *ballistic value* of the meteorological element can be calculated as:

$$\mu_b = \int_0^{Y_S} \mu(y)\times r'(y)\times dy = \int_0^{Y_1} \mu(y)\times r'(y)\times dy +$$
$$+\int_{Y_1}^{Y_S} \mu(y)\times r'(y)\times dy = tg\beta_1 \int_0^{Y_1} \mu(y)\times dy + tg\beta_2 \int_{Y_1}^{Y_S} \mu(y)\times dy$$

and it can be written that:

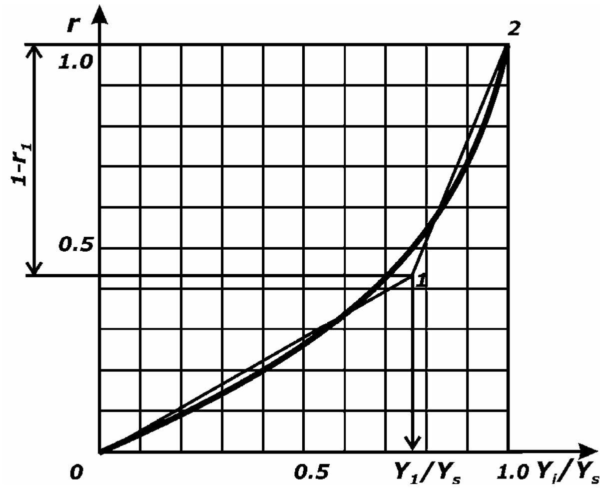

Figure 5. Weight function substitution with two lines

$$\mu_b = \frac{1}{Y_1}\int_0^{Y_1} \mu(y)\times dy \qquad (13)$$

as $tg\beta_1 = \frac{1}{Y_1}$; $tg\beta_2 = 0$ (Fig.6). Equation (13) enables determining the *ballistic value* μ_b of a meteorological element μ for trajectory with the summit altitude Y_S, where μ_b is the average value μ_{sr} of the meteorological element for some other altitude, Y_1 i.e.

$$(\mu_b)_{Y_S} = (\mu_{sr})_{Y_1} \qquad (14)$$

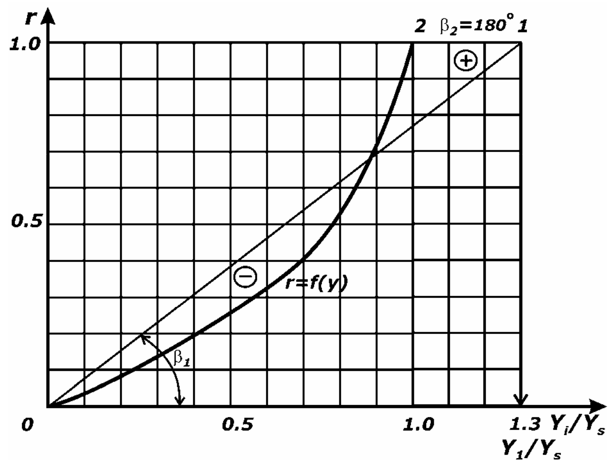

Figure 6. Weight function substitution with one line

So, for wind and temperature the following relations are valid:

$$(W_b)_{Y_s} = \left(\overline{W}\right)_{Y_1}$$
$$(\Delta\tau_b)_{Y_s} = \left(\overline{\Delta\tau}\right)_{Y_1} \qquad (15)$$

where the short line above the value denotes average of the actual meteorological element from the gun level to the altitude Y_1, not equal to the summit altitude Y_S. If the variation of meteorological elements is linear with altitude, then the *ballistic value* of the actual meteorological element can be calculated if it is equal to the real value of the actual meteorological element in the middle of altitude Y_1. Altitude Y_1 is called CONDITIONAL ALTITUDE – [1-5] and between it and the summit altitude Y_S exists an appropriate correspondence. This correspondence is not mathematical, but it can be given as a nomograms in the data sheets or in the *Firing tables* as one of the trajectory characteristics. *Conditional altitudes* Y_1 are not the same for all meteorological elements, and for that reason they are given as average of three in the *Firing tables*:

$$(Y_1)_{SR} = \frac{(Y_1)_{W_x} + (Y_1)_{W_z} + (Y_1)_{\Delta\tau}}{3} \qquad (16)$$

where are: $(Y_1)_{W_x}$, $(Y_1)_{W_z}$, $(Y_1)_{\Delta\tau}$ are *conditional altitudes* obtained by *weight function* substitution with one line. From previously established relations, *conditional altitudes* can be obtained as follows:

$$(Y_1)_{W_x}, (Y_1)_{W_z}, (Y_1)_{\Delta\tau} = (K)_{W_x,W_z,\Delta\tau} \times Y_s \quad (17)$$

where $(K)_{W_x,W_z,\Delta\tau}$ are *altitude coefficients*. In references, the actual value of the altitude obtained in such a way is called *conditional altitude*.

After carefully examination, a formula for *altitude coefficients* calculation is established, on condition that the surface S under the curve (integral $r = f(y)$) – Fig.6, subtracted from one is equal to surface 0-1-2-1-0. Integral $r = f(y)$, S, can be numerically calculated if the curve $r = f(y)$,i.e. $r = f(Y_i/Y_S)$ is known. The formula for *altitude coefficient* calculation is

$$(K)_{W_x,W_z,\Delta\tau} = 2(1-S) \quad (18)$$

for each of the meteorological elements.

Numerical results

Table 1 the results of *altitude coefficients* calculations obtained by formula (18) are given. Ballistic projectile characteristics are given for law of resistance $c_{x"1943"}$ and ballistic coefficient $C = 0.5$, muzzle velocities 100-900 m/s with increase of 100 m/s, angles of departure 10, 25, 45, 60, 80 degrees and standard atmosphere (Vencelj) - [6]. Surface S is calculated by Simpson′s rule for *layer weights* using equation (11) with difference method and the procedure given inFig.3. *Altitude coefficients* are calculated for both wind components and temperature, longitudinal W_x and lateral W_z and temperature τ. Average values are calculated using (16).

Table 1. Altitude coefficients

θ_0 / V_0 $K_{W_{x,z,t}}$	10	25	45	60	80
100 W_x	1.264	1.260	1.263	1.242	1.075
W_z	1.328	1.312	1.263	1.195	1.060
Te	1.244	1.170	1.130	1.061	0.931
Aver	1.279	1.247	1.219	1.166	1.022
200 W_x	1.257	1.264	1.262	1.236	1.075
W_z	1.328	1.308	1.250	1.175	1.033
Te	1.192	1.175	1.118	1.048	0.921
Aver	1.259	1.249	1.210	1.153	1.010
300 W_x	1.131	1.134	1.165	1.160	1.032
W_z	1.131	1.286	1.218	1.136	0.991
Te	1.077	1.027	0.987	0.926	0.813
Aver	1.113	1.149	1.123	1.074	0.945
400 W_x	1.478	1.255	1.001	0.939	0.825
W_z	1.259	1.106	1.010	0.925	0.789
Te	1.359	1.161	0.871	0.752	0.641
Aver	1.365	1.174	0.961	0.872	0.752
500 W_x	1.411	1.521	1.220	1.041	0.806
W_z	1.333	1.179	1.006	0.893	0.755
Te	1.282	1.357	1.087	0.927	0.794
Aver	1.342	1.352	1.104	0.954	0.785
600 W_x	1.379	1.553	1.336	1.107	0.820
W_z	1.352	1.244	1.045	0.910	0.767
Te	1.242	1.348	1.174	1.008	0.866
Aver	1.324	1.382	1.185	1.008	0.818
700 W_x	1.365	1.520	1.397	1.142	0.832
W_z	1.360	1.284	1.080	0.929	0.780
Te	1.218	1.293	1.197	1.029	0.877
Aver	1.308	1.366	1.225	1.033	0.830
800 W_x	1.354	1.471	1.420	1.158	0.882
W_z	1.362	1.306	1.108	0.941	0.779
Te	1.199	1.231	1.185	1.012	0.831
Aver	1.305	1.336	1.238	1.037	0.831
900 W_x	1.352	1.424	1.414	1.143	0.787
W_z	1.365	1.317	1.128	0.936	0.751
Te	1.190	1.179	1.153	0.945	0.744
Aver	1.302	1.307	1.232	1.008	0.761

Altitude coefficients for X_x, W_z and $\Delta\tau$ are calculated by (18) for S and 20 node points, meaning that the projectile trajectory was partitioned into 19 layers according to the procedure shown in Fig.3.

Conclusion

In the ballistic practice, influences of individual meteorological elements are determined by differential meteorological elements coefficients and ballistic values of meteorological elements. Ballistic values of meteorological elements are constants whose influence is the some as that of the actual meteorological elements varying with altitude. Ballistic values of meteorological elements depend on the ballistic characteristic of the projectile and the flight conditions. Traditionally, ballistic values of meteorological elements are calculated by the so called *layer weights*. For the purpose of *layer weights* calculations, projectile trajectory summit is divided into layers and calculation performed: the unit of meteorological element deviation is calculated and divided with total meteorological element deviation, resulting in the *layer weights* of the actual layer. Curves of *layer weights* can not be obtained as analytical functions, but as discreet values that serve as node points for graphical presentation of the set of values obtained. The same discreet values serve for ballistic values calculation. It is understandable that accuracy depends on the number of discreet values, i.e. node points. This method in not suitable for practical application and in [1] approximation of an imagined curve of *layer weights* by a straight line is suggested. The consequence of this approximation is that the ballistic value of an individual meteorological element can be obtained as average value of all node points of meteorological elements from the ground to a certain altitude, not equal to the summit of the projectile trajectory. In references, the actual value of such altitude is called *conditional altitude*. For the sake of simplifications the calculation is made for obtaining *conditional altitude* relation with *projectile trajectory summit*, and that relation is called *altitude coefficient*. *Conditional altitudes* are usually given in the *Firing tables* as trajectory data, that depend on the ballistic projectile characteristics. If *weight function* is calculated by any difference method, *layer weights* can easily be obtained, as well as the *ballistic values* of individual meteorological elements using (8). Approximating partial *weight function*, a graphical construction that provides *conditional altitude* $(Y_1)_{SR}$ is obtained. A method for calculating *altitude coefficients* using equation (18) introduced by the author is put to practice. On the basis of integral $r = f(y)$, S, there is no need to draw $r = f(y)$, $r = f(Y_i/Y_S)$ by node points (r_i, f_i).

References

[1] SAVKIN,L.S., LEBEDEV,B.D.: *Meteorologija i strelba artillerii*, Moskva 1974.

[2] PETROVIĆ,D.: *Analiza meteoroloških uticaja u spoljnoj balistici*, Magistarski rad, Mašinski fakultet Beograd, Beograd, 1982.

[3] PETROVIĆ,D.: *Metode za korišćenje meteoroloških podataka u ZA*, VTG, broj 4, Beograd, 1981.

[4] PETROVIĆ,D.: *Uticaj meteoroloških elemenata na let projektila*, Elaborat VTI 02-27-068, Beograd, 1982.

[5] PETROVIĆ,D.: *Metode za proračun težina slojeva i koeficijenata uslovnih visin*, Elaborat VTI 02-27-069, Beograd, 1982.

[6] JANKOVIĆ,S.: *Spoljna balistika*, Beograd, VIZ, 1977.

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Automatizovan postupak nalaženja koeficijenata uslovnih visina

Izložen je autorov automatizovan postupak proračuna koeficijenata uslovnih visina. Ovi koeficijenti smeštaju se u rubrike tablica gađanja i služe za potrebe artiljerijskih jedinica koje koriste bilten meteo-srednji. Dati su numerički rezultati test primera.

Ključne reči: **spoljna balistika, balistička merenja, meteorološka merenja, tablice gađanja artiljerijskih projektila, artiljerijska gađanja, numerički rezultati.**

Автоматизированный поступок нахождения коэффициента условной высоты

Здесь растолкован автоматизированный поступок расчёта коэффициентов условных высот. Эти коэффициенты записываются в графы таблиц стрельбы и пользуются для надобности артиллерийских частей, которые пользуются средним метеорологическим бюллетенем. Здесь приведены цифровые результаты испытательных экспериментов.

Ключевые слова: внешняя баллистика, баллистическое измерение, метеорологическое измерение, таблицы стрельбы, артиллерийская стрельба, артиллерийский снаряд, полёт снаряда, численные результаты

Le procédé automatisé pour trouver le coefficient des altitudes de condition

Dans ce papier on a exposé le procédé automatisé du calcul des coefficients des altitudes de condition. Ces coefficients se situent dans les rubriques des tableaux de tir et ils sont utilisés pour les besoins des unités d'artillerie qui se servent du bulletin météo-moyen. On a donné aussi les résultats du test exemple.

Mots clés: **balistique extérieure, mesurement balistique, tableaux de tir, tir d'artillerie, projectile d'artillerie, vol de projectile, résultats numériques.**